Summary of a study on the TM01 radiation pattern of corrugated conical horn antennas with small flare angle


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Summary of a study on the TM$_{01}$ radiation pattern of corrugated conical horn antennas with small flare angle.

by

ir.Th. Scharten
Summary of a study on the $TM_{01}$ radiation pattern of corrugated conical horn antennas with small flare angle

Th. Scharten

This summary is based on the technical reports Nr ETA-3-1969 and Nr ETA-30-1969 (Eindhoven University of Technology)

1. The configuration and its electromagnetic description

1.1 Corrugated conical horn antenna

We assume a simple harmonic time dependence $e^{j\omega t}$. We suppose $t_1, t_2 \ll \lambda_{guide}$ and $R_o \approx a \gg \lambda_{guide}$. Furthermore, we suppose the flare angle to be so small that if a $TM_{0n}$ mode is excited in the horn, the aperture field can be approximated by the $TM_{0n}$ field of a circular cylindrical, corrugated waveguide with inner radius $a$, but now with a quadratic phase distribution in the radial aperture direction. For that case the longitudinal phase factor of the aperture field, $e^{i\beta_n z}$, has to be replaced by the factor $e^{i\beta_n (R_f - R_o)}$, so this factor reads

$$e^{i\beta_n z}$$
2. Fields on the horn surfaces

2.1 Outer surface. On the outer surface we assume the wall currents to be zero; so we have

\[
\hat{n} \times \vec{E} = 0 \\
\hat{n} \times \vec{H} = 0 
\]
on \( S_c \).

2.2 Inner surface. The surface impedance on the inner horn surface is determined by the relation

\[
Z_r = j X_r = \frac{t_1 Z_{\text{groove}} + t_2 Z_{\text{dam}}}{\epsilon_1 + t_2} \\
= \frac{t_1}{\epsilon_1 + t_2} Z_{\text{groove}}, \quad \text{on} \ S_r,
\]

where

\[
Z_{\text{groove}} = j X_{\text{groove}} = j \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{J_0(ka) Y_0(ka+kd) - J_0(ka+kd) Y_0(ka)}{J_1(ka) Y_0(ka+kd) - J_0(ka+kd) Y_1(ka)},
\]

being the input impedance of a shortened radial waveguide, \( a \leq \varphi \leq a + \alpha \), propagating a TEM wave.

2.3 Aperture. The aperture fields are found by considering a circular cylindrical waveguide (radius \( a \)) having the reactance wall condition, given above, and propagating a TM\(_{0n}\) mode (exp[+\(i\beta_z z\)]. Taking into account the quadratic phase distribution we find

\[
E_{\rho n} = -i \beta_n \lambda_n J_1(\lambda_n \rho) \exp(\rho^2/2 R_0) \\
H_{\phi n} = -i \omega \epsilon_0 \lambda_n J_1(\lambda_n \rho) \exp(\rho^2/2 R_0), \quad \text{on} \ S_a
\]

where \( \lambda_n \) is the \( n \)th root of the TM\(_{0n}\) characteristic equation

\[
\lambda J_0(\lambda a) - \omega \epsilon_0 X_n J_1(\lambda a) = 0.
\]

In our case \( ka \gg 1 \); so the surface reactance can be approximated by

\[
X_r = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{t_1}{\epsilon_1 + t_2} \tan kd.
\]

It is shown that the characteristic equation has two imaginary solutions (surface wave solutions) if

\[
k \tan kd > \frac{2(t_1 + t_2)}{\epsilon_1 a},
\]

and real, single valued solutions if

\[
k \tan kd < \frac{2(t_1 + t_2)}{\epsilon_1 a}.
\]
3. The radiation pattern
The energy-flow density can now be evaluated in the usual way, giving
\[
S_n(R,\Theta) = (k \cos \Theta + \beta_n)^2 \left| \int_0^\infty J_n(\lambda_n \rho) J_1(k \rho \sin \Theta) e^{i \beta_n \rho^2} \rho d\rho \right|^2
\]

4. Results
Horn dimensions
- \( a = 0.132 \) (m)
- \( d = 0.009 \) (m)
- \( t_1 = 2.5 \) (mm)
- \( t_2 = 4.0 \) (mm)
- \( R = 0.493 \) (m)
- \( \phi = 15^\circ \)

Dimensions of the 'equivalent' waveguide

Figure 3  Dispersion curves for the circular cylindrical, corrugated waveguide
Figure 4-10 Theoretical and experimental TM\(_{01}\) radiation patterns in the frequency range 8.5 - 11.5 GHz
Figure 11  Half beamwidth characteristics

-o-o-o-o-o-o-
\( a = 0.132 \, (m) \)

\( d = 0.009 \, (m) \)

\( t = 0.385 \)

Dispersion curves

Figure 3
TM\textsubscript{01} radiation patterns
theoretical ---
experimental ----
frequency: 8.5 GHz

Figure 4
$\text{Tm}_{01}$ radiation patterns
theoretical ---
experimental ----
frequency: 9 GHz

Figure 5
Figure 6

TM$_{01}$ radiation patterns
theoretical ——
experimental ----
frequency: 9.5 GHz
Figure 7

$T_{M01}$ radiation patterns

theoretical

experimental

frequency: 10 GHz
$T_{M_{01}}$ radiation patterns
theoretical ———
experimental ———
frequency: 10.5 GHz

Figure 8
Figure 9

TM$_{01}$ radiation patterns
theoretical ---
experimental ----
frequency: 11 GHz
Figure 10

$T_{01}$ radiation patterns

- theoretical
- experimental

frequency: 11.5 GHz
Half beamwidth at 0, 10, 20, 30 db level respectively, as a function of frequency.