

# Discussion about: Regimes of traction in concentrated contact lubrication / Bair, S. and Winer, W.O. [Journal of lubrication technology 104 (1982), p. 382-386]

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#### I. Andersson<sup>2</sup> and H. van Leeuwen<sup>2</sup>

During the past decade the definition of lubrication regimes in concentrated contacts through its film thickness has received much attention, especially in the full film EHD regime. In this paper the authors attempt to determine lubrication regimes through some characteristic traction, which may open new views on the lubrication of this type of contact.

The discussers think that there is a need for a discrimination in lubrication regimes by traction. It provides a basis for comparison, both experimentally and theoretically, and gives a check on assumptions and formulas when performing calculations. When these regimes become well defined, they will be useful for designers.

By this paper the authors have invoked a very interesting subject. They should be commended for their effort to extend the scope of the work in nondimensional representations of frictional traction to the non-Newtonian EHD lubrication.

What follows is a rather long commentary on the basic idea and a query on the authors' opinion about this.

In their paper the authors make clear that some transitions in the lubrication regime are quite closely bound with transitions in the so-called reduced traction coefficient (RTC) under isothermal conditions. This RTC is defined as the ratio of the measured traction coefficient  $\tau$  at 5 percent slip to the quotient of the limiting shear stress  $\tau_L$  at the averaged Hertzian pressure  $\bar{p}$  and the same temperature, and the mean Hertzian pressure.

Going from the hydrodynamic (HD) to the "classical" elastohydrodynamic (EHD) range, the RTC increases considerably from a value between 0.3 and 1.0, to 1.0, while the transition from EHD to mixed lubrication conditions is marked by a salient increase from 1.0 to over 6.0. This also supports the authors viewpoint that the traction is controlled by the limiting shear stress  $\tau_L$ .

In order to have a clear discussion, first the lubrication regimes have to be defined.

Lubrication regimes are boundary, mixed, and full film lubrication, as is defined more or less through the Stribeck curve (hence through the traction) or the  $\Lambda$  ratio (which are correlated, as can be concluded from this paper).

Lubrication subregimes can be defined for the full film lubrication regime. Up till now, this has been obtained using the film thickness. E.g., see Johnson [A1]. Four regimes are generally accepted, viz. isoviscous-rigid (IR), viscous-rigid (VR), isoviscous-elastic (IE), and viscous-elastic (VE), which are distinguished by viscosity and elasticity effects. The classical EHD work is done in the viscous-elastic subregime.

A transition from the mixed to the full film region is marked by a decrease in the traction coefficient and a much lower wear rate, while the film thickness increases.

Transitions within the full region are characterized by changes in the viscous and elastic parameters, and in the film thickness. It seems reasonable to assume that traction in these subregimes is controlled by different mechanisms.

#### Transition Mixed—Full Film Lubrication Regime

The discussers also believe that the  $\Lambda$  ratio is one of the parameters which can be used successfully when describing the transition from mixed to full film lubrication. Other variables, showing a considerable change when changing regimes, are the frictional traction and the specific wear rate (Begelinger and De Gee, [A2]. In reference also an attempt

is made to discriminate lubrication regimes by measured discontinuous changes in friction and wear as a function of normal load and sliding speed, resulting in a kind of transition diagram.

In [A2] it is concluded that, under mixed lubrications conditions, as long as an adsorbed layer on the contact surface exists, the coefficient of friction  $\tau_m$  for the part of the load supported by mechanic contact  $N_m$  has about a constant value. If the hydrodynamically supported load is designated  $N_h$ , and the corresponding coefficient of friction  $\tau_h$ , then the averaged traction over the contact is

$$\tau = (\tau_m N_m + \tau_h N_h) / (N_m + N_h)$$

or, if we call the ratio  $N_m/(N_m + N_h) = m$ 

$$\tau = m\tau_m + (1-m)\tau_h$$

where  $\tau_m$  is a constant below the desorption temperature of the surface layer. This  $\tau_m$  can be determined through Abbott's curve, the approach of the contacting bodies ( $\Lambda$  ratio),  $\tau$  measurements and  $\tau_h$  calculations. For known  $\tau_m$  values the traction in the mixed lubrication regime can now be predicted.

