

## Analysis of a milling machine : computed results versus experimental data

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# ANALYSIS OF A MILLING MACHINE: COMPUTED RESULTS VERSUS EXPERIMENTAL DATA

by

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## SUMMARY

The paper describes the transformation of a structure of a horizontal milling machine into a beam model. From the comparison of the static results of the model and the experimental values of the milling machine some corrections in the model are carried out. The dynamic results are compared by means of the direct and cross receptances between table and spindle of the machine tool. To this end an arbitrary value for the overall relative damping is introduced into the model.

## INTRODUCTION

In order to calculate the static and dynamic properties of a machine tool structure, it is necessary to define a suitable topological model of that structure. The details of such a model depend on the structure itself, on the available program facilities and on the person who defines the model.

The machine tool structure under discussion is a universal milling machine, equipped for horizontal milling. A computer program based on the 'finite-element' method was available. A description of this program is given in ref. 1.

When computing the static properties of the model, any kind of load and combination of loads may be placed in any station point of the model. The displacements and rotations of all the station points can be calculated. For the dynamic properties the natural frequencies with corresponding modal shapes are computed. It is also possible to calculate the modal flexibilities and—according to Cowley<sup>2</sup>—the frequency response between two points of the structure. Viscous damping is assumed to occur.

## THE MODELLING

In the computer program the relevant stiffness quantities of the beam elements are—in general—calculated from the length of the element, the cross-sectional area, the second moments of area with respect to the principal axes, the second polar moment of area and the material properties. The program also offers the possibility to characterize some elements by direct input of the stiffness quantities. Thus, it is possible to put in other elements than 'beams'.

Figure 1 shows how the Jaspar milling machine is reduced to a mathematical model of elastic and stiff elements. The length axis of the elastic elements is taken through the centre of gravity of the machine-parts cross-section. For the connection between the

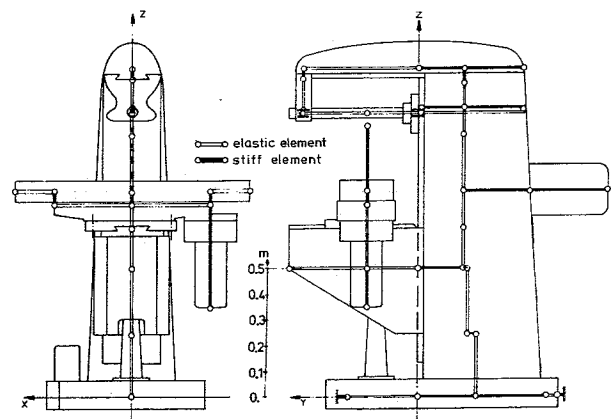


Figure 1. The Jaspar milling machine and its mathematical model.

elastic elements, stiff elements can be used. A stiff element is defined as an element for which the rotations and displacements of one of the two station points of the element depend upon those of the other station point. For the computer program the numbering of the station points is of importance. To decrease the bandwidth of the stiffness matrix, and with it the computing time, the difference between two independent station points of an element must be as small as possible. It has to be noticed that the dependent point of a stiff element is not important

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for the stiffness matrix. With the numbering shown in Fig. 2 the largest difference is only two. For the preparation of the input data from elements and station points, two systems of axes are applied. A global system is used for the input of the coordinates

