

Models, information and control

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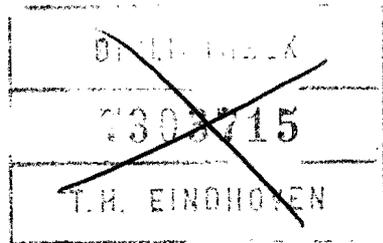
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Models, information and control.

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Models, information and control.

1. Introduction.

One of the aims of systems research is the entanglement of the terminological jungle. For that reason one should be happy to see that there are already interdisciplinary concepts such as system, model, information, adaptivity, stability and control. This offers a strong argument for considering some of these concepts together. There is however another reason for choosing the three concepts of model, information and control as main building blocks of a systemtheoretical article. First of all the concept of control is so pervasive in our culture that its importance hardly can be denied. Secondly the concepts of control, information and model are intimately related: one cannot control without a model of the system to be controlled and without feedback information.

In this article we explore the relationships between the three concepts and will formulate some conclusions.

2. On systems.^x

Elsewhere (de Leeuw, 1971) we called a set of objects W a system if and only if for all non-zero real subsets A of W there is a relation between A and the complement $W \setminus A$.

We may define the environment E of the system as the set of objects which are not a member of the set of objects W but are related to W . The set of relations within W may be called the internal structure R_{WS} and the set of relations between E and W is called the external structure R_{ES} .

Now a system can be characterized as a four-tuple S .

$$S = \langle W, E, R_{ES}, R_{WS} \rangle$$

A very special but important system is a black-box. A black-box is a system with only one object in W . Thus:

$$S = \langle \{ \omega_0 \}, E, R_{ES} \rangle$$

The internal structure of a black-box is not known. When considering the behavior of a black-box it is convenient to decompose the external structure into two sets: the input and the output.

This leads to the familiar picture of a black-box shown in figure 1.

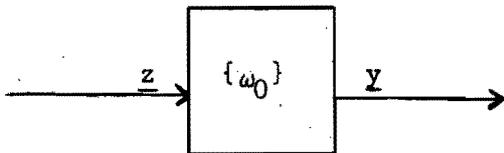


figure 1. A black-box

The input is an element of the input set $D(\underline{z})$. In the same way we state $\underline{y} \in D(\underline{y})$. Now the input set $D(\underline{z})$ and the output set $D(\underline{y})$ may consist of vectors, of functions of time, may or may not be finite and so on. In general the behavior of S has the form

$$f \subset D(\underline{z}) \times D(\underline{y})$$

where f is everywhere defined. For stochastic systems the behavior of S may further be specified by a probability density function. In

^x More elaborate statements of the following concepts may be found in "Landeweerd" a.o. (1970).

the case of deterministic systems f is a mapping.

$$f: D(\underline{z}) \longrightarrow D(\underline{y})$$

3. On control.

To introduce the concept of control it is necessary to decompose the input \underline{z} in a controllable input \underline{u} and a noncontrollable input \underline{x} . Then we get the picture of figure 2.

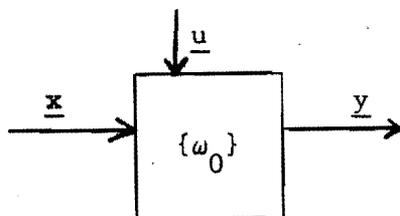


figure 2.

The behavior of such a system for the deterministic case is specified by the mapping f :

$$f: D(\underline{x}) \times D(\underline{u}) \longrightarrow D(\underline{y})$$

Now in general a goal^x is a weak ordering \mathcal{V}

Thus:

$$\mathcal{V} \subset (D(\underline{x}) \times D(\underline{u}) \times D(\underline{y}))^2$$

This means in fact that one prefers some combinations of \underline{x} , \underline{u} and \underline{y} above others. So the mapping f may also be seen as a constraint upon \mathcal{V} : f depicts the realisable combinations of \underline{x} , \underline{u} and \underline{y} .

