

## Modal identification of vibration system

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**Modal Identification  
of Vibration System**

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MODAL IDENTIFICATION OF VIBRATION SYSTEM

— A USAGE OF HP3566A MEASUREMENT DATA

abstract

The work on modal identification realized a method based on the maximum likelihood principle for the estimation of parameters of a single- input multi-output linear vibration system. The identification procedure includes modes judgement, initial parameters set up, non-linear curve fitting, and results verification, Gauss-Newton methods is applied to deal with the non-linearity of the curve fitting. Cyclic adjustment of parameters and step size optimization are proposed to improve the efficiency of the iterative algorithms — reduce the memory requirement and time cost for the curve fitting. The fruit of the work is a program that can be executed in a PC computer where the measurement data are stored.

**Nomenclature**

- b**: modal parameters of a vibration system;
- $\beta$ : an estimation of **b**;
- $\beta^L$ : the maximum likelihood estimation of **b**;
- N**: number of degree of freedom of the system;
- m**: indication of a mode,  $m=1,2,\dots,N$ ;
- l**: number of frequency response functions (FRFs);
- v**: indication of a FRF,  $v=1,2,\dots,l$ ;
- s**: number of spectral lines of each FRF;
- k**: indication of **s** spectral line of a FRF,  $k=1,2,\dots,s$ ;
- $\omega_m$ : natural frequency of mode **m**;
- $\xi_m$ : damping factor of mode **m**;
- $M_m$ : generalized mass of mode **m**;
- $K_m$ : generalized stiffness of mode **m**;
- $\phi_{mv}$ : model shape of mode **m** at point **v**;
- $\phi_{mp}$ : model shape of mode **m** at input point **p**,  $p \in v$ ;
- $\psi_{mv}$ : model shape when  $M_m=1$  is supposed;
- $h_{kv}$ : a value of  $v_{th}$  measured FRF,  $k_{th}$  spectral line;
- H**: matrix of  $h_{kv}$ ;
- $g_{kv}$ : a value of FRF constructed by modal parameters **b** or  $\beta$ ;
- G**: matrix of  $g_{kv}$ ;
- $\Delta$ : difference between **G** and **H**;
- $\rho_{kv}^2$ : ordinary coherence function;
- $w_{kv}$ : weighting coefficient;
- W**: matrix of  $w_{kv}$ ;
- $P(\mathbf{H}, \mathbf{b})$ : probability density function of FRF;
- $L(\mathbf{H}; \beta)$ : likelihood function;
- $\Gamma(\beta, \mathbf{H})$ : error function;

$\alpha_v$ : number of averages taken when  $v_{th}$  FRF was measured;  
 $R$ : correlation function;  
 $x$ : input in time domain;  
 $y_v$ :  $v_{th}$  output in time domain;  
 $Y$ : matrix of  $y_v$ ;  
 $n$ : measurement noise;  
 $n_f$ : noise in frequency domain;  
 $\sigma_{kv}^2$ : variance of  $n_f$ ;  
 $D$ : Jacobian matrix;  
 $\delta$ : a change of parameter estimation  $\beta$ ;  
 $\mu$ : step size factor of changing parameter estimation;  
 $\epsilon$ : rate of error reduction while iterating;  
 $I$ : iteration times;  
 $r$ : radius of Nyquist circle;  
 $\Re$ : real part of ( $\cdot$ );  
 $\Im$ : imaginary part of ( $\cdot$ );  
 $t$ : transpose a matrix;

## 1. INTRODUCTION

Experimental modal analysis technology has advanced significantly with the development of random FRF measurements and curve fitting. It becomes quite necessary to determine modal parameters of mechanical structures in order to facilitate and optimize mechanical designs.<sup>[3]</sup> Since the measurement Frequency Response Function(FRF) data, acquired by HP3566A, can be read and reorganized<sup>[7]</sup>, it is reasonable to use these data for modal analysis — modal identification, for example.

The work on modal identification is to realize a curve fitting technique, which is based on the maximum likelihood principle, for modal parameters estimation of a single-input multi-output linear vibration system. The Gauss-Newton method is applied for the non-linear curve fitting.<sup>[5]</sup> To improve efficiency of the iteration algorithm, some skills are proposed. Using the technique of cyclic adjustment, instead of global curve fitting, rapidly reduced the memory requirement for the curve fitting. Step size optimization of parameter adjustment while iterating accelerates the convergence of the algorithm obviously.

Determination of the degree of The freedom of the system and initial values of the parameters are important for the non-linear curve fitting. The work pays much attention to fulfil the task. Curves of measured FRFs and ordinary coherence functions, Nyquist figures, and fitted Nyquist circles are simultaneously used for mode judgment and initial values set up.

After curve fitting, the reliability of the estimated parameters can be verified according to the fitting of curves from measurement data and curves from the data constructed by estimated parameters. The results are saved in files for output and another

usages.

Discussions on the proposed identification method are presented at the end.

## 2. THE MAXIMUM LIKELIHOOD ESTIMATOR

A linear vibration system can be described in the frequency domain. If the input is 1, the outputs are the frequency response functions. The vibration system with modal parameters  $\mathbf{b}$  and the identification system are shown as following.

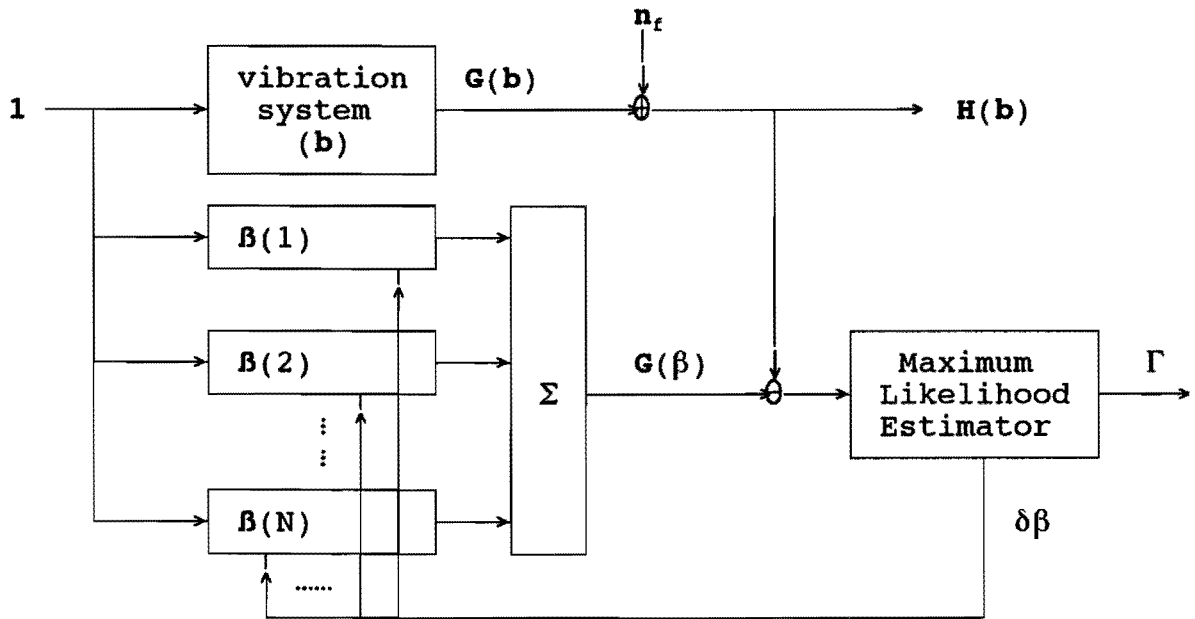


Fig.[1] Identification System

Estimating  $\mathbf{b}$  is a procedure of finding a  $\beta$  which minimizes the error

$$\Gamma = \Gamma(\beta, H) \quad (1)$$

$\Gamma$  is a non-negative function. The type of estimator depends on the definition of  $\Gamma$ .

Because of the noise  $n_f$ , the measured  $H(\mathbf{b})$  is a random function. The probability-density function of  $H$  can be written as follows.<sup>[5]</sup>

$$P(H, \mathbf{b}) = \left( \prod_{k,v} \frac{1}{\pi \sigma_{kv}^2} \right) \text{EXP} \left( - \sum_{k,v} \frac{|h_{kv} - g_{kv}(\mathbf{b})|^2}{\sigma_{kv}^2} \right) \quad (2)$$

The priori knowledge is obtained under the assumption that the experimentally measured FRFs contain no biased errors and the errors follow the complex normal distribution. The likelihood function takes the form:<sup>[1]</sup>

$$L(\mathbf{H}; \boldsymbol{\beta}) = \left( \prod_{k,v} \frac{1}{\pi \sigma_{kv}^2} \right) \text{EXP} \left( - \sum_{k,v} \frac{|h_{kv} - g_{kv}(\boldsymbol{\beta})|^2}{\sigma_{kv}^2} \right) \quad (3)$$

The maximum likelihood estimation is the root of the equation<sup>[1]</sup>

$$\frac{\partial}{\partial \boldsymbol{\beta}} L(\mathbf{H}; \boldsymbol{\beta}) \Big|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} = 0 \quad (4)$$

which maximizes the value of L.

