Measurement over rolls or balls for involute spur and helical gears

De Ingenieur

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Measurement over rolls or balls for involute spur and helical gears

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Samenvatting: In dit artikel wordt een methode aangegeven om steeds terugkerend omslachtig rekenwerk te vervangen door een zeer eenvoudige rekenwijze. Deze methode wordt hier toegepast op een kogel- of rollenmaat voor evolvente tandwielen.

Symbols

A	value dependent on z, x, d^*_M, α_n
B	coefficient dependent on z, x, C, β, α_n
C	value dependent on d^*_M, α_n
d_M	roll or ball diameter (kogel- of rolmiddellijn)
d^*_M	roll or ball diameter for $m_n = 1$
h_M	height of contact point (hoogte van het contactpunt)
h^*_M	height of contact point for $m_n = 1$
m_n	normal module (normaalmodulus)
M	measurement quantity over rolls or balls (kogel- of rollenmaat)
M^*	measurement quantity over rolls or balls for $m_n = 1$
q	auxiliary value for $\beta = 0°$
q_t	auxiliary value for $\beta \neq 0°$
Q	auxiliary quantity dependent on z, x
x	addendum modification coefficient (profielverschuivingscoëfficiënt)
z	number of teeth (aantal tanden)
α_n	normal pressure angle (normaaldrukhoek)
α_t	transverse pressure angle (omtreksdrukhoek)
α_M	pressure angle of the roll or ball centre for $\beta = 0°$
α_{Mt}	transverse pressure angle of the roll or ball centre
α_h	pressure angle of the contact point for $\beta = 0°$
α_{ht}	transverse pressure angle of the contact point
β	helix angle (tandhoek)
λ	auxiliary angle for $\beta = 0°$
λ_t	auxiliary angle for $\beta \neq 0°$

Introduction

The measurement over rolls or balls (kogel- of rollenmaat) placed in diametrically opposite tooth spaces is an accurate method of determining the tooth thickness of cylindrical involute gears. With the measurement of the base tangent length (tandwijdte) it has in common that the tooth thickness is measured independently of the tip cylinder and the axis of rotation of the gear. Both measurements have their advantages and drawbacks, and consequently they have their typical applications.

On the European continent the measurement of the base tangent length is the preferred method. It can be carried out easily and quickly, and calculation requires no great effort since modern tables [5] are available which give the basic values for a number of commonly used helix angles. Addendum modification (profielverschuiving) can be taken into account in a very simple way. However, the measurement of the base tangent length fails when the face width of a helical gear is smaller than the limiting value that can also be found in the tables mentioned above. Practical difficulties are also encountered when applying the method to finest pitch gears. In these cases the tooth thickness can only be determined by measurement over rolls or balls.

The disadvantages of measurement over rolls or balls are the cumbrous calculation, the uncertainty as to the accuracy achieved and the absence of clear indications as to the choice of the roll or ball diameter. The existing tables [1, 2, 3, 4], can be used neither for helical gears nor for modified gears. The purpose of this paper is the diminution or the elimination of these difficulties by a very simple calculation with a known accuracy and with a distinct choice of the ball diameter.

Original formulae and factors

The formulae below and their deduction are supposed to be known (fig. 1).

$$M = d_M + m_n \left\{ \frac{z}{\cos\beta} \cdot \frac{\cos\alpha_t}{\cos\alpha_{Mt}} \right\} \quad \text{for } z \text{ even} \quad (1)$$

$$M = d_M + m_n \left\{ \frac{z}{\cos\beta} \cdot \frac{\cos\alpha_t}{\cos\alpha_{Mt}} \cdot \cos\left(\frac{\pi}{2z}\right) \right\} \quad \text{for } z \text{ odd} \quad (2)$$

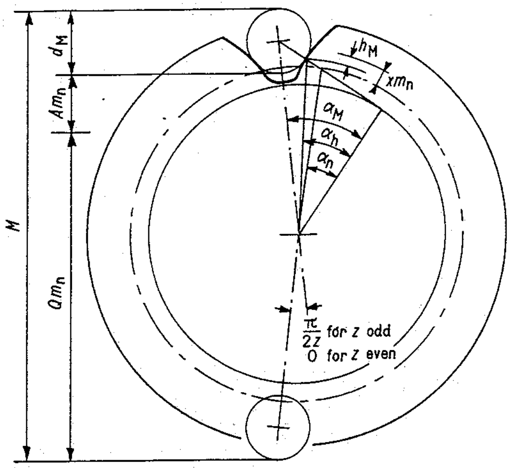

Fig. 1. Measurement over rolls or balls.

