A single server queue with mixed types of interruptions : application to the modelling of checkpointing and recovery in a transactional system

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EUT Report 83-E-138
ISBN 90-6144-138-2
ISSN 0167-9708
July 1983

Eindhoven University of Technology Research Reports

EINDHOVEN UNIVERSITY OF TECHNOLOGY

Department of Electrical Engineering
Eindhoven The Netherlands

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CIP-gegevens

Nicola, V.F.

A single server queue with mixed types of interruptions:
application to the modelling of checkpointing and recovery
in a transactional system / by V.F. Nicola. -
Eindhoven: University of Technology. -
(Eindhoven University of Technology research reports,
ISSN 0167-9708; 83-E-138)
Met lit. opg., reg.
ISBN 90-6144-138-2
SISO 517.1 UDC 519.24 UGI 650
Trefw.: wachttijden.

Abstract

This paper demonstrates a brief derivation of the average number of customers in an M/G/1 queue with mixed types of Poisson interruptions. We note the relation to the Pollaczek-Khintchine formula. The analysis is based on the definition of the effective service time and the completion time associated with a customer's service.
The results are applied to the modelling of checkpointing and recovery in a transactional system.

Nicola, V.F.
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Department of Electrical Engineering, Eindhoven University of Technology, 1983.
EUT Report E-83-138

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Contents

1. Introduction

The analysis of queueing systems with mixed types of interruptions is important in many computer systems modelling applications such as systems operating in different modes, systems subject to breakdowns and priority queueing systems.

The M/G/1 queue with a single type of Poisson interruptions was dealt with extensively by Gaver [2] for a variety of service-interruption interactions. The analysis was based on the definition of the completion time. He derived the Laplace transform of the density function of the completion time and used the method of imbedded Markov chain and the renewal theory to obtain the generating function of the distribution of the number of customers in the system.

In this paper we show that if one is only interested in the average number of customers in the system, then it is possible to derive it using probabilistic arguments which can be proved rigorously. The present analysis is based on the definition of the effective service time which is meaningful and significant in subsequent discussions. We further extend the results of Gaver to include the case of M/G/1 queue with different types of Poisson interruptions present simultaneously. Section 2 contains a description of the system and the different types of service-interruption interaction and introduces basic definitions. In section 3 we establish the relations between the moments of the completion time, the effective service time and the customer's service time. The average number of customers in the system is obtained in section 4 when different types of Poisson interruptions are present simultaneously . In section 5, application of theory to the modelling of checkpointing and recovery in a transactional database system is considered.

2. Definitions; the effective service time and the completion time

Consider the M/G/1 queue subject to different sources of Poisson interruptions of different types. Customers receive service according to the FCFS discipline.

It is necessary to distinguish different types of interruptions. Independent interruptions may arrive when the system is idle or when the system is servicing a customer. Active interruptions may arrive only when the system is servicing a customer. No interruptions may arrive when the system is servicing an interruption.
The following is a classification of the different types of service-interruption interactions considered in this paper (See fig. 1).

I) Preemptive interruption (pmv)

Customer's service is preempted immediately on arrival of an interruption. After servicing the interruption there are two possibilities; namely:

a) preemptive-resume (prs): the customer's service is resumed from the point at which it was preempted.

b) preemptive-repeat (prt): the customer's service is repeated from its beginning. In preemptive-repeat-identical interruption (pri), the same identical customer's service time is repeated. In preemptive-repeat-different interruption (prd), a corresponding customer's service time from the same distribution is repeated.

II) Postponable interruption (psp):

Customer's service continues upon the arrival of an interruption. The interruptions accumulated during the customer's service are serviced immediately after servicing the customer.
Any of the interruptions classified above may be active (a) or independent (i).
We define the subsets of interruption sources: aprd, apri, aprt, aprs, apmv, apsp and a, corresponding to the different types of active interruption, and the subsets ipri, iprd, iprt, iprs, ipmv, ipsp and i, corresponding to the different types of independent interruption. It follows for the subsets of active interruption sources

aprt = either apri or aprd

apmv = aprt U aprs

a = apmv U apsp

and for the subsets of independent interruption sources

iprt = either ipri or iprd

ipmv = iprt U iprs

i = ipmv U ipsp

Define also the subsets pri, prd, prt, prs, pmv and psp such that

pri = apri U ipri

prd = aprd U iprd

prt = either pri or prd

prs = aprs U iprs

pmv = apmv U ipmv = prt U prs

psp = apsp U ipsp

The total set of interruption sources (T) which may be present in the system is given by

T = pmv U psp = a U i

Note that interruptions of the subsets pri and prd may not be present simultaneously in the system.