If the error function is defined as

$$\Gamma(\boldsymbol{\beta}, \mathbf{H}) = \sum_{k,v} \frac{|h_{kv} - g_{kv}(\boldsymbol{\beta})|^2}{\sigma_{kv}^2} \quad (5)$$

The solution  $\boldsymbol{\beta}$  which minimizes  $\Gamma(\boldsymbol{\beta}, \mathbf{H})$  is just the root of (4) according to (3). The maximum likelihood estimation is then equalized to a weighted least squares estimation:

$$\Gamma(\boldsymbol{\beta}, \mathbf{H}) \Big|_{\boldsymbol{\beta}=\hat{\boldsymbol{\beta}}} = \Gamma_{\min} \quad (6)$$

The variance  $\sigma_{kv}^2$  is decided by  $h_{kv}$ ,  $\rho_{kv}^2$ , and measurement average times  $\alpha_v$ .

$$\sigma_{kv}^2 = \frac{1}{\alpha_v - 1} |h_{kv}|^2 \frac{1 - \rho_{kv}^2}{\rho_{kv}^2} \quad (7)$$

The representation (7) is yielded under the conditions:<sup>[2]</sup>

- a) Frequency response function and ordinary coherence are obtained from correlation measurement:

$$[h_{kv}] = \frac{\mathbf{R}(\mathbf{Y}, \mathbf{X})}{\mathbf{R}(\mathbf{X}, \mathbf{X})}, \quad [\rho_{kv}^2] = \frac{\mathbf{R}(\mathbf{Y}, \mathbf{X}) \cdot \mathbf{R}(\mathbf{X}, \mathbf{Y})}{\mathbf{R}(\mathbf{Y}, \mathbf{Y}) \cdot \mathbf{R}(\mathbf{X}, \mathbf{X})}. \quad (8)$$

There  $\mathbf{x}$  and  $\mathbf{Y} = \{y_1, \dots, y_L\}$  are the input and output measurement in time domain.

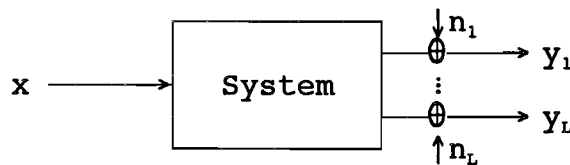


Fig.[2] input and output in time domain

- b) The noise is concerned with only in the output signals. They are independent.

Actually, HP3566A system measures frequency response function and ordinary coherence by use of (8).<sup>[8]</sup>

### 3. METHOD OF CURVE FITTING

According to the real modal theory, the frequency response function of a linear vibration system with N degree of freedom is

$$g_{kv}(\mathbf{b}) = \sum_{m=1}^N \frac{\Phi_{mv} \Phi_{mp}}{M_m ((\omega_k^2 - \omega_m^2) + 2j \omega_k \omega_m \xi_m)} \quad , (j = \sqrt{-1}) \quad (9)$$

Letting

$$\Psi_m = \Phi_m / \sqrt{M_m} \quad , \quad (10)$$

we have

$$g_{kv}(\mathbf{b}) = \sum_{m=1}^N \frac{\Psi_{mp}}{(\omega_k^2 - \omega_m^2) + 2j \omega_k \omega_m \xi_m} \Psi_{mv} \quad (11)$$

$\mathbf{b}$  and  $\beta$  are  $((1+2)*N) \times 1$  matrixes, and they take same forms:

$$\mathbf{b} = \{\mathbf{b}_1^t, \dots, \mathbf{b}_m^t, \dots, \mathbf{b}_N^t\}^t,$$

$$\beta = \{\beta_1^t, \dots, \beta_m^t, \dots, \beta_N^t\}^t,$$

$$\mathbf{b}_m = \beta_m = \{\omega_m, \xi_m, \Psi_{mp}, \dots, \Psi_{m1}, \dots, \Psi_{m(p-1)}, \Psi_{m(p+1)}, \dots, \Psi_{m1}\}^t.$$

The error function (5) can be written in matrix form.

$$\Gamma = \Gamma(\beta, \mathbf{H}) = \{\mathbf{H} - \mathbf{G}\}^t \mathbf{W}^2 \{\mathbf{H} - \mathbf{G}\} \quad (12)$$

where:

$$\mathbf{H} = \{\mathbf{H}_1^t, \mathbf{H}_2^t, \dots, \mathbf{H}_k^t, \dots, \mathbf{H}_s^t\}^t,$$

$$\mathbf{H}_k = \{\mathbf{h}_{k,1}^t, \mathbf{h}_{k,2}^t, \dots, \mathbf{h}_{k,v}^t, \dots, \mathbf{h}_{k,1}^t\}^t,$$

$$\mathbf{h}_{k,v} = \{\Re(\mathbf{h}_{k,v}), \Im(\mathbf{h}_{k,v})\}^t,$$

$\mathbf{G} = \mathbf{G}(\beta)$  is of the same form of  $\mathbf{H}$ ,

$\mathbf{H}$  and  $\mathbf{G}$  are  $(s*1*2) \times 1$  matrices.

$$\mathbf{W} = \text{diag}[\mathbf{W}_1, \mathbf{W}_2, \dots, \mathbf{W}_k, \dots, \mathbf{W}_s],$$

$$\mathbf{W}_k = \text{diag}[\mathbf{W}_{k,1}, \mathbf{W}_{k,2}, \dots, \mathbf{W}_{k,v}, \dots, \mathbf{W}_{k,1}],$$

$$\mathbf{W}_{k,v} = (\text{diag}[\sigma_{k,v}, \sigma_{k,v}])^{-1},$$

$\mathbf{W}$  is a  $(s*1*2) \times (s*1*2)$  diagonal matrix.

$\mathbf{G}(\beta)$  is not a linear function. Gauss-Newton method can be applied to solve the non-linear least square problem (or non-linear curve fitting). Consequently, iterative algorithm is used for the fitting. The algorithm beginning with a initial point  $\beta(0)$ , determines successive approximations  $\beta(I)$ ,  $I=1,2,\dots$ , as follows:<sup>[4]</sup>

- (1) For  $\beta(I)$  compute a minimum point  $\delta(I)$  for the linear squares problem

$$\min \|W(H-G) - WD\delta(I)\|^2 \quad (13)$$

that is

$$\delta(I) = (D^t W^2 D)^{-1} D^t W^2 (H-G(I)) \quad (14)$$

where:  $G(I)$  are constructed by pre-estimated  $\beta(I)$ ;  
 $D$  is a  $(s*1*2) \times ((1+2)*N)$  Jacobian matrix:

$$D = \left[ \frac{\partial G}{\partial \beta} \right]$$

(2) Let  $\lambda(\mu) = \Gamma(\beta(I) + \mu\delta(I))$ , and further, let  $\tau$  be the smallest integer  $\tau \geq 0$  with

$$\lambda(2^{-\tau}) < \lambda(0) = \Gamma(\beta(I)) \quad (15)$$

(3) Define

$$\beta(I+1) = \beta(I) + \mu\delta(I) \quad (16)$$

The solution of the non-linear least squares problem

$$\beta^L = \lim_{I \rightarrow \infty} \beta(I)$$

is the maximum likelihood estimation of  $b$ .

#### 4. CYCLIC ADJUSTMENT OF PARAMETERS

Considering the fact that the scales of matrix  $D, W, G, H$  are quite large and the heap size of a PC computer is limited, cyclic adjustment skill is adopted to meet the purposes of avoidance of heap overflow, large scale matrix inversion, and reduction of time spent for each iteration.

It is natural to divide  $D$  into  $N$  parts according to the structure of  $b$  and  $\beta$ .

$$D = [D_1, D_2, \dots, D_m, \dots, D_N] \quad (17)$$

$D_m = \partial G / \partial \beta_m$  is a  $(s*1*2) \times (1+2)$  matrix;

When  $\beta_m$  is adjusted without changing  $\beta_u$  ( $u=1, \dots, N; u \neq m$ ). There are  $\delta_u = 0$ , and

$$\begin{aligned} \delta(I) &= \{\delta_1^t(I), \delta_2^t(I), \dots, \delta_m^t(I), \dots, \delta_N^t(I)\}^t \\ &= \{0, \dots, 0, \delta_m^t(I), 0, \dots, 0\}^t \end{aligned} \quad (18)$$

The linear squares problem (13) becomes



$$\min \|W(H-G) - WD_m \delta_m(I)\|^2 \quad (19)$$

where

$$D_m = \frac{\partial G}{\partial \beta_m} = [D_{m1}^t \ \dots \ D_{mk}^t \ \dots \ D_{ms}^t]^t \quad (20)$$

$D_{mk} = [d_{m,k}]$  is a  $(1*2) \times (1+2)$  matrix. ( $d_{m,k}$  represent the elements of  $D_{mk}$ )

$$D_{mk} = \frac{\partial G_k}{\partial \beta_m} = \begin{bmatrix} \frac{\partial g_{k,1}}{\partial \omega_m} & \frac{\partial g_{k,1}}{\partial \xi_m} & \frac{\partial g_{k,1}}{\partial \psi_{m,p}} & \vdots & \frac{\partial g_{k,1}}{\partial \psi_{m,1}} & \dots & \frac{\partial g_{k,1}}{\partial \psi_{m,u}} & \dots & \frac{\partial g_{k,1}}{\partial \psi_{m,l}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial g_{k,v}}{\partial \omega_m} & \frac{\partial g_{k,v}}{\partial \xi_m} & \frac{\partial g_{k,v}}{\partial \psi_{m,p}} & \vdots & \frac{\partial g_{k,v}}{\partial \psi_{m,1}} & \dots & \frac{\partial g_{k,v}}{\partial \psi_{m,u}} & \dots & \frac{\partial g_{k,v}}{\partial \psi_{m,l}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial g_{k,l}}{\partial \omega_m} & \frac{\partial g_{k,l}}{\partial \xi_m} & \frac{\partial g_{k,l}}{\partial \psi_{m,p}} & \vdots & \frac{\partial g_{k,l}}{\partial \psi_{m,1}} & \dots & \frac{\partial g_{k,l}}{\partial \psi_{m,u}} & \dots & \frac{\partial g_{k,l}}{\partial \psi_{m,l}} \end{bmatrix} \quad (21)$$

Referring to (11) and (12), it is not difficult to find that when  $k$  takes some values that  $(\omega_k^2 - \omega_m^2)$  is large enough, all  $|d_{m,k}|$  are near zero. Suppose that  $k(m)$  and  $q(m)$  satisfy

$$\omega_{k(m)} \approx \omega_m \quad \text{and}$$

$$|d_{m,k}| \approx 0, \quad \text{for all } k: k < (k(m) - q(m)) \text{ or } k > (k(m) + q(m)).$$

Then,  $D_{m,k} |_{k < k(m) - q(m) \text{ or } k > k(m) + q(m)} = 0$  exists approximately.

