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Mathematical Representation of Friction in Metal Forming Analysis

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As recently stressed by Kalpakjian [1], Schey [2] and many other authors [3-8] the development of an improved friction model is an important topic in metal forming analysis.

In the present study a fresh friction approach is suggested. It connects the friction shear stress not only with the normal stress but also with the relative displacement between tool and workpiece and the relative surface increase. The model presented is based on the physical conception of the friction phenomenon as described by Schey [2,4], Wanheim [7] and Dautzenberg [8,9].

Connections with the well known Coulomb model ($\tau_{Fr} = \mu p$) and the so called constant friction model ($\tau_{Fr} = m \sigma_F / \sqrt{3}$) are discussed.

Theoretical findings are verified with experimental data from different tests. Although a few questions are still open, some progress in technical understanding of friction in forming tools could be achieved.

1. INTRODUCTION.

The importance and complexity of tribology in metal forming has recently been outlined by Kalpakjian [1] and Kawai-Dohda [6]. The present contribution is dedicated to the mathematical representation of friction under mixed conditions.

Besides the well known Coulomb- and constant friction model an extended model, relating the friction shear stress to the normal stress, the relative displacement between tool and workpiece and the increase of the nominal area of contact between tool and workpiece (surface-extension) is suggested. It is based on physical ideas as presented by many authors (for example [1-10]).

Three plane strain forming processes - upsetting, ironing and continuous sheet bending - served for the experimental support. The friction concept suggested explains a number of observations and covers several often used notions of friction. Although more experience with the application is needed, some practical conclusions are already possible.

2. THE FRICTION MODEL.

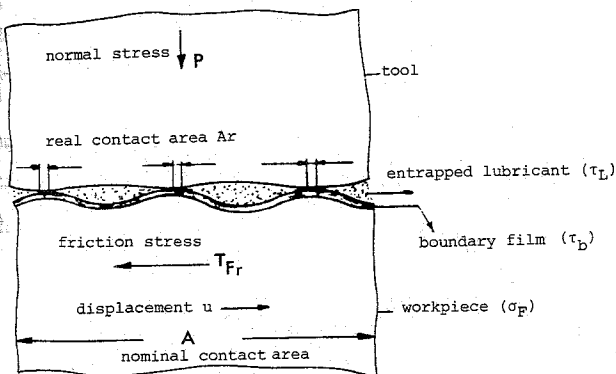


Fig. 2.1. Mixed film lubrication.

The friction mechanism considered is the mixed film lubrication (Fig. 2.1). Assuming the shear strength of the entrapped lubricant negligible compared with to the shear strength τ_F of the boundary film, the equilibrium of shear forces on a surface element A dictates:

$$(2.1) \quad \tau_{Fr} = \frac{A_r}{A} \tau_F$$

During a forming operation the shear strength τ_F of the boundary film and the real contact area A_r will continuously change. If an increase of these quantities with the normal stress p , the relative displacement u and the "surface strain" (relative increase of the nominal surface) A/A_0 is assumed, the following first approach could be defined:

$$(2.2) \quad \tau_{Fr} = \alpha p u \frac{A}{A_0}$$

where A_0 is the initial nominal contact area.

The friction model expressed herewith will be examined by means of the plane strain forming processes: ironing (u-shape), sheet bending and upsetting. Because only forces were measured the mean values of the quantities are defined. Furthermore the friction hill in the upsetting test will be calculated with the aid of Eq. (2.2) and be compared with the result from the Coulomb model

$$(2.3) \quad \tau_{Fr} = \mu p$$

and the constant friction model

$$(2.4) \quad \tau_{Fr} = m/\sqrt{3} \sigma_F.$$

It can easily be seen that all the above mentioned dependencies have a physical background. The relative displacement, for example, effects the surface roughness profile in a way that the tops are flattened. This results in an increasing real contact area. The increase of the nominal contact area results in a decreasing film thickness of the bound lubricant layer which likely effects its shear resistance.