This illustrates the particular important part of the design of a system and the intimate relation between the goal and the design process. Therefore it is understandable that particularly in the engineering literature one often find "system" defined as something "having a goal" or "designed to fulfill a specific function" (See f.i. Jenkins, 1970).

One could call "function" a particular type of goal. This becomes clear if one accept a definition of function like: "a function is a statement of the desired behavior of a system". This means that a function f^x (in the non mathematical sense) may be depicted as a function f^x (in the mathematical sense).

^x More detailed information may be found elsewhere (de Leeuw, 1969)

$$f^x: D(\underline{x}) \times D(\underline{u}) \longrightarrow D(\underline{y})$$

We call $\Omega = D(\underline{x}) \times D(\underline{u}) \times D(\underline{y})$

Now clearly f^x can be written as

$$f^x \subset \Omega$$

And remembering f^x is a function (in the non mathematical sense) we may call f^x the subset of the desired combinations of and output.

So $\Omega = D(\underline{x}) \times D(\underline{u}) \times D(\underline{y})$ is split up in two disjunct subsets: the set f^x consisting of the desired elements, and the set $D(\underline{x}) \times D(\underline{u}) \times D(\underline{y}) \setminus f^x$ consisting of the non desired elements. Thus f^x induces a weak ordering \mathcal{V}_{f^x}

$$\mathcal{V}_{f^x} \subset ((D(\underline{x}) \times D(\underline{u}) \times D(\underline{y}))^2$$

$$\text{Or } \mathcal{V}_{f^x} \subset \Omega^2$$

Because \mathcal{V}_{f^x} is a goal in the sense of our earlier definition we prefer conceiving a function as a particular type of goal.

Now we restrict ourselves to systems already designed. There is given a statement of the behavior f and a goal \mathcal{V}

$$f: D(\underline{x}) \times D(\underline{u}) \longrightarrow D(\underline{y})$$

$$\mathcal{V} \subset \Omega^2$$

Our restriction means that we do not further consider those elements of \mathcal{V} not belonging to f . So we write for the system:

$$f: D(\underline{x}) \times D(\underline{u}) \longrightarrow D(\underline{y})$$

$$\mathcal{V}_f \subset f \times f$$

where

$$\mathcal{V}_f = \mathcal{V} \cap f^2$$

And when the system is designed well we have $\mathcal{V}_f = \mathcal{V}$ (also $\mathcal{V} \subset f^2$)

Suppose further that the weak ordering \mathcal{V}_f is such that there exists a mapping V :

$$V: f \longrightarrow R_e$$

with

$$\forall a, b \in f (V(a) \geq V(b) \iff \langle a, b \rangle \in \mathcal{V}_f \iff a \succcurlyeq b)$$

(Where $a \succcurlyeq b$ means: "b is not preferred above a".)

We now may define an optimal control action \underline{u}^x with respect to \mathcal{V} and f , as that $\underline{u}^x \in D(\underline{u})$ which maximizes^x

$$V(\underline{x}, \underline{u}, f(\underline{x}, \underline{u}))$$

^x This includes also the concept satisficing.

One often defines a goal as related only to the output of a system. This may not in general be true. It is however possible to define a new inputvariable and give a description of a system such that the goal may be defined as related only to the output of that system.

Namely we can define on the basis of f:

$$g: D(\underline{x}) \times D(\underline{u}) \longrightarrow D(\underline{x}) \times D(\underline{u}) \times D(\underline{y})$$

$$\text{Or } g: D(\underline{x}) \times D(\underline{u}) \longrightarrow \Omega$$

$$\text{Where } g(\underline{x}, \underline{u}) = \langle \underline{x}, \underline{u}, f(\underline{x}, \underline{u}) \rangle$$