Define

$$D_{(m)} = \begin{bmatrix} D_{m, (k(m) - q(m))} \\ \vdots \\ D_{m, (k(m))} \\ \vdots \\ D_{m, (k(m) + q(m))} \end{bmatrix}$$

There is

$$D_m = \begin{bmatrix} \vdots \\ 0 \\ \vdots \\ D_{(m)} \\ \vdots \\ 0 \\ \vdots \end{bmatrix}. \quad (22)$$

The linear squares problem (19) is approximately equivalent to

$$\min \|W_{(m)} (H_{(m)} - G_{(m)}) - W_{(m)} D_{(m)} \delta_{(m)}(I)\|^2 \quad (23)$$

Where:

$$\mathbf{H}_{(m)} = (\mathbf{H}_{k(m)-q(m)}^t, \dots, \mathbf{H}_{k(m)}^t, \dots, \mathbf{H}_{k(m)+q(m)}^t)^t$$

$$\mathbf{G}_{(m)} = (\mathbf{G}_{k(m)-q(m)}^t, \dots, \mathbf{G}_{k(m)}^t, \dots, \mathbf{G}_{k(m)+q(m)}^t)^t$$

$$\mathbf{W}_{(m)} = \text{diag}[\mathbf{W}_{k(m)-q(m)}, \dots, \mathbf{W}_{k(m)}, \dots, \mathbf{W}_{k(m)+q(m)}]$$

The minimum point for the linear squares problem (23) is

$$\delta_m(I) = [\mathbf{D}_{(m)}^t \mathbf{W}_{(m)}^2 \mathbf{D}_{(m)}]^{-1} \mathbf{D}_{(m)}^t \mathbf{W}_{(m)}^2 (\mathbf{H}_{(m)} - \mathbf{G}_{(m)}(I)) \quad (24)$$

$\mathbf{G}_{(m)}(I)$  is constructed by  $\beta(I)$ . Referring to (11), there is

$$\mathbf{G}_{(m)} = \sum_{o=1}^N \mathbf{G}_{(m)}^o \quad (25)$$

and the elements of  $\mathbf{G}_{(m)}^o$  are:

$$g_{(m)k,v}^o = \begin{bmatrix} \omega_k^2 - \omega_o^2(I) \\ -2\omega_k \omega_o(I) \xi_o(I) \end{bmatrix} \frac{\psi_{op}(I)}{(\omega_k^2 - \omega_o^2(I))^2 + 4\omega_k^2 \omega_o^2(I) \xi_o^2(I)} \psi_{ov}(I) \quad (26)$$

(24) and (25) reflect the consideration of the fact that any period of FRF data, though the data are used for estimating parameters of only one mode at a time, are influenced by all modes of the vibration system. However, while adjusting parameters of a mode without changing parameters of another modes, (24) can be written as follows.

$$\delta_m(I) = [\mathbf{D}_{(m)}^t \mathbf{W}_{(m)}^2 \mathbf{D}_{(m)}]^{-1} \mathbf{D}_{(m)}^t \mathbf{W}_{(m)}^2 (\Delta_{(m)}(I) - \mathbf{G}_{(m)}^m(I)) \quad (27)$$

$$\Delta_{(m)}(I) = \mathbf{H}_{(m)} - \sum_{\substack{o=1 \\ (o \neq m)}}^N \mathbf{G}_{(m)}^o(I)$$

$\Delta_{(m)}$  may be called "Single-mode" FRF data, because the influences of all the another modes are picked out.

Observing the right part of the matrix in (21), one can find that there are only two non-zero elements in each column of the part. The elements in these columns are

$$\frac{\partial g_{k,u}}{\partial \psi_{m,u}} = \begin{bmatrix} \frac{\partial \Re(g_{k,u})}{\partial \psi_{m,u}} \\ \frac{\partial \Im(g_{k,u})}{\partial \psi_{m,u}} \end{bmatrix}, \text{ and } \frac{\partial g_{k,v(v \neq u)}}{\partial \psi_{m,u}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Referring to (26), all non-zero elements are not the function of  $\psi_{mu}$  ( $u \neq p$ ). Because of the linearity of  $\psi_{mu}$  ( $u \neq p$ ) and the sparseness of the right part of  $\mathbf{D}_{(m)}$ , it is suggested that linear parameters and non-linear parameters be estimated separately.<sup>[5]</sup>

Write

$$\beta_m = \{ \beta_{(m)\omega}^t ; \beta_{(m)\psi}^t \}^t$$

$$D_{(m)} = [ D_{(m)\omega} ; D_{(m)\psi} ]$$

where  $\beta_{(m)\omega} = \{ \omega_m, \xi_m, \psi_{mp} \}^t$  is the non-linear part of  $\beta_m$ , and  $\beta_{(m)\psi} = \{ \psi_{m1}, \dots, \psi_{m(p-1)}, \psi_{m(p+1)}, \dots, \psi_{m1} \}^t$  is the linear part.  $D_{(m)\omega}$  and  $D_{(m)\psi}$  are left and right parts of  $D_{(m)}$ . Estimating  $\beta_{(m)\psi}$  needs no iteration or initial values.

$$\beta_{(m)\psi}^L = [ D_{(m)\psi}^t W_m^2 D_{(m)\psi} ]^{-1} D_{(m)\psi}^t W_m^2 \Delta_m(0) \tag{28}$$

(28) implies that  $D_{(m)\omega}$  is nonfunctional as  $\beta_{(m)\omega}$  are kept unchanged while estimating  $\beta_{(m)\psi}$ .

Referring to (20), (21), and (22), the matrix  $[ D_{(m)\psi}^t W_m^2 D_{(m)\psi} ]$  in (28) is a diagonal matrix. So, the linear parameters can be estimated one by one without inverting the matrix.

$$\psi_{mu}^L(u+p) = \left( \left\{ \frac{\partial g_{k,u}}{\partial \psi_{m,u}} \right\}^t W_{k,u}^2 \left\{ \frac{\partial g_{k,u}}{\partial \psi_{m,u}} \right\} \right)^{-1} \left\{ \frac{\partial g_{k,u}}{\partial \psi_{m,u}} \right\}^t W_{k,u}^2 \Delta_m(0) \tag{29}$$

Estimating  $\beta_{(m)\omega}$  needs iteration. The algorithms goes as follows.

(1)  $\delta_{(m)\omega}(I) = [ D_{(m)\omega}^t W_m^2 D_{(m)\omega} ]^{-1} D_{(m)\omega}^t W_m^2 \{ \Delta_m(I) - G_m^m(I) \}$  (30)

(2) Let  $\lambda(\mu) = \Gamma_m(H; \beta_{(m)\omega}(I) + \mu \delta_{(m)\omega}(I))$   
 $= \{ \Delta_m(I) - G_m^m(I) \}^t W_m^2 \{ \Delta_m(I) - G_m^m(I) \}$ ,  
 and  $\tau$  be the smallest integer  $\tau \geq 0$  with  $\lambda(2^{-\tau}) < \lambda(0)$  (31)

(3) define  $\beta_{(m)\omega}(I+1) = \beta_{(m)\omega}(I) + \mu \delta_{(m)\omega}(I)$ . (32)

$\mu$  is called the adjustment step size factor (step-size). The scheme of cyclic adjustment is illustrated in Fig.[2].

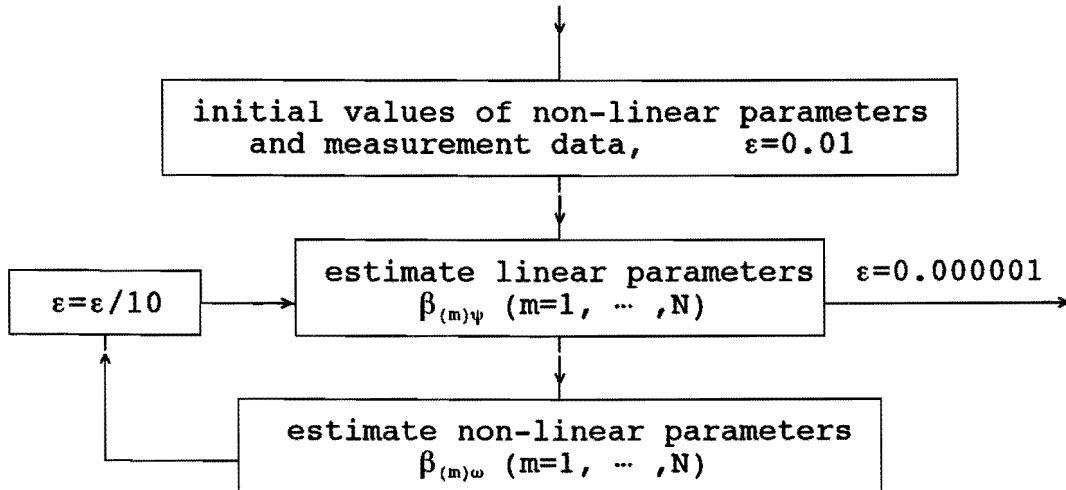


Fig.[3] Cyclic adjustment

The part of estimating non-linear parameters is as follows.

