A way to generalize Peaucellier's inversor

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A WAY TO GENERALIZE PEAUCELLIER'S INVERSOR

(O metodă de a generaliza "Inversorul Peaucellier")

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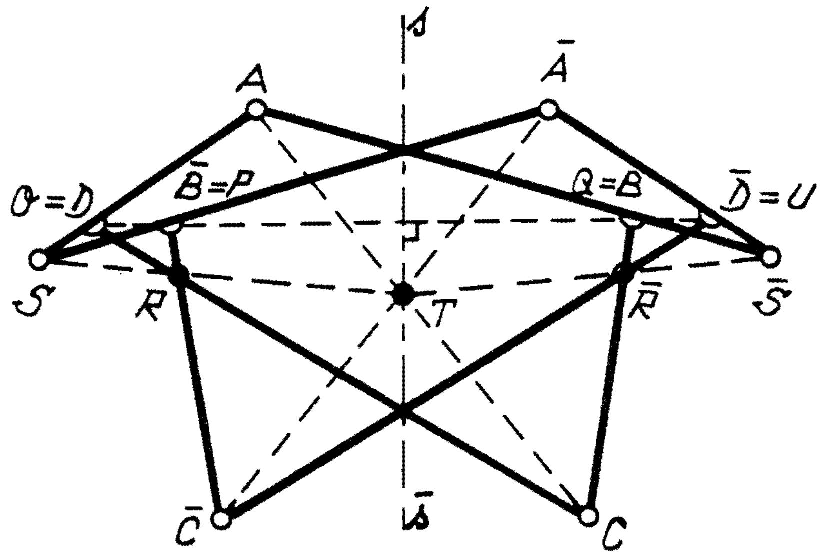
Introduction. In the past much has been written about inversion mechanisms. They were all based on the principle of *geometric inversion.*, transforming a point, or a locus of points, into other points by way of inversion with respect to a unit circle. Such an inversion deals with three points,say O, P and Q, meeting the relationship $\overline{OP} \bullet \overline{OQ} = \overline{OR}^2 =$ constant. This relation transforms any point P into its inverse point Q ,and conversely.
To materialize this,one needs a (linkage) mechanism - usually named a cell - containing these three points. The mechanism ,or the cell, must have the mobility *two* if O represents a fixed turning-joint. The curve traced by point P of this cell then transforms into the inverted curve, traced by point Q ,and conversely. Famous cells, meeting these requirements, are Hart's contraparallelogram chain and Peaucellier's cell. It is possible to derive the one from the other through *cognate theory* as proved in ref. [1].
In that paper, this cognation also lead to new inversion mechanisms.
They all have the property to turn, for instance, a circle into another one, sometimes having an infinite radius, enabling the designer to create exact straight-line mechanisms.
Some of these new cells are shown in chapter 8 of author's book "Motion Geometry of Mechanisms" , (See ref.[2]).
With the occasion of the 7th World Congress on the Theory of Machines and Mechanisms, held in september of 1987, the author found a way to interconnect two contraparallelogram-cells of Hart in an overconstrained way, (See ref.[3]). He did this by adjoining them through *four* turning-joints being the intersections of corresponding sides of these cells, all four of them joining a random line running parallel to all diagonals of the two contraparallelograms, (See figure 1).



Two *perspective* contraparallelograms interconnected by the aligned joints D, $\bar{B}$, B and $\bar{D}$, resulting into the *reflected* four-bars ABCD and $\bar{A}\bar{B}\bar{C}\bar{D}$, being interconnected by the joints S, R, $\bar{R}$ and $\bar{S}$