$$\operatorname{inv}\alpha_{Mt} - \operatorname{inv}\alpha_t = \frac{1}{z}\left\{\frac{d_M}{m_n}\cdot\frac{1}{\cos\alpha_n} - \frac{\pi}{2} + 2x\tan\alpha_n\right\} \tag{3}$$

$$\tan\alpha_t = \frac{\tan\alpha_n}{\cos\beta} \tag{4}$$

The normal pressure angle has a standardised value $\alpha_n = 20°$. The quantities M and d_M are proportional to the normal module m_n.

$$d_M = m_n \cdot d_M^* \tag{5}$$

$$M = m_n \cdot M^* \tag{6}$$

Diameter of the rolls or balls

The ball diameter has to be chosen in such a way that the ball projects well above the tip cylinder, even if the basic addendum (kophoogte in het theoretische heugelprofiel) should be 1.1 m_n. It should also touch the tooth flanks within an area that is not influenced by tip or root relief (intreespeling). As to this requirement, the middle third part of the working depth is a good assumption.

In the diametrical pitch system values of $d_M^* = 1.92$, 1.728, 1.68 and 1.44 are recommended or standardised. The value of 1.92 is chosen because of its favourable divisibility, which results in convenient roll or ball diameters in the diametrical pitch system. The other values are obtained by subtracting 10%, 12.5% and 25% from 1.92. As the same roll or ball cannot be used in the diametrical pitch system and the metric module system simultaneously, there is obvious no obligation to use exactly the same values.

A preliminary graphical analysis, fig. 2, revealed that for external gears two values of d_M^*, viz. 1.8 and 2.0 together meet the requirements in all possible cases. Especially the value 2.0 results in convenient roll or ball diameters in the metric module system.

Therefore, contrary to what is said in [3] but partly in accordance with [4] these two values are chosen as standards in the exposition to follow.

The requirement that the roll or ball rises above the teeth is easy to check: M compared with the tip diameter (middellijn van de topcirkel) is sufficiently long in all cases presented here. The second requirement is met if

$$-0.30 \leqq h_M^* \leqq +0.30 \tag{7}$$

where h_M^* is the height of the contact point of the roll or ball above the 'middle' of the tooth.

$$h_M^* = \frac{z}{2\cos\beta}\left(\frac{\cos\alpha_t}{\cos\alpha_{ht}} - 1\right) - x \tag{8}$$

$$\tan\alpha_{ht} = \tan\alpha_{Mt} - \frac{d_M^*\cos\alpha_n\cos^2\beta}{z\cos^2\alpha_t} \tag{9}$$

Fig. 3 represents the values of the number of teeth z and the addendum modification coefficient x for which $d_M^* = 1.8$ and $d_M^* = 2.0$ can be used. The upper boundary line for tip thickness

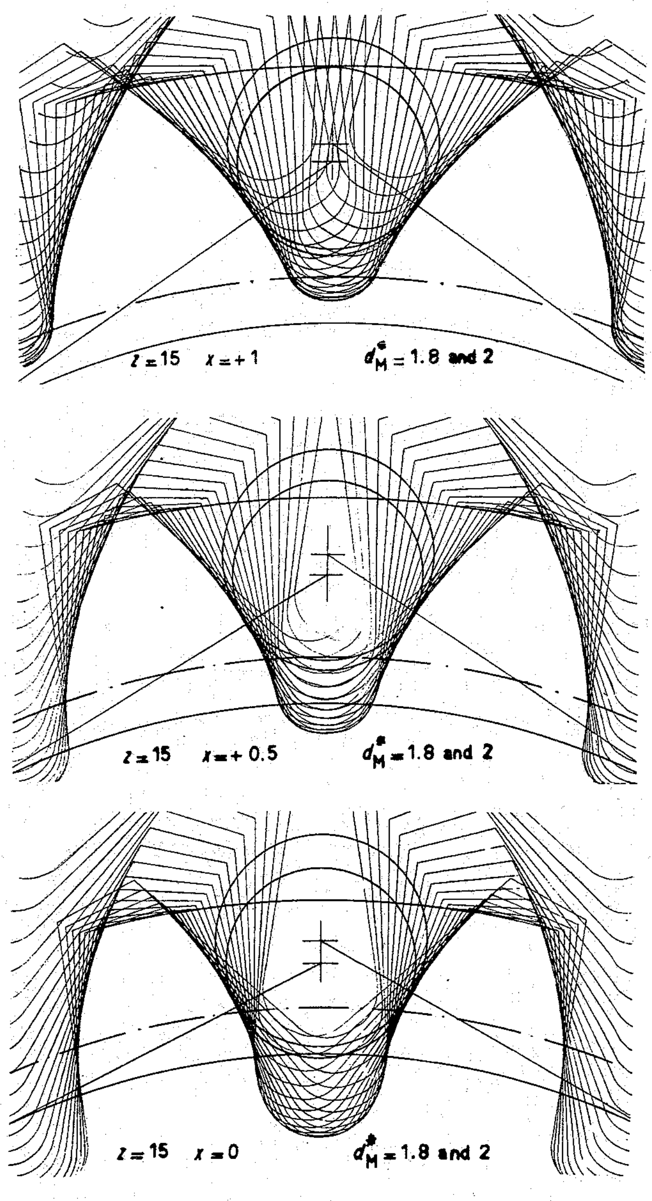

Fig. 2. Example of graphical analysis.

