

Telescope mappings

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N.G. Bourne 26 Jan 1974.

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Telescope mappings

~~LP notation~~

Abstractor string height k $[x_1, A_1][x_2, A_2(x_1)] \dots [x_k, A_k(x_1, \dots, x_{k-1})]$

An LP is an abstractor string mod α -reduction on the bound variables

A vector of height k is a column $\begin{pmatrix} v_1 \\ \vdots \\ v_k \end{pmatrix}$, where the v_i are AUT-expressions

We use three lambda-like ~~operator~~ quantifiers:

$\lambda_{x \in Q} v(x)$ will be a vector-valued function

(Q is an LP, v(x) is a vector)

$\pi_{x \in Q} R(x)$ is the concatenation of the LP's Q and R

$\eta_{x \in Q} R(x)$ will be the LP of the mappings of Q into R

if v and Q have height k, then

$v \in Q$ (v is a foler for Q)

says that $v_1 \in A_1, v_2 \in A_2(v_1), v_3 \in A_3(v_1, v_2), \dots$

(where $v = \begin{pmatrix} v_1 \\ \vdots \\ v_k \end{pmatrix}$, $Q = [x_1, A_1] \dots [x_k, A_k(x_1, \dots, x_{k-1})]$)

Later notation:

ε
v is foler for Q
LP

repl. by

\vdash
v fits into Q
Telescope

Folgers on higher level: in context $x \in R$ we have $\forall x \in R (v(x) \in Q(x))$, ~~for all $x \in R$~~

write $\forall_{x \in R} (v(x) \in Q(x))$ ~~or~~ $\forall_{x \in R} (v(x) \in Q(x))$ *)

Here it is assumed that $\forall_{x \in R} Q(x)$ is an acceptable LP.
 \uparrow
 π

Book lines: R in context R we write a line as

$$\forall_{x \in R} (f := v(x) \in Q(x))$$

or $\forall_{x \in R} (f := PN \in Q(x))$

We need not write block opening lines.

We can write LP-defining lines:

~~$\forall_{x \in R} (S := \prod_{x \in Q} R(x))$~~

~~$\forall_{x \in R} (T := \prod_{x \in Q} R(x))$~~

Folging: if $v = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$, $w = \begin{pmatrix} w_1 \\ \vdots \\ w_k \end{pmatrix}$ then

$\langle v \rangle w$ denotes $\begin{pmatrix} \langle v_1 \rangle \cdot \langle v_1 \rangle w_1 \\ \vdots \\ \langle v_n \rangle \cdot \langle v_1 \rangle w_k \end{pmatrix}$

*) Remark (1913): If $\boxed{x \in R} v(x) \in Q(x)$ then $\boxed{x \in R} x \circ v(x) \in \leq \pi_{y \in R} Q(y)$

was 15
dat 9

$$\text{If } Q = [x_1, A_1] \cdots [x_k, A_k(x_1, \dots, x_{k-1})] \pmod{\alpha_{x_1, \dots, x_k}}$$

$$\text{Then } v(x) = \begin{pmatrix} f_1(x_1, \dots, x_k) \\ \vdots \\ f_n(x_1, \dots, x_k) \end{pmatrix}$$

then

$$\lambda_{x \in Q} v(x)$$

denotes

$$\begin{pmatrix} [x_1, A_1] \cdots [x_k, A_k(x_1, \dots, x_{k-1})] f_1(x_1, \dots, x_k) \\ \vdots \\ [x_1, A_1] \cdots [x_k, A_k(x_1, \dots, x_{k-1})] f_n(x_1, \dots, x_k) \end{pmatrix}$$

$$\text{If } Q = [x_1, A_1] \cdots [x_k, A_k(x_1, \dots, x_{k-1})] \pmod{\alpha_x}$$

$$R(x) = [y_1, B_1(x)] \cdots [y_m, B_m(x, y_1, \dots, y_{m-1})] \pmod{\alpha_y}$$

then

$$\pi_{x \in Q} R(x) = [x_1, A_1] \cdots [x_k, A_k(\dots)] [y_1, B_1(x)] \cdots [y_m, B_m(\dots)] \pmod{\alpha_{x,y}}$$

$$\mu_{x \in Q} R(x) = [s_1, \lambda_{x \in Q} B_1(x)] [s_2, \lambda_{x \in Q} B_2(x, \langle x \rangle s_1)] \cdots$$

$$\cdots [s_m, \lambda_{x \in Q} B_m(x, \langle x \rangle s_1, \dots, \langle x \rangle s_{m-1})]$$