# Traction Transitions Within the Full Film Lubrication Regime

The parameter M according to Moes and Bosma [2], or, which is almost the same, the parameter  $g_3$  (also called  $g_E$ ) according to Johnson [A1], is a correct choice for discriminating between elastic effects of the contact surfaces. It allows for a test whether the pressure distribution can be considered Hertzian or not.

However, the discussors feel that when different traction regimes in full film lubricated concentrated contact have to be distinguished, more (dimensional) parameters are needed than merely M and RTC.

The authors conclude that experimental evidence supports the limiting shear stress model for traction under high slip and high loads. On the other hand, the classical EHD regime also contains low slip or pure rolling, and lower loads conditions. Hence, next to M,  $\tilde{p}$ , and  $\tau_L$ , other variables like  $\eta_0$ ,  $\alpha$ , G, and the slip ratio have to be used.

One dimensional parameter which shows a remarkable sensitivity to changes from viscous to elastic behavior in the linear part of the traction curve is the Barus viscosity  $\eta_0$  exp  $(\alpha \bar{p})$ , see Hirst and Moore [A3]. A dimensionless number which controls the behavior in the linear part is the Deborah number D (see Johnson and Tevaarwerk [A4]), defined as  $(\eta U/Ga)$ , where  $\eta$ , U, G and a represent the local viscosity, the rolling velocity, the elastic shear modulus of the fluid, and the Hertzian semi contact width, respectively.

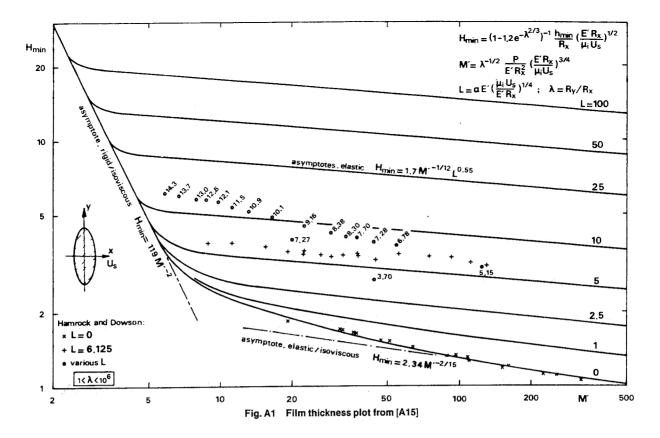
Under low slip conditions, the fluid behaves elastic when D is high, while it behaves viscous Newtonian under low D numbers.

For Newtonian fluids representations have been determined by several authors. For example, see ten Napel, Moses, and Bosma [A5], which gives the nondimensional traction for low slip as a function of L and M (so  $g_E$  and  $g_v$ ). Archard and Baglin [A6] also provide nondimensional traction coefficients for the IR, IE and VE subregime, under low slip and low D conditions.

A generalized approach for elliptical contacts in the VE subregime (Assumed Hertzian pressure distribution) for Newtonian fluids can also be performed. In this case the number for the sliding friction, minimum film thickness, elasticity, and viscosity parameters can be corrected for ellipticity effects, resulting in only one expression for all ellipticity values.

For the nonlinear part of the traction curve the shear stress distribution and hence the traction is described by using

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nonlinear viscosities including some representative stress, e.g., the limiting shear stress  $\tau_L$ , or the characteristic shear stress  $\tau_0$ , which concept is developed by Johnson and Tevaarwerk [A4].

However, the strain rate distribution is a function of both the slip ratio and the amount of spin in the contact, and this has to be taken into account if and when choosing a characteristic slip value as the authors have done. Different test rigs will yield different characteristic slip values depending on their amount of spin.

If the limiting shear stress, the slip ratio and the spin is used, it is possible to predict the nonlinear part of the traction curve (see e.g. Tevaarwerk [A7] which also shows traction curves for spin and zero spin). The number of dimensionless groups should probably be too high for plotting regime charts if all these three variables should be included.

Since the derivation of new dimensionless numbers is complicated, it will be more easy to attack these problems first under line contact and no spin conditions.

#### Evidence for Transitions in the Full Film Lubrication Regime

The authors found a bend in the RTC curve in their Fig. 7 at about  $\Lambda = 20$ , which describes the transition from EHD to HD lubrication. Similar transitions should be found in [1] and in [2].