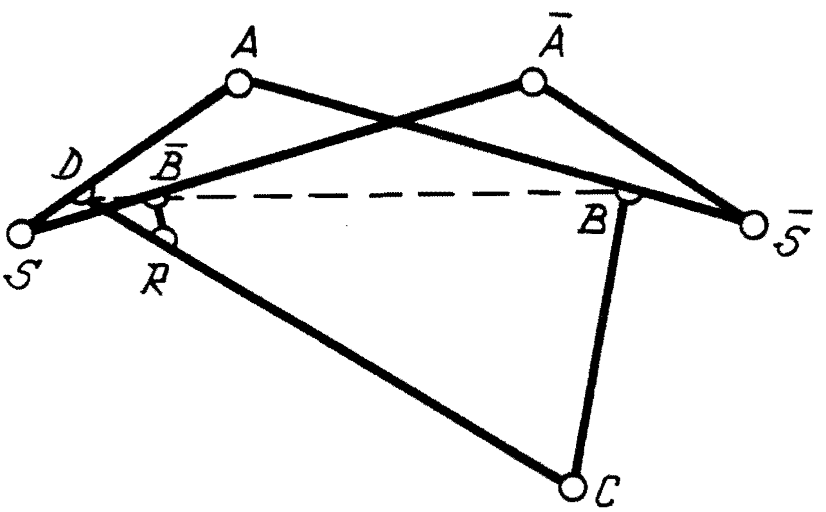
$$\square\, AS\bar{A}\bar{S} \overset{T}{\underset{\wedge}{\equiv}} \square\, CR\bar{C}\bar{R}$$

Figure 1.

Design:
- a. Start from a *random* four-bar ABCD,
- b. *Choose* joint S at AD,
- c. Find $\bar{S}$ at AB such that S$\bar{S}$ // DB,
- d. Reflect ABCD into its image four-bar $\bar{A}\bar{B}\bar{C}\bar{D}$ with respect to the perpendicular bisector of S$\bar{S}$,
- e. Determine R and $\bar{R}$ at the respective sides (CD, $\bar{B}\bar{C}$) and ($\bar{C}\bar{D}$, BC).

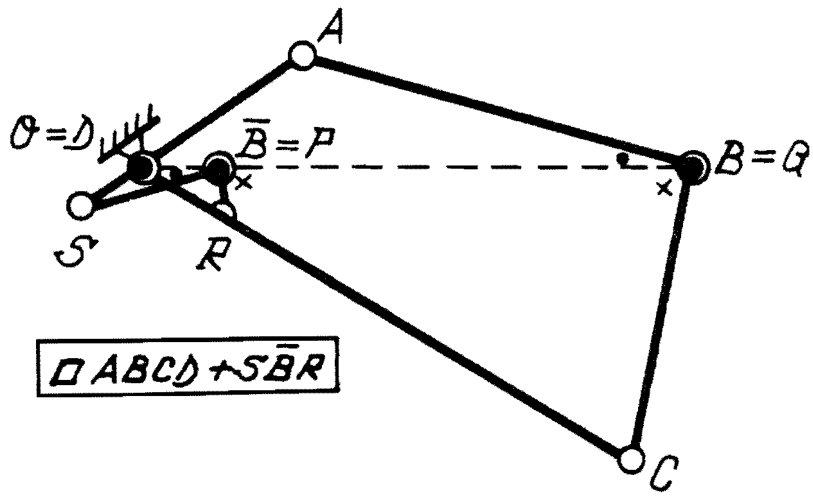
The method to derive this linkage-composition of cells was based on *perspectivity* and *reflection*. The result was a mobility-1 double overconstrained linkage chain with 8 bars and 12 turning-joints, namely 2 times 4 turning-joints of the contraparallelograms and the 4 adjoined ones. At the time, it represented a mechanical way to avoid a contraparallelogram turning into a parallelogram through its folded position: when one of the two contraparallelograms was folded or stretched, the other was not, thus avoiding an abrupt change into an unwanted mode of the stretchable four-bar. In practice, the linkage-composition gave any contraparallelogram the possibility to restrict its *complete* motion to a permanent and continious motion of only the *contra*parallelogram. The other half of the motion, namely the parallelogram one, was thus subtracted. (Note that something similar was done to maintain the parallelogram-motion: then a singular bar was adjoined creating a 5-bar assembly with three linkage-parallelograms in order to avoid transition into the contraparallelogram-mode.)
Observing our 8-bar configuration of contraparallelograms, one may observe that it remains still possible to omit one of its ternary bars without changing

its capability to maintain its contraparallelogram mode. One then factually creates a 7-bar chain with 9 turning-joints, still being overconstrained to overcome the stretched position(s) of the contraparallelogram left in the chain, (See figure 2) .