Then we define a mapping h from Ω onto $D(\underline{z})$

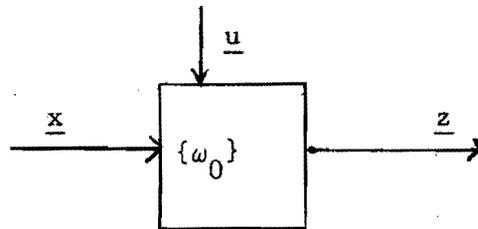
$$h: \Omega \longrightarrow D(\underline{z})$$

Where

$$\forall \underline{x}_1, \underline{x}_2, \underline{u}_1, \underline{u}_2, \underline{y}_1, \underline{y}_2 (h(\underline{x}_1, \underline{u}_1, \underline{y}_1) = h(\underline{x}_2, \underline{u}_2, \underline{y}_2)) \iff (\langle \underline{x}_1, \underline{u}_1, \underline{y}_1 \rangle \approx \langle \underline{x}_2, \underline{u}_2, \underline{y}_2 \rangle \wedge \underline{y}_1 = \underline{y}_2)$$

Herein $a \approx b$ denotes "a and b belongs to the same equivalence class of the weak ordering \mathcal{V}_f ". So we defined a new output variable \underline{z} who enables us to speak of the goal as a preference ordering of $D(\underline{z})$ because the weak ordering \mathcal{V} of Ω is induced in $D(\underline{z})$ by the mapping h.

Summarizing thus far we may describe a black box with a goal as follows.



$$f: D(\underline{x}) \times D(\underline{u}) \longrightarrow D(\underline{z})$$

$$\mathcal{V}_f \subset (D(\underline{z}))^2$$

And we know that the restriction of the goal concept to the output is not a real restriction because the general case can be translated in the foregoing form.

In the same way we can measure the value of the output when there exists a mapping $V: D(\underline{z}) \rightarrow R_e$ such that

$$\forall z_1, z_2 \in D(\underline{z}) (V(z_1) \geq V(z_2) \iff (z_1 \succ z_2))$$

And an optimal control action, \underline{u}^x with respect to \mathcal{V} and f is that

$$\underline{u} \in D(\underline{u}) \text{ which maximizes } V(\underline{z}) = V(h(\underline{x}, \underline{u}, \underline{y})).$$

We call a system controllable with respect to a specific goal \mathcal{V}_f if and only if for every \underline{x} there exists an \underline{u} such that $\langle \underline{x}, \underline{u}, \underline{z} \rangle \in f$ and $V(\underline{z})$ is maximal. This surely is not always so. In practice one often cannot find an optimal control action in this sense. Then we may say that the system is not optimally designed with respect to \mathcal{V} .

But the system being designed we can only select a suboptimal control action as that $\underline{u}^* \in D(\underline{u})$ which maximizes $V(\underline{z}) = V(h(\underline{x}, \underline{u}, \underline{y}))$ with respect to \mathcal{V}_f and f .

In the following we restrict ourselves to this suboptimal control in order not to intermingle control problems at one hand and design problems at the other. We may also suppose that $\mathcal{V} = \mathcal{V}_f$.

We define the class $G \subset D(\underline{z})$ as the class of the most desired outputs with respect to \mathcal{V}_f and f .

$$G = \{ \underline{z} \mid \underline{x} \in D(\underline{x}) \wedge \langle \underline{x}, \underline{u}, \underline{z} \rangle \in f \wedge V(\underline{z}) \text{ is maximal} \}$$

Now an optimal controller of the system is a controller which selects for every $\underline{x} \in D(\underline{x})$ an $\underline{u} \in D(\underline{u})$ such that $f(\underline{x}, \underline{u}) \in G$.

4. On optimal controllers.

Having elaborated on the concept of control and the relations with the concepts of goal and function we now turn to the problem of the requirements for systems to be able to function as optimal controllers. Tocher formulates four conditions for a precise control problem to exist:

1. There must be a specific set of times at which a choice of action is possible.
2. At each such time, there must be a specified set of actions from which to choose.
3. A model must exist which can predict the future behavior of the system under every possible choice.
4. There must be a criterion or objective on which the choice of action is based by a comparison of predicted behavior of the system with the objective.