If R has height 1, $R(x) = ([y, \Lambda(x)] \pmod{\alpha_y})$, then

$$\mu_{x \in Q} R(x) = ([t, \lambda_{x \in Q} \Lambda(x)] \pmod{\alpha_t})$$

Composition $Q \rightarrow R_1 \circ R_2$

$$\mu_{x \in Q} \left(\prod_{y \in R_1(x)} R_2(x, y) \right) =$$

(with some notes)

$$= \prod_{f \in \mu_{x \in Q} R_1(x)} \left(\mu_{x \in Q} R_2(x, f(x)) \right)$$

Compare this to

$$\forall_{x \in A} \exists_{y \in B} T(x, y) \iff \exists_{f \in (A \rightarrow B)} \forall_{x \in A} T(x, f(x))$$

under Ax. of Choice

Rules ~~#~~

$$f \in \mu_{x \in M} R(x), \quad x \in M$$

$$\langle v \rangle f \in R(v)$$

$$\langle a \rangle \left(\lambda_{x \in M} f(x) \right) \stackrel{D}{=} f(a)$$

$$\forall_{x \in M} f(a) \in R(x)$$

$$\lambda_{x \in M} f(a) \in \mu_{x \in M} R(x)$$

Contra

These rules also hold with extra \forall 's in front, as
 $\forall_{x \in A} \left(f(x) \in \mu_{x \in M(x)} R(x, y) \implies \forall_{y \in M(x)} \dots \right)$

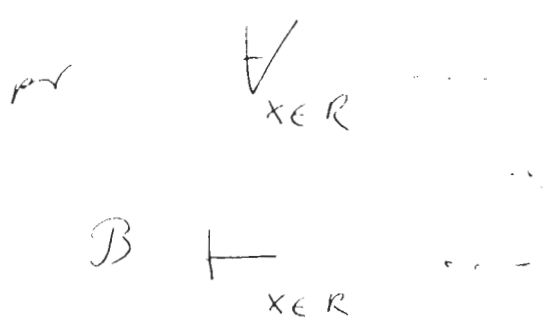
Compositie $R_1 \otimes R_2 \rightarrow w$

$$\lambda_{u \in \prod_{x \in R_1} R_2(x)} w(u) = \prod_{x \in R_1} \left(\prod_{y \in R_2(x)} w \left(\begin{pmatrix} x \\ y \end{pmatrix} \right) \right)$$

Compositie $R_1 \otimes R_2 \rightarrow Q$

\mathcal{P} voor deze Σ

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} =: f_1 \otimes f_2$$



schrijft aan afgevoerde Φ 's en v's.

$\mathcal{P} \mathcal{L} =$ isg met som - prod.

$$\Sigma \{ \dots \prod_{i \in S} \prod_{j \in T(i)} \dots$$

Compositie $\lambda_{x \in M} f(x) \leftarrow \prod_{x \in M} R(x)$

Compositie

$$\prod_{u \in \prod_{x \in Q_1} Q_2(x)} R(u) = \prod_{x \in Q} \prod_{y \in Q_2(x)} R(y)$$

Die Kompositionregel im $Q \rightarrow R, \text{ @ } R_1$ mit R_1 in length 1,

$$R_1 = ([X, \Lambda(x)])_{\text{mit } \alpha_x}$$

$$\mu_{x \in Q} \left(\prod_{y \in R_1(x)} R_2(x,y) \right) = \left[f, \prod_{x \in Q} \Lambda(x) \right] \left(\mu_{x \in Q} R_2(x, \langle x \rangle f) \right)$$

Nur hieraus im deduktion der regel $\forall_{x \in Q} f(x) \in R_1 \dots$ lassen
 $\lambda_{x \in Q} f(x) \in \mu_{x \in Q} R(x)$

$$\text{mit } \begin{cases} f \in \mu_{x \in Q} R(x) & , x \in Q \\ \langle x \rangle f \in R(x) \end{cases}$$

Vorbereitung von mappings LP

$$\mu_{\substack{[E, \text{type} \\ (P)]}} \left(\begin{matrix} E \\ P \end{matrix} \right) \in \left([q, \langle y, u \rangle] [q, [t, y] \text{ prop}] \right)_{\text{mit } u(y, u)} \quad ([y, E] [X, \langle y \rangle P])_{\text{mit } \alpha(y, u)}$$

Reine Vorz: um ein parametris E, P in ein set $S(E, P)$

hat set LP hat, d.h. ~~[E, P]~~ hat eng mit de typen

mit $\begin{pmatrix} u \\ v \end{pmatrix} \in S(E, P)$ bilden $\begin{cases} u \in E \\ v \in \langle u \rangle P \end{cases}$

Notation $\chi(P) = ([t, P])_{\text{mit } t}$

of sets unen - just!