Contraparallelogram-linkage adjoined with
a linkage-dyad BCD and a bar $\bar{B}R$ to overcome
its *stretched* positions
Figure 2

Designing a new inversion cell. Omitting *two* ternary links instead of one from our double overconstrained 8-bar linkage chain , turns the 8-bar into a 6-bar mobility-1 chain of the Watt-type. The two ternary links to obliterate, are those having one of the four aligned turning-joints in common. The result of this procedure is demonstrated in figure 3, in which the bars $\overline{CR}$ and $\overline{AS}$ are omitted. Since there are four of these aligned turning-joints, to wit the points O, P, Q and U, there are also four possibilities to omit two bars having one of these points in common. In all cases,however, we are left with a cell containing three crucial points meeting the condition of inversion. When, for instance, the two links having their common turning-joint Q are obliterated , instead of the ones having U as their common turning-joint, we simply observe the condition $\overline{PO}.\overline{PU}$ = constant, instead of $\overline{OP}\bullet\overline{OQ}$ = constant.
The original 8-bar contained a random linkage four-bar, ABCD, from which it was possible to start the design with of the 8-bar containing the two contraparallelograms, see ref.[3]. Even then the designer was left with an infinite row of solutions by the random choice of, for instance, point S at the side AD of the random four-bar.
The Watt-linkage, obtained by obliterating two ternary links, still contains this random four-bar, then adjoined with the linkage-dyad $S\bar{B}R$. Like with the 8-bar, we still find an infinite row of solutions with any random four-bar. This may be done by choosing point S at AD, or, alternatively, and also more practical as shown later on, by choosing point $\bar{B}$ at the diagonal DB of the random four-bar.



New inversion-cell, for which

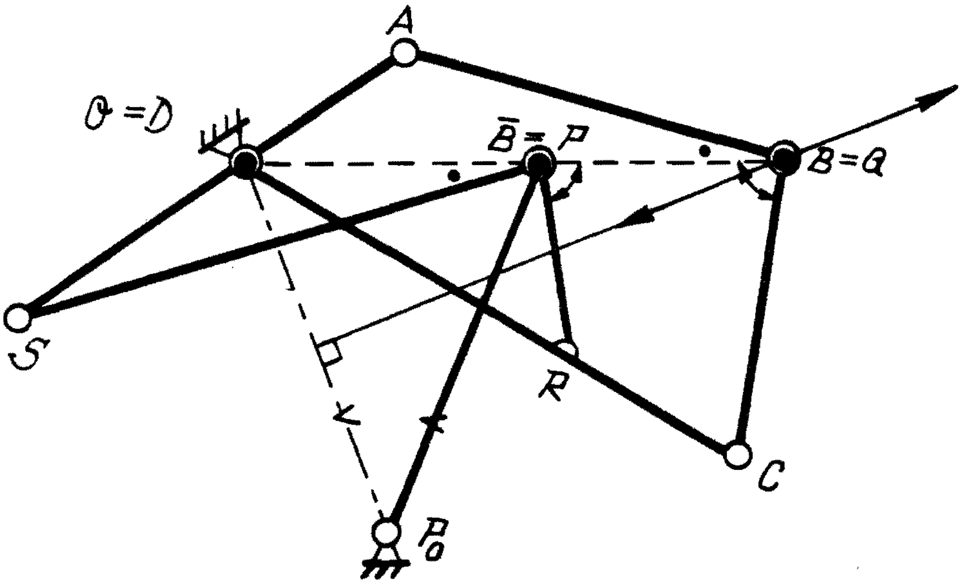
$\overline{OP}\cdot\overline{OQ}$ = constant = $\overline{DA}\cdot\overline{SD}\{(\overline{S\bar{B}}^2/\overline{SD}^2 - 1\}$
(the cell contains a *random* four-bar ABCD
as well as a *random* turning-joint $\bar{B}$ at its
diagonal DB.)