3. PLANE STRAIN IRONING.

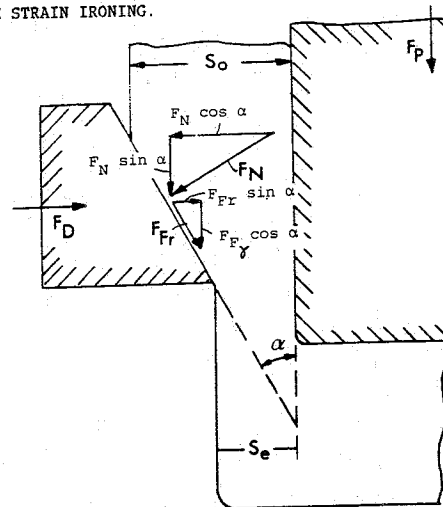


Fig. 3.1. The plane strain ironing test.

Equilibrium conditions provide the relations between the friction force F_{Fr} and the normal force F_N and the tool forces F_p and F_D (Fig. 3.1) respectively:

$$(3.1) \quad F_{Fr} = 0.5 F_p \cos \alpha - F_D \sin \alpha$$

$$(3.2) \quad F_N = 0.5 F_p \sin \alpha + F_D \cos \alpha$$

Figs. 3.2 and 3.3 show some experimental values from Eqs. (3.1) and (3.2) in conventional representations.

From Figs. 3.2. and 3.3 it could be concluded that the Coulomb model is correct. From Fig. 3.3 it follows that the constant friction model is also useful to some extent.

In Fig. 3.4 the representation according to Eq. (2.2) is given. The extended model seems to be rather correct. From a practical point of view the constant friction model is more favourable because of its easier handling.

4. CONTINUOUS SHEET BENDING.

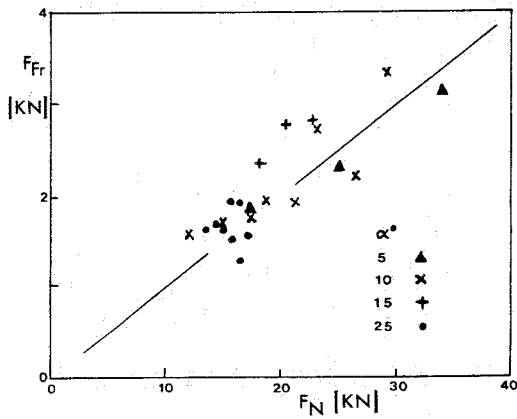


Fig. 3.2. Friction force and normal force in the ironing test (see Table 1).

material	C N/mm ²	n	ε ₀	lubricant	w [mm]	s ₀ [mm]	Fig.
SPO	540	0.19	0	E.P.50	24.8	1.54	3.2 3.3 3.4
SPO	540	0.19	0	lanoline	24.8	1.54	4.2
Aluminum	123	0.24	0.35	li-stear	50	4	5.2 5.3 5.4

Table 1. Test conditions.

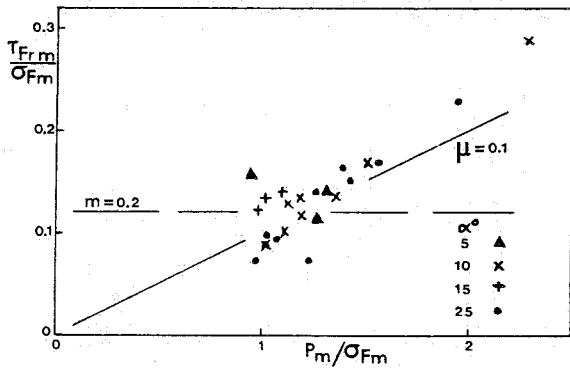


Fig. 3.3. Mean values of friction stress $\tau_{Fr,m}$ and normal pressure P_m , related to the yield stress σ_{Fm} from the ironing test.