In subsequent discussion the index $t(t \in T)$ indicates the source of interruption.

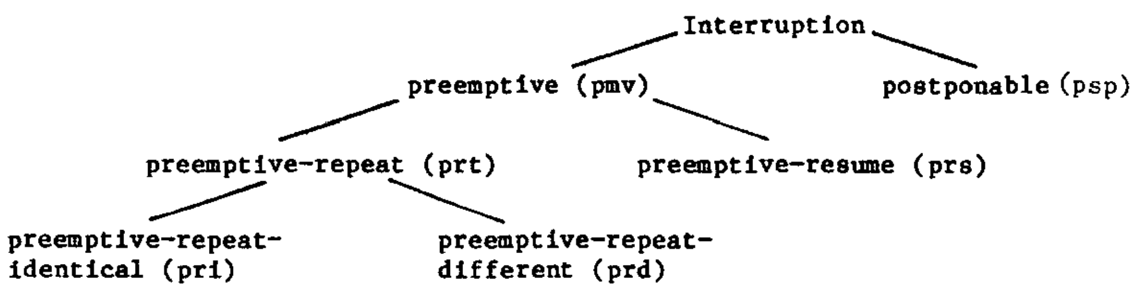

Fig. 1 Classification of different types of service - interruption interaction

The following notations and definitions are related to the system.

λ is the customer's arrival rate.

S is the customer's service time, a random variable, E(S) and $E(S^2)$ are the first and second moments, respectively.

v_t is the arrival rate of interruptions from source t.

I_t is the service time of source t interruption, a random variable $E(I_t)$ and $E(I_t^2)$ are the first and second moments, respectively.

We define the effective service time (S_e) to be the random interval of time spent by the system in servicing a customer, including the repetitions due to interruptions during the customer's service and excluding the time spent in servicing the interruptions. Thus

$$(2.1) \qquad S_e = \begin{cases} S & \text{, if no prt interruptions are present} \\ \sum_{k=1}^{N_{pri}} S_{pri}(k) + S & \text{, if pri interruptions are present} \\ \sum_{k=1}^{N_{prd}} S_{prd}(k) + S' & \text{, if prd interruprions are present} \end{cases}$$

N_{pri} *is the random number of pri interruptions* (possibly from different sources) which arrived during the customer's service before an interval S identical to the customer's service time is elapsed without pri interruption.

N_{prd} is the random number of prd interruptions (possibly from different sources) which arrived during the customer's service before an interval S' corresponding to the customer's service is elapsed without prd interruptions (Note that S' is a random interval of time with a probability distribution different to that of S).
$S_{pri}(k)$ (or $S_{prd}(k)$) is the random interval of time spent in servicing the customer between the k-th and the (k-1)-th pri (or prd) interruptions (k=0 corresponds to the beginning of the customer's service).

It is important to note that when different types of interruptions are present, S_e is *merely determined by the pri (or prd) interruptions.* If there are no prt interruptions, then S_e is identical to the customer's service time S.
The completion time (C, as defined by Gaver) is the random interval of time between the instant at which a customer service begins and the instant at which the service of the next customer may begin (does begin, provided that a customer is present). It follows that

(2.2) $$C = S_e + \sum_{t\in T} \sum_{k=1}^{N_t} I_t(k)$$

N_t is the random number of source t interruptions which arrived during the customer's service and $I_t(k)$ is the random time interval spent in servicing the k-th interruption of source t. S_e is as given by equation (2.1). It is important to note that the completion times as defined above are independent and identically distributed random variables.

3. The first moment and the expected residual time

In this section we relate the first moment and the expected residual time of the effective service time (S_e) and the completion time (C) to the probability distribution function of the customer's service time (S), the rates of interruptions (v_t, $t\in T$), the first and the second moments of the service time of interruptions (I_t, $t\in T$).