Figure 3

The actual design of the 6-bar cell of the Watt-type now runs as follows:

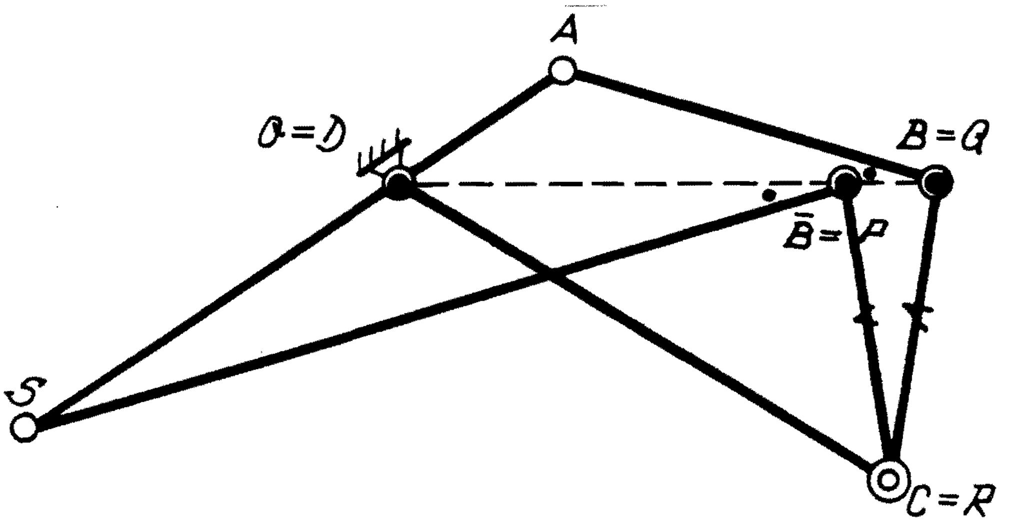
a. Start with a *random* linkage four-bar, indicated as the quadrilateral ABCD in figure 3.
b. Choose a *random* turning-joint $\bar{B}$ at the diagonal DB of the four-bar.
c. Run the bar $\bar{B}$S parallel to the image of the side AB into the diagonal DB of the four-bar. (whence $\measuredangle$ S$\bar{B}$D = $\measuredangle$ DBA)
d. Change the intersection-point S of AD and $\bar{B}$S, into a turning-joint between AD and $\bar{B}$S.
e. Run the bar $\bar{B}$R parallel to the image of the side BC into the diagonal DB of the four-bar. (whence $\measuredangle$ B$\bar{B}$R = $\measuredangle$ CBD)
f. Change the intersection-point R of CD and $\bar{B}$R into a turning-joint of these bars

The random four-bar has now been adjoined with a linkage-dyad S$\bar{B}$R ,running parallel to the image of the linkage-dyad ABC with respect to the diagonal DB of the four-bar. The turning-joints D, $\bar{B}$ and B are still identical with the respective points O, P and Q ,the crucial points of our inversion cell. Thus, taking D = O as fixed, a curve traced by $\bar{B}$ = P will be inverted by the curve then traced by B = Q.
As $\bar{B}$ is a random point of DB, we have an infinite row of solutions.
We will further show, that the Peaucellier cell is just a very particular case, which is the reason why the cells of this type are in fact a *generalized* form of Peaucellier's cell.