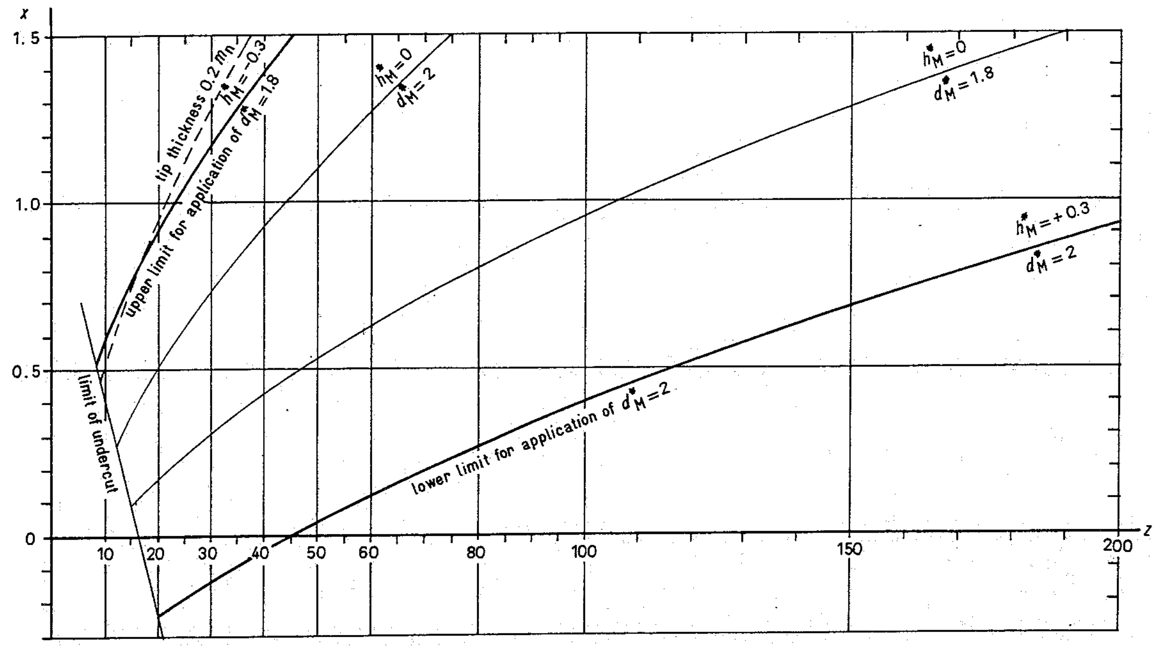

Fig. 3. Application areas for $d^*_{\mathrm{M}} = 1.8$ and $d^*_{\mathrm{M}} = 2.0$.

(toptanddikte) runs very close to the boundary line $h^*_{\mathrm{M}} = -0.30$ for $d^*_{\mathrm{M}} = 1.8$. Therefore, the only important boundary line is the line $h^*_{\mathrm{M}} = +0.30$ for $d^*_{\mathrm{M}} = 2.0$. This line separates an area in which $d^*_{\mathrm{M}} = 1.8$ and $d^*_{\mathrm{M}} = 2.0$ are both applicable, and an area, in which $d^*_{\mathrm{M}} = 1.8$ is recommended.

Existing tables

After fixing d^*_{M}, three independent quantities are left: the helix angle β, the number of teeth z, and the addendum modification coefficient x. Existing tables are restricted to $\beta = 0°$ and $x = 0$ [1, 2, 3], or some fixed value of x [4]. Consequently, these tables are of no use for general application and the original formulae must be applied frequently. Also for this reason the departure from these tables and the ball diameter coefficients on which they are based is fully justified.

The quantities β, z and x enter the formulae in a complicated way requiring detailed auxiliary tables. The calculation errors are multiplied by the number of teeth z, so the calculation has to be extended to a considerable number of decimal places.

New point of view

A radical simplification can be achieved by checking which independent quantities determine the function, by examining their influence and by splitting up the function. The calculation of the addendum modification coefficient was modernised in the same way, as developed in [6, 7] and adopted in [5, 8, 9, 10].

The independent quantities were β, z and x. For helix angle $\beta = 0°$ it could be possible to make a chart for direct reading of M^* from the parameter z and the abscissa x. Obviously, such a chart would have an unacceptable height.

