The effect of workload control on order flow times

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THE EFFECT OF WORKLOAD CONTROL ON ORDER FLOW TIMES

Approvisionnement d'atelier et durée d'exécution des tâches

National Contribution of THE NETHERLANDS

J.W.M. BERTRAND
Department of Industrial Engineering, Eindhoven University of Technology
Eindhoven, The Netherlands

Abstract: In many production situations where production is a disturbed variable, the flow times of jobs can be controlled by controlling the workload of the shop. New jobs are released or production is varied such that the actual workload is kept in accordance with a predetermined workload norm. Such a procedure stabilizes the job flow times and reduces the job flow-time variance over time.
In this paper, we restrict attention to situations where single components are manufactured with a relatively high frequency, where production capacity can be considered as homogeneous and where the manufacturing of a component requires the performance of a relatively large number of successive operations.
A model is presented of the dynamic behaviour of the system on an aggregate level. For two control modes, the variance and the steady state deviation of the order flow time from the norm is calculated. Typical differences are revealed between workload control by means of capacity variations and by means of order release variations. It is shown that the flow-time norm is a major determinant of the control effectiveness.

Résumé: Dans bien des problèmes de production où les perturbations sont importantes, la durée d'exécution des tâches peut être maîtrisée en agissant sur l'approvisionnement de l'atelier. On enclenche de nouvelles tâches, on modifie la production de façon à ce que la charge d'atelier soit compatible avec des plans de charges préétablis. Une telle procédure permet une réduction de la moyenne et de la variance du temps d'exécution des tâches.
On s'intéresse ici aux cas où une seule pièce est fabriquée avec une fréquence élevée, où la capacité de production est homogène, et où la fabrication d'une pièce nécessite un nombre élevé d'opérations successives.
On présente un modèle dynamique de comportement du système de production à un niveau agrégé. On calcule la variance et l'écart aux normes des dates d'exécution des tâches pour deux modes de gestion et pour lesquels on met en évidence les différences.

1. INTRODUCTION

In many production control situations, ordering policies are used which assume that the production lead times of the various products are constant over time, or, at least, that they obey *stationary* probability density functions. We mention the various techniques of Statistical Inventory Control [3] and Materials Requirement Planning [5]. Also production progress signalling techniques, like the Line of Balance [6], assume constant or at least stationary production lead times. Thus, for the proper functioning of these techniques it is necessary that the lead times per product type are to some extent under control.

The lead time of an order for a product type mainly consists of the order-back-log time, the time from order arrival until order release, plus the order flow time, the time from order release until order completion. The order flow time mainly consists of operation processing times and operation waiting times; the operation waiting times depend, among other things, on the capacity utilizations of the production facilities that are required for the manufacturing of that product type. The capacity utilizations, in turn, depend on the worksupply to the production facilities which in the end is determined by the orders released to the production system, and on the capacity available at the facilities.

It follows that the order release during some period, which was based on an assumption regarding the order flow times, may create a capacity utilization during that period which conflicts with realizing order flow times equal to the assumed order flow times. This deficiency obviously is due to the fact that these ordering policies do not take into account the available production capacity; implicitely it is assumed that the capacity has been tuned to the orders in an earlier phase, or that the capacity can be easily adapted to the capacity requirements. Suppose that such an additional control system exists and that, for each production period, the available capacity and the incoming order are tuned to each other beforehand. But even then, the order flow time may go out of hand because of production disturbances which may realize during the production period. The production disturbances of successive periods will cumulate over time and will lead to random-walk behaviour of the workload in the system, and consequently to random behaviour of the flow time of the orders through the system, unless *feedback* exists from the workload to the periodic order-release or to the available production capacity [4]. We conclude that such a feedback is necessary in order to keep the actual order flow time according to the flow-time norms implied by the lead time assumptions used by the ordering system. Figure 1 shows the global structure of the required control system.