(Tocher, 1970).

And, as he states at another place "At the heart of every control system there is a model". Thus there must be moments to choose an $\underline{u} \in D(\underline{u})$; to ~~be able~~ to choose $D(\underline{u})$ must contain more than one element. Furthermore we must have a model and a goal of the

system.

In the system-theoretical literature in particular Ashby has turned himself to these problems with his famous "Law of the requisite variety". Conant studied with him the relationships between model, information and control. (Ashby, 1956; Ashby and Conant, 1970; Conant, 1969). We shall draw heavily on their findings and insights in the following.

In the foregoing paragraph we studied a black-box $\{\omega_0\}$ whose behavior was specified by a mapping f .

$$f: D(\underline{x}) \times D(\underline{u}) \longrightarrow D(\underline{z})$$

and a set of "good events" $G \subset D(\underline{z})$. To be sure that all events are good ($f(\underline{x}, \underline{u}) \in G$) the controller must assure that always

$$\langle \underline{x}, \underline{u} \rangle \in f^{-1}(G)$$

where

$$f^{-1}(G) = \{ \langle \underline{x}, \underline{u} \rangle \mid \langle \underline{x}, \underline{u}, \underline{z} \rangle \in f \wedge \underline{z} \in G \}$$

We may call this set $f^{-1}(G)$ the set of good pairs $\langle \underline{x}, \underline{u} \rangle$

Evidently

$$f^{-1}(G) \subset D(\underline{x}) \times D(\underline{u})$$

In other words there must be a relation between the non-controllable input \underline{x} and the control input \underline{u} . Now the controller for a given $\underline{x} \in D(\underline{x})$ must choose an $\underline{u} \in D(\underline{u})$ such that $\langle \underline{x}, \underline{u} \rangle \in f^{-1}(G)$

For controllable systems we have

$$\{ \underline{x} \mid \langle \underline{x}, \underline{u} \rangle \in f^{-1}(G) \} = D(\underline{x})$$

This means that the relation $f^{-1}(G)$

$f^{-1}(G) \subset D(\underline{x}) \times D(\underline{u})$ is everywhere defined but is in general not single valued.

In other words for a certain $\underline{x} \in D(\underline{x})$ there exists several different but equally good control actions $\underline{u} \in D(\underline{u})$.

So we may, without loss of effectivity choose for every \underline{x} just one \underline{u} from the set $D(\underline{u}(\underline{x}))$

$$\{ \underline{u} \mid \langle \underline{x}, \underline{u} \rangle \in f^{-1}(G) \}$$

Then we get a function h .

$$h: D(\underline{x}) \longrightarrow D(\underline{u})$$

with $h \subset f^{-1}(G)$

This function h depicts what Ashby and Conant (1970) call the simplest optimal regulator. Three remarks seem in order.

First we may say that the controller uses a model of the system to be controlled because of the use of the function f in its inverse

form. Secondly this model is optimal in the sense of containing only the information relevant for the given goal. Thirdly the "Law of the requisite variety" of Ashby is implicitly shown in the argument. The requisite variety of \underline{u} , i.e. the number of elements in $D(\underline{u})$ depends on f and G . The latter is quite understandable if one takes the extreme case that $G = D(\underline{z})$. In that case, even when f is one-one, the requisite variety is not equal to the number of elements of $D(\underline{x})$ but is smaller, namely one.

To see if a certain controller behaves effectively it is not necessary to see all the different events $\underline{z} \in D(\underline{z})$. It is necessary and sufficient to evaluate an output \underline{z}^1 related to \underline{z} by the mapping r :

$$r: D(\underline{z}) \longrightarrow D(\underline{z}^1) = \{\text{Good}, \text{Bad}\}$$

with

$$\cup r(\underline{z}) = \{\text{Good}\}$$

$$\underline{z} \in G$$

$$\cup r(\underline{z}) = \{\text{Bad}\}$$

$$\underline{z} \in D(\underline{z}) \setminus G$$

Now an effective controller is also characterized by the fact that the entropy $H(\underline{z}^1)$ is zero, at least for controllable systems. For partly controllable systems we may so describe the function of a controller as the minimisation of $H(\underline{z}^1)$ where \underline{z}^1 is a one-one function defined on the set of equivalence classes of $D(\underline{z})$ under the weak ordering (the goal) \mathcal{V}_f .

