

Stuiken en pletten

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STUIKEN EN PLETTEN

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Stuiken en Pletten

JAH Ramaekers

W-WPT-Omvormen
maart 1982

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1. Cylinder stikken uniforme deformatie

1.1 Energie methode ($\sigma_v = \text{konst}$)

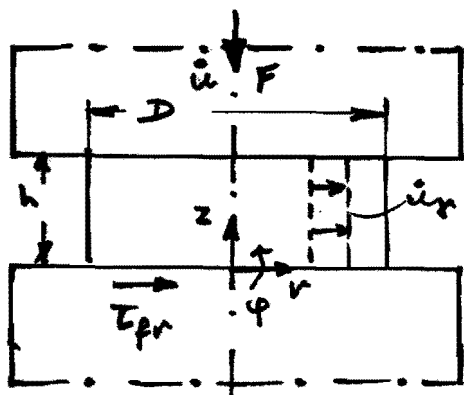


fig 1.1 stukproef rot. symm.

h - momentane hoogte
D - - - - - diameter

F - pers kracht
u-dot - stempelsnelheid

$\tau_{\phi,r} = m \frac{\sigma_v}{\sqrt{3}}$ wrijving

$\sigma_v = \text{konst}$, eg. $\sigma_v = C \bar{\epsilon}^n$

snelheidsveld : $\dot{u}_r = \frac{1}{2} \dot{u} \frac{r}{h}$ $\dot{u}_z = - \dot{u} \frac{z}{h}$ $\dot{u}_\phi = 0$

reksnelheden : $\dot{\epsilon}_r = \dot{u}_{r,r} = \frac{1}{2} \frac{\dot{u}}{h}$ $\dot{\epsilon}_\phi = \frac{\dot{u}_\phi}{r} = \frac{1}{2} \frac{\dot{u}}{h}$
 $\dot{\epsilon}_z = - \frac{\dot{u}}{h}$ $\dot{\bar{\epsilon}} = \frac{\dot{u}}{h}$ (uniform!)

def. vermogen : $P_D = \int_V \sigma_v \dot{\bar{\epsilon}} dV = \sigma_v \frac{\dot{u}}{h} \cdot \frac{\pi}{4} D^2 h = \frac{\pi}{4} D^2 \dot{u} \sigma_v$

wrijvingsverm : $P_{fr} = \int_S \tau_{\phi,r} \dot{u}_r dS = \frac{\pi}{4} D^2 \dot{u} \tau_{\phi,r} \frac{m}{\sqrt{3}} \frac{D}{h}$

energie balans : $F \cdot \dot{u} = \Sigma P$

$$F^* = \frac{F}{\sigma_v \frac{\pi}{4} D^2} = 1 + \frac{m}{3\sqrt{3}} \frac{D}{h}$$

(1.1)

1.2 Energie methode ($\sigma_v = C \bar{\epsilon}^n$)

eindige rek : $\bar{\epsilon} = \int \dot{\bar{\epsilon}} dt = \int_{h_0}^h - \frac{dh}{h} = \ln \frac{h_0}{h}$

vloei spanning : $\sigma_v = C \left(\ln \frac{h_0}{h} + \bar{\epsilon}_0 \right)^n$

kracht :

$$F^* = \frac{F}{C \frac{\pi}{4} D^2} = \left(\ln \frac{h_0}{h} + \bar{\epsilon}_0 \right)^n \left\{ 1 + \frac{1}{3} \frac{\tau_{\phi,r}}{\sigma_v} \frac{D}{h} \right\}$$

(1.2)

NB : m ≠ konstant

Met $s = h_0 - h =$ stempelweg kan de kracht-weg kromme uitgerekend worden (zie § 2.2 fig 2.3).

1.3 Schillenmethode ($\nu = \text{konst}$)

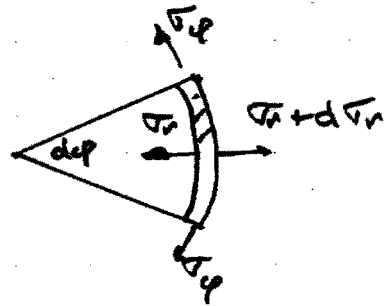
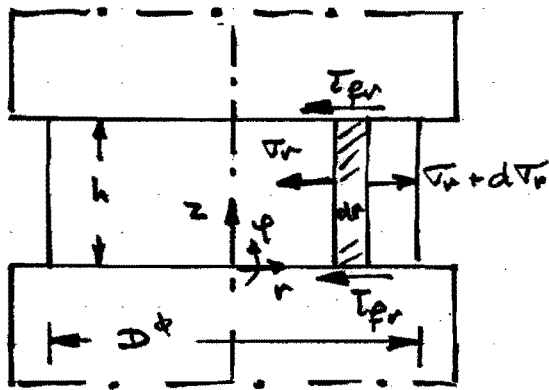


fig 1.2 stukproef rot. symm. met evenwicht op schil.