However, when tyring to locate the data from Fig. 7 in the Hamrock and Dowson plot for k=1 [1], the discussers found that all data obtained are well outside the IR region, that is, far in the VE region near the transition VE-VR, mostly outside the depicted area in [1]. The diagram presented by Moes and Bosma [2] only applies to elliptical contacts having higher ellipticity ratios than about 5. So it is questionable whether this diagram may be used for circular contact or not.

As the discussers believe that the authors' measurements are correct, and no experimental data is given in [1], the precision of the transition lines in the Hamrock and Dowson plot need some more attention.

From Brewe, et al. [A8] the maximum reduced pressure

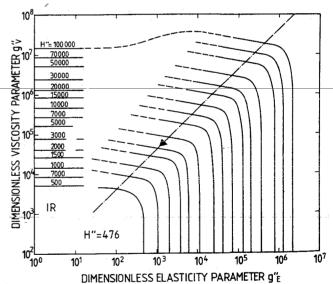


Fig. A2 Map of lubrication regimes with dimensionless film parameter contours on log-log grid of dimenionless viscosity and elasticity parameters. Solid lines in IE and VE regimes are transformed from Fig. A1. Broken straight lines represents VE asymptotes from Fig. A1 ( $H''=2.76~g_E^{-0.10}~g_V^{-0.55}$ ). Solid lines in VR regime are from [1] ( $H''=1.66~g_V^{-2/3}$ ). These lines should be connected to the lines in other regimes as indicated for H''=100.000. Also shown are the location of the authors experimental points, with the bend marked by .

under IR conditions can be obtained. This can be done for the Kapitza solution, or for the solution at Reynolds boundary conditions and a parabolic film shape. In the last case it can be found that for k=1:

$$q'_{\text{max},k=1} = 0.663 \cdot 10^{-3} \ q'_f$$

where  $q'_{\text{max}}$  and  $q'_f$  as defined in [A1].

Using the same arguments for determining IR-VR boun-

dary as Archard and Baglin [A6], viz. at this border the IR pressure maximum should equal  $1/\alpha$ , it is found that the IR-VR transition line is found at  $\alpha q_f' = g_v' = 1.5 \cdot 10^3$ , which is almost one decade more than in [1].

Dalmaz and Godet [A9] performed calculations and experiments for a circular contact in the same subregimes. From their work it can be concluded that the transiton takes place at  $g'_v = 10^3$ , which is still a factor 5 beyond the transition according to [1].

It is striking to find that, when expressing the minimum film thickness formula of Archard and Cowking [A10], Cheng [A11], and Hooke [A12] for the EV regime in the Johnson parameters  $g'_E$  and  $g'_v$ , the lines for constant film thickness H all have a positive gradient, while the Hamrock and Dowson formula as given in [8] always results in a negative slope. We feel that they can both be correct in different parts of the EV subregime, as is shown by Hooke [A13] for line contacts.

For the line contact case, Johnson [A1] obtained the transition boundaries from the Moes plot [A14]. Very recently Moes completed a generalized film thickness chart for elliptical contacts, which holds for a very wide ellipticity ratio range. This chart is soon to be published elsewhere [A15]. By kind permission of the authors we publish this chart here, see Fig. A1. The diagram is a large improvement on the previous one [2], because ellipticity ratios go down to 1, and the numerical results of Hamrock and Dowson for high and low modulus of elasticity have been carefully curve fitted. In Fig. A1 also the computational results of [8] have been inserted. They all occur in a rather narrow band, due to the small changes in the materials choice.

It is possible to transform the Moes and Droogendijk parameters to the parameters as employed by Hamrock and Dowson. To retain its generality with respect to the ellipticity ratio it is better to use

$$g''_{E} = g'_{E} \lambda^{4/3}$$

$$g''_{v} = g'_{v} \lambda^{-3/2}$$

$$H'' = \hat{H} \cdot (1 - 1.2 e^{/\lambda^{2/3}})^{-1} \lambda^{-1}$$
where  $\lambda = R_{v}/R_{x} > 1$ 

The transformed plot, which is analogous to [1], but for all ellipticity ratios (k) between 1 and about 104, is shown in Fig. A2.

Also shown are the experimental data of the authors. From this, it can be concluded that this new regime chart gives an explanation for the observations done by the authors, since the points with  $\Lambda > 20$  are in the lower part of VE, or even in the IR Regime.