Because it is not necessary to be able to discern \underline{z}_1 and \underline{z}_2 when they belong to the same equivalence class it is convenient to speak further of $D(\underline{z}^1)$ and forget about $D(\underline{z})$. Then we may say that the task of the controller is to minimize $H(\underline{z}^1)$.

$$H(\underline{z}^1) = - \sum_{\underline{z}^1 \in D(\underline{z}^1)} p(\underline{z}^1) \log p(\underline{z}^1)$$

6. On information and control.

Now we turn to the relation between information and control and will follow Conant (1969). There is a definite relationship between uncertainty or entropy in the information theoretical sense at one hand and control at the other.

In the foregoing paragraph we said that the task of the controller can be formulated as the minimization of $H(\underline{z}^1)$. Furthermore we stipulate that the controller selects for any disturbing factor \underline{x} at any time one and the same control action out of the set of equally good control actions and may state the task of the regulator as the minimization of $H(\underline{z})$. The starting point of Conant's discussion is a black-box described ⁽¹⁾ by a mapping

$$f: D(\underline{x}) \times D(\underline{u}) \longrightarrow D(\underline{z})$$

\underline{x} is a variable comprising all the factors necessary and sufficient to make \underline{z} a determinate function of \underline{x} and \underline{u} . So \underline{x} may be thought of as consisting of external and internal factors.

We assume also that \underline{x} is not dependent on \underline{u} . So $p(\underline{x}|\underline{u}) = p(\underline{x})$.

This means that the distribution of the "disturbing" variable is not affected by the control action. Conant now defines:

$H(\underline{z})$ as the entropy in \underline{z} when the controller functions optimally

$H_i(\underline{z})$ as the entropy in \underline{z} when the controller at any time selects the same $\underline{u}_i \in D(\underline{u})$

$$H_{\underline{x}}(\underline{z}) = \min_i \{H_i(\underline{z})\}$$

$V = H_{\underline{x}}(\underline{z}) - H(\underline{z})$ as the amount of entropy the controller suppresses by being active.

Now:

$$H_{\underline{x}}(\underline{x}, \underline{z}) = H_{\underline{x}}(\underline{z}) + H_{\underline{x}}(\underline{x}|\underline{z})$$

$$H_{\underline{x}}(\underline{z}) = H_{\underline{x}}(\underline{x}, \underline{z}) - H_{\underline{x}}(\underline{x}|\underline{z})$$

$$H_{\underline{x}}(\underline{z}) = H_{\underline{x}}(\underline{x}) + H_{\underline{x}}(\underline{z}|\underline{x}) - H_{\underline{x}}(\underline{x}|\underline{z})$$

From $p(\underline{x}|\underline{u}) = p(\underline{x})$ follows that $H_{\underline{x}}(\underline{x}) = H(\underline{x})$ and from $\underline{z} = f(\underline{x}, \underline{u})$

we know that $H_{\underline{x}}(\underline{z}|\underline{x}) = 0$

Thus

$$H_{\underline{x}}(\underline{z}) = H(\underline{x}) - H_{\underline{x}}(\underline{x}|\underline{z})$$

Furthermore

$$H_{\underline{x}}(\underline{u}, \underline{z}) = H(\underline{z}) + H(\underline{u}|\underline{z})$$

$$H(\underline{z}) = H_{\underline{x}}(\underline{u}, \underline{z}) - H(\underline{u}|\underline{z})$$

(1) We adapted the notation.