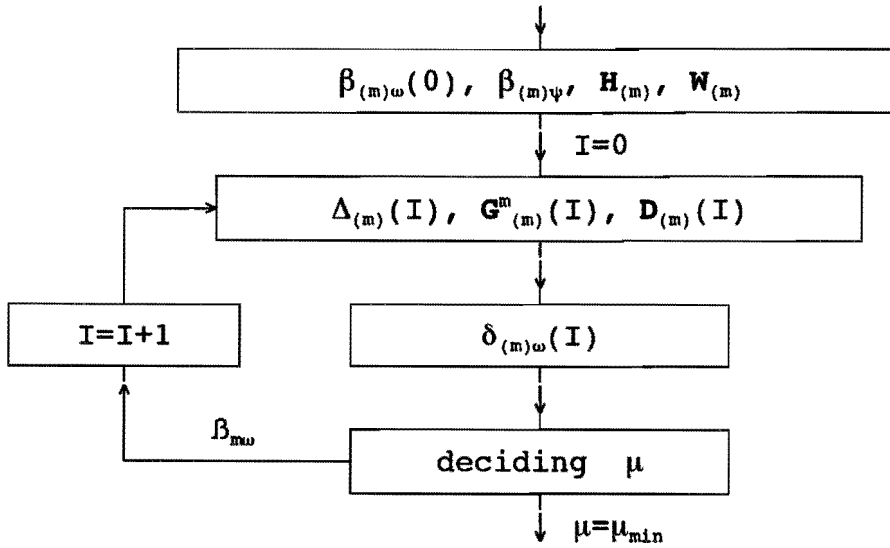


Fig.[4] Non-linear parameters estimation

For  $I=0,1,2,\dots$ , computing  $\delta_{(m)\omega}(I)$ , by use of (30), needs much less memory - about 1% of that if (14) is used because of the difference of size between  $D_{(m)\omega}$  and  $D$ . And, inverting a only  $(3 \times 3)$  matrix is then needed for the computing.

5. STEP SIZE OPTIMIZATION FOR NON-LINEAR PARAMETERS ESTIMATION

In (32), the step size  $\mu$  is decided by

$$\mu = 2^{-\tau},$$

under the condition of (31).

The Gauss-Newton method guarantees the convergence of the iterative algorithm<sup>[4]</sup>. However, the algorithm is not efficient, especially when  $\beta_{(m)\omega}(I)$  is near  $\beta_{(m)\omega}^L$ . The phenomenon, which is illustrated in Fig.[5], is that the error is reduced slowly and parameters are oscillating.

Optimization of the step-size is that, after finding a  $\tau$  which satisfies (31), let  $\tau'=\tau+1, \tau+2, \dots$ , until a  $\tau'$  satisfies the condition:

$$\|\Delta_m - G_{(m)}^m (\beta_{(m)\omega}(I) + 2^{-\tau'} \delta_{(m)\omega}(I))\| > \|\Delta_m - G_{(m)}^m (\beta_{(m)\omega}(I) + 2^{-(\tau'-1)} \delta_{(m)\omega}(I))\| \quad (33)$$

and decide

$$\mu = 2^{-(\tau'-1)}. \quad (34)$$

(33), (34) mean that  $\mu$  is the best step-size because adjusting the non-linear parameters by use of (32) will mostly reduce the error  $\Gamma$  (either  $\mu=2^{-(\tau'-2)}$  or  $\mu=2^{-(\tau')}$  will cause larger error).

As the step-size optimization skill applied, average times of iteration reduced about 40%. The improvement of efficiency can

also be illustrated in Fig.[5].

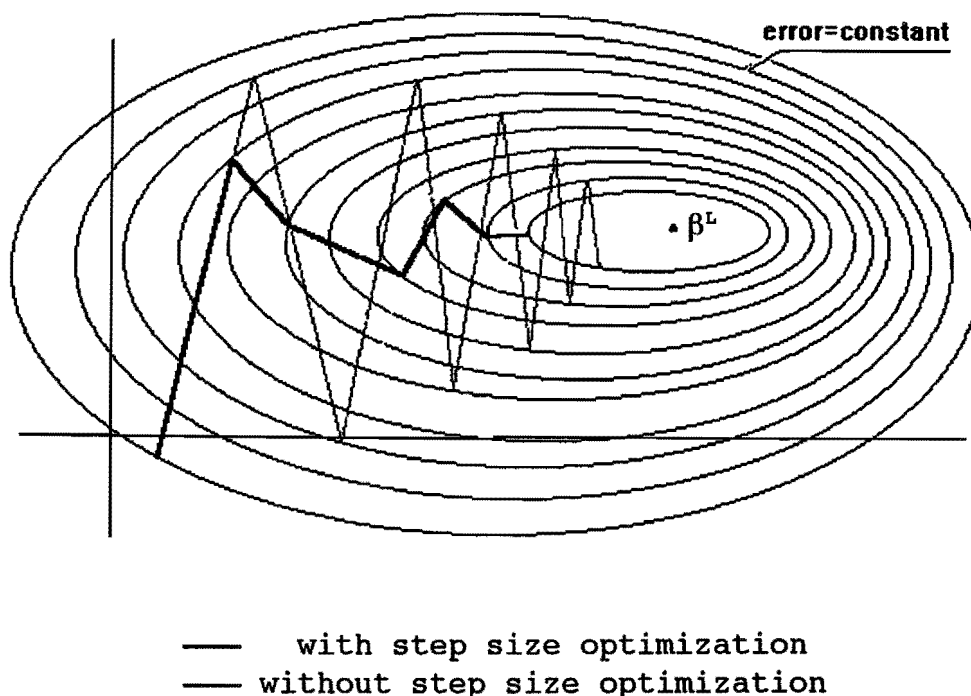


Fig.[5] Parameters Adjustment

## 6. MODE CHOICE AND INITIAL VALUES SET UP

The proposed method needs a set of initial values of non-linear parameters of all modes(Fig.[3]). As the cyclic adjustment skill is applied, only periods of measurement data are used for curve fitting to estimate model parameters. One should pay much attention to

- a) preliminary guess of modes — for each mode, choosing a periods of measurement data which make (22) approximately true and  $q(m)$  small enough,
- b) setting up the initial values of non-linear parameters,
- c) making judgement if a chosen mode is physically meaningful.

The program provides some facilities for fulfilling the tasks. They are:

- 1) Curves of all measured FRFs (real and imaginary parts) and ordinary coherence functions for preliminary guess,
- 2) Nyquist figures of each period data chosen for verification of the guess and the reference vector,
- 3) Nyquist circle fitting for initial values set up,<sup>[7]</sup>
- 4) Averaging  $\omega_m$ ,  $\xi_m$ , which can be estimated by use of data from different vectors, for better initial values,
- 5) Any possible changes to reflect the judgment before curve fitting.

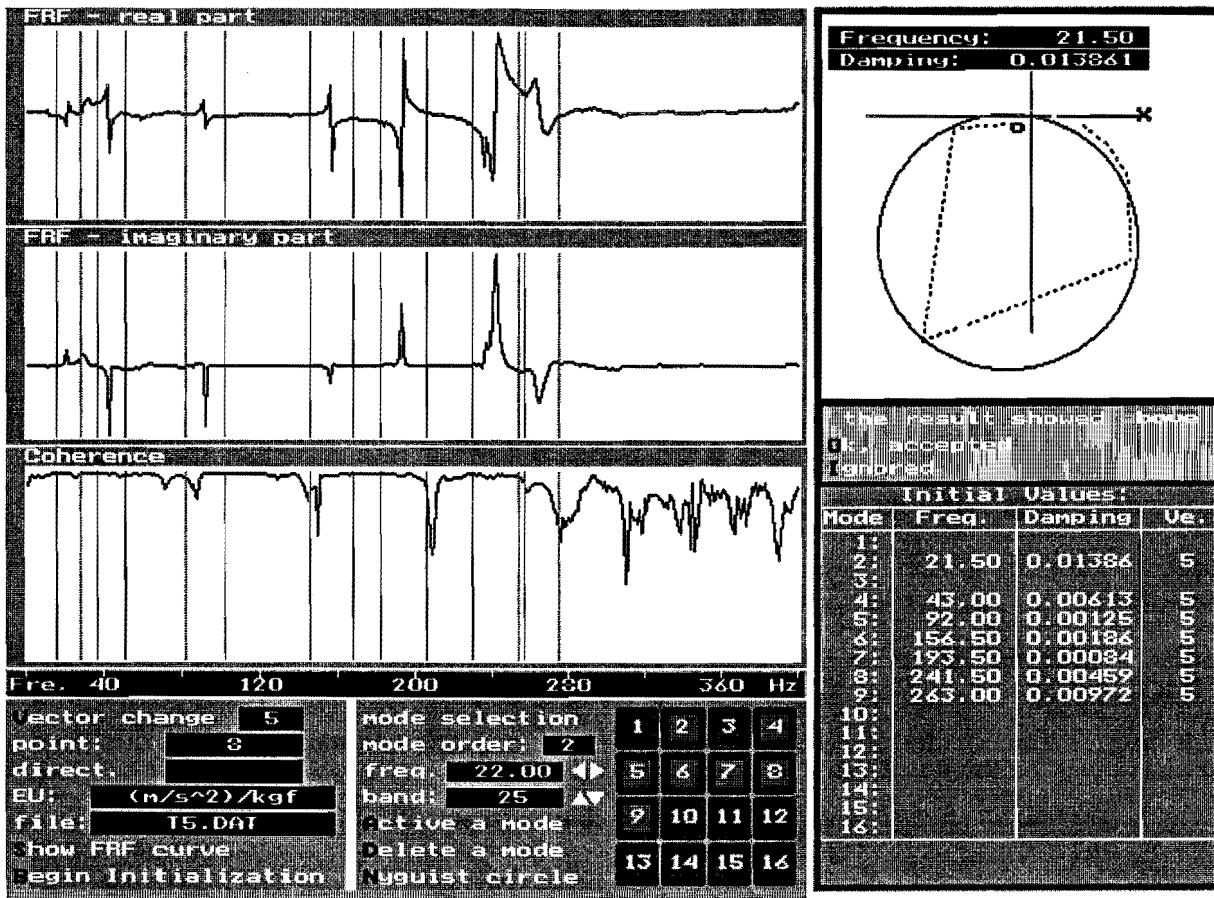


Fig.[6] Screen for mode judgement

A tabulating method is used for the Nyquist circle fitting as the method needs not any initial values.