#### Suggestions and Questions

If the film thickness formulas from [1] are accepted it is possible to make a film thickness chart where the occurence of mixed lubrication conditions easily can be tested in the VR, VE, and IE subregimes. This is achieved by using the new parameters

$$H''' = h_{\min}/R_x$$

$$g_E''' = g_E'' (\eta_0 U R_x / F)^2$$

$$g_n''' = g_n'' (\eta_0 U R_x / F)^2$$

and expressing

$$H''' = H''' (g_E''', g_v''')$$

which can be done because of the common exponent 2/3 in

the formulas of [1]. This will be a great advantage, since given geometry and surface roughness immediately give the boundary for mixed lubrication.

The discussers believe that many papers on traction would gain in strength if all test conditions are reported. This should make it easier to distinguish and compare different experimental results.

- —Can the authors elaborate on this?
- -Do the authors agree that you need more dimensionless numbers to distinguish traction regimes?
- What is the authors' opinion on the different lubrication regime charts for elliptical contacts?

#### **Additional References**

Al Johnson, K. L., "Regimes of Elastohydrodynamic Lubrication," Jl.

Mech. Engng. Sci., Vol. 12, No. 1, 1970, pp. 9-16.

A2 Begelinger, A., and de Gee, A. W. J., "Failure of Thin Film Lubrication-The Effect of Running-In on the Load Carrying Capacity of Thin-Film Lubricated Concentrated Contacts," ASME JOURNAL OF LUBRICATION

TECHNOLOGY, Vol. 103, No. 2, Apr. 1981, pp. 203-210.

A3 Hirst, W., and Moore, A. J., "Elastohydrodynamic Lubrication at High Pressures," "II. Non-Newtonian Behavior," Proc. R. Soc. Lond. A.,

Vol. 365, 1979, pp. 537-565.

A4 Johnson, K. L., and Tevaarwerk, J. L., "Shear Behavior of Elastohydrodynamic Oil Films," *Proc. R. Soc. Lond. A.*, Vol. 356, 1977, pp.

A5 ten Napel, W. E., Moes, H., and Bosma, R., "Traction in Elastohydrodynamic Lubrication at Low Sliding Speeds," Instn. Mech. Engrs., Paper C82/71, Tribology Conv. 1971, pp. 142-150, and Discussion, pp. 193-194 and p. 220.

A6 Archard, J. F., and Baglin, K. P., "Nondimensional Presentation of Frictional Tractions in Elastohydrodynamic Lubrication-Part I: Fully Flooded Conditions," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 97, No. 3, July 1975, pp. 398-411.

A7 Tevaarwerk, J. L., "A Simple Thermal Correction for Large Spin Traction Curves," ASME Journal of Mechanical Design, Vol. 103, No. 2, Apr. 1981, pp. 440-446.

A8 Brewe, D. E., Hamrock, B. J. and Taylor, C. M., "Effect of Geometry on Hydrodynamic Film Thickness," ASME JOURNAL OF LUBRICATION TECHNOLOGY, Vol. 101, No. 2, Apr. 1979, pp. 231-239.

A9 Dalmaz, G., and Godet, M., "Traction, Load and Film Thickness in Lightly-Loaded Lubricated Point Contacts," Journal of Mechanical

Engineering Science, Vol. 15, No. 6, 1973, pp. 400-409.
A10 Archard, J. F., and Cowking, E. W., "Elastohydrodynamic Lubrication at Point Contacts," Proc. Instn. Mech. Engrs. 1965-1966, Vol.

Lubrication at Point Contacts, \*Proc. Instr. Mech. Engls. 1903-1906, Vol. 180, Part 3B, Paper 3, pp. 47-56.

All Cheng, H. S., "A Numerical Solution of the Elastohydrodynamic Film Thickness in an Elliptical Contact," ASME Journal of Lubrication Technology, Vol. 92, No. 1, Jan. 1970, pp. 155-162.

All Hooke, C. J., "The Elastohydrodynamic Lubrication of Heavily Loaded Point Contacts," Journal of Mechanical Engineering Science, Vol. 22,

No. 4, 1980, pp. 183-187.

A13 Hooke, C. J., "The Elastohydrodynamic Lubrication of Heavily Loaded Contacts," *Journal of Mechanical Engineering Science*, Vol. 19, No. 4, 1977, pp. 149-156.