An acceptable height of the chart is obtained when it is set up for reading the relatively small and moderately changing difference A between the second term of the formula for M^* and a suitably chosen quantity Q which is easy to calculate from z and x

So we get $M^* = d^*_{\mathrm{M}} + Q + A$

It must be emphasised here that this procedure does not affect the exactness of the formulae, for it does not imply an approximation. It only concerns the representation of the function as a sum of a quantity to be accurately read from a chart and a quantity easy to calculate.

The remaining independent quantity β can be dealt with in different ways. A vigorous way is to fix a number of values and to draw up a chart for each of them. If only few helix angles are standardised, this is a convenient method, indeed. The other way is to deduce a conversion factor. In our case, such a conversion factor exists as an exact function. Therefore, the complete new method is exact. The formula in the new presentation reads

$$M^* = d^*_{\mathrm{M}} + (Q + A)B$$

The quantity $Q = 0.999\,z + 1.75\,x$

has been chosen to obtain a convenient chart for the term A (Figs. 4 to 7) and is easy to calculate when written:

$$Q = z - \frac{z}{1000} + \frac{7}{4}x$$

The factor B for spur gears is $B = 1$. The factor B for helical gears is to be read from a table.

Accuracy of the new method

The term A in the diagram and the factor B in the table were determined by

$$A = z\left\{\frac{\cos\alpha_{\mathrm{n}}}{\cos\alpha_{\mathrm{M}}} - 0.999\right\} - 1.75x \qquad \text{for } z \text{ even} \quad (8)$$

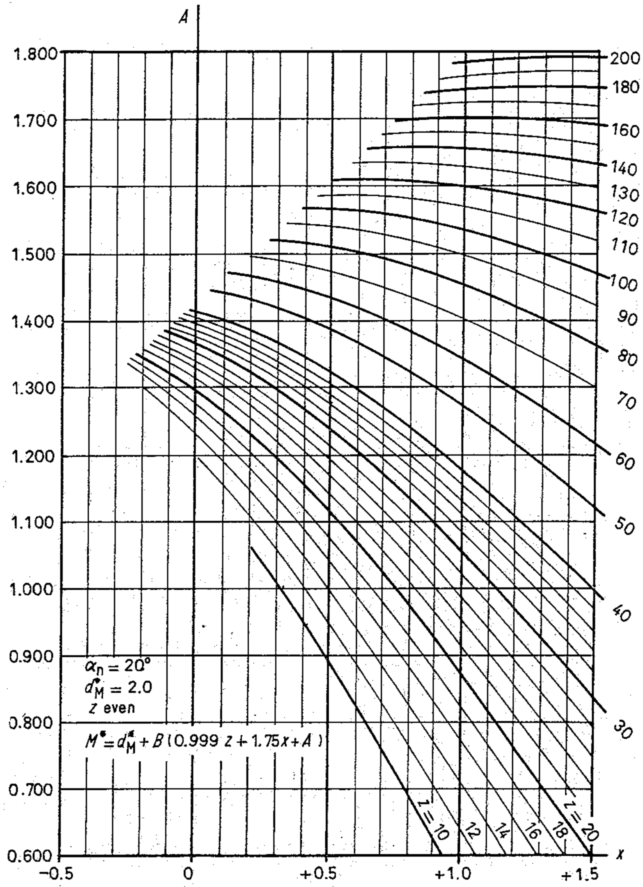

Fig. 4. Term A for $d^*_M = 2.0$ and z even.

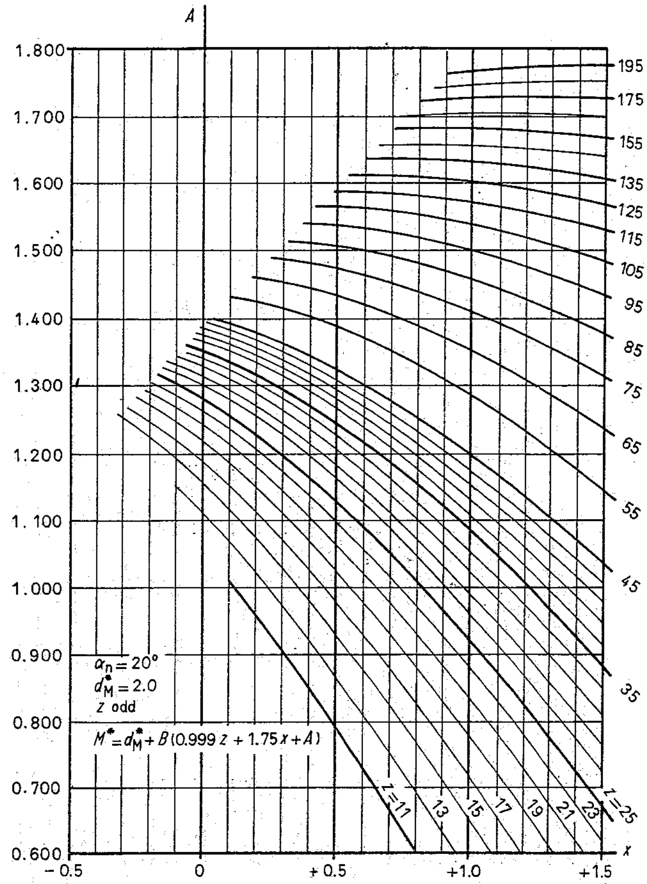

Fig. 5. Term A for $d^*_M = 2.0$ and z odd.