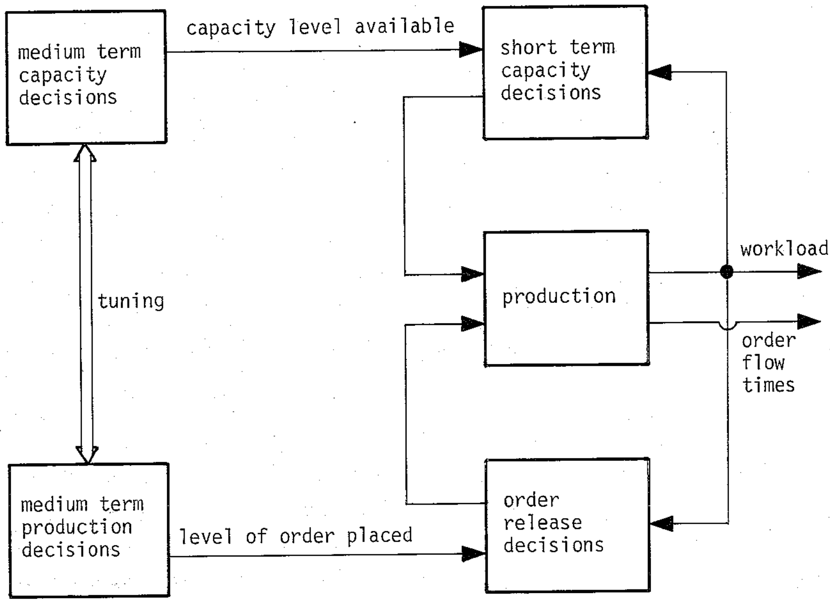

Figure 1: Global structure of the required production control system

Notice that also other important reasons may exist to control the workload of the production system. For instance, existing operator flexibility [2] can only be fully utilized if the number of orders in the system is kept at a specific level.

This paper analyzes the behaviour of the order flow times if the workload in the system is controlled, either by controlling the short term available capacity or by controlling the order release, or by controlling both. We assume that a control system exists which beforehand roughly harmonizes the arriving orders with available capacity (cf. figure 1). In particular we are interested in the variance and the steady state deviation of the order flow time which results from production disturbances, for the various ways to control the workload.

2. THE MODEL

We consider the behaviour of the production system at a high level of aggregation; it is assumed that the available capacity can be considered as homogeneous with respect to their use and that the orders can be considered as homogeneous with respect to their capacity requirements and routings. These assumptions would for instance be fairly valid for a component manufacturing shop where the majority of the orders requires about the same number of operations and operator capacity and where operator capacity is the main bottleneck.

We introduce the following variables:

- let $P(t)$ denote the production during period t, measured in capacity hours,
- let $R(t)$ denote the number of orders released to the system during period t,
- let $U(t)$ denote the number of orders that leave the system during period t,
- let $B(t)$ denote the number of orders in the shop at the start of period t,
- let F denote the flow-time norm of the order through the system,
- let k denote the amount of capacity required for the production of an order.

Thus, for an order to flow according to its flow-time norm, on the average k/F capacity units should be consumed by the orders per time unit, as soon as the order is released to the system. For each released order, this requirement implies a production progress schedule, expressed in cumulative capacity units to be consumed by the order at time t. For each individual order in the system, the actual production progress may differ from this schedule.

Let $D_i(t)$ denote the production progress deviation of order i in the system, expressed in capacity units; $D_i(t)$ is possitive if the order is ahead of schedule. Then, the aggregate production deviation in the system $D_i(t)$ can be defined as:

$$D(t) = \sum_{i \in I_t} D_i(t)$$

, where I_t is the set of orders in the system at time t.

We further assume that the orders in the system are sequenced over the available capacity such that there is a strong tendency for all orders to have the same progress deviation. Such behaviour can be obtained for instance by assigning adequate operation due dates to all operations of the orders and by giving priority to the order with the lowest operation due date [1]. It is realistic to assume the existence of such a sequencing rule for the type of production control problem considered, because control of the flow times is important here and this type of sequencing rule strongly reduces the deviations of the order flow times from the flow-time norms.

An order which is about to leave the system gradually looses its progress deviation, expressed in capacity units, as the last operations are performed; if the last operation is done, the progress deviation of the order is zero again. It follows that each period, $D(t)$ vanishes *automatically* in proportion to the number of orders that leave the system, at least if, due to the sequencing behaviour, all orders tend to have the same progress deviation, expressed in time units. Let $Z(t)$ denote the amount of deviation which vanished in this way during period t; then:

$$Z(t) = U(t) * D(t)/N(t) \tag{1}$$

Let $H(t)$ denote the total amount of capacity which, according to the order flow-time norms, is required for the production of the orders in the system during period t:

$$H(t) = k/F * N(t) \tag{2}$$

The number orders which leave the shop during period t can be related to the production as:

$$U(t) = P(t)/k \tag{3}$$

From (1) and (2) follows the balance equation for $D(t)$:

$$D(t) = D(t-1) - (1/k)(P(t-1)/N(t-1))D(t-1) + P(t-1) - (k/F)N(t-1) \tag{4}$$

From (3) follows the balance equation for $N(t)$:

$$N(t) = N(t-1) - (1/k)P(t-1) + R(t-1) \tag{5}$$

In the tuning of the production capacity to the required production (cf. figure 1) measures are taken such that the agreed production level, P_0, can be realized with order flow times equal to the flow-time norms, F.