It is necessary that the Nyquist figures and the fitted Nyquist circles are under the real axis on the Nyquist plane if the data are from the reference vector. The necessity, which is also used for the verification of reference vector, can be deduced from (11).

Observing mode  $m$  only and supposing  $k(m)=k$ ,  $q(m)=q$  and  $\omega_k=\omega_m$ , there is

$$g_{kp} = \begin{Bmatrix} 0 \\ -\psi_{mp}^2 \\ 2\omega_m^2 \xi_m \end{Bmatrix} \quad (34)$$

When  $g_{ip}$  ( $i=k-q, \dots, k-1, k, k+1, \dots, k+q$ ) form a Nyquist figure, the figure should be under the real axis and the point  $g_{kp}$  on the imaginary axis with distance  $|g_{kp}|$  to the zero point. After Nyquist circle fitting, the centre should also be under the real axis and the radius equals to  $(0.5) * |g_{kp}|$ .

From (34), there is

$$\Psi_{mp}^2 = 4\omega_m^2 \xi_m (-r_{mp})$$

where:  $r_{mp}$  is the radius of the Nyquist circle.

Define  $r_{mp} > 0$ , if the centre is above the real axis and vice versa.

$$\Psi_{mp} = 2\omega_m \sqrt{\xi_m (-r_{mp})} \quad (35)$$

It is necessary for (35) that  $r_{mp} < 0$ .  $r_{mp} > 0$  indicates that:

- 1) The mode may be physically meaningless, or
- 2) definitions of positive direction of input and output measurements at reference point did not match, or
- 3) the vector is not the reference vector.

To solve these possible problems, one can

- 1) delete the meaningless mode, or
- 2) tell the program that the definitions did not match, or
- 3) select correct reference vector.

The first correction can be done at any time before curve fitting. But, the other two corrections can only be done before the mode choice — reentering the program is necessary.

## 7. RESULTS CHECK AND ENGINEERING UNIT STANDARDIZATION

The parameters estimated by the curve fitting can be checked by

- 1) Comparing the curves of measured data  $H(\mathbf{b})$  and constructed data  $G(\beta)$ :
  - a) global:  $|g_{kv}|$  and  $|h_{kv}|$  ( $k=1, \dots, s$ ), ( $v=1, \dots, l$ ),
  - b) detail: real parts and imaginary parts of  $g_{kv}$  and  $h_{kv}$  ( $k=k(m)-25, \dots, k(m)+25$ ), ( $v=1, \dots, l$ ).
- 2) studying the digital results of estimation.
- 3) Studying history records of parameters adjustment and errors reduction during the curve fitting. These records are saved in ASCII form (if need). The records can only be read after exiting the program.

In (10) the function  $g_{kv}$  are displacement frequency response. If measured FRF -  $h_{kv}$  is velocity or acceleration response, the data used for Nyquist figures, circle fitting, and curve fitting are modified as if the displacement is measured.

And, whatever the engineering units are used for input and output measurement in time domain, all units are changed to the SI Units with suitable correcting to the FRF data. The only one exception, for the sake of custom, is that the unit of frequency is Hz instead of  $\text{Sec}^{-1}$ . When one uses the estimated frequency for some calculation, the unit should be changed to  $\text{Sec}^{-1}$  — timing the

frequency by  $2\pi$ . For example, The generalized stiffness  $K_m$  is calculated by

$$K_m = M_m(2\pi\omega_m)^2$$

Corresponding modal shape vectors are all normalized to

$$\|\Phi_m\|^2 = \phi_m^2 + \phi_m^2 + \dots + \phi_m^2 = 1$$

### 8. CONCLUSION AND DISCUSSION

The presented method for modal identification is realized to a user-friendly program which can be executed in a PC computer efficiently.

To examine the program, a set of experiment data, totally 13 FRFs, measured by HP3566A, are used to estimate modal parameters of a structure, which is shown in fig[7].

The results of the curve fitting are printed in Fig.[8].

The change of error, corresponding to a mode, is shown in Fig.[9].

The fitted curves with experiment data points are shown in Fig.[10].

From the results, figures, and curves, it can be concluded that the method is practically reliable.

The estimated parameters are also saved in a record with information of vectors. The record can be used for vibrating animation.

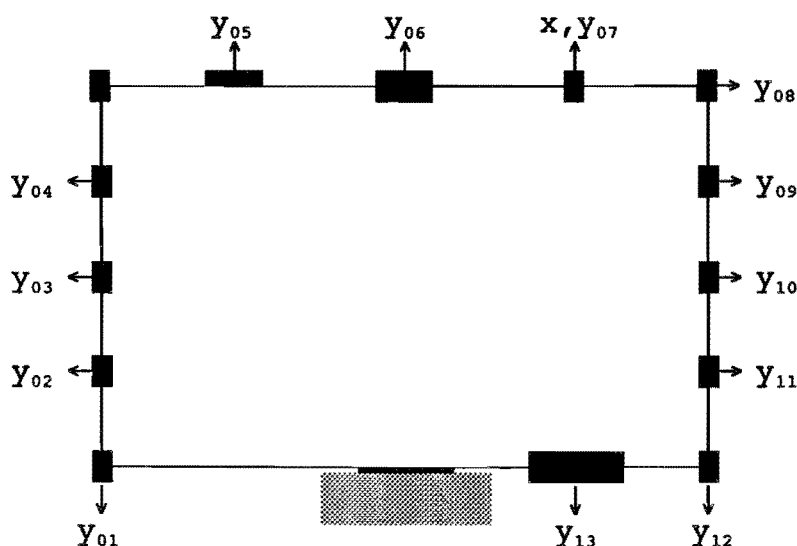


Fig.[7] Experiment Structure



MODEL PARAMETERS

	mode 1	mode 2	mode 3	mode 4	mode 5	mode 6	mode 7	mode 8	mode 9
$\Omega$	10.92	21.52	29.02	42.72	91.94	156.65	193.41	241.50	264.19
$\xi\%$	5.980	2.615	6.507	2.790	0.193	0.164	0.096	0.519	0.778
M	2.504	3.321	2.338	3.236	2.357	1.282	1.346	1.308	1.345
$\phi_{01}$	-.2938	+.3633	+.2721	-.5627	-.2664	-.0358	+.1440	+.2536	-.1493
$\phi_{02}$	+.1053	-.0570	-.0791	+.0884	-.1054	+.3691	-.3363	-.0316	-.3516
$\phi_{03}$	+.2782	+.0198	-.1482	-.0434	-.1958	+.4992	-.3830	+.1863	-.7135
$\phi_{04}$	+.4275	+.0707	-.1569	-.1550	-.1670	+.3079	-.1940	+.2501	-.4247
$\phi_{05}$	-.1649	+.4654	+.4373	-.5126	+.3146	-.3945	-.1153	-.2730	-.0950
$\phi_{06}$	-.0130	+.5031	+.5440	-.1639	+.7499	-.0059	-.3339	-.0381	-.1656
$\phi_{07}$	<u>+.1644</u>	<u>+.4272</u>	<u>+.5218</u>	<u>+.3725</u>	<u>+.0932</u>	<u>+.1605</u>	<u>+.1100</u>	<u>+.1699</u>	<u>+.1428</u>
$\phi_{08}$	-.5434	-.1106	+.1433	+.3350	-.0369	+.0453	-.0078	-.0592	-.0687
$\phi_{09}$	-.4237	-.1361	+.0247	+.2806	-.1813	-.2647	-.3523	+.2681	+.0583
$\phi_{10}$	-.2393	-.0467	-.0057	+.0483	-.1846	-.3762	-.5152	+.6260	+.2178
$\phi_{11}$	-.0969	-.0123	-.0848	-.0231	-.0671	-.2010	-.3391	+.4791	+.2072
$\phi_{12}$	-.1959	-.3616	-.2660	-.1376	+.2868	+.2063	+.0603	+.1684	+.0688
$\phi_{13}$	-.0910	-.2078	-.1206	-.0923	+.1632	+.2010	+.1915	-.0863	-.0674