Let D be a renewal time interval with E(D) and $E(D^2)$ the first and the second moments, respectively. The expected residual time R(D) of the renewal interval is given by the well-known relation

(3.1) $$R(D) = \frac{E(D^2)}{2E(D)}$$

Consider the M/G/1 queue with mixed types of interruptions, $t \in T$.

Denote by v_I (or v_D) the merged rate of all pri (or prd) interruption sources. Thus

$$v_I = \sum_{t\in pri} v_t, \text{ and}$$

$$v_D = \sum_{t\in prd} v_t$$

It can be shown using equation (2.1) (see appendix) that $E(S_e)$ is given by

$$(3.2)\quad E(S_e) = \begin{cases} E(S) & \text{, if no prt interruptions are present} \\ \frac{1}{v_I}\left[E(e^{Sv_I})-1\right] & \text{, if pri interruptions are present} \\ \frac{1}{v_D E(e^{-Sv_D})}\left[1-E(e^{-Sv_D})\right] & \text{, if prd interruptions are present} \end{cases}$$

and that the expected residual time $R(S_e)$ is given by

$$(3.3)\quad R(S_e) = \begin{cases} \frac{E(S^2)}{2E(S)} & \text{, if no prt interruptions are present} \\ E(S_e) - \frac{1}{E(S_e)}\left[\frac{dE(S_e)}{dv_I} - \frac{E(e^{2Sv_I})-(E(e^{Sv_I}))^2}{v_I^2}\right] & \text{, if pri interruptions are present} \\ E(S_e) - \frac{1}{E(S_e)}\,\frac{dE(S_e)}{dv_D} & \text{, if prd iterruptions are present} \end{cases}$$

Let A be the expected fraction of completion time spent by the system in actually servicing the customer. From the Poisson property of interruptions it follows that the expected fraction of completion time A_t spent by the system in servicing source t interruptions is given by $A\ v_t\ E(I_t)$. This yields

$$(3.4)\qquad A = \left[1 + \sum_{t\in T} v_t\ E(I_t)\right]^{-1}$$

$$(3.5)\qquad A_t = v_t\ E(I_t)\ \left[1 + \sum_{t\in T} v_t\ E(I_t)\right]^{-1}\ ,\ \text{for all } t\in T$$

From the definition of A we have for E(C) the following

$$(3.6)\qquad E(C) = \frac{E(S_e)}{A}$$

$$= E(S_e)\left[1 + \sum_{t\in T} v_t\ E(I_t)\right]$$

with $E(S_e)$ as given by equation (3.2).

The expected residual time R(C) is given without a rigorous proof, but it can be interpreted by probabilistic arguments.

$$(3.7) \qquad R(C) = \sum_{t \in prs \cup psp} \left[R(I_t) + \frac{R(S_e)}{A}\right]A_t + \sum_{t \in prt} \left[R(I_t) + E(c)\right]A_t + R(S_e)$$

with A and A_t as given by equations (3.4) and (3.5). $R(I_t)$ is the expected residual time of source t interruption given by equation (3.1). $R(S_e)$ is given by equation (3.3) and E(C) is given by equation (3.6).

Note that in the former equations the interruptions $t \in prt$ may be either all of type pri or all of type prd but not mixed and that the interruptions may be of any type - active or independent.

The formula (3.7) is intuitively clear for cases with prs and prt interruptions. For cases with psp interruptions, one should notice that the probability distribution of the completion time C is identical to the distribution that one would obtain if the psp interruptions are treated as prs interruptions.

4. The average number of customers in the system

In this section we derive the average number of customers in the system with mixed types of interruptions by probabilistic arguments [5,8].
It is shown that when all interruptions are of the active-preemptive type, the average number of customers in the system is determined by the Pollaczek-Khintchine formula.
It is convenient at this point to introduce the concept of a virtual customer associated with each real customer; its service time is identical to the completion time defined in section 2. This implies

that a virtual customer leaves the system only when its "own" postponable interruptions, if any, are serviced, while a real customer leaves the system just before servicing the postponable interruptions. In any case, it is obvious that the system is stable if $\lambda E(C)$ is less than unity.