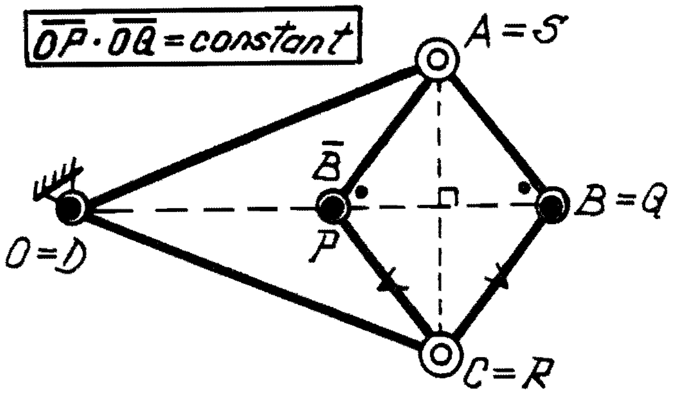
New *Inversor* with Q tracing a
straight-line -segment.
Figure 4.

Particular cells A particular case of the generalized cell occurs when R coincides with turning-joint C of the four-bar. Then, the cell has a *double* turning-joint, whereas in addition, the length of $\bar{B}C$ = $\overline{BC}$, see figure 5.



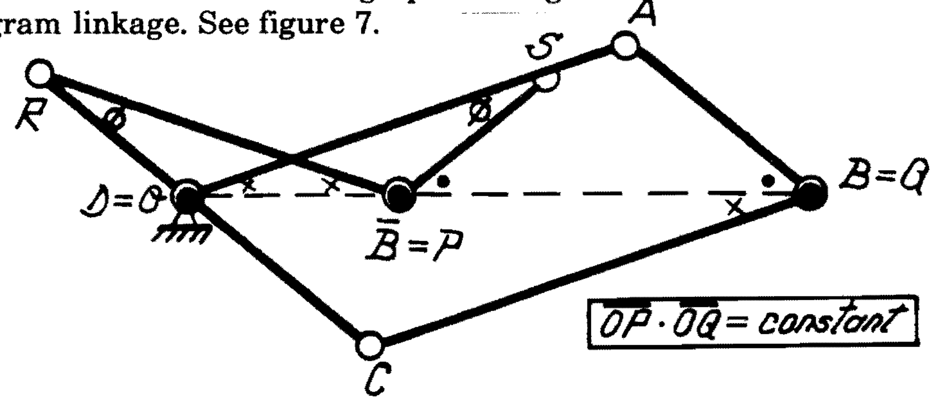
Particular Inversion-cell
($\overline{OP} \cdot \overline{OQ}$ = constant)
Figure 5.

A further specialization occurs by turning the random four-bar into a particular one. If ABCD would be a *kite,* for instance, then the diagonals are mutually perpendicular, whereas if R = C, also S = A, whence $ABC\bar{B}$ represents a rhombus. One so obtains the well-known inversion-cell of Peaucellier again. In other words: Peaucellier's inversion cell is just a very particular case, see figure 6.



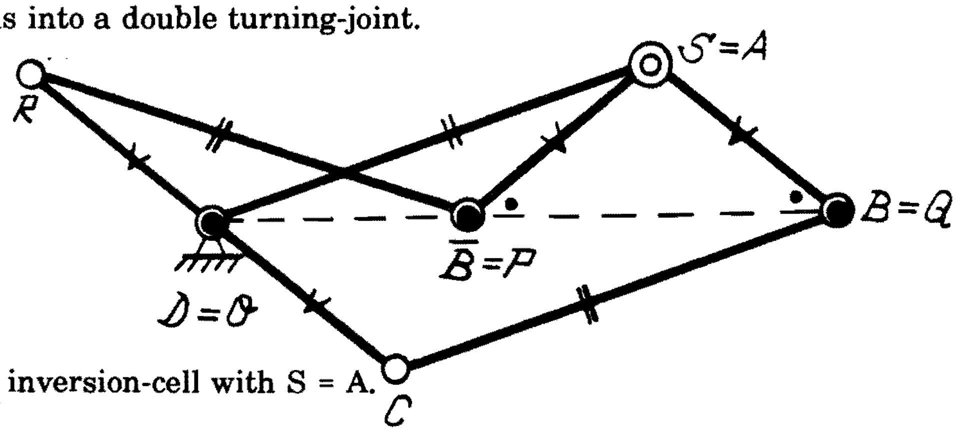
Peaucellier's inversion-cell
as a very particular case.
(ABCD represents a *kite.*)
Figure 6.