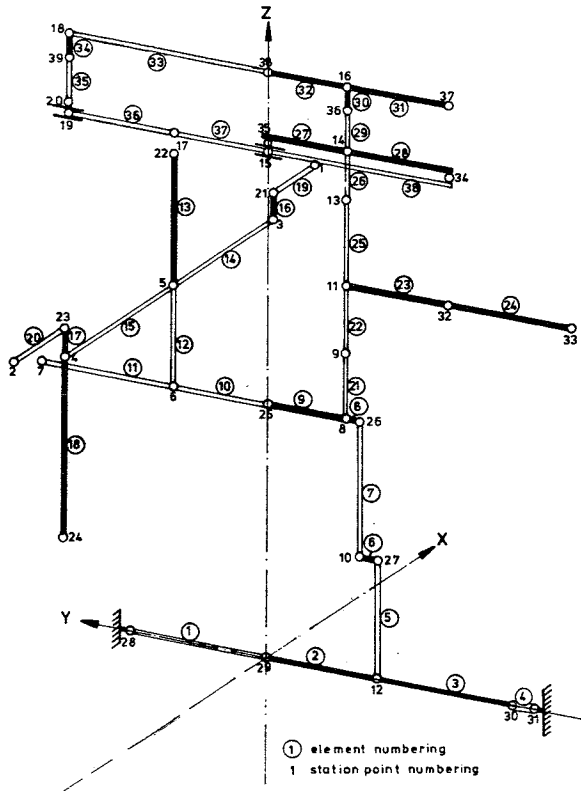


Figure 2. The numbering of the elements and the station points in the mathematical model.

of the station points. This global system is indicated in the Figs 1 and 2 as XYZ. For each element separately a local system of axes is defined. The local X-axis coincides with the length axis of the element. The local Y- and Z-axis must coincide with the principal axes of the cross-sectional area of the element.

PROPERTIES OF THE ELEMENTS

As already mentioned, the following characteristics of every element are to be known in order to compose the stiffness matrix of that element:

- the length and the cross-sectional area of the element,
- the second moments of area with respect to the local Y- and Z-axis,
- the effective second polar moment of area with respect to the local X-axis,
- the modulus of elasticity and the shear modulus of the material.

To calculate the second moments of area we can use auxiliary programs. Some examples of cross-sections whose characteristics are calculated with one of these programs are shown in Figs 3(a), (b), (c) and (d). In general, the shape of the cross-sections is obtained from the drawings of the machine tool. The auxiliary program used in this case, calculates the second

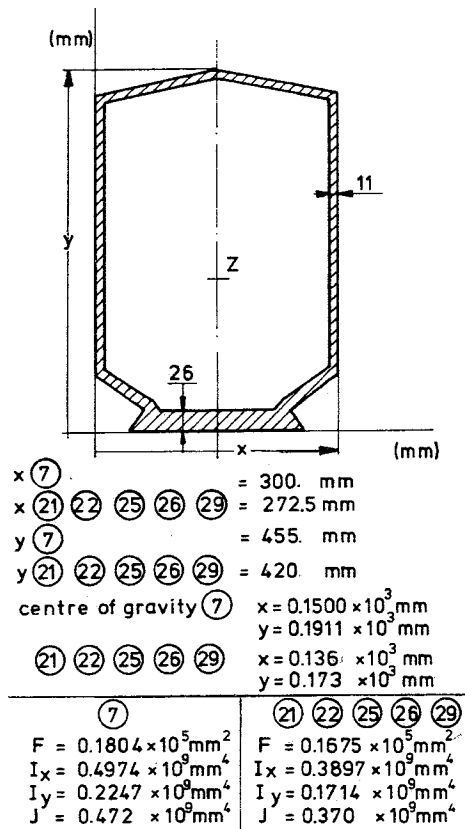


Figure 3(a). Cross-section of elements 7, 21, 22, 25, 26 and 29.