Denote by P_I the expected fraction of time the system is idle (i.e. neither customers nor interruptions in the system). From the Poisson property of interruptions it follows that the expected fraction of time the system is servicing independent interruptions from source $t \in i$ that start busy periods is given by $P_t = P_I v_t E(I_t)$. This is also the expected number of independent interruptions from source $t \in i$ that start busy periods in service.

P_I can be determined from the normalizing relation

$$\lambda E(C) + P_I + \sum_{t \in i} P_t = 1$$

It follows that

$$P_I = [1-\lambda E(C)] \, [1 + \sum_{t \in i} v_t \, E(I_t)]^{-1} \tag{4.1}$$

and

$$P_t = v_t \, E(I_t) \, [1 - \lambda E(C)] \, [1 + \sum_{t \in i} v_t \, E(I_t)]^{-1} \tag{4.2}$$

Let W' be the mean response time of a virtual customer and denote by N' the average number of virtual customers seen by an arrival; it is identical to the time average since arrivals are Poisson [10].

The expected number of virtual customers in service seen by an arrival is $\lambda E(C)$ and hence the expected number of waiting virtual customers is $(N' - \lambda E(C))$. The mean response time W' of a virtual customer is made up of the following terms

$$W' = W_1' + W_2' + W_3' + W_4'$$

W_1' is the expected remaining service of independent interruptions that start busy periods, found in service

$$W_1' = \sum_{t \in i} P_t \, R(I_t)$$

with $R(I_t)$ from equation (3.1) and P_t from equation (4.2).

W_2' is the expected remaining service of virtual customers found in service

$$W_2' = \lambda E(C)\ R(C)$$

with E(C) and R(C) from equations (3.6) and (3.7).
W_3' is the expected time spent in servicing the waiting virtual customers found in the system

$$W_3' = \left[N' - \lambda E(C)\right] E(C)$$

W_4' is the expected service time of the arriving virtual customer

$$W_4' = E(C)$$

Substituting for N' from Little's formula ($N' = \lambda W'$) yields an explicit expression for W'

$$(4.3) \qquad W' = \left[1 + \sum_{t \in i} v_t\ E(I_t)\right]^{-1} \sum_{t \in i} v_t\ E(I_t)\ R(I_t)$$

$$= \left[1 - \lambda E(C)\right]^{-1} \lambda E(C)\ R(C) + E(C)$$

The mean response time W of a real customer is determined by subtracting the expected time spent in servicing the postponable interruptions, if any

$$(4.4) \qquad W = W' - E(S_e) \sum_{t \in psp} v_t\ E(I_t)$$

The average number of real customers in the system N follows by using Little's formula

$$(4.5) \qquad N = \lambda\left[1 + \sum_{t \in i} v_t\ E(I_t)\right]^{-1} \sum_{t \in i} v_t\ E(I_t)\ R(I_t)$$

$$+ \left[1-\lambda E(C)\right]^{-1} \lambda^2 E(C)\ R(C) + \lambda E(C) - \lambda E(S_e) \sum_{t \in psp} v_t\ E(I_t)$$

When all interruptions are of the active-preemptive type, N reduces to

$$(4.6) \qquad N = \lambda E(C) + \left[1 - \lambda E(C)\right]^{-1} \lambda^2 E(C)\ R(C)$$

This is the well-known Pollaczek-Khintchine fomula. This result could be anticipated since when all interruptions are active-preemptive, the system can be viewed as an M/G/1 queue with the customer's service time replaced by the virtual customer's service time (or, equivalently, the completion time).

5. Applications to the modelling of checkpointing and recovery in a transactional database system

5.1 Introduction

Periodical checkpointing is a common technique for maintaining the integrity of information in database systems subject to failures. During a checkpoint a copy of the system files is saved in a secondary storage device. When a failure occurs, a recovery operation is initiated. It starts with reloading a copy of the system files that were saved at the last checkpoint into primary memory. This is followed by reprocessing all those transactions that have been processed since the last checkpoint. The system is unavailable for processing transactions during checkpointing and recovery operations. Too frequent checkpoints cost much time in making unnecessary copies, and too infrequent checkpoints cost much time in recoveries after failures. Therefore it is of interest to determine the checkpointing frequency that optimizes certain performance measures such as system availability (the fraction of time that the system is available for processing) or mean response time of a transaction.