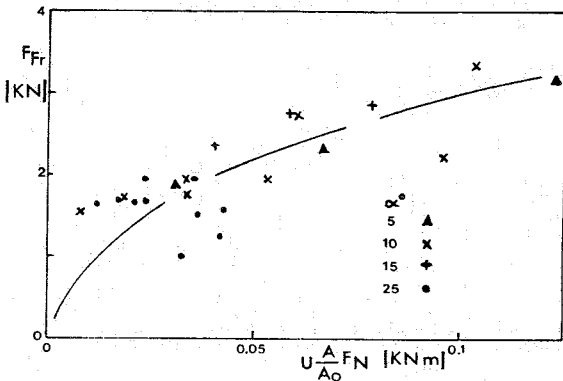


Fig. 3.4. The friction force F_{Fr} in relation with the displacement u , the increase of the contact area A/A_0 and the normal load F_N .

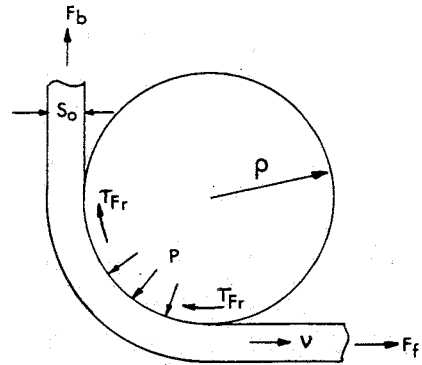


Fig. 4.1. Continuous sheet bending test.

A strip with width w and initial thickness s_0 is pulled along a cylinder. A backpull F_b can be introduced to increase the normal pressure p . The test can be carried out in two different ways: with a fixed cylinder or with a freely rotating cylinder. In the first mode the influence of the double bending plus the friction is measured. In the second mode only the effect of the bending is measured.

So the mean values of the normal pressure and the friction stress are obtained from the following expressions:

$$(4.1) \quad P_m = \frac{F_f + F_b}{2 \cdot \rho \cdot w}$$

$$(4.2) \quad \tau_{Fr,m} = \frac{(F_f - F_b)_{fixed} - (F_f - F_b)_{free}}{w/2 \cdot \rho \cdot w}$$

When carrying out the test with only one cylinder and neglecting a minor surface increase of the sheet ($s, w = konst$), according to Eq. (2.2) a linear relation between $\tau_{Fr,m}$ and p_m should be found. The results of the experiments are represented in Fig. (4.2). However, the presumed connection with the displacement $-U_m = \rho \cdot \pi/4$ - is not confirmed with these experiments.

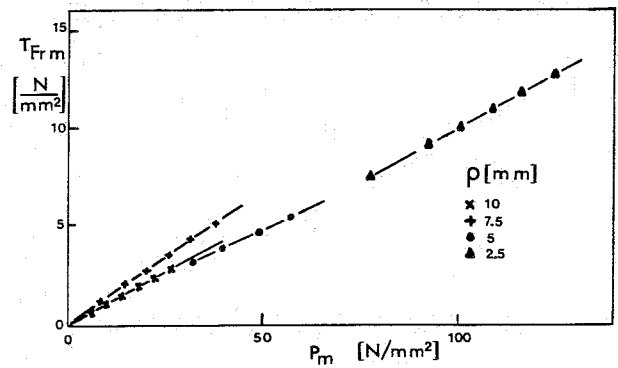


Fig. 4.2. Friction shear stress $\tau_{Fr,m}$ and normal pressure p_m in the sheet bending test.

5. PLANE STRAIN UPSETTING.

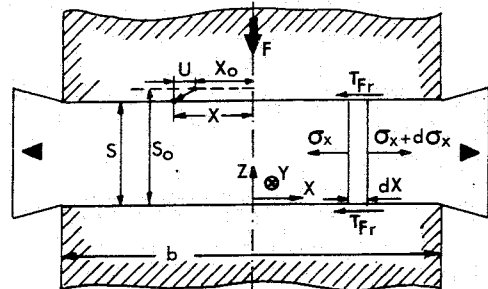


Fig. 5.1. Plane strain upsetting ($\epsilon_y = 0$).