Al4 Moes, H., Discussion on Paper R1 by D. Downson, "Elastohydrodynamic Lubrication: An Introduction and a Review of Theoretical Studies," Proc. Instn. Mech. Engrs. 1965-1966, Vol. 180, Part 3B, pp. 244-245.

A15 Moes, H., and Droogendijk, L., "Elastohydrodynamic Lubrication of Human Joints," soon to be published.

#### L. D. Wedeven<sup>3</sup>

The authors have contributed a great deal to the understanding of fluid transition and shear strength in concentrated contacts. This paper puts this into perspective in connection with mixed film friction and thick film friction by "normalizing" the traction using the fluid limiting shear strength as shown in Fig. 7. This discusser has two comments on this figure.

First, the left-hand side of the curve, in the mixed-film region, is constructed with only the data of the diester which apparently is unformulated. If measurements were made of all the fluids in this region we would expect a band of data

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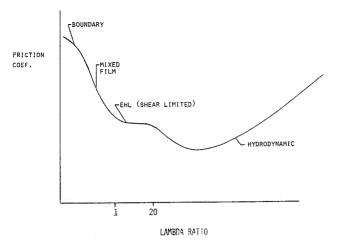


Fig. A3 Modified "Streibeck" friction curve

depending on the boundary lubricating characteristics of the fluids. For example, the traction with 5P4E would be expected to rise more rapidly as lambda decreases because of its poor boundary characteristics, whereas the formulated Santotract 50 or mineral oil R620-15 would rise more slowly.

Second, the right-hand side of the curve moves into a regime of lower maximum pressures which is less characteristic of Hertizian conditions. As we move further to the right the pressure (and friction) becomes dominated by the hydrodynamic pressure generated over a larger area. If the right-hand side of Fig. 7 were extended we would have a modified "Streibeck" friction curve as shown in Fig. A3.

#### S. H. Loewenthal4

The authors' results are instructive in pointing out the complicated nature of traction behavior and its dependence on the lubrication regime. A good example is that shown in the Authors' Fig. 4 where increasing temperature can benefit traction when asperity interaction is prevalent (thin film) while having the opposite effect in the EHD regime due to reduced limiting shear strength.

Plotting traction coefficient against film thickness or lambda ratio, as in Fig. 6, can lead to misinterpretation without additional clarification. The trends shown in Fig. 6 suggest that an increase in film thickness will cause an improvement in traction in the case of three of the lubricants studied. Although not specifically mentioned, this increase in traction is presumably the result of lower inlet temperatures which enhance the lubricant's limiting shear strength. Had the film thickness been increased by increasing the mean rolling velocity rather than lowering the temperature, a loss in the traction coefficient would probably result. Such is the case for most fluids. This is illustrated in the Discussor's figure for Santotrac 50 where the measured peak traction coefficient gradually drops with increased surface velocity while the lubricant's temperature hence limiting shear stress remain ostensibly constant. Strictly speaking, the shear strength of a fluid is not completely independent of surface velocity because of internal heating and its subsequent effects on the film temperature. Would the Authors care to comment on the effects that rolling velocity might have on the relationship between their measured traction coefficient and that deduced from limiting shear stress data? Do the authors have traction data at other rolling speeds which might help clarify this point?

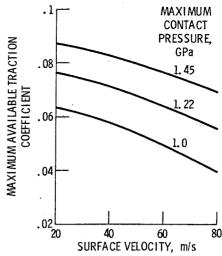


Fig. A Effect of surface velocity and contact pressure on maximum traction coefficient for Santotrac 50 at 70 C from a twin disk test machine

#### **Authors' Closure**

The authors wish to express their appreciation to the discussors for adding value to the paper by the thoughtful discussions especially the extensive contribution of Anderson and van Leeuwen. With respect to the specific questions raised by the discussors we offer the following responses:

Anderson and van Leeuwen:

We agree that authors should present more details of their operating conditions when presenting traction data so more thorough comparisons can be made in this complex field. We also agree that more dimensionless groups than we have discussed are necessary to fully map out the traction regimes of concentrated contacts. It is clear to us that the needed dimensionless groups will include quantities not included in the parameters used in film thickness regime definition. Our intent in this paper was to initiate and present some aspects of the discussion and introduce some previously unconsidered ideas but not to present the final word on the subject. Regarding their last question as to our opinion of the different film thickness regime charts for elliptical contacts, we have not had much opportunity to study the newly presented method of Moes but we believe it looks very promising and useful.