Fig.[8] Estimated Parameters

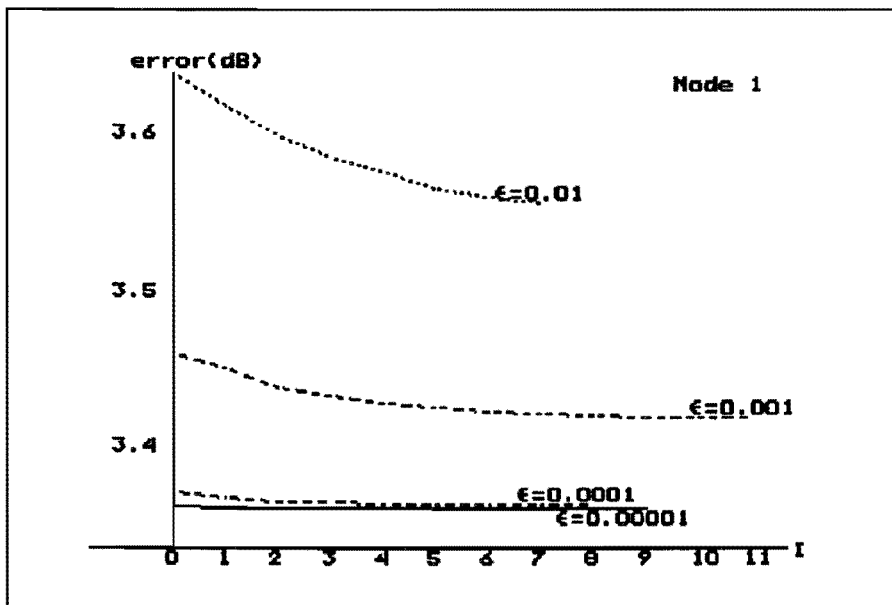


Fig.[9] Error Changing

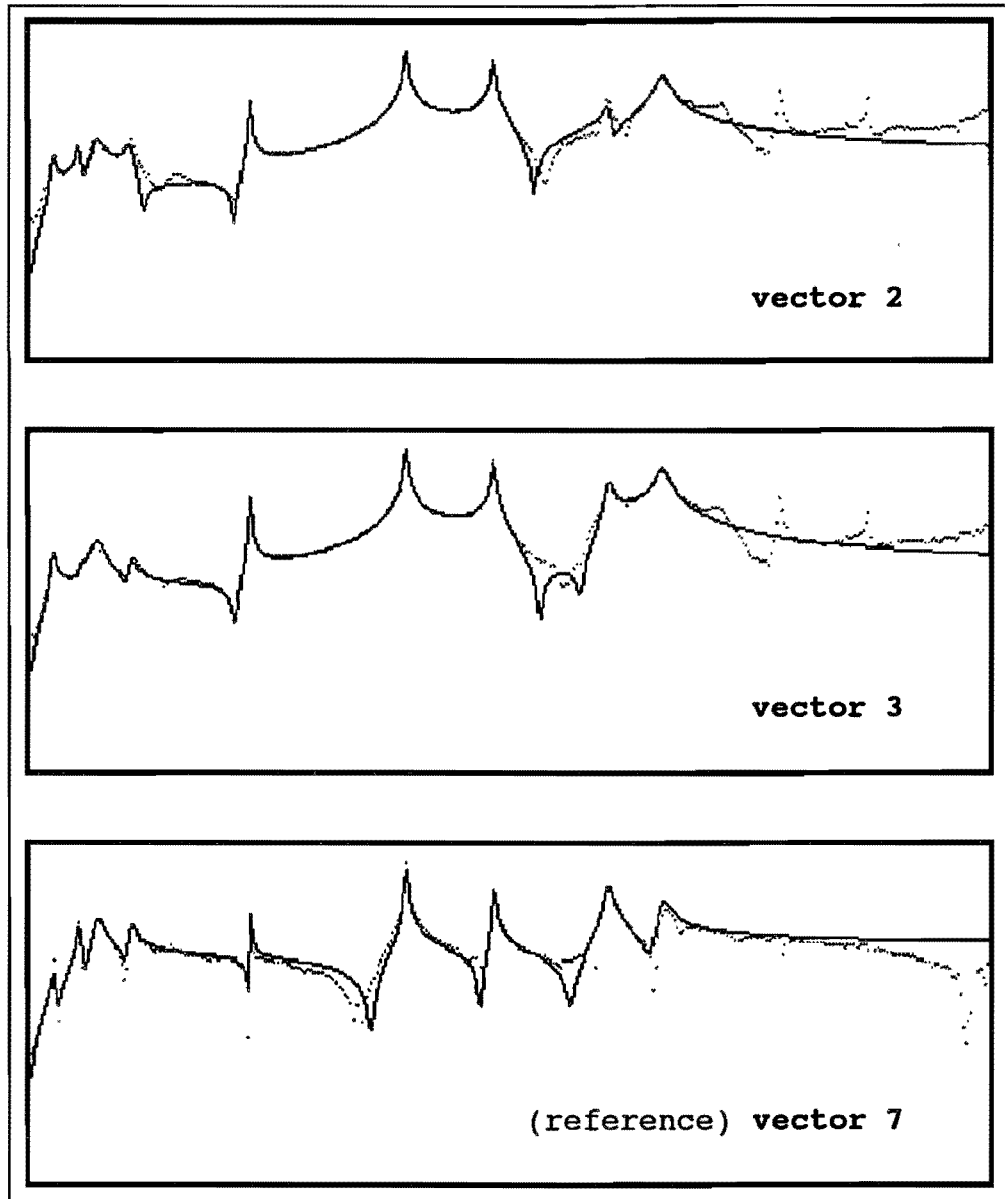


Fig.[10] Fitted Curves

For a complicated mechanical structure, some modes may be very close to one another. The cyclic adjustment skill can be modified to deal with the situation by making all modes in groups (each group consists of  $Z$  modes ( $Z=1,2,3,\dots; Z < Z_{\max}$ )) without increasing the heap size requirement because the size of  $D_{(m)}$  and corresponding  $H_{(m)}, G_{(m)}, W_{(m)}$  for group of modes will not be enlarged a lot as these modes are near to one another. And, the matrix inversion will not be a serious problem because  $Z_{\max}$  is limited in practice.

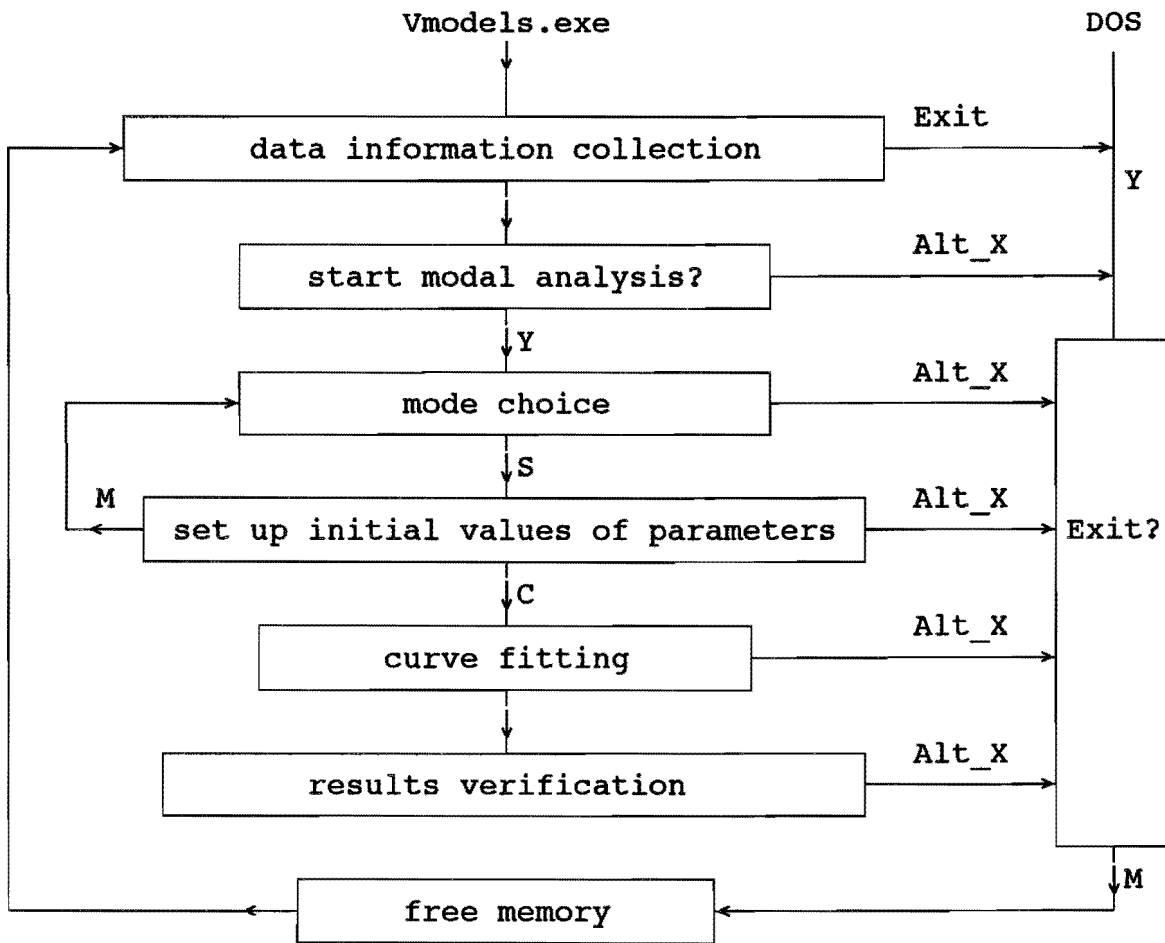
If viscous damping of a vibration system can not be decoupled in undamped modal space (the base vectors of the space are  $[\Phi]$ ). The complex modal theory should be applied. The presented method can be generalized to estimate complex modal parameters, according

to the structure of the program. The charts of the program are shown in Appendix A-F.