And because

$$I(\underline{u}; \underline{z}) = H_{\underline{x}}(\underline{u}, \underline{z}) - H(\underline{u} | \underline{z}) - H(\underline{z} | \underline{u})$$

We get

$$H(\underline{z}) = I(\underline{u}; \underline{z}) + H(\underline{z} | \underline{u})$$

And with

$$H(\underline{z} | \underline{u}) + H(\underline{x} | \underline{u}, \underline{z}) = H(\underline{x} | \underline{u}) + H(\underline{z} | \underline{u}, \underline{x})$$

We get

$$H(\underline{z}) = I(\underline{u}; \underline{z}) + H(\underline{x} | \underline{u}) + H(\underline{z} | \underline{u}, \underline{x}) - H(\underline{x} | \underline{u}, \underline{z})$$

And also

$$H(\underline{z} | \underline{u}, \underline{x}) = 0$$

Thus

$$H(\underline{z}) = I(\underline{u}; \underline{z}) + H(\underline{x} | \underline{u}) - H(\underline{x} | \underline{u}, \underline{z})$$

Now

$$\begin{aligned} V &= H_{\underline{x}}(\underline{z}) - H(\underline{z}) \\ &= H(\underline{x}) - H_{\underline{x}}(\underline{x} | \underline{z}) - I(\underline{u}; \underline{z}) - H(\underline{x} | \underline{u}) + H(\underline{x} | \underline{u}, \underline{z}) \end{aligned}$$

Using the fact that

$$I(\underline{u}; \underline{x}) = H_{\underline{x}}(\underline{u}, \underline{x}) - H(\underline{u} | \underline{x}) - H(\underline{x} | \underline{u})$$

$$I(\underline{u}; \underline{x}) = H(\underline{x}) - H(\underline{x} | \underline{u})$$

We have

$$V = I(\underline{u}; \underline{x}) - I(\underline{u}; \underline{z}) + [H(\underline{x} | \underline{u}, \underline{z}) - H_{\underline{x}}(\underline{x} | \underline{z})]$$

The first term of V , $I(\underline{u}; \underline{x})$ gives the transinformation between \underline{u} and \underline{x} .

The second one, $I(\underline{u}; \underline{z})$ is the information in \underline{z} about \underline{u} . When the controller functions better $H(\underline{z})$ becomes smaller and because $I(\underline{u}; \underline{z}) \leq H(\underline{z})$ also $I(\underline{u}; \underline{z})$ becomes smaller.

Now the terms $H(\underline{x} | \underline{u}, \underline{z})$ and $H_{\underline{x}}(\underline{x} | \underline{z})$ have to do with the uncertainty about \underline{x} , \underline{u} and \underline{z} . In other words they are a characteristic of $p(\underline{x})$ and the mapping f .

When for every $\underline{u} \in D(\underline{u})$, \underline{z} uniquely determines \underline{x} then both terms are zero.

To have an easier expression we write

$$V \leq I(\underline{u}; \underline{x}) + H(\underline{x} | \underline{u}, \underline{z})$$

which we get by the non negativity of $I(\underline{u}, \underline{z})$ and $H_{\underline{x}}(\underline{x} | \underline{z})$.

Also $H(\underline{x} | \underline{u}, \underline{z})$ is bounded above.

To see this we define $g_{\underline{u}}$ as follows

$$g_{\underline{u}} = \{ \langle \underline{z}, \underline{x} \rangle | \underline{z} = f(\underline{u}, \underline{x}) \}$$

In general $g_{\underline{u}}$ is everywhere defined but not unique.

Define further

$$R_{\underline{u}}(\underline{z}) = \{ \underline{x} | \langle \underline{z}, \underline{x} \rangle \in g_{\underline{u}} \}$$

Now $H(\underline{x}|\underline{u},\underline{z})$ is maximal if the probabilities of the elements of $R_{\underline{u}}(\underline{z})$ are equal.