Grid measurements showed that, with sufficient lubrication, plane sections remain plane. So with the help of the slab method it can be written:

$$(5.1) \quad d\sigma_x = 2 \tau_{Fr} \frac{dx}{s}$$

substituting Eq. (2.2) it follows (see App. A):

$$(5.2) \quad P = \frac{2}{\sqrt{3}} \sigma_F \left(1 + \frac{1}{2} \frac{s}{b}\right) \exp\left(\frac{1}{4} q \frac{b^2}{s} \left(\frac{s_0}{s} - 1\right) \left(1 - 4 \left(\frac{x}{b}\right)^2\right)\right)$$

whereas Coulombs' law leads to:

$$(5.3) \quad P = \frac{2}{\sqrt{3}} \sigma_F \left(1 + \frac{1}{2} \frac{s}{b}\right) \exp\left(\mu \frac{b}{s} \left(1 - 2 \frac{x}{b}\right)\right)$$

With the constant friction model Eq. (2.4) the expression:

$$(5.4) \quad P = \frac{2}{\sqrt{3}} \sigma_F \left(1 + \frac{1}{2} \frac{s}{b} + \frac{\mu}{2} \frac{b}{s} \left(1 - 2 \frac{x}{b}\right)\right)$$

can be derived.

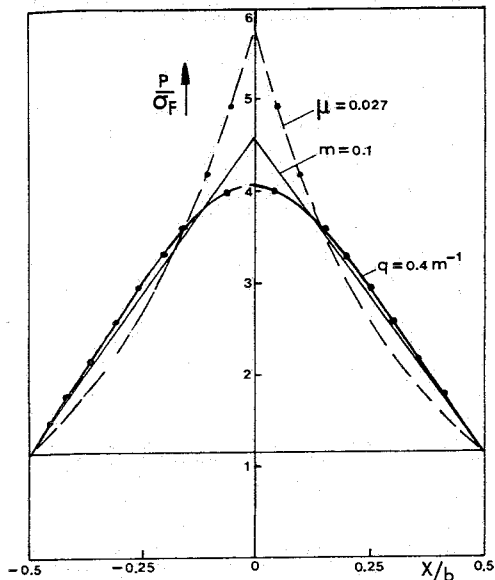


Fig. 5.2. The friction hills calculated according to Eqs. (5.2), (5.3) and (5.4) respectively.

For an arbitrary case - $F = 680 \text{ kN}$, $\sigma_F = 157 \text{ N/mm}^2$, $w = 50 \text{ mm}$, $b = 30 \text{ mm}$, $s_0 = 4 \text{ mm}$, and $s = 0.5 \text{ mm}$ - the calculated friction hills according to the equations (5.2), (5.3) and (5.4) are represented in Fig. (5.2). The friction coefficients are: $q = 0.4 \text{ m}^{-1}$, $\mu = 0.027$ and $m = 0.1$ respectively.

Apart from the consideration that the "q"-curve (Eq. 5.2) seems to be the most acceptable one, this kind of pressure profile is nearest to measured pressure distributions [16,17].

From a measured force - displacement diagram mean values of the normal pressure, the friction stress, the displacement and the increase of the nominal contact area can be achieved:

$$(5.5) \quad p_m = \frac{F}{b \cdot w}$$

$$(5.6) \quad \tau_{Fr m} = 2 \frac{s}{b} \left(p_m - \frac{2}{\sqrt{3}} \sigma_F\right)$$

$$(5.7) \quad u_m = \frac{1}{4} b \left(1 - \frac{s}{s_0}\right)$$

$$(5.8) \quad \frac{A}{A_0} = \frac{s_0}{s}$$

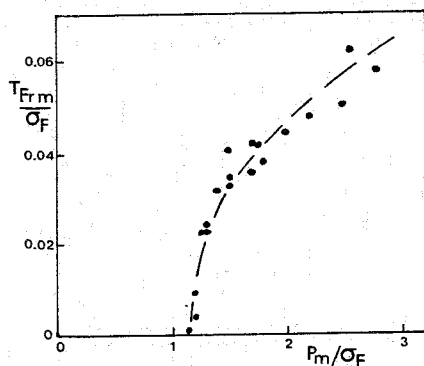


Fig. 5.3. Friction stress $\tau_{Fr m}$ in relation with the normal stress p_m during upsetting.