### Reference

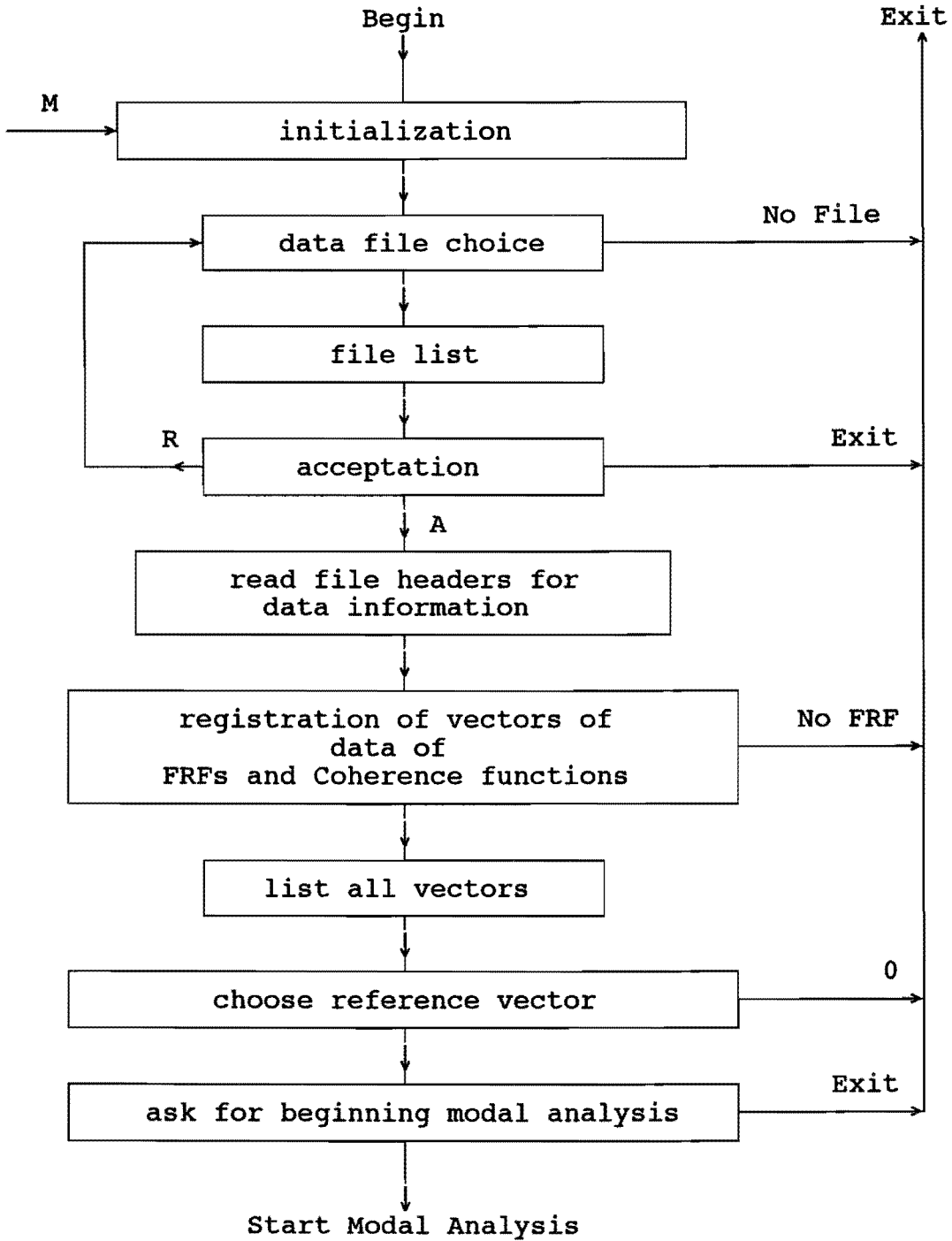
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Appendix A