Now the actual production, $P(t)$, may differ from P_0 for two reasons:

- first, in each period incidental deviations may occur, caused by absenteeism, technical problems etc.; we model this type of production disturbance with a white noise random variable with a normal distribution function:

$$S_1(t) = N(0,\sigma^2) \tag{6}$$

- second, the tuning may not succeed or may have been based on an erroneous model of the relation between capacity utilization and order flow times; as a result the production will systematically deviate from P_0. We model this type of production disturbance with a step function:

$$S_2(t) = c \tag{7}$$

The control actions regarding the short term production capacity will be taken on the basis of the difference between the actual workload and a workload norm. This workload norm can be expressed in the number of orders in the system. From the tuning process, the workload norm, N_0, follows as:

$$N_0 = R_0 F \tag{8}$$

where: $R_0 = P_0/k$

Now suppose we can model the results of the short term capacity control actions as:

$$P(t) = P_0 + \psi(N(t) - N_0) \tag{9}$$

That is, if the workload at the start of period t deviates from the norm, then the production during that period will vary such that this deviation decreases; the parameter ψ represents the extent to which such corrections are possible within one period.

The control of the workload by means of order release variations (cf. figure 1) can simply be modelled as:

$$R(t) = R_0 + \theta(N_0 - N(t)) \tag{10}$$

The equations (4), (5), (9) and (10) together constitute a model of the dynamic behaviour of the controlled production system.

3. ANALYSIS

The variance of the flow time deviations

First we determine the variance-amplifying property of the controlled system. Equation (4) is non-linear. Approximation of this relation in the set-point defined by (N_0, R_0, U_0, P_0) by a linear equation and redefining all variables as variations around the set-point leads to the following set of equations:

$$n(t) = n(t-1) - (1/k)p(t-1) + r(t-1) \tag{11}$$

$$d(t) = d(t-1) - (1/F)d(t-1) - (k/F)n(t-1) + p(t-1) \tag{12}$$

$$p(t) = \psi n(t) + s_1(t) \tag{13}$$

$$r(t) = -\theta n(t) \tag{14}$$

Application of the positive z-transform,

$$f(z) = \sum_{t=0}^{\infty} f(t) z^t$$

to the above set of equations and eliminating variables yields the relation between $d(z)$ and $s_1(z)$:

$$d(z) = \frac{z\theta/(\theta+\psi/k-1/F)}{1-(1-1/F)z} + \frac{z(\psi/k-1/F)/(\theta+\psi/k-1/F)}{1-(1-\psi/k-\theta)z} \quad s_1(z) \tag{15}$$

If both capacity variations and order-release variations are used to control the workload, these two control actions should be co-ordinated, for instance to prevent overshoot. For convenience we will restrict attention to two cases:

I. the case where only capacity variations are used: $0 \leq \psi/k \leq 1$, $\theta = 0$.

II. the case where also order release variations are used such that the workload deviation at the start of period t is entirely corrected during period t:

$$0 < \psi/k < 1 \; ; \quad \theta + \psi/k = 1$$

In case I, equation (15) reduces to:

$$d_I(z) = \frac{z}{1-(1-\psi/k)z} s_1(z) \tag{16}$$

, which leads to a variance of $d(t)$, denoted by V_I, equal to:

$$V_I = k^2/(2k\psi-\psi^2)\delta^2 \tag{17}$$

In case II, equation (15) reduces to:

$$d_{II}(z) = \frac{z(1-(\psi/k-1/F)z)}{1-(1-1/F)z} \tag{18}$$

, which leads to a variance of $d(t)$, denoted by V_{II}, equal to:

$$V_{II} = [1+F^2(1-\psi/k)^2/(2F-1)]\delta^2 \tag{19}$$

Comparing the equations (16) and (18) reveals that in case I, the dynamics of $d(t)$ is determined by the factor ψ/k; in this case the flow-time norm, F, does not play any role. In the case II however, the dynamic behaviour is entirely determined by the flow-time norm, F, in this case ψ/k only plays a minor role in that it influences the magnitude of $d(t)$. Equation (17) shows that the variance of the production deviation indeed grows without bound if no workload control exists at all ($\psi = 0$); equation (18) shows that for large values of F, the control via the order release is not very effective. Furthermore, a very remarkable point is that for any positive value of ψ/k, there can always be found a value of F such that $V_{II} > V_I$. Or, stated otherwise, for any value of F, there always exists a value of ψ/k, say δ, such that, for $\psi/k > \delta$, using order release variations in addition to capacity variations only makes the control performance worse.

The order flow-time standard deviation, SD_F, can simply be deduced from (17) and (19) as:

$$SD_F = F/(kN_0)V^{\frac{1}{2}} \tag{20}$$

SD_F can be used to calculate the safety stock required to absorb the aggregate variations in the order flow times.