Fig. (5.3) shows some experimental values from Eq. (5.6) and (5.5) in a conventional representation. Both the Coulomb and constant friction model are not in accordance with these measurements. In Fig. (5.4) the friction stress is given in relation with the present friction model. However the suggested dependency in Eq. (2.2) appears to be a nonlinear one.

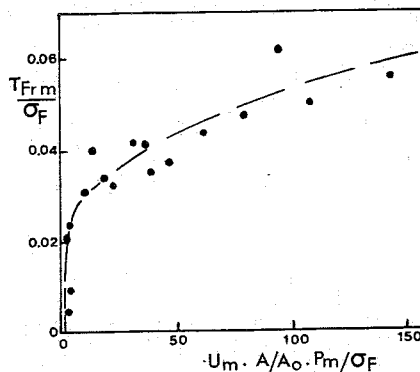


Fig. 5.4. Experimental results of the upsetting test.

CONCLUSIONS.

- None of the presented friction models covers all experiments perfectly. In the domain of relatively low values of the normal pressure ($p/\sigma_F < 1$) the Coulomb model has obviously a certain validity, whereas for higher values of $p/\sigma_F (> 1)$ the extended model fits best and provides a clear progress with respect to the predicted pressure distribution.
- The result of the bending test (Fig. 4.2) can be explained by assuming a thick film lubrication regime. Because the viscosity of the lubricant increases with increasing pressure [2] the Coulomb model describes the friction process satisfactory.
- Compared with the other investigated parameters the normal pressure obviously has a dominating influence on the global level of the friction stress. Nevertheless in upsetting operations the effect of the displacement on the pressure distribution is clear and essential. The role of the increase of the contact area, being the minor influence, became not yet very clear.
- The direct connection between "stick" ($u = 0$) and maximum friction stress, as often put forward is controversial:
 - It is easy to prove that an upsetting test with artificial stick ($u = 0$) in the entire contact area leads to a maximum value of the friction stress ($\tau_{Fr} = 1/3 \cdot \sigma_F$).
 - Contrary to the often accepted idea of stick in coining [1] the displacements u are very small and in accordance with the extended model (Eq. 22) the friction stress is negligible [15].
- From a practical point of view it is advantageous to apply the constant friction model. However, the relation between the friction shear stress and the yield stress of the workpiece material is physically incorrect. A better approach is obtained by the Coulomb model and choosing the value of the friction stress in relation with the value of the mean normal pressure.

APPENDIX A: CALCULATION OF THE FRICTION HILL.

$$(A1) \quad d\sigma_x = 2 \tau_{Fr} \frac{dx}{s}$$

$$(A2) \quad \tau_{Fr} = q p u \frac{A}{A_0}$$

$$(A3) \quad \epsilon_Y = 0 \dots \sigma_x - \sigma_z = \frac{2}{\sqrt{3}} \sigma_F \dots P = \frac{2}{\sqrt{3}} \sigma_F \sigma_x$$

$$(A4) \quad x_0 s_0 w = x s w \dots \frac{A}{A_0} = \frac{x \cdot w}{x_0 \cdot w} = \frac{s_0}{s}$$

$$(A5) \quad u = x - x_0 = x \left(1 - \frac{s_0}{s}\right)$$

$$(A6) \quad \tau_{Fr} = q \left(\frac{2}{\sqrt{3}} \sigma_F - \sigma_x\right) \left(\frac{s_0}{s} - 1\right) x$$

From (A1) to (A6) follows:

$$(A7) \quad \frac{d\sigma_x}{2\sqrt{3}\sigma_F - \sigma_x} = 2 \frac{q}{s} \left(\frac{s_0}{s} - 1 \right) x dx$$