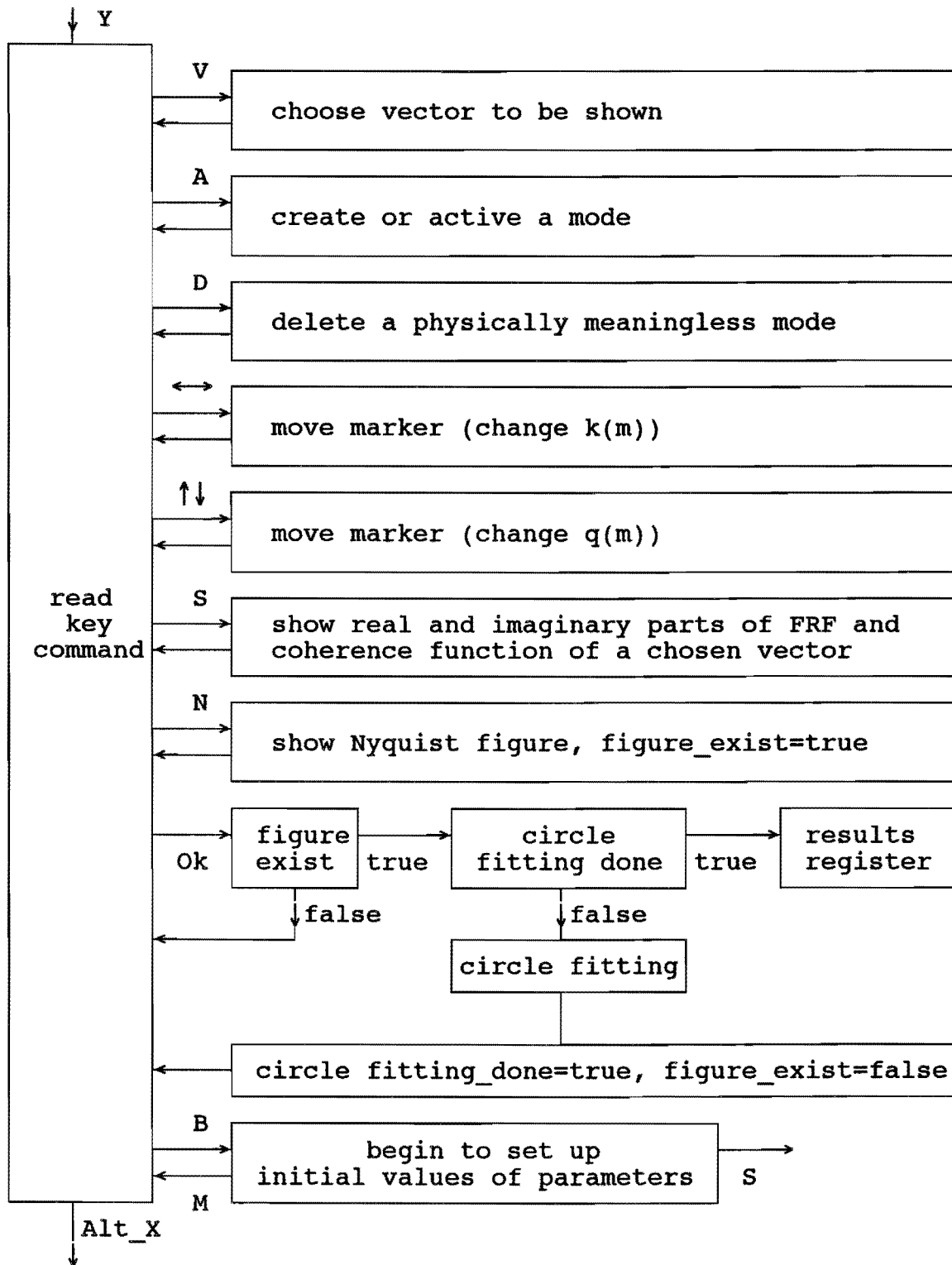
Program for Modal Analysis

Appendix B



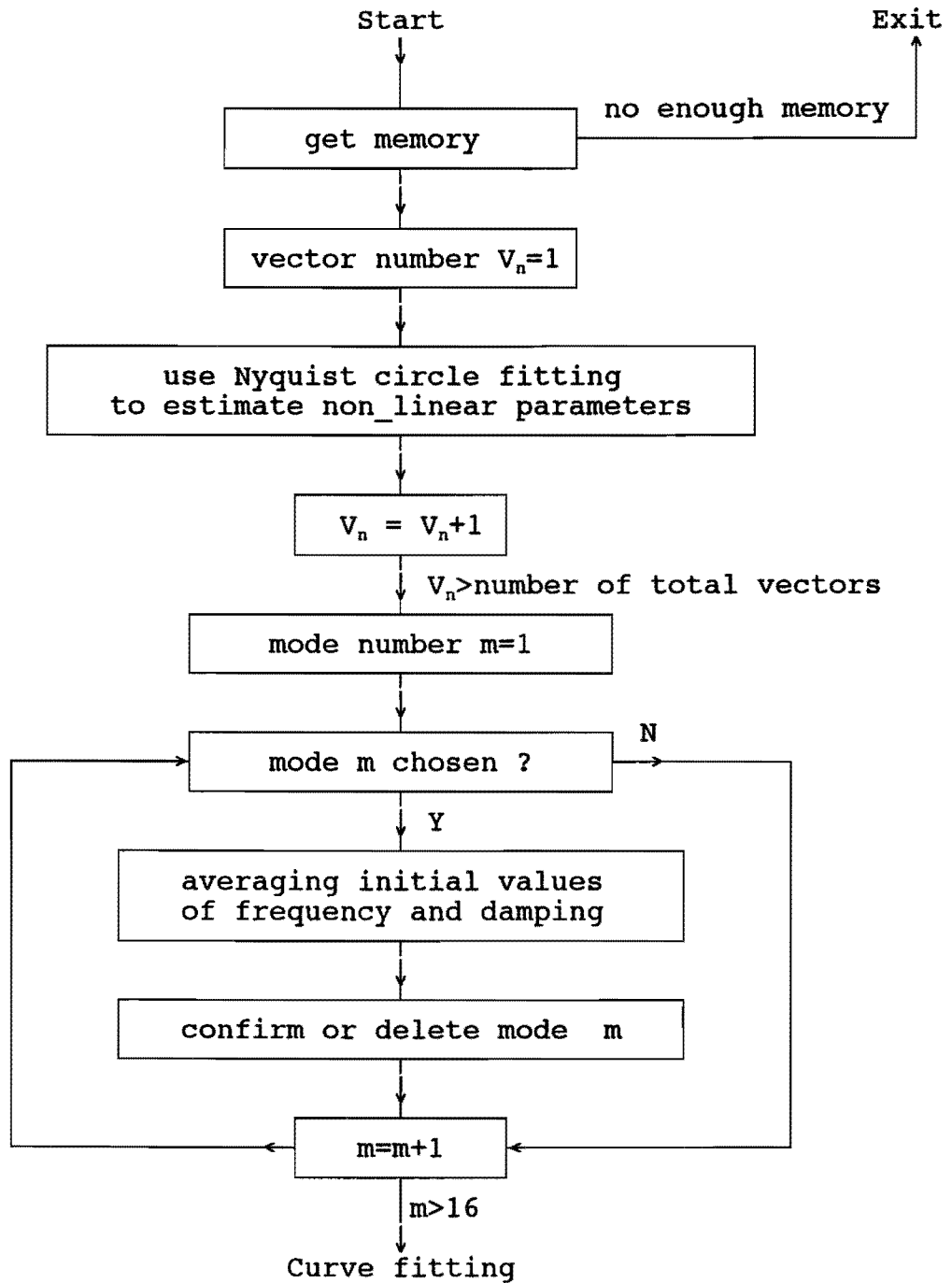
Data Information Collection

Appendix C



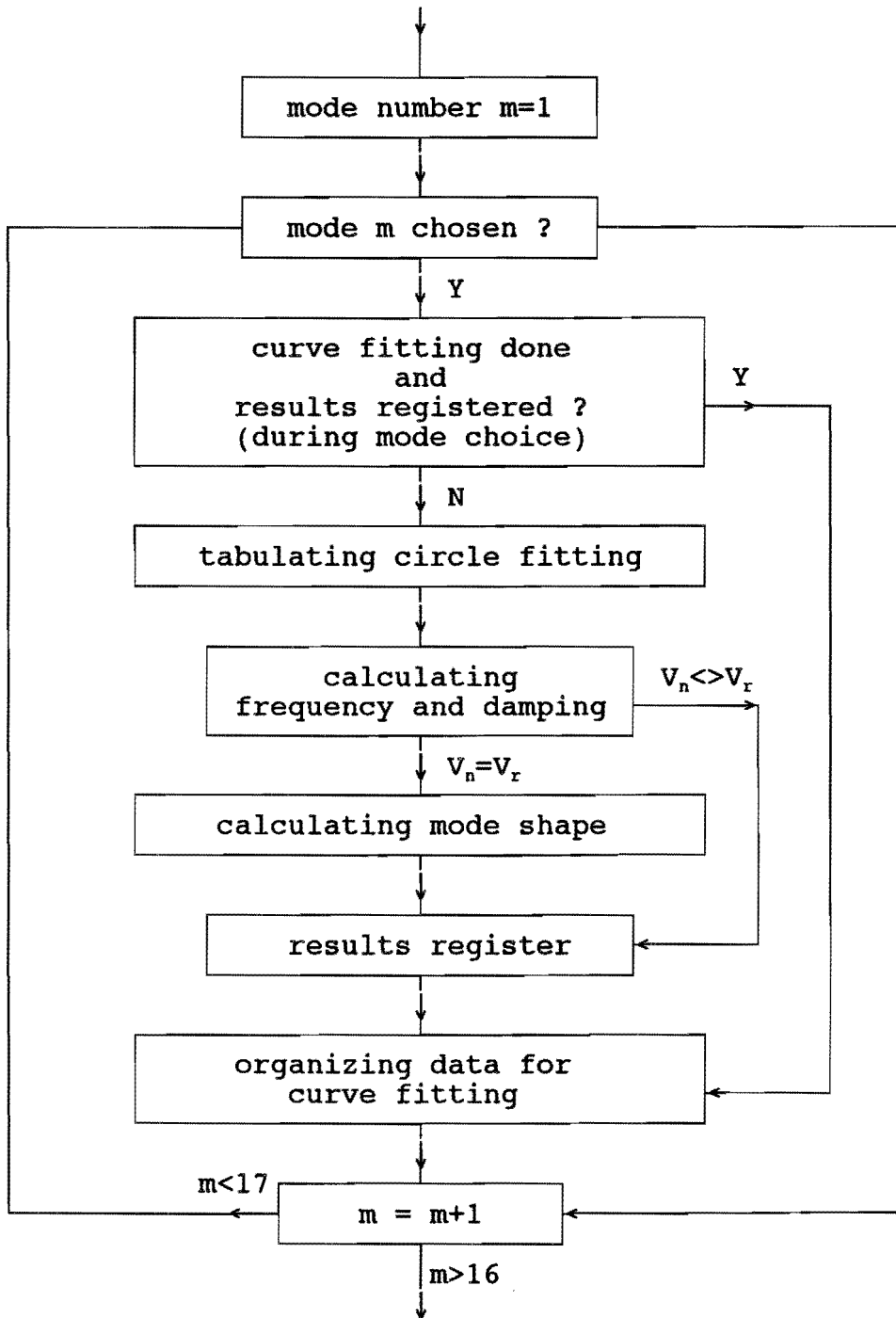
Mode choice

Appendix D



Set up initial values

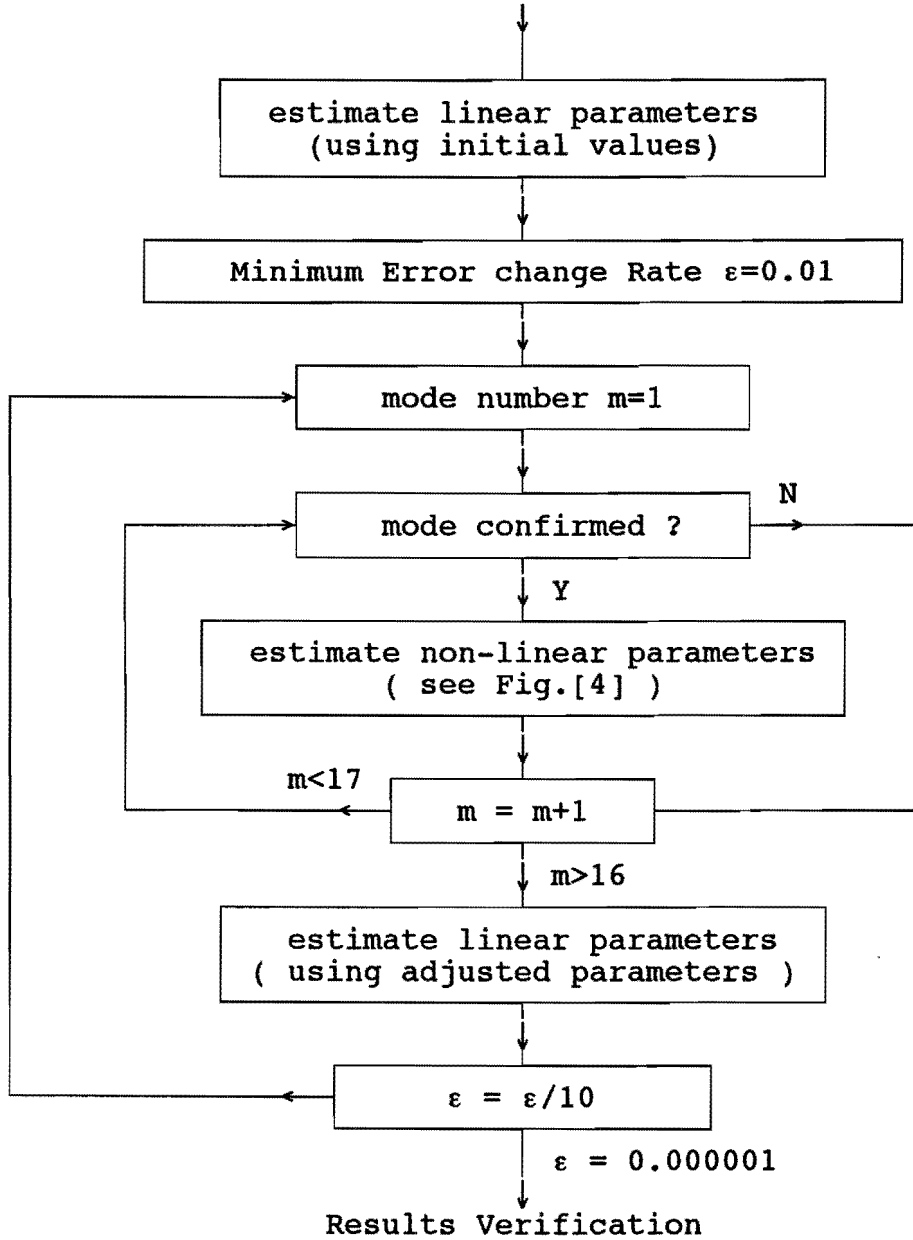
Appendix E



Nyquist Circle Fitting



Appendix F



Curve Fitting