The five percent slip criteria to give maximum traction may be too low for high temperature and/or low viscosity situations but it was sufficient for this work. The effect of spin in our equipment was overshadowed by slip at the chosen five percent value of slide-roll ratio.

The lubricant shear modulus,  $G_{\infty}$ , is believed to not be important to the lower slip traction. Metal shear elasticity and the thin lubricant film cause the linear portion to be controlled by metal creep when the Barus viscosity becomes sufficiently large, and non-Newtonian  $(\tau_L)$  effects are also important in the linear part of traction curves. We found in applying our model that the linear part of the curve is produced by the non-Newtonian  $(\tau_L)$  effect spreading from the center-high pressure region of the concentrated contact to the lower pressure perimeter as fluid strain rate or strain is increased.

Wedeven:

The discussor is correct that the left hand side of the curve in Figure 7 will be a band of points for different fluids and solids because of the dependence of the boundary friction portion of the traction on materials chemistry. However, we do not agree that the friction in this system will increase at

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higher Lambda ratios as shown in the classical Stribeck chart (Figure A3). That increase will only occur in the Stribeck case of a journal bearing where the shaft is confined, hence, the film thickness and Lambda ratios have maximum values and as the rotational speed is increased the viscous drag increases causing the friction coefficient to increase. In the case of the crowned roller on a flat (or almost any traditional EHD contact) there is no limit on the film thickness and the ratio of the viscous drag to the load will continue to decrease as the film thickness increases.

Loewenthal:

The data for peak traction presented by Loewenthal are consistent with those of the authors in that they are a value of the speed parameter of 10<sup>-10</sup> using viscosity data obtained in this laboratory at 70C. Therefore the traction decreases with increasing speed as was found. We have also found the traction decrease with increasing speed at the speed parameter value of about 10<sup>-10</sup> which is in agreement with Loewenthal's data and our data in this paper where the speed parameter increase resulted from a decrease in temperature.

# Paper and Publications of Special Interest to Lubrication **Engineers**

## Journal of Applied Mechanics, June, 1982

"An Integral Equation Approach to the Inclusion Problem of Elastoplasticity," W. C. Johnson and J. K. Lee, p. 312.

"Rotational Sliding of Rubber: Second-Order Stresses, Seizure, and Buckling," A. N. Gent and R. L. Henry, p. 336.

"Rough Contact Between Elastically and Geometrically Identical Curved Bodies," M. D.

Bryant and Y. T. Chou, p. 345. "Disturbance at a Frictional Interface Caused by a Plane Elastic Pulse," M. Comninou, J.

R. Barber, and J. Dundurs, p. 361. "On the Instability of Rotating Shafts Due to Internal Damping," L. L. Bucciarelli, p. 425.

"Analysis of the Weibull Distribution," K. T. Chang, p. 450.

# Journal of Engineering Materials and Technology, April, 1982

"A Review of the Abrasive Wear of Metals," Ambrish Misra and Iain Finnie, p. 94.

### Journal of Mechanical Design, April, 1982

"Bearing Fault Detection Using Adaptive Noise Cancelling, G. K., Chaturvedi and D. W.

Thomas, p. 280

"Effect of Inter-Modulation and Quasi-Periodic Instability in the Diagnosis of Rolling Element Incipient Defect," C. C. Osuagwu and D. W. Thomas, p. 296.

"Iterative Determination of Squeeze Film Damper Eccentricity for Flexible Rotor Systems,"

L. M. Greenhill and H. D. Nelson, p. 334.

"Optimum Journal Bearing Parameters for Minimum Rotor Unbalance Response in Synchronous Whirl," R. B. Bhat, J. S. Rao, and T. S. Sankar, p. 339.

"Transverse Vibrations of a General Cracked-Rotor Bearing System," T. Inagaki, H. Kanki, and K. Shiraki, p. 345.

"A Numerical Approach to the Stability of Rotor-Bearing Systems," K. Athre, J. Kurian, K. N. Gupta, and R. D. Garg, p. 356. "Finite Journal Bearing With Nonlinear Stiffness and Damping. Part 1: Improved Mathematical Models," E. Hashish, T. S. Sankar, and M. O. M. Osman, p. 397.

"Finite Journal Bearing With Nonlinear Stiffness and Damping. Part 2: Stability Analysis," E. Hashish, et al., p. 406.