

On the distribution of $[\theta]$ modulo 1

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On the distribution of θ modulo 1

by

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In this note we report the following results, joint work with R. L. Graham, which will be published in the Canadian Journal of Mathematics.

If θ is an irrational number and $0 = a_0 < a_1 < \dots < a_n < a_{n+1} = 1$ is the sequence of points $\{1\theta\}$, $1 \leq i \leq n$ we define

$$d_\theta(n) = \max_{1 \leq i \leq n+1} (a_i - a_{i-1}).$$

$$\text{THEOREM 1: } \sup_{\theta} \liminf_{n \rightarrow \infty} n d_\theta(n) = \frac{1+\sqrt{2}}{2}$$

(this sup is a maximum taken if $\theta = 1+\sqrt{2}$).

$$\text{THEOREM 2: } \inf_{\theta} \limsup_{n \rightarrow \infty} n d_\theta(n) = 1 + \frac{2}{5}\sqrt{5}$$

(this inf is a minimum taken if $\theta = \frac{1+\sqrt{5}}{2}$).

$$\text{THEOREM 3: } \limsup_{n \rightarrow \infty} n d_\theta(n) = \infty \text{ iff } \limsup_{n \rightarrow \infty} b_n = \infty$$

where $[b_1, b_2, \dots]$ is the regular continued fraction for θ .

REMARK: It was remarked by V. Turán-Sós that she had proved theorem 3 in one of her papers.

The proofs depend upon the somewhat surprising and apparently little-known fact that the set of numbers $\{a_{i+1}-a_i : 0 \leq i \leq n\}$ always consists of at most 3 numbers. This enables one to find an explicit expression for $d_\theta(n)$.