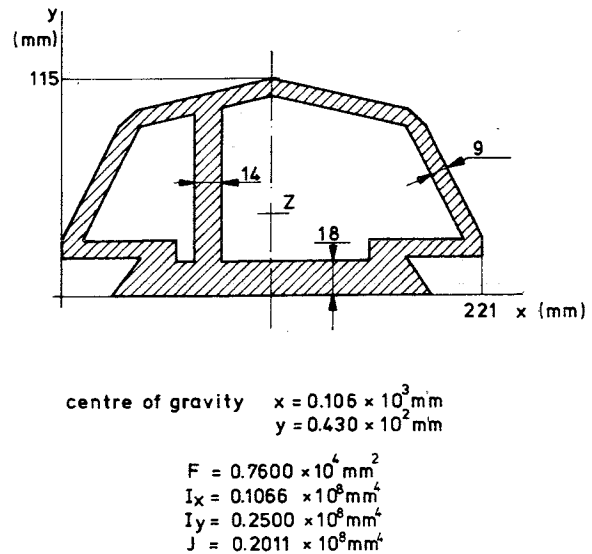
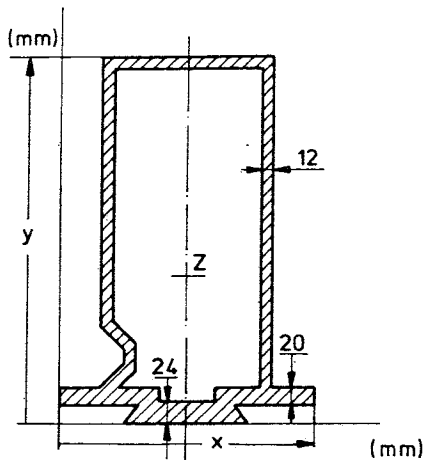


Figure 3(b). Cross-section of element 33.

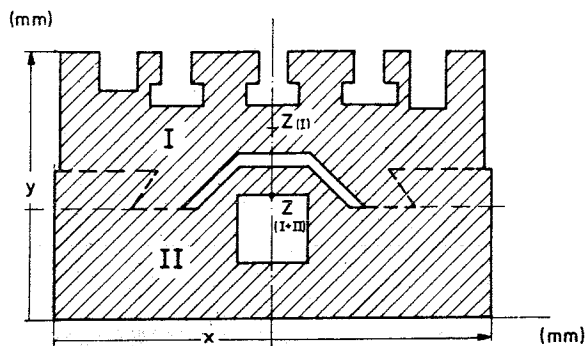
moments of area with respect to the principal axes of the cross-section. For this the cross-section is fed into the program by means of coordinates. For closed thin-walled cross-sections the polar moment of area is calculated according to Bredt's relation and for thin-walled open cross-sections De St Venant's relation is used. The output of the auxiliary program also gives the coordinates of the centre of gravity and the magnitude of the cross-sectional area.

Finally, for calculating the dynamic behaviour of the structure, the uniformly distributed mass per element and the lumped masses in the station points are to be known.



$x$ (10) = 245 mm	$y$ (10) = 420 mm
$x$ (11) = 245 mm	$y$ (11) = 310 mm
centre of gravity (10)	
$x = 0.1410 \times 10^3$ mm	$y = 0.1778 \times 10^3$ mm
" " " (11)	$x = 0.1405 \times 10^3$ mm
	$y = 0.1291 \times 10^3$ mm
$F$ (10) = $0.1727 \times 10^5$ mm <sup>2</sup>	$F$ (11) = $0.1463 \times 10^5$ mm <sup>2</sup>
$I_x = 0.3928 \times 10^9$ mm <sup>4</sup>	$I_x = 0.1835 \times 10^9$ mm <sup>4</sup>
$I_y = 0.9743 \times 10^9$ mm <sup>4</sup>	$I_y = 0.7646 \times 10^8$ mm <sup>4</sup>
$J = 0.2304 \times 10^9$ mm <sup>4</sup>	$J = 0.1523 \times 10^9$ mm <sup>4</sup>

Figure 3(c) Cross-section of elements 10 and 11.



$x$ I = 223 mm	$y$ I = 82 mm
$x$ I + II = 226 mm	$y$ I + II = 140 mm
centre of gravity I	
$x = 0.1117 \times 10^2$ mm	$y = 0.4322 \times 10^2$ mm
" " " I + II	$x = 0.1124 \times 10^2$ mm
	$y = 0.6467 \times 10^2$ mm
$F$ I = $0.1211 \times 10^5$ mm <sup>2</sup>	$F$ I + II = $0.2654 \times 10^5$ mm <sup>2</sup>
$I_x = 0.4739 \times 10^7$ mm <sup>4</sup>	$I_x = 0.4073 \times 10^8$ mm <sup>4</sup>
$I_y = 0.4832 \times 10^8$ mm <sup>4</sup>	$I_y = 0.1200 \times 10^9$ mm <sup>4</sup>
$J = 0.5316 \times 10^8$ mm <sup>4</sup>	$J = 0.7000 \times 10^8$ mm <sup>4</sup>

Figure 3(d). Cross-section of elements 14 (I + II), 15 (I + II), 19 (I) and 20 (I).