Therefore we state

$$H(\underline{x}|\underline{u},\underline{z}) \leq -^2 \log \frac{1}{\text{Max}_{\underline{u},\underline{z}} |R_{\underline{u}}(\underline{z})|}$$

Where

$|R_{\underline{u}}(\underline{z})|$ denotes the number of elements of $R_{\underline{u}}(\underline{z})$

Evidently therefore we may write

$$H(\underline{x}|\underline{u},\underline{z}) \leq K$$

Where K is a constant dependent only on the function f .

Thus we may write

$$V \leq \underline{I}(\underline{u},\underline{x}) + K$$

This last expression indicates that the value of the regulator is essentially bound by the amount of transinformation between \underline{u} and \underline{x} . In his paper Conant also gets some related results in the area of information and control, but we only used that part which clarified the relation between the concepts of information and control.

6. On the quality of models.

We wish to define the quality of a model as a measure of the quality of its predictions.

We argued that an essential requirement for a controller was having a model of the system. It is not difficult to see that a system whose behavior changes in time can only be effectively controlled when the model used by the controller is adapted. "Updating" the model is necessary for holding the quality of the model high enough. This gives a second argument for the necessity of information transfer between controller and controlled system.

Now a model with high quality, we could call the model effective, not always is also efficient. It certainly could have too much detail. A model is effective and efficient if and only if it has high quality and a minimum of details. We must specify what is meant by the phrase "minimum of details".

Suppose we have a system described by

$$f: D(\underline{x}) \times D(\underline{u}) \longrightarrow D(\underline{z})$$

$\mathcal{V}_2 \subset (D(\underline{z}))^2$ is a weak ordering of $D(\underline{z})$

call $D(\underline{z}^1)$ the set of equivalence classes of $D(\underline{z})$. Then we could describe the system with

$$g: D(\underline{x}) \times D(\underline{u}) \longrightarrow D(\underline{z}^1)$$

$\mathcal{V}_2^1 \subset D(\underline{z}^1)$ is a linear ordering of $D(\underline{z}^1)$

where

$$h: D(\underline{z}) \longrightarrow D(\underline{z}^1)$$

and g is the composition of h and f :

$$g(\underline{x}, \underline{u}) = h(f(\underline{x}, \underline{u}))$$

Thus we see that the function g has less detail. But only information, irrelevant for the given goal is thrown away.

To shorten a long story we think by analogy that also the minimum necessary information about \underline{x} depends on the given goal. More research should be done to further clarify the relationship between goal and necessary and sufficient detail of the model and necessary and sufficient information about the disturbing variables.

7. On efficient controllers.

An efficient and effective controller is a controller which is optimal with respect to a given goal and the system's mapping f , uses an efficient model of the system and does his work with a minimum of handling of information. One could suppose that, if for any $\underline{x} \in D(\underline{x})$ the appropriate $\underline{u} \in D(\underline{u})$ is found, the controller could operate as a deterministic, stationary black box described by $h: D(\underline{x}) \longrightarrow D(\underline{u})$

So \underline{x} is recognized and $h(\underline{x})$ is chosen.

Now suppose that the probability distribution on $D(\underline{x})$ is such that there is a subset $D_0(\underline{x}) \subset D(\underline{x})$ with relatively often appearing elements.

We then could suppose that it is efficient to store the mapping $h_0 \subset h$ where

$$h_0 = \{ \langle \underline{x}, \underline{u} \rangle \mid \langle \underline{x}, \underline{u} \rangle \in h \wedge \underline{x} \in D_0(\underline{x}) \}$$

Also perhaps not the whole $D(\underline{x})$ is known and therefore h cannot be stored.

We could further suppose that an efficient controller uses two types of models. One is the model implicit in the function h . The other is a much more detailed model used for searching an appropriate $\underline{u} \in D(\underline{u})$ for an $\underline{x} \in D(\underline{x}) \setminus D_0(\underline{x})$ not known before.

Then we could imagine that an efficient controller consists of a patternrecognizer or perceptual mechanism followed by two different control subsystems, one for the routine events $\underline{x} \in D_0(\underline{x})$ and using the function h_0 and one for the non routine events and using a much more complicated model.