In previous work [1, 3, 4, 7] models were presented in which checkpoints and recoveries were modelled as independent-preemptive Poisson interruptions of exponentially distributed durations. In [1] Baccelli considered an M/G/1 system with two types of independent Poisson interruptions; namely preemptive-resume (for checkpointing) and preemptive-repeat-different (for recovery). In these models the mean of the recovery period is assumed to be proportional to the mean available time between checkpoints.

In this section we consider an M/G/1 system. Checkpoints may occur when the system is idle or when it is processing. If the system is processing then the checkpoint operation is postponed until the end of the transaction being processed. Therefore checkpoints are modelled as

independent-postponable Poisson interruptions. Checkpoint durations are independent and of identical general distribution.
Failures may occur only when the system is processing. A recovery operation preempts the transaction being processed. When recovery is completed the preempted transaction is reprocessed. Therefore recoveries are modelled as active-preemptive-repeat-identical Poisson interruptions. We use a more accurate recovery model than those considered previously. It is assumed that a random number of transactions should be reprocessed in a recovery operation. The distribution of this number is identical to that of the random number of processed transactions between failure occurrence and the last checkpoint. This yields independent recovery durations of identical distribution. The mean and variance of this distribution can be determined as functions of the checkpointing frequency as will be shown later.

5.2 Performance measures

First we define the parameters and the random variables associated with the model described in section 5.1.

Consider the M/G/1 system in which transactions arrive at rate λ. They are processed according to the FCFS discipline. The processing time of a transaction (S) is a random variable of general distribution, its Laplace Stieltjes Transform is $S^*(s)$ and its first and second moments are $E(s)$ and $E(S^2)$, respectively.
Checkpoints are independent-postponable Poisson interruptions. They occur at rate α. Checkpoint duration (B) is a random variable of general distribution, its first and second moments are $E(B)$ and $E(B^2)$, respectively.
Recoveries are active-preemptive-repeat-identical Poisson interruptions. They occur at rate γ (failure rate). Recovery duration (Q) is a random variable of general distribution, its first and second moments are $E(Q)$ and $E(Q^2)$, respectively.
We proceed to determine the first moment and the expected residual time of the effective service time (S_e) and the completion time (C) as defined in sections 2 and 3. From equations (3.2) and (3.3) we have

$$(5.1) \qquad E(S_e) = \frac{1}{\gamma}\left[S^*(-\gamma)-1\right]$$

$$(5.2) \qquad R(S_e) = \frac{1}{\gamma} \; S^*(-\gamma) - \left[S^*(-\gamma)-1\right]^{-1} \frac{dS^*(-\gamma)}{d\gamma}$$

where we make use of the identity $E(e^{\gamma S}) \equiv S^*(-\gamma)$.

Let A be the expected fraction of completion time spent by the system in processing the transaction (thus excluding the time spent in servicing interruptions). From equation (3.4) we have

$$(5.3) \qquad A = \left[1 + \alpha \, E(B) + \gamma \, E(Q)\right]^{-1}$$

Let A_B and A_Q be the expected fraction of completion time spent by the system in servicing checkpoints and recoveries, respectively. It follows that

$$(5.4) \qquad A_B = \alpha \, E(B) \, A$$

$$(5.5) \qquad A_Q = \gamma \, E(Q) \, A$$

Equations (3.6) and (3.7) give for E(C) and R(C) the following

$$(5.6) \qquad E(C) = \frac{E(S_e)}{A}$$

$$(5.7) \qquad R(C) = A_B \left[R(B) + \frac{R(S_e)}{A}\right] + A_Q\left[R(Q) + \frac{E(S_e)}{A}\right] + R(S_e)$$

Let P_I be the probability that the system is idle. It is determined from equation (4.1). The system availability (A*) is given by

$$(5.8) \qquad A^* = \lambda E(S_e) + P_I$$

$$= (1-\lambda E(S_e) \; \gamma E(Q)) \left[1 + \alpha E(B)\right]^{-1}$$

The mean response time of a transaction (W) is determined from equation

(4.5) and Little's formula

$$(5.9) \qquad W = [1 + \alpha E(B)]^{-1} \alpha E(B) R(B) + [1 - \lambda E(C)]^{-1} \lambda E(C) R(C) + E(C) - E(S_e) \alpha E(B)$$

The first term is the contribution of checkpoints that start busy periods and the last term is due to the postponement of checkpoints. The middle terms correspond to the Pollaczek-Khintchine form (see equation (4.6)).