Another particular case will be found by turning the random four-bar into a linkage *parallelogram*. The adjoined linkage dyad SB̄R then forms a contraparallelogram with RDS. The particular inversion-cell, therefore, then simultaneously contains a linkage-parallelogram as well as a contraparallelogram linkage. See figure 7.



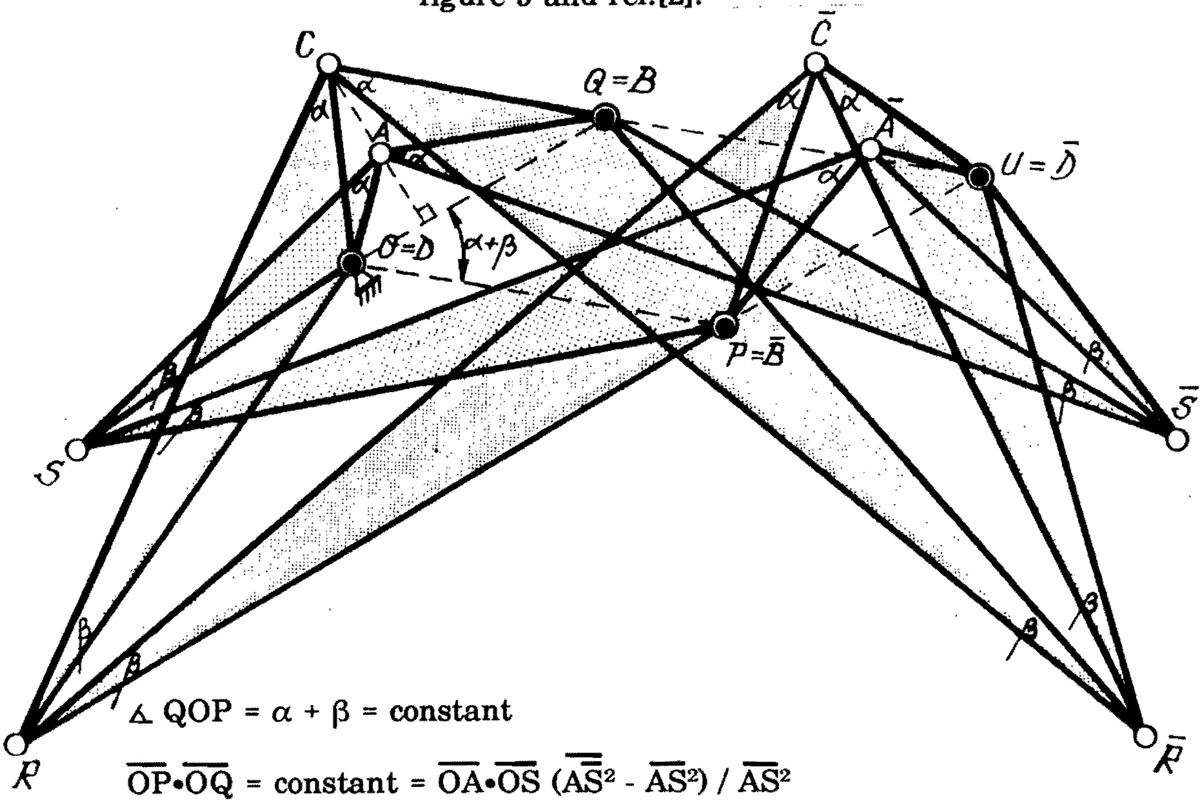
Particular *inversion-cell* with ABCD being a parl.gram-linkage and RDSB̄ representing a contraparallelogram-linkage.
Figure 7.

A further specialization is demonstrated in figure 8, in which S and A coincide. Point A then turns into a double turning-joint.



Very particular inversion-cell with S = A.
Figure 8.

Further Generalization One may generalize the procedure by starting from the basic inversion cell of Sylvestor and Kempe, see figure 9 and ref.[2].