### THE STATIC RESULTS

First of all, the measurements of the milling machine in the laboratory are used as a feedback for the original model by comparing the results of the measurements with those of the calculations. Actually, this feedback was necessary in

two places. The first correction was carried out in the part where the column is connected with the basis of the machine-tool frame. In the first model this connection was considered to be stiff. However, the measurements showed relative large rotations of the column with respect to the basis. This is corrected by adapting the stiffness of the elements 1 and 4 in the basis of the frame, in a way that the calculated rotations are in agreement with the measurements. The second point of correction is element 12. In the beginning this element was also considered to be stiff because of the fact that during the measurements

TABLE 1 Examples of static loading

Loading case	Station point	Loading	Direction	
1	36	1000 N	+X	
2	36	1000 N	+Y	
3	17	1000 N	+X	
	22	1000 N	-X	
4	22	50 Nm	-Y	
	20	1000 N	+X	
	7	1000 N	-X	
	7	600 Nm	-Y	
	7	55 Nm	-Z	
5	20	1000 N	-Y	
	20	60 Nm	+Z	
	22	1000 N	+Y	
	22	50 Nm	-X	
	22	60 Nm	-Z	
6	17	1000 N	+Z	
	22	1000 N	-Z	
	7	20	1000 N	+Z
		7	1000 N	-Z
	7	55 Nm	+X	

the supports are fixed. However, it appeared that the table exhibited relative large displacements and rotations. The reason for this flexibility is the possibility to rotate the longitudinal carriage with respect to the cross carriage. These carriages are fixed with two bolts and this joint happened to be very flexible in horizontal direction. This effect is only partially taken into account in the latest model. Table 1 shows the static loading cases. The deflections of the station points caused by these loadings are listed in Table 2. In addition to this, Table 2 gives the deviation of the calculated deflection with respect to the measured value if the latter is larger than 5  $\mu$ m.

A detailed analysis of the results<sup>3</sup> shows that the causes of the deviations originate mainly from:

- the stiffnesses of the connection between column and basis of the structure,
- the stiffnesses of the connection of the carriages,
- the stiffnesses of the spindle-bearing system,
- the stiffnesses of the connection between over-arm and column.

Within the scope of this paper we accept the model and conclude from Table 2 that the average deviation is about 24 per cent.

TABLE 2 Deflections due to static loading

Loading case	Station point	Direction	Measured displacement $\mu\text{m}$	Calculated displacement $\mu\text{m}$	Deviation %
1	8	-X	45	35	22.2
	13	-X	90	78	13.3
	36	-X	130	99	23.8
	20	-X	120	88	26.6
2	8	+Y	19	19.8	4.2
	13	+Y	43	43.5	1.2
	36	+Y	54	54.5	0.9
3	2	-X	26	22.5	13.4
	13	+X	(3)	(3.1)	-
	14	+X	(4)	(4.1)	-
	34	+X	(3)	(0.7)	-
	35	+X	(4.5)	(6.6)	-
	36	+X	5.5	5.4	1.8
	17	+X	35	31	11.4
	20	+X	19	23	21.1
	39	+X	16	20	25
4	7	-X	20	3.1	84.5
	13	+X	(4)	(3.1)	-
	14	+X	(3.5)	(4.1)	-
	34	-X	(2)	(1.5)	-
	35	+X	8	8.3	3.8
	36	+X	7.5	5.4	28
	17	+X	21	23	9.5
	20	+X	53	48	9.4
	39	+X	41	38	7.3
5	35	-Y	(4.5)	(1.8)	-
	36	-Y	(4.0)	(2.4)	-
	39	-Y	5.0	6.2	24
6	39	-Z	20	14	30
	17	+Z	40	23	42.5
	20	+Z	19	12	36.8
7	20	+Y	(1)	(0.3)	-
	39	-Y	(1.5)	(1.6)	-
	7	-Z	8	1	87.5
	17	+Z	20	12	40
	20	+Z	60	36	40
	20	+Y	15	14	6.7
	39	-Y	(2.5)	(0.0)	-

THE DYNAMIC RESULTS

The natural frequencies with corresponding mode shapes are calculated with the aid of the latest model. To this end, the mass distribution per

element and per station point has to be introduced additionally in the model.