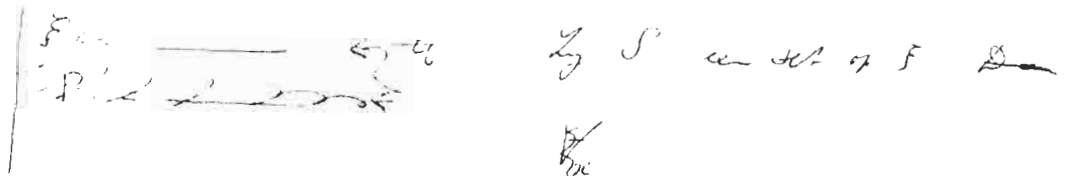
mit $\chi \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \left([t, v_n] \quad [t, v_1, \dots, v_n] \right)_{\text{mit } t_1, \dots, t_n}$

Nur mit $u_1 \in v_1$ da $\langle u \rangle P \iff u \in \chi(v)$

Beispiel

$$\tau_0 := ([L, \text{LPM}])_{\text{maximal}}$$

Nach τ_0 ist F ein τ_0 -
Schritt $F \in \tau_0$
prop: $\text{PW} \in \tau_0$
 $\text{PROP} := \chi(\text{prop})$



Definiere, ob $\forall x \in S (P(x) \in \text{PROP})$,

$$\text{ALL}_{x \in S} P(x) := \mu_{x \in S} P(x)$$

Nachfolgend (zu Regel 4)

$$f \in \text{ALL}_{x \in S} P(x), \quad v \in S$$

$$\langle v \rangle f \in P(v)$$

Nachweis der μ -Reduktion von Schritt τ_0 auf PROP

Als P ist die Menge S fest ist, dann gilt

$$\forall x \in S \quad P^* := P(\text{inj. von } x)$$

$$\text{ein } \text{ALL}_{x \in S} P^*(x)$$

Compositio (v. 2015 mod)

31

$$R \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = [y_1, B_1 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}] [y_2, \dots] [y_3, B_3 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, y_1, y_2]$$

$$\prod_{Q \in Q_1(x)} R \begin{pmatrix} x_1 \\ u \end{pmatrix} = [s_1, \lambda_{a \in Q_1(x)} B_1 \begin{pmatrix} x_1 \\ a \end{pmatrix}] \dots [s_3, \lambda_{a \in Q_3(x)} B_3 \begin{pmatrix} x_1 \\ a \end{pmatrix}, \langle a \rangle s_1, \dots]$$

$$\prod_{Q \in Q_1} \prod_{u \in Q_1(x)} R \begin{pmatrix} v \\ u \end{pmatrix} =$$

$$= [w_1, \lambda_{v \in Q_1} \lambda_{a \in Q_1(x)} B_1 \begin{pmatrix} v \\ a \end{pmatrix}] \dots [w_3, \lambda_{v \in Q_3} \lambda_{a \in Q_3(x)} B_3 \begin{pmatrix} v \\ a \end{pmatrix}, \langle a \rangle \langle v \rangle w_1]$$

$$= [w_1, \lambda_{\begin{pmatrix} v \\ a \end{pmatrix} \in Q} B_1 \begin{pmatrix} v \\ a \end{pmatrix}] \dots [w_3, \lambda_{\begin{pmatrix} v \\ a \end{pmatrix} \in Q} B_3 \begin{pmatrix} v \\ a \end{pmatrix}, \langle \begin{pmatrix} v \\ a \end{pmatrix} \rangle w_1]$$

wh. $Q = \sum_{v \in Q_1} \sum_{a \in Q_1(x)} Q_1(v, a)$; Note: $\langle \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rangle^s = \langle x_1 \rangle \langle x_2 \rangle^s$

31. kept.