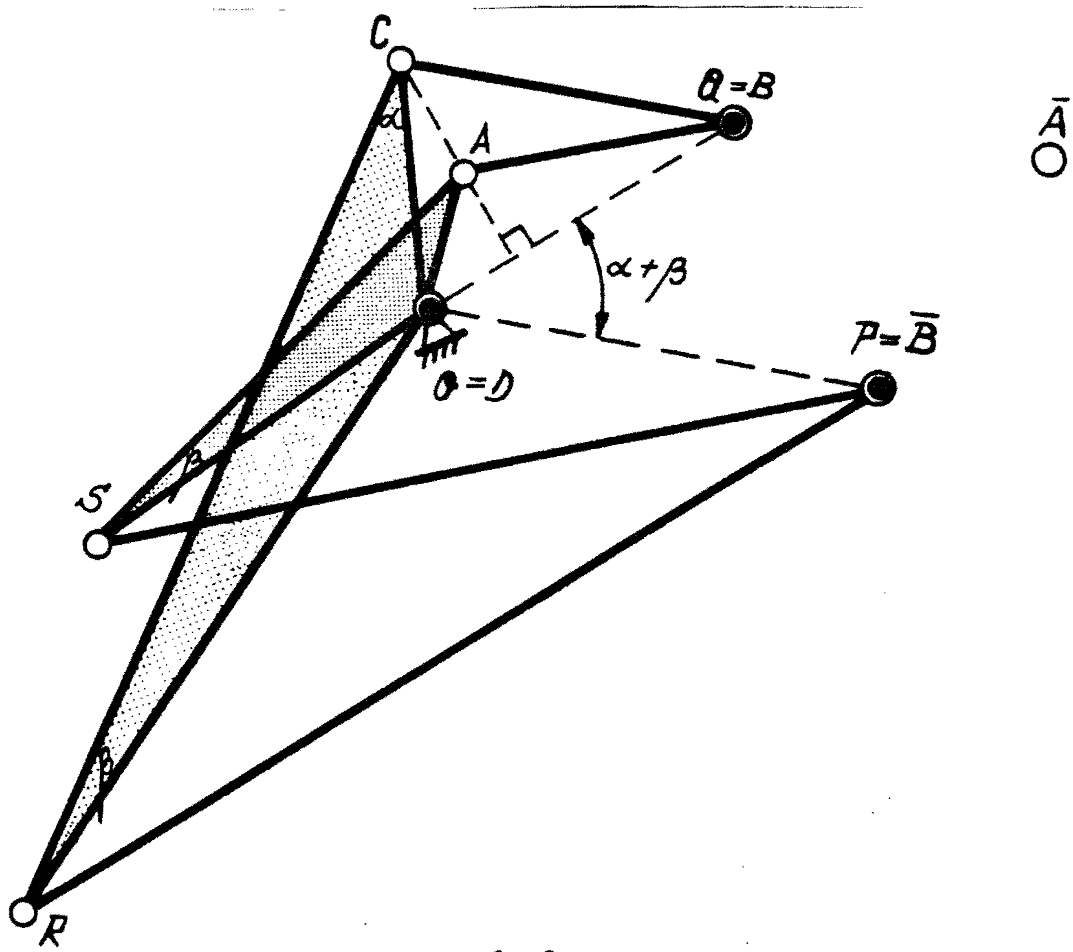
$\measuredangle\ QOP = \alpha + \beta = \text{constant}$

$\overline{OP}\bullet\overline{OQ} = \text{constant} = \overline{OA}\bullet\overline{OS}\ (\overline{A\bar{S}}^2 - \overline{AS}^2)\ /\ \overline{AS}^2$

$= \overline{OC}\bullet\overline{OR}\ (\overline{C\bar{R}}^2 - \overline{CR}^2)\ /\ \overline{CR}^2$

Two different quadruplanar *inversion-cells* of Sylvester and Kempe *interconnected* through 4 turning-joints Q, O, P and U, forming a parallelogram with variable lengths.
Double overconstrained 8-bar linkage with two contraparallelogram-linkages, to wit $AS\bar{A}\bar{S}$ and $CR\bar{C}\bar{R}$.
Figure 9.