It must be clear that we especially in this paragraph tried to express ourselves carefully and in terms of "suppose" and "perhaps". This is necessary because of the lack of relevant theoretical research to enable more definite statements. However at the other hand we believe that it is important to point at further theoretical research which is relevant in our opinion because the formulated hypotheses does seem to fit the experimental findings of Landeweerd and Kragt (1972).

8. On human controllers.

We shall summarise the conclusions and hypotheses suggested from the foregoing theoretical paragraphs as far as they seem relevant for the problem of the human control of production processes. We will even add some speculations which can shed some new light on the problem of automation of the control of production processes.

1. We suppose a human controller to be efficient.
2. Because a human controller is a controller we suppose that he has a model, we call it a "mental model" of the system he controls.
3. This "mental model" must not have more than the minimum detail necessary.
4. This model will be updated when the behavior of the system changes.
5. Therefore a flow of information between the human controller and the system is necessary.
6. More information is not always better because of the information processing.
7. Perhaps an efficient controller uses two models; one for routine and one for non-routine events.
8. Automation of control may reduce the flow of information between human controller and controlled system, may therefore lead to a lower quality of his "mental-model" and may thus lead to a decrease of the effectiveness of the controller with respect to the not automated control tasks.

9. On organizations.

In the study of organizations one may discern two types of approaches crudely described by the following two questions.

- "How an organization must be structured in order to fulfill a specific goal?" We call this the synthetic or normative approach.
- "What are the structures of organization found in reality?". This we will call the analytic or describing approach.

Now the latter decade there is growing a type of research and theory (exemplified f.i. by Thompson (1967)) wherein the normative and describing approach are "thrown together" by means of a universal trick. This "trick" is founded at the reasonable assumption that when a result of the normative approach is stated like "it is smart to do so and so" there is no reason to assume "nature" to be stupid. So f.i. Thompson (1) speaks of "under norms of rationality" (Thompson, 1967). One may contradict that in this way there ~~infact~~ only is one hypothese making all others tautological. This is true in a sense. When the condition of "rationality" is met in an empirical situation, we use this condition as axiom and do not err in our use of the calculus, then any statement derived from this axiom necessarily is true. But there are at least two other ways of looking at this.

First we may view all derived hypotheses as potential ways of falsifying the main hypothese of "rationality". Confirmation of the derived hypotheses thus indirectly confirms the "rationality hypothese". Secondly by skipping the rationality conditions we have a set of hypotheses capable of being tested. The normative approach, we might say, generated the hypotheses. We need not to argue the importance of hypotheses generation.

The foregoing arguments make plausible the use of the normative approach for the descriptive approach.

- (1) The following is meant as a sketchily methodological evaluation of work like Thompsons. We are not certain if Thompson directed himself to all the aspects. However one gets the impression that he in any event tries to build a bridge between the normative - and the descriptive approach.

There is however another main goal of the normative approach. Science should give an aid for the solution of practical problems. In the realm of organizations we wish to know the answer to question of "How an organization must be structured in order to fulfill a specific goal". Application is normative, asks for which alternative must be chosen and depends essentially upon the empirical content in the normative statements. (See: f.i. Grochla, 1969)

We will not continue this rather sketchily argument concerning the value of the normative and the descriptive approach but simply state that our discussion of control in the foregoing paragraphs might be seen as normative. Its use is therefore at least threefold.

First we may learn something about how organizations should be structured (Organizations, at least partly, being describable as control systems). This for instance has to do with the practical implication of the concept of effective and efficient controller.

Secondly we may use them as a more explicit theory "behind" such statements as f.i. Thompson (1967) or the work in the area of Technology and organization (See f.i. van der Zwaan, 1971; Harvey, 1968; Perrow, 1967; Woodward, 1965).

Thirdly the control paradigm is interpretable in many directions; is therefore useable in disciplines corresponding to these directions and may thus correspond to unity of science (the aim of systems research) and the integral approach of organizations which is an urgent, practical and scientific problem.

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