For optimization of performance measures, we need to establish a model for the dependence of recovery duration Q on the checkpointing rate α. It can be shown [3], for exponential available time interval between checkpoints and Poisson failure occurrences, that the available time interval F between failure occurrence and the last checkpoint is exponentially distributed with a mean α^{-1}. Assume that the completion process of transactions (in the available time) is Poisson with rate λ/A^*. The mean and variance of the random number NF of completed transactions between failure occurrence and the last checkpoint can easily be determined, and is given by

$$(5.10) \qquad E(NF) = \frac{\lambda}{A^*} E(F) = \frac{\lambda}{\alpha A^*}$$

$$(5.11) \qquad var(NF) = (\frac{\lambda}{A^*})^2 var(F) + \frac{\lambda}{A^*} E(F) = \frac{\lambda}{\alpha A^*} [1 + \frac{\lambda}{\alpha A^*}]$$

where var(.) denotes the variance of a random variable. It is assumed that a random number, corresponding to NF and of identical distribution, should be reprocessed in a recovery operation. This yields a recovery duration Q of mean and variance [9] given by

$$(5.12) \qquad E(Q) = E(NF) E(S)$$

$$= \frac{\lambda}{\alpha A^*} E(S)$$

(5.13) $$var(Q) = E(NF)\ var(S) + var(NF)\ [E(S)]^2$$

$$= \frac{\lambda}{\alpha A^*} \left[E(S^2) + \frac{\lambda}{\alpha A^*} (E(S))^2\right]$$

The expected residual time R(Q) follows

(5.14) $$R(Q) = R(S) + \frac{\lambda}{\alpha A^*} E(S)$$

The optimization of performance measures with respect to the checkpointing rate α can be carried out analytically or numerically after substituting for E(Q) and R(Q) from equations (5.12) and (5.14). In general the maximization of the system availability A* and the minimization of the mean response time of a transaction W yield different values for the optimum checkpointing rate.

6. Conclusions

We have defined the effective service time and the completion time associated with a customer's service in an M/G/1 queue with mixed types of Poisson interruptions.

The first moment and the expected residual time of the effective service time and of the completion time are expressed in terms of the probability distribution function of the customer's service time, the rates of interruptions and the moments of the service time of interruptions.

When all interruptions are active-preemptive, the average number of customers in the system can be determined by the Pollaczek-Khintchine formula, with the customer's service time replaced by the completion time. When other types of interruptions are present the average number of customers in the system follow by probabilistic arguments.

The theory developed is relevant in many computer systems performance modelling applications. One such application, namely the modelling of checkpointing and recovery in a transactional database system is studied. The theory enables us to model the interaction of checkpointing and recovery with transaction processing under more realistic assumptions than used in previous work.

Acknowledgement

Discussions with prof. ir. F.J. Kylstra had a pronounced effect on the final version of this manuscript, for which I am very grateful.
It is also a pleasure to thank Dr. E.A. van Doorn for his interest, comments and encouragement.

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Appendix

Derivation of the Laplace Stieltjes Transform (LST) of the effective service time $\left(S_e^*(s)\right)$

i) For preemptive-repeat-identical (rpi) Poisson interruptions:

The effective service time (S_e) can be written as follows

$$(i.1) \qquad S_e = \sum_{i=1}^{N} S'(i) + S$$

S is the customer's service time with a probability distribution function S(x) and LST S*(s).

S(i) is the part of service expended between the (i-1)-th and i-th interruptions. Obviously S'(i) < S, for i = 1,2,...,N. N is the random number of interruptions during the customer's service . For a given customer's service S we have

$$\Pr.\{S'(i) \leq x \mid S'(i) < S\} = \frac{1-e^{-v_I x}}{1-e^{-v_I S}} \;, \quad 0 \leq x \leq S$$

where v_I is the interruption rate. It follows that

$$(i.2) \qquad E\left(e^{-sS'(i)} \mid S\right) = \frac{1}{1-e^{-v_I S}} \int_0^S e^{-sx}\, v_I\, e^{-v_I x}\, dx$$

$$= \left(\frac{v_I}{s+v_I}\right)\left[\frac{1-e^{-(s+v_I)S}}{1-e^{-v_I S}}\right]$$