$$A = z\left\{\frac{\cos\alpha_n}{\cos\alpha_M}\cdot\cos\left(\frac{\pi}{2z}\right) - 0.999\right\} - 1.75x \qquad \text{for } z \text{ odd} \quad (9)$$

$$B = \frac{\cos\alpha_M \cos\alpha_t}{\cos\alpha_{Mt}\cos\alpha_n\cos\beta} \qquad (10)$$

$$C = \frac{d^*_M}{2\sin\alpha_n} - \frac{\pi}{4\tan\alpha_n} \qquad (11)$$

$$\text{inv}\,\alpha_M - \text{inv}\,\alpha_n = 2\,\frac{x+C}{z}\,\tan\alpha_n \qquad (12)$$

$$\text{inv}\,\alpha_{Mt} - \text{inv}\,\alpha_t = 2\,\frac{x+C}{z}\,\tan\alpha_n \qquad (13)$$

$$\tan\alpha_t = \frac{\tan\alpha_n}{\cos\beta} \qquad (14)$$

The calculations carried out by the authors to achieve accurate diagrams for the term A answered high requirements. Therefore, an auxiliary angle λ was introduced,

$$\lambda = \alpha_M - \alpha_n, \qquad (15)$$

and iterated from

$$2\frac{x+C}{z}\tan\alpha_n = \left\{\frac{\sin(\alpha_n+\lambda)}{\cos(\alpha_n+\lambda)} - \frac{\sin\alpha_n}{\cos\alpha_n} - \lambda\right\} =$$

$$= \frac{\sin\lambda}{\cos(\alpha_n+\lambda)\cos\alpha_n} - \lambda =$$

$$= \frac{\lambda}{q}\{1 - q + \lambda^2(C_2 + \lambda^2(C_4 + \lambda^2(C_6 + \lambda^2(C_8 + C_{10}\lambda^2))))\} \qquad (16)$$

where [11]
C_2 = —0.16666 66664
C_4 = + 0.00833 33315
C_6 = —0.00019 84090
C_8 = + 0.00000 27526
C_{10} = —0.00000 00239

and

$$q = \cos(\alpha_n + \lambda)\cos\alpha_n \qquad (17)$$

Next, A was found by (8) or (9) following substitution of:

$$\frac{\cos\alpha_n}{\cos\alpha_M} = \frac{\cos(\alpha_n+\lambda-\lambda)}{\cos(\alpha_n+\lambda)} = \cos\lambda + \sin\lambda\left(\tan\alpha_n + \frac{\sin\lambda}{q}\right) \qquad (18)$$

