

On some big closed polygons

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On Some Big Closed Polygons

by N.G. de Bruijn

Eindhoven University of Technology
 Department of Mathematics and Computing Science
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Recently the following problem was proposed to me:

Show that there exists a polygon with 1990 equal angles, of which the edges have length $1^2, 2^2, \dots, 1990^2$ (in some order).

This result is generalized in the following theorem (the special case is $N = 1990, k = 3, q_1 = 2, q_2 = 5, q_3 = 199, P(x) = (1 + x)^2$).

Theorem. Let N and k be positive integers, and let

$$N = q_1 \cdots q_k,$$

where q_1, \dots, q_k are integers > 1 , pairwise relatively prime. Furthermore, let P be a polynomial of degree $< k$, and let ζ be the primitive N -th root of unity

$$\zeta = e^{2\pi i/N}.$$

Then there exists a permutation π of the integers $0, \dots, N - 1$ such that

$$(1) \quad \sum_{n=0}^{N-1} P(\pi(n))\zeta^n = 0.$$

Proof. In order to avoid complicated notation, we restrict ourselves to the case $k = 3$; the general case is completely analogous.

Let the integer a run through the range $0, \dots, q_1 - 1$, b through $0, \dots, q_2 - 1$, c through $0, \dots, q_3 - 1$. Then the combination

$$(2) \quad a \cdot \frac{N}{q_1} + b \cdot \frac{N}{q_2} + c \cdot \frac{N}{q_3}$$

runs through a complete system of residues mod N , and the combination

$$(3) \quad aq_2q_3 + bq_3 + c$$

represents each one of the integers n with $0 \leq n < N$ exactly once.

We define the permutation π as follows. For given n with $0 \leq n < N$ we take a, b, c such that (2) is congruent to $n \pmod{N}$, and with these a, b, c the value of (3) is called $\pi(n)$.

We now evaluate the sum (1). In that sum we have to deal with

$$P(aq_2q_3 + bq_3 + c);$$

as a function of a, b, c this is a polynomial of degree < 3 . We write it as a sum

$$\sum_{\lambda=0}^2 \sum_{\mu=0}^2 \sum_{\nu=0}^2 C_{\lambda, \mu, \nu} a^\lambda b^\mu c^\nu.$$

The left hand side of (1) can now be expressed as follows:

$$(4) \quad \sum_{n=0}^{N-1} P(\pi(n)) \zeta^n = \sum_{\lambda=0}^2 \sum_{\mu=0}^2 \sum_{\nu=0}^2 C_{\lambda, \mu, \nu} W(\lambda, \mu, \nu),$$

where

$$W(\lambda, \mu, \nu) = \sum_{a=0}^{q_1-1} \sum_{b=0}^{q_2-1} \sum_{c=0}^{q_3-1} a^\lambda b^\mu c^\nu \zeta^h,$$

where h stands for the value of (2). So

$$(5) \quad W(\lambda, \mu, \nu) = \sum_{a=0}^{q_1-1} a^\lambda \zeta^{aN/q_1} \cdot \sum_{b=0}^{q_2-1} b^\mu \zeta^{bN/q_2} \cdot \sum_{c=0}^{q_3-1} c^\nu \zeta^{cN/q_3}.$$

Since the degree of P is less than 3, we have $\lambda + \mu + \nu < 3$, so at least one of λ, μ, ν is zero. The corresponding factor in the product (5) vanishes, and therefore $W(\lambda, \mu, \nu) = 0$. The theorem now follows from (4).