For a dynamic analysis of the milling machine it is often of importance to calculate the modal flexibilities between two points in several directions. In this respect the points 17 and 22 (see Fig. 2) deserve special consideration with respect to the X- and Z-direction. On the basis of the magnitude of these modal flexibilities it is possible to examine which modes contribute mainly to the dynamic transfer function.

In order to obtain a correct impression of the transfer function in a certain frequency range, it is necessary to introduce a value for the damping ratio  $\zeta$ . Especially at an initial stage of the analysis, it is proposed to introduce for every mode the same overall damping ratio. In our case, we chose for this arbitrary value  $\zeta = 0.03$ . After this, the transfer function can be calculated for a number of frequencies.

If the experimental transfer curves are available, it is possible to obtain from these the natural frequencies and the damping ratios of the several modes. With the aid of the modal flexibilities, the calculated natural frequencies can be matched with the experimental ones. In this way we can attach to a number of modes an experimental damping value, while the remaining modes can be suppressed by means of a large  $\zeta$ -value. Again the transfer function can be calculated.

The results of the analysis of the Jaspur milling machine are shown in Figs 4(a), (b) and (c). These figures show the result of the calculation of the receptances between point 17 (the tool) and 22 (the workpiece). The experimental values are also plotted in these figures. From the results we might conclude that the natural frequencies can be found with this analysis. Furthermore, the

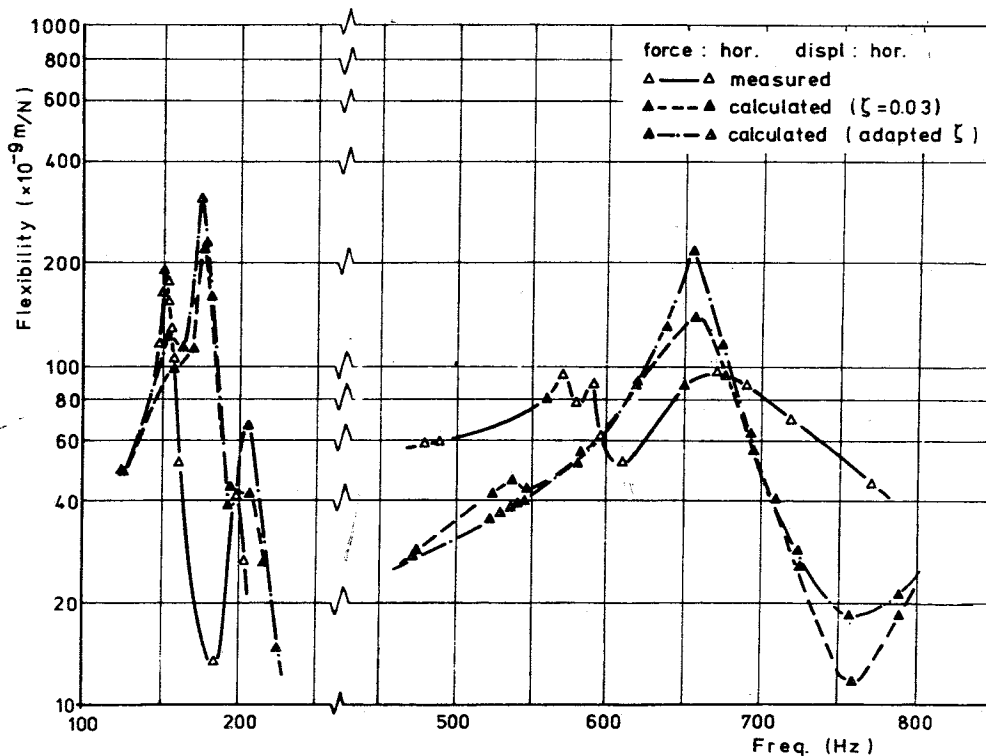


Figure 4(a). Direct receptance between points 17 (the tool) and 22 (the workpiece) for X-direction.