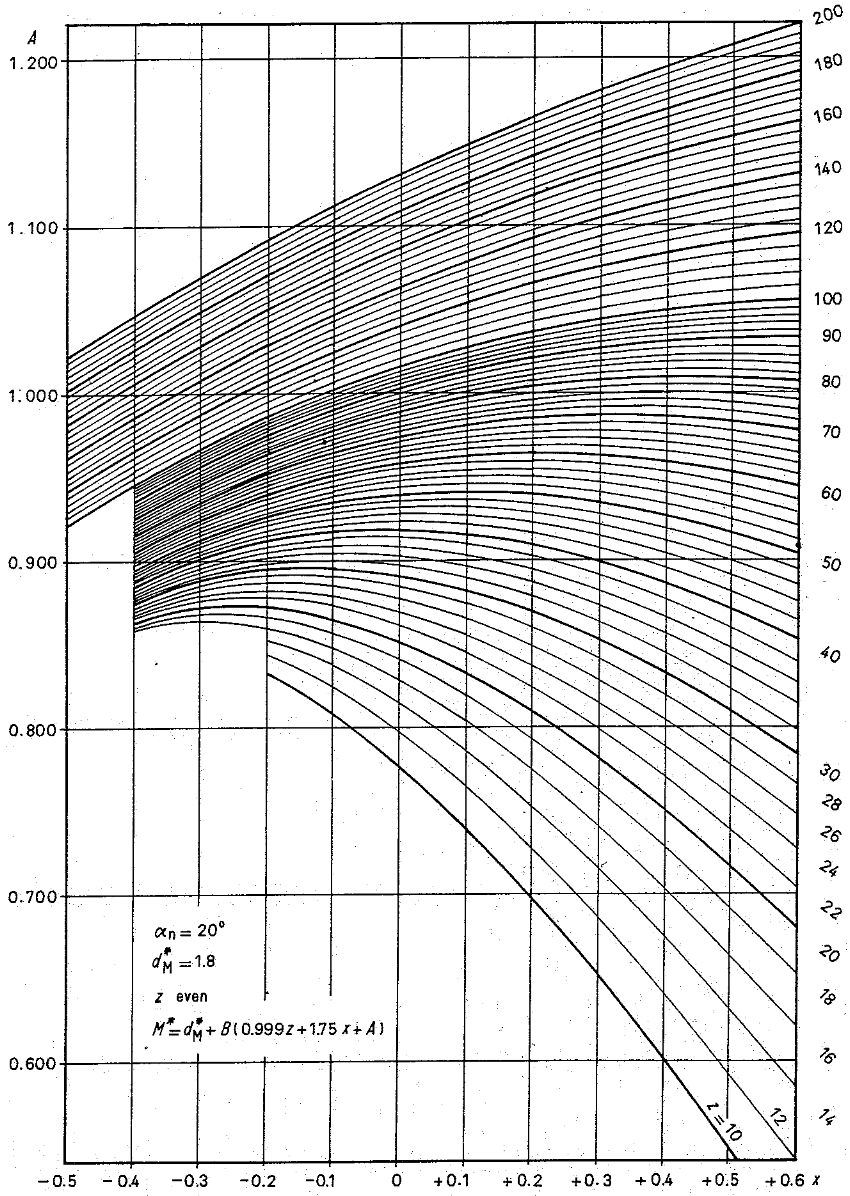

Fig. 6. Term A for $d^*_M = 1.8$ and z even.

The factor B was found after a second iteration with α_t, α_{Mt}, λ_t, q_t replacing α_n, α_M, λ, q in (15) to (18).

Finally, the accuracy of the method depends on the conscientiousness in the drawing and reading of the charts. The authors made the charts accurate to three decimal places. The charts in this paper are less accurate than suggested above because of the limited space available.

The table of the factor B

The factor B depends on the normal angle α_n, the helix angle β, the roll or ball diameter coefficient d^*_M, the number of teeth z and the addendum modification coefficient x. However, three of them cluster together in one factor $(x + C)/z$, as appears from (12) and (13). So, for a standardised α_n, the factor B depends on β and $(x + C)/z$ only. The constant C from (11) is given in the table below for $\alpha_n = 20°$.

Table 1. The constant C

d^*_M	C (*for* $\alpha_n = 20°$)
2.0	0.76594
1.8	0.47356

Table 2 contains the factor B for frequently used helix angles. In order to cover a vast area for $(x + C)/z$ and to offer good interpolation, we have split up the entry of the table into a linear part and a logarithmic part. The differences are marked with a minus sign because B is a decreasing function. Notice that $B = \sec\beta$ for $(x + C)/z = 0$.