This cell represents a generalization of Hart's contraparallelogram cell. However, the crucial points of this cell no longer join a straight-line: then, they appear to be the vertices of similar triangles attached to the respective sides of a contraparallelogram linkage. The four vertices P, O, Q and U form a parallelogram with variable lengths of its sides, though enclosing constant angles. When the cell is used as an exact straight-line mechanism, one obtains the so-called quadruplanar Inversor of Sylvestor and Kempe.
In view of the foregoing, it is only natural to assume that two of these generalized cells may be interconnected through common turning-joints at P, O, Q and U. Like before, we then omit the ternary links having point U as a common turning-joint. One so obtains a generalized 6-bar cell, containing the crucial points O,P and Q, for which $\overline{OP}\bullet\overline{OQ}$ = constant as well as $\measuredangle$ POQ = constant. (See figure 10).



f = 2

$\overline{OP} \bullet \overline{OQ}$ = constant =

= $\overline{OA} \bullet \overline{OS}\ \{\overline{PS}^2\ (\ \sin^2(\alpha+\beta)\ /\ \sin^2\alpha\ )\ -\ \overline{AS}^2\ \}\ /\ \overline{AS}^2$.
Further generalized *inversion-cell* with two *triangular* links and four binary ones.
Figure 10.

Design:
a. Start from a *random* 4-bar ABCD with perpendicular diagonals,
b. *Choose* a random point S, attached to AD,
c. Make $\triangle$ DCR similar to $\triangle$ DAS,
d. Determine point W with $\measuredangle$ BAW = α = $\measuredangle$ BDW,
e. Draw $S\overline{S}$ // DW, point $\overline{S}$ joining AW,
f. Determine the perpendicular bisector of SS,
g. Determine the image $\overline{A}$ of A into this perpendicular bisector
h. Determine point P with $\triangle$ $\overline{A}$SP ~ $\triangle$ ASD,
i. Adjoin the linkage-dyad SPR.

The so-obtained inversion-cell is of the Watt-type, and represents a highly generalization of Peaucellier's cell. Naturally, all particular cases are derivable from this highly generalized inversion-cell.

Rezumat: *O metodă de a generaliza "Inversorul Peaucellier"*

Două lanţuri antiparalelograme diferite, pot fi legate cu 4 puncte comune de rotaţie, fără a-şi schimba gradul de mobilitate. Aceasta se poate realiza în felul următor: Laturile concomitente ale două antiparallelograme se întretaie reciproc în aceste 4 puncte comune de rotaţie, situate pe o linie dreaptă, aflate paralel faţă de fiecare diagonală ale celor două antiparalelograme. Compoziţia ce rezultă din aceste două antiparalelograme, legate între ele, reprezintă un lanţ dublu determinat. Dacă înlăturăm două laturi ale acestei compoziţii, ce au unul din aceste 4 puncte comune de rotaţie, va rezulta un lanţ cu 6 laturi de tip - Watt - , apărînd astfel ca o generalizare a celulei Peaucellier.
O generalizare mai departe este deasemeni posibilă, dacă se începe cu generalizarea celulei Hart, la fel cum au procedat Sylvester şi Kempe, şi apoi acestă aplicare prin procedura cu celula obţinută, cum a fost menţionată mai sus.

Summary: *A way to generalize Peaucellier's Inversor*

Any two different contraparallelogram-chains may be linked together with *four* turning-joints being located at the common intersection-points of a random-line running parallel to the diagonals of these two contraparallelograms.
The composition of these contraparallelograms represents a double overconstrained linkage chain having the same mobility as each contraparallelogram alone. By erasing two ternary links having one of these four turning-joints in common, a six-bar linkage of Watt's type remains that appears to be a generalization of Peaucellier's cell.
A further generalization is possible by first generalizing Hart's cell, as done by Sylvester and Kempe, and then applying the same procedure as above with this so-called contraparallelogram cell of Sylvester and Kempe.

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