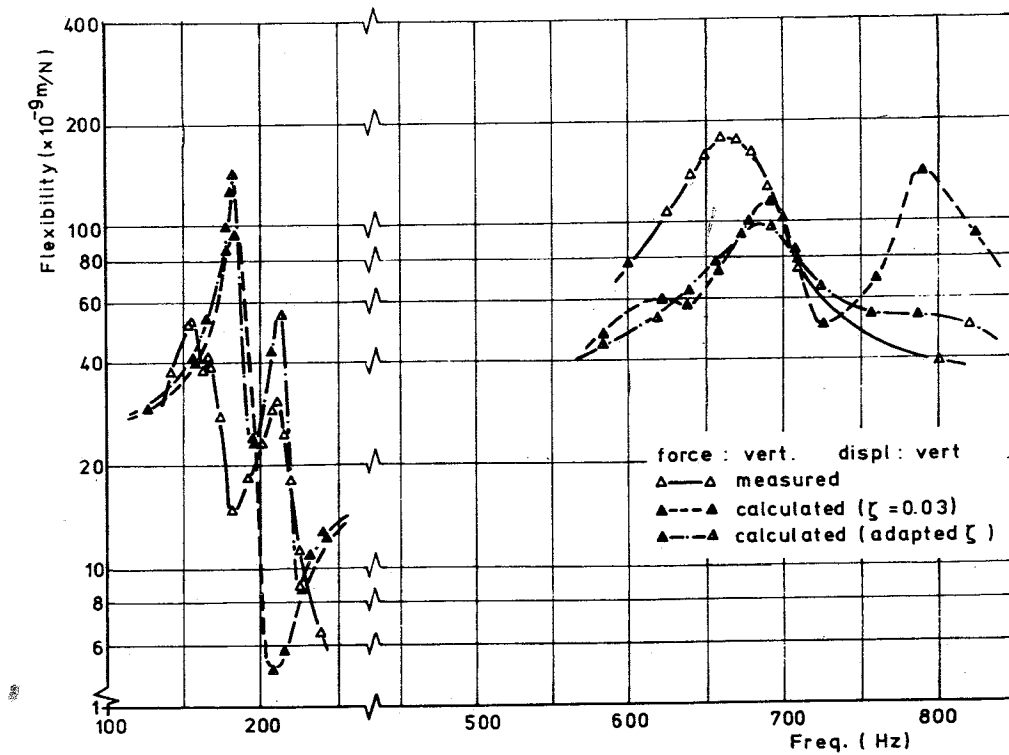


Figure 4(b). Direct receptance between points 17 (the tool) and 22 (the workpiece) for Z-direction.