The boundary condition for σ_x can be calculated from:

$$\sigma_x(x=b/2) \cdot \dot{u}_x(x=b/2) \cdot s \cdot w = \int_0^{\frac{s}{2}} \frac{1}{\sqrt{3}} \sigma_F \dot{u}_z dz w$$

So:

$$(A8) \quad \sigma_x(x=b/2) = - \frac{1}{\sqrt{3}} \sigma_F \frac{s}{b}$$

Solving Eq. (A7) leads to:

$$(A9) \quad p = \frac{2}{\sqrt{3}} \sigma_F \left(1 + \frac{1}{2} \frac{s}{b} \right) \exp \left(\frac{1}{2} q \frac{b^2}{s} \left(\frac{s_0}{s} - 1 \right) \left(1 - 4 \left(\frac{x}{b} \right)^2 \right) \right)$$

REFERENCES.

- [1] Kalpakjian, S.: Recent Progress in Metal forming Tribology. Ann. CIRP 34,2 (1985), 1-8.
- [2] Schey, J.A.: Tribology. Handbook of Metal Forming, chapter 6; ed. K. Lange; Mc-Graw - Hill (1985).
- [3] Bartz, W.J.; Wolff, J.; eds.: proc. 3rd International Colloquium "Lubrication in Metal Working". Esslingen (1982).
- [4] Schey, J.A.: Metal Deformation Processes. M. Dekker (1970).
- [5] Wilson, W.R.D.: Friction and Lubrication in sheet Metal Forming; Mechanics of Sheet Metal Forming, p 157-177; eds. D.P. Koistinen and N.M. Wang; Plenum Press (1978).
- [6] Kawai, N. and K. Dohda: Tribology in Metal Forming Processes. Advanced Technology of Plasticity, p 145-156. Proc. 1e Int. Conf. Technol. Plast., Tokyo (1984).
- [7] Gräbener, Th.: Entwicklungen und Anwendung neuer Schmierstoffprüfverfahren für die Kaltmassivumformung, Universität Stuttgart. Springer Verlag (1983).
- [8] Wanheim, T.; Bay, N.: A Model for Friction in Metal Forming Processes. Ann. CIRP 27,1(1978), p. 189-194.
- [9] Dautzenberg, J.H.: Reibung und Gleitverschleiß bei Trockenreibung. Ph.D. Thesis, Eindhoven University of Technology, (1977).
- [10] Dautzenberg, J.H.; Kals, J.A.G.: Metallverformung bei trockener Reibung. Zeitschrift für Metallkunde 79(1985), p. 635-639.
- [11] Bay, N.: Tool/Workpiece Interface in Cold Forward Extrusion. Advanced Technology of Plasticity, p259-266. Proc. 1e Intern. Conf. Technol. Plast., Tokyo (1984).
- [12] Nine, H.D.: Drawbead Forces in Sheet Metal Forming. Mechanics of Sheet Metal Forming, p179-211. eds. D.P. Koistinen and N.M. Wang; Plenum Press (1978).
- [13] Dautzenberg, J.H. and J.A.G. Kals; Stress state and Surface Roughness. Advanced Technology of Plasticity, p186-191; Proc. 1st Int. Conf. Technol. Plast., Tokyo (1984).
- [14] Ramaekers, J.A.H. and M.J.H. Smeets: Contactverschijnselen bij het omvormen (in Dutch). Metaalbewerking no. 3 (1984) p54-58.
- [15] Ramaekers, J.A.H. and S. Hoogenboom: The Prediction of Tool Pressures in Coining; Advances in Manufacturing Technology 1985; ed. P.P. Miller; Proc. 2nd Conf. I.M.C., Jordanstown (1985).
- [16] De Jong, J.E.: Numerical Analysis of Metalforming Processes. Ph.D Thesis, Eindhoven University of Technology (1983).
- [17] Bay, N. and G. Gerved: Friction and Pressure Distribution in Disk Forging. 17th Int. Cold Forging Group. Nagoya (1984).