From equations (i.1) and (i.2) and the independence of S'(i), for $1 \leq i \leq N$, and S, we can write

$$(i.3) \qquad E\left(e^{-sS_e} \mid S, N = n\right) = e^{-sS} \left(\frac{v_I}{s + v_I}\right)^n \left[\frac{1-e^{-(s+v_I)S}}{1 - e^{-v_I S}}\right]^n$$

The conditional probability that n interruptions will precede an uninterrupted service interval S is given by

$$(i.4)\qquad Pr.\{N = n \mid S\} = (1-e^{-v_I S})^n \, e^{-v_I S}$$

Removing the condition on N, it follows that

$$(i.5)\qquad E(e^{-sS_e} \mid S) = \frac{(s+v_I)\, e^{-(s+v_I)S}}{s + v_I\, e^{-(s+v_I)S}}$$

Removing the condition on S we finally get

$$(i.6)\qquad S_e^*(s) \triangleq E(e^{-sS_e}) = \int_o^\infty \left(\frac{s+v_I}{v_I}\right) \left[1 - \frac{s}{s + v_I e^{-(s+v_I)x}}\right] dS(x)$$

ii) <u>For preemptive-repeat-different (rpd) Poisson interruptions</u>:

The effective service time (S_e) can be written as follows

$$(ii.1)\qquad S_e = \sum_{i=1}^{N} S'(i) + S'$$

S' is the last interval; it is the first interval drawn from the customer's service time distribution that elapsed without interruptions. Its probability distribution function is different to that of S.
S'(i) is the part of service expended between the (i-1)-th and the i-th interruptions. N is the random number of interruptions during the customer's service.

$$Pr.\{S' < x\} = \frac{\int_o^x e^{-v_D y}\, dS(y)}{S^*(v_D)}$$

where v_D is the interruption rate. S(y) and S*(s) are the probability distribution function of customer's service time (S) and its LST, respectively. We notice that $S^*(v_D)$ is the probability of uninterrupted customer's service.
It follows that

$$(ii.2)\qquad E(e^{-sS'}) = \frac{\int_o^\infty e^{-sx}\, d \int_o^x e^{-v_D y}\, dS(y)}{S^*(v_D)}$$

$$= \frac{S^*(s+v_D)}{S^*(v_D)}$$

$$Pr.\{S'(i) < x\} = \frac{\int_o^x (1-S(y))(v_D e^{-v_D y} dy)}{1-S^*(v_D)}$$

It follows that

$$(ii.3) \qquad E(e^{-sS'(i)}) = \frac{\int_o^\infty e^{-sx} d \int_o^x v_D(1-S(y))e^{-v_D y} dy}{1 - S^*(v_D)}$$

$$= \frac{v_D}{1-S^*(v_D)} \left[\int_o^\infty e^{-(s+v_D)x} (1-S(x))dx \right]$$

$$= \left(\frac{v_D}{s+v_D}\right) \left[\frac{1-S^*(s+v_D)}{1-S^*(v_D)}\right]$$

From equations (ii.1), (ii.2) and (ii.3) and the independence of $S'(i)$, for $1 < i < N$, and S' we can write

$$(ii.4) \qquad E(e^{-sS_e} \mid N = n) = \frac{S^*(s+v_D)}{S^*(v_D)} \left[\frac{v_D(1-S^*(s+v_D))}{(s+v_D)(1-S^*(v_D))}\right]^n$$

The probability of n interruptions during customer's service is given by

$$Pr.\{N = n\} = (1 - S^*(v_D))^n S^*(v_D)$$

Removing the condition on N, we finally get

$$(ii.5) \qquad S_e^*(s) \triangleq E(e^{-sS_e})$$

$$= \frac{(s+v_D)\ S^*(s\ v_D)}{s + v_D\ S^*(s+v_D)}$$

Making use of the relation

$$E(S_e^i) = (-1)^i \left. \frac{d^i S_e^*(s)}{ds^i} \right|_{s = 0}$$, the i-th moment of S_e

it is left to the reader to verify the relations in equations (3.2) and (3.3) of the main text.

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