The steady state deviation

The steady state production deviation can simply be calculated from the equations (4), (5), (9) and (10) by setting $N(t) = N(t-1)$, $D(t) = D(t-1)$, $P(t) = P(t-1)$ and $R(t) = R(t-1)$; first, the disturbance term $S_2(t)$ is added to (9), which yields:

$$P(t) = P_0 + \psi(N(t) - N_0) + c \tag{21}$$

For case I, the steady state deviation in $D(t)$, E_I, can be calculated as:

$$E_I = k(N_0-c/\psi) - k^2/(FP_0)(N_0-c/\psi)^2 \tag{22}$$

Because $N_0 = FP_0/k$, (22) reduces to:

$$E_I = k(N_0-c/\psi)(kc/(\psi FP_0))$$

The steady state deviation in the order flow times, S_I, can be calculated as:

$$S_I = (F/kN)E_I = (Fkc)/(\psi FP_0) = c/(\psi R_0) \tag{23}$$

Similarly, the aggregate steady state production deviation for case II can be calculated as:

$$E_{II} = ck(R_0F-c/k)(1+1/F-\psi/k)/(kR_0-\psi c/k+c) \tag{24}$$

The steady state order flow-time deviation, S_{II}, follows as:

management judged that the mean flow time, and therefore the workload, could be reduced to some extent without much loss of production. The relations (19) and (25) can be used to estimate the improvement of the flow-time performance which would result from such reduction.

The total possible improvements can be evaluated in terms of stock reductions allowed by the flow-time improvement.

In the end, department management decided to introduce workload control by means of variations of order release, and to set the flow-time norms to F = 25 days. As a result, the flow-time control performance indeed improved substantially [7].

5. CONCLUSIONS

A model is presented of the dynamics which determine the behaviour of the aggregate order flow-time deviations from the flow-time norm. For two specific control policies, the variance and the steady state deviation of the aggregate flow time, resulting from production disturbances, have been investigated. The analysis revealed that workload control is a necessary condition for order flow-time control. Furthermore, typical differences between the control effectiveness of order release variations and capacity variations for workload control have been shown. In particular, it is shown that the aggregate flow-time norm determines the control effectiveness of order release variations. Furthermore the analysis shows that both for the variance and for the steady state deviations, there exist combinations of flow-time norm and short-term capacity variability such that using order release variations in addition to capacity variations worsens the control performance.

Finally, the model presented can yield valuable information about the performance of possible control policies, which information can be used in the design of a control system.

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$$S_{II} = (F/k)(R_0F-c/k)E_{II} = Fc(1+1/F-\psi/k)/(kR_0+c(1-\psi/k)) \qquad (25)$$

Equation (23) shows that for case I the steady state deviation grows without bound if $\psi = 0$; again it is shown that workload control is necessary. Furthermore the steady state deviation in case I is independent of F and k and does depend on R_0; the larger R_0, the smaller the resulting steady state deviation.

Equation (25) shows that for case II, the steady state deviation is an increasing function of the flow-time norm F and, similar to case I, a decreasing function of R_0. Furthermore, for any positive value of ψ/k, there can always be found a value of F such that $S_{II} > S_I$. Thus, also for the steady state deviations, using order release variations in addition to capacity variations may make the control performance worse.

Based on estimates of the disturbance term c and given the workload control policy used, equations like (23) and (25) can be used to estimate possible values of the order flow-time steady state deviations, which leads to additional safety stocks required to absorb such deviations.

Equations like (17), (19), (23) and (25), derived from the basic model of the systems dynamic for various control policies can be used to evaluate the performance and the total costs of possible combinations of flow-time norm, short term capacity variations, short term order release variations and safety stocks.

4. APPLICATION

The above model has been used for evaluating the possible benefits of improving the workload control in a diffusion department of a servi-conductor plant. Details regarding the design of the workload control system are given in [7]. The following model parameters were estimated to be valid for the production system: $k = 4$ manshift per order, $R_0 = 10$ orders per day, $F = 30$ days, $\delta = 4$ manshift per day and $c = 4$ manshift per day. Short term capacity variations were not possible. However, due to the queueing character of the production process the daily production to some extent depended on the workload in the system; thus there exists a kind of 'automatic correction of the workload'. In the set point given by $N_0 = FR_0 = 300$ orders, this relation was estimated to result in a value for ψ equal to $\psi = 0.08$ capacity units per order.

According to (17) and (19) the corresponding values of V_I and V_{II} are $V_I = 433.6$ manshift^2 and $V_{II} = 222.4$ manshift^2. Introducing workload control by means of order release variations with reaction parameter $\theta = 1 - \psi/k$ leads, according to (22) and (25), to steady state deviations equal to $S_I = 5$ days and $S_{II} = 2.77$ days.

These reductions were considered quite satisfactory. However, the department

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