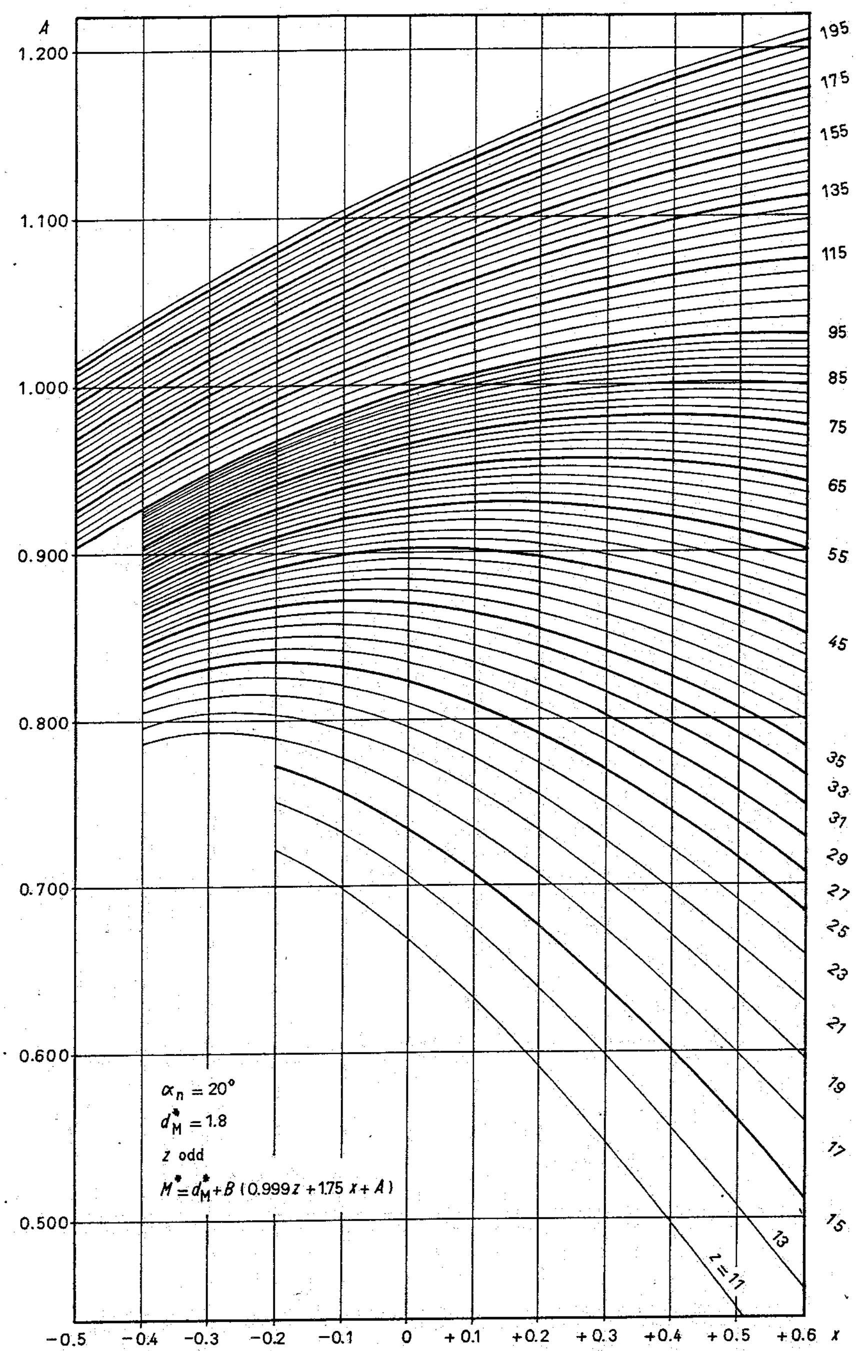

Fig. 7. Term A for $d^*_M = 1.8$ and z odd.

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Table 2. Factor B for frequently used helix angles β ($\alpha_n = 20°$)

$\frac{x+C}{z}$	$\log(200\frac{x+C}{z})$	β 6°	8°	10°	12°	15°	20°	30°
0.0000		1.005508	1.009828	1.015427	1.022341	1.035276	1.064178	1.154701
		−10	−19	−30	−44	−68	−125	−302
0.0010		1.005498	1.009809	1.015397	1.022297	1.035208	1.064053	1.154399
		−10	−18	−28	−41	−65	−118	−286
0.0020		1.005488	1.009791	1.015369	1.022256	1.035143	1.063935	1.154113
		−10	−17	−27	−38	−61	−111	−273
0.0030		1.005478	1.009774	1.015342	1.022218	1.035082	1.063824	1.153840
		−9	−16	−25	−37	−58	−106	−260
0.0040		1.005469	1.009758	1.015317	1.022181	1.035024	1.063718	1.153580
		−8	−16	−24	−34	−55	−101	−248
0.0050	0.0000	1.005461	1.009742	1.015293	1.022147	1.034969	1.063617	1.153332
		−11	−18	−29	−43	−67	−124	−306
	0.1000	1.005450	1.009724	1.015264	1.022104	1.034902	1.063493	1.153026
		−12	−22	−34	−49	−79	−145	−362
	0.2000	1.005438	1.009702	1.015230	1.022055	1.034823	1.063348	1.152664
		−14	−25	−40	−58	−91	−169	−424
	0.3000	1.005424	1.009677	1.015190	1.021997	1.034732	1.063179	1.152240
		−16	−29	−45	−66	−105	−194	−488
	0.4000	1.005408	1.009648	1.015145	1.021931	1.034627	1.062985	1.151752
		−18	−32	−51	−74	−118	−218	−556
	0.5000	1.005390	1.009616	1.015094	1.021857	1.034509	1.062767	1.151196
		−20	−36	−56	−82	−130	−243	−623
	0.6000	1.005370	1.009580	1.015038	1.021775	1.034379	1.062524	1.150573
		−22	−38	−61	−88	−143	−265	−685
	0.7000	1.005348	1.009542	1.014977	1.021687	1.034236	1.062259	1.149888
		−23	−42	−66	−96	−152	−285	−742
	0.8000	1.005325	1.009500	1.014911	1.021591	1.034084	1.061974	1.149146
		−24	−43	−68	−99	−160	−300	−790
	0.9000	1.005301	1.009457	1.014843	1.021492	1.033924	1.061674	1.148356
		−25	−45	−71	−104	−165	−312	−827
0.0500	1.0000	1.005276	1.009412	1.014772	1.021388	1.033759	1.061362	1.147529
		−25	−45	−72	−105	−170	−319	−853
	1.1000	1.005251	1.009367	1.014700	1.021283	1.033589	1.061043	1.146676
		−26	−46	−72	−106	−170	−322	−867
	1.2000	1.005225	1.009321	1.014628	1.021177	1.033419	1.060721	1.145809
		−25	−46	−72	−106	−169	−321	−869
	1.3000	1.005200	1.009275	1.014556	1.021071	1.033250	1.060400	1.144940
		−25	−44	−71	−103	−167	−317	−863
	1.4000	1.005175	1.009231	1.014485	1.020968	1.033083	1.060083	1.144077
		−24	−44	−69	−101	−163	−309	−847
	1.5000	1.005151	1.009187	1.014416	1.020867	1.032920	1.059774	1.143230