γ : ket lambda

Neune notatus, us in NF lambda

$(Q \rightarrow F)$	$\lambda_{u \in Q} f(u)$:	$L(Q, \gamma f(u))$
$Q \rightarrow R$	$\mu_{u \in Q} R(u)$:	$M(Q, \gamma R(u))$
$Q \rightarrow R \otimes R$	$\sum_{u \in Q} R(u)$:	$S(Q, \gamma R(u))$

Compositio regule

$(Q \rightarrow (R_1 \otimes R_2)) = (Q \rightarrow R_1) \otimes (Q \rightarrow R_2)$:
ll 4 top

$M(Q, \gamma S(R_1, \gamma R_2)) = S(M(Q, \gamma R_1), \gamma M(Q, \gamma R_2))$

$(Q_1 \otimes Q_2) \rightarrow f = Q_1 \rightarrow (Q_2 \rightarrow f)$
ll 5 top

$L(S(Q_1, \gamma Q_2), \gamma f) = L(Q_1, \gamma L(Q_2, \gamma f))$

$(Q_1 \otimes Q_2) \rightarrow R = Q_1 \rightarrow (Q_2 \rightarrow R)$
ll 5 bottom, ll 0

$M(S(Q_1, \gamma Q_2), \gamma R) = M(Q_1, \gamma M(Q_2, \gamma R))$

$(Q_1 \otimes Q_2) \otimes Q_3 \quad Q_1 \otimes (Q_2 \otimes Q_3) \quad S(S(Q_1, \gamma Q_2), \gamma Q_3) = S(Q_1, \gamma S(Q_2, \gamma Q_3))$

Skema notasi etia komone an γ

\Rightarrow Veranone an notasi so' dat an γ 's verting.

$L(Q, \gamma f(u))$ notasi us $L(Q, F)$ e an dat f iati is ut an $\gamma f(u)$

En an direct an konfix notasi utia ?

$Q \circ F$ hitahat $\lambda_{u \in Q} F(u)$

F is an an $\lambda_x F(x)$

$Q \rightarrow R$ hitahat $\mu_{u \in Q} R$

R is an an $\lambda_x R(x)$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \quad y = \begin{pmatrix} y_1 \\ \vdots \\ y_k \end{pmatrix} \quad (NW)$$

$$f\left(\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}\right) = f(x_1, \dots, x_n)$$

$$\langle \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \rangle \oplus = \langle x \rangle \oplus \langle y \rangle$$

$$[x_1, A_1(x)] \dots [y_k, A_k(x, y_1, \dots, y_{k-1})]$$

besch. als een B met ~~van~~ free var. list x , h var. list y .

(B, x) is het LP het NFLP ~~is~~ NF-calculus

(B, x, y) is een abstractor string

Bij B ~~kan men~~ kan men hoofst in de base var. list compon.
 sup met B_i ~~het de voor~~ B_i ~~met x heeft i , y heeft j~~

$$\begin{pmatrix} x_1 \\ \vdots \\ x_j \\ \vdots \\ x_n \end{pmatrix} \oplus \begin{pmatrix} x_{j+1} \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

v : vector NF vector
 (v, x) is een expr. vector.
 \uparrow free var list

\mathcal{B}_i : vector met length i

Fulcrum $\Leftrightarrow \langle \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \rangle \langle \begin{pmatrix} w_1 \\ \vdots \\ w_k \end{pmatrix} \rangle$ denk in $\begin{pmatrix} \langle v_1 \rangle \dots \langle v_i \rangle v_i \\ \vdots \\ \langle v_n \rangle \dots \langle v_i \rangle v_k \end{pmatrix}$

linear met B_j ~~reken~~ \oplus de B_j

Zij B een NFLP, v een NF-vector.

Na $B \circ v$ een vector.

$$B \text{ is } \in B_j \quad v \text{ is } \in v_j$$

Als x, y var. lists zijn van same length met B , dan is

$$(B \circ x, v) = [x_1, A_1(x)] \dots [x_n, A_n(\dots)]$$

$$(v, x) = \begin{pmatrix} v_1(x) \\ \vdots \\ v_k(x) \end{pmatrix}$$

dan is $B \circ v$ B.v ~~het string~~ by var. list $x \oplus y$ is

$$\begin{pmatrix} [x_1, A_1(x)] \dots [x_n, A_n(x, y_1, \dots, y_{k-1})] v_1(x \oplus y) \\ \vdots \\ v_k(x \oplus y) \end{pmatrix}$$