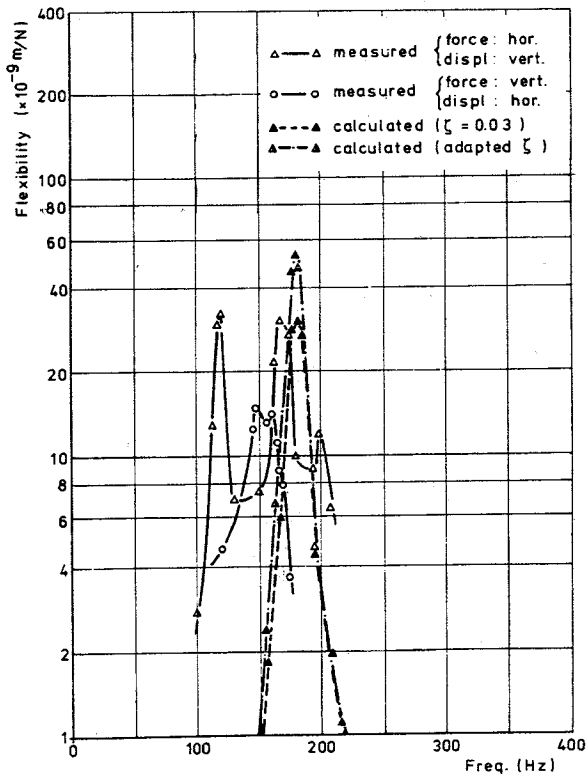


Figure 4(c). Cross receptance between points 17 (the tool) and 22 (the workpiece) for X- and Z-direction.

calculated values give a fair indication of the order of magnitude of all important modal flexibilities. Finally, it has to be remarked that carefully adapting the  $\zeta$ -value per mode has not so many advantages for this machine tool;

the introduction of an arbitrary value for the overall damping in the model seems to be equally satisfactory.

### CONCLUSIONS

In spite of the complicated nature of a machine tool structure such as a horizontal milling machine, it is possible to transform that structure into a relatively simple beam model which can be considered to be representative to a certain extent of that structure.

At this moment feedback to the model via measurements cannot be avoided and is essential in order to obtain more insight into the technique of modelling.

In order to give this computer aided design method more reliability in the design stage of machine tools, it is of vital importance to have more fundamental information concerning the stiffness and the damping of machine elements such as bolted joints, spindle-bearing systems etc. This information should be made available in such a numerical form, that those specific machine elements can be introduced into the model as elements with well defined stiffness and damping matrices.

### ACKNOWLEDGMENT

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## REFERENCES

1. J. D. JANSSEN, N. A. L. TOUWEN, F. E. VELDPAUS and A. C. H. van der WOLF (1970). Computation of a model milling machine, note presented to C.I.R.P. Group Ma, Tirrenia.
2. F. KOENIGSBERGER and J. TLUSTY (1970). *Machine Tool Structures*, vol. 1, Pergamon Press, 485.
3. P. R. M. VAN DIJK (1971). Analyse van een freesbankstructuur. Report WT 0282, University of Technology, Eindhoven.

## DISCUSSION

*Q.* G. J. McNulty, Sheffield Polytechnic. Why did the authors use a uni-model viscous damping, when the structure (especially at higher modes) will be predominantly hysteretic? Certainly the analysis is simplified using the former by accuracy may be sacrificed.

The authors stated that modal clamping was obtained by suppressing adjacent modes using a large damping coefficient. This would seem to neglect modal cross coupling: please comment.

*A.* Because the damping ratios in machine tools are relatively low ( $S = 0.03$  satisfies often), it does not matter whether you use viscous or hysteretic damping in order to calculate the frequency response. Furthermore, using Cowley's method for calculating the dynamic response (see ref. 2), only viscous damping can be applied.

Modal damping was not obtained by suppressing adjacent modes, but was calculated from experimental transfer functions. It is quite obvious that in this way only damping ratios can be attached to those modal shapes which are indeed experimentally found.

*Q.* S. Taylor, University of Birmingham. With reference to the previous speaker on damping: in the case of a lightly damped structure, the difference between computed response loci based on viscous and hysteretic damping is extremely small. Further, it is not possible to deduce from experimental measurements the nature of damping which is present in a machine tool. Consequently either assumption is satisfactory for purposes of computer aided design. This comment is valid when, as is usual in machine tools, the damping in any work is appreciably less than the critical damping.

*A.* No comment.

*Q.* F. M. Stansfield, MTIRA. What method was used for calculating the shear flexibility of the beam elements? MTIRA has experience of using a similar program and finds that it is important to estimate the shear flexibilities of beam elements carefully—they can be of the same order of magnitude as the bending flexibilities.

*A.* No shear flexibility, other than caused by torsional loading, was taken into account in this program. The authors feel that it is only necessary to introduce shear when using a fairly short element, loaded by two opposite shear forces, working at both ends of that particular element.