Korte technische berichten

Nieuwe ontwikkelingen in de werkplaatstechniek

In een rede die dr. John Convey als scheidend President van de 'American Society of Metals' hield voor de '49th Materials Engineering Exposition and Congress' te Ohio, signaleerde hij een aantal ontwikkelingen, die naar verwachting in de naaste toekomst tot industriële toepassingen zullen leiden. Het volgende is een resumé van hetgeen Convey naar voren bracht:

— Het aaneenlassen van delen van aandrijfassen en stangen door middel van de wrijvingswarmte, die vrijkomt als een roterend onderdeel tegen een vastgezet onderdeel wordt geperst, betitelde Convey als een staaltje van geavanceerde engineering. Het concept van het wrijvingslassen was enige jaren geleden nog geheel nieuw. Thans worden drie miljoen stuurstangen van auto's op deze wijze gemaakt. Bij deze toepassing wordt een goedkope koolstofstalen stang gelast aan een uit gelegeerd staal vervaardigd en gehard asdeel met glijspieën.

— Het lassen met elektronen - de techniek werd ontwikkeld voor ruimtevaarttoepassingen - vindt in de automobielindustrie reeds toepassing voor het fabriceren van tandwielen. Afwijkende materialen en dunne metaalfolies kunnen worden samengevoegd met behulp van een laserbundel.

— De superplasticiteit, die op laboratoriumschaal werd verkregen met aluminium-zinklegeringen en met roestvrije stalen, lijkt een veel belovende ontwikkeling. Met behulp hiervan zou het mogelijk kunnen worden om thermo-vormtechnieken toe te passen vergelijkbaar met die voor kunststoffen. Van sommige gelegeerde stalen kunnen de mechanische eigenschappen en de ductiliteit aanzienlijk worden opgevoerd als tijdens plastische deformatie een martensitische structuur - ontstaan door vervorming - kan worden verkregen.

— Naast deze metaalontwikkelingen staan die van de kunststoffen voor toepassingen op gebieden waar van oudsher de metalen domineerden. Kunststoffen nemen in tonnage, als constructiematerialen na staal de tweede plaats in. De nieuwe bumper, die thans op de Pontiac auto's wordt geïnstalleerd en die in belangrijke mate energie kan absorberen, was volgens hem een duidelijke aanwijzing van de mogelijkheden van plastics. In deze toepassing wordt de bumper gevormd uit een dikke laag urethaanschuim met staalwapening. Het urethaanschuim heeft het vermogen om aanzienlijke botsingsenergieën te absorberen en te verspreiden.

— Tenslotte wees Convey op het belang van de vonk-erosietechniek voor het vervaardigen van onderdelen uit moeilijk te bewerken metaallegeringen. Hij maakte speciaal melding van het gebruik van deze techniek bij het maken van onderdelen voor gasturbines, waarbij deze techniek reeds als standaardmethode ingang begint te vinden.

Op het gebied van non-destructief testen werd een speciale voordracht gehouden door G. R. Woody van National Airlines. Hoewel hij in zijn lezing slechts in grote lijnen inging op de problemen, die de luchtvaartmaatschappijen in hun bedrijf ondervinden, viel er een interessant detail te vernemen. Woody maakte melding van het gebruik van de 'sonic diagnostic engine analyzer', een instrument, dat gemaakt wordt door Curtiss-Wright Corporation, Electronics Division. National Airlines begon in 1966 te experimenteren met dit instrument, dat veertig verschillende geluiden, gemaakt door de bewegende delen van de motor, kan 'beluisteren'. Het instrument heeft drie microfoons, die tegen de motorgondel worden bevestigd. Vervolgens stemt men het instrument achtereenvolgens af op veertig afzonderlijke geluidsbronnen, die zich in de stationair draaiende turbinemotor bevinden. Een vergelijking van de signalen met die, verkregen uit een standaard functionerende motor, levert een indicatie op over de toestand van de verschillende bewegende delen en dus van de conditie, waarin de motor zich bevindt. National Airlines heeft het testen van motoren met dit instrument tot een standaardprocedure gemaakt. (Paper 50.)