evenwicht: $-\sigma_r r d\phi h + (\sigma_r + d\sigma_r)(r + dr) d\phi h +$
 $- 2 \tau_{\phi r} r d\phi \cdot dr - 2 \sigma_{\phi} \frac{dr}{2} d\phi h = 0$

$\Leftrightarrow (\sigma_r - \sigma_{\phi}) dr h + d\sigma_r r h - 2 \tau_{\phi r} r dr = 0$

Met $\dot{\epsilon}_r = \dot{\epsilon}_{\phi} \Rightarrow \sigma_r = \sigma_{\phi} \Leftrightarrow d\sigma_r = 2 \tau_{\phi r} \frac{dr}{h}$

Met $\sigma_r(r=0) = 0$ volgt $\sigma_r = 2 \tau_{\phi r} \frac{r - D/2}{h}$

Met v. Mises $\sigma_v = \sigma_r - \sigma_z$ volgt:

Sturzbürg $\frac{\sigma_z}{\sigma_v} = - \left(1 + \frac{\mu}{1-\mu} \frac{D-2r}{h} \right)$ (1.3)

Met $F = \int_0^{D/2} |\sigma_z| 2\pi r dr$ en $F^* = \frac{F}{\nu \pi h D}$ volgt

$F^* = 1 + \frac{\mu}{3(1-\mu)} \frac{D}{h}$ (1.4)

zie (1.1) dit betekent dat binnen deze benadering
 f schil $\sigma_r \neq \sigma_r(z)$ en $\dot{\epsilon}_r \neq \dot{\epsilon}_r(z)$ -
 boven- en ondergrens oplossing
 tot hetzelfde resultaat leiden.
 Dus de "beste" zijn!

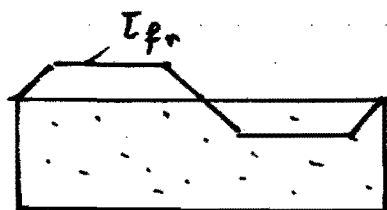


fig 1.3 $\tau_{\phi r} = \tau_{\phi r}(r)$ en drukber.

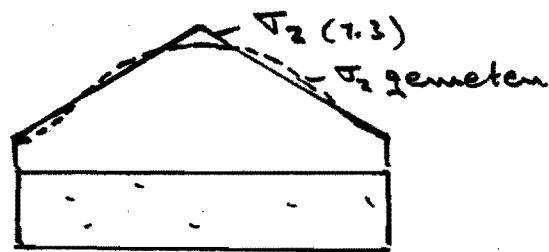


Fig 1.3 geeft juistere $\tau_{\phi r}$ verdeling
 echter vaak is dit een
 technisch niet zo relevant
 verfijning mbt. de belasting
 op het gereedschap

2. strip pletten (vlakke def. $\dot{\epsilon}_y = 0$)

2-1

2.1 Energimethode ($\sigma_v = \text{konst.}$)

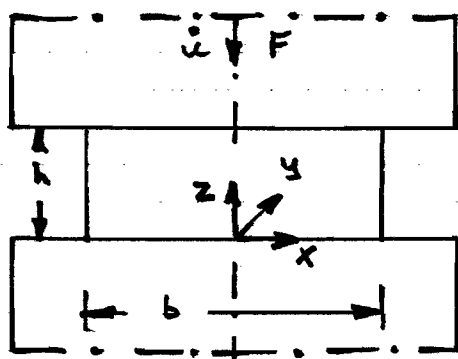


fig 2.1 strip pletten
h - momentane hoogte
b - - - - - breedte

De berekeningen worden uitgevoerd per eenheid van lengte l.

snelheidsveld: $\dot{\epsilon}_x = \dot{\epsilon} \frac{x}{h}$ $\dot{\epsilon}_y = 0$ $\dot{\epsilon}_z = -\dot{\epsilon} \frac{z}{h}$

reksnelheden: $\dot{\epsilon}_x = \dot{\epsilon}_{x,x} = \frac{\dot{\epsilon}}{h}$ $\dot{\epsilon}_y = 0$ $\dot{\epsilon}_z = \dot{\epsilon}_{z,z} = -\frac{\dot{\epsilon}}{h}$
 $\dot{\epsilon} = \frac{2}{\sqrt{2}} \frac{\dot{\epsilon}}{h}$

def. vermogen: $P_D = \frac{2}{\sqrt{2}} b l \dot{\epsilon} \sigma_v$

wrijvings verm.: $P_{fr} = b l \dot{\epsilon} \tau_v \frac{\mu}{2\sqrt{2}} \frac{b}{h}$

$$F^* = \frac{F}{b \cdot l \cdot \sigma_v} = \frac{2}{\sqrt{2}} \left(1 + \frac{\mu}{4} \frac{b}{h} \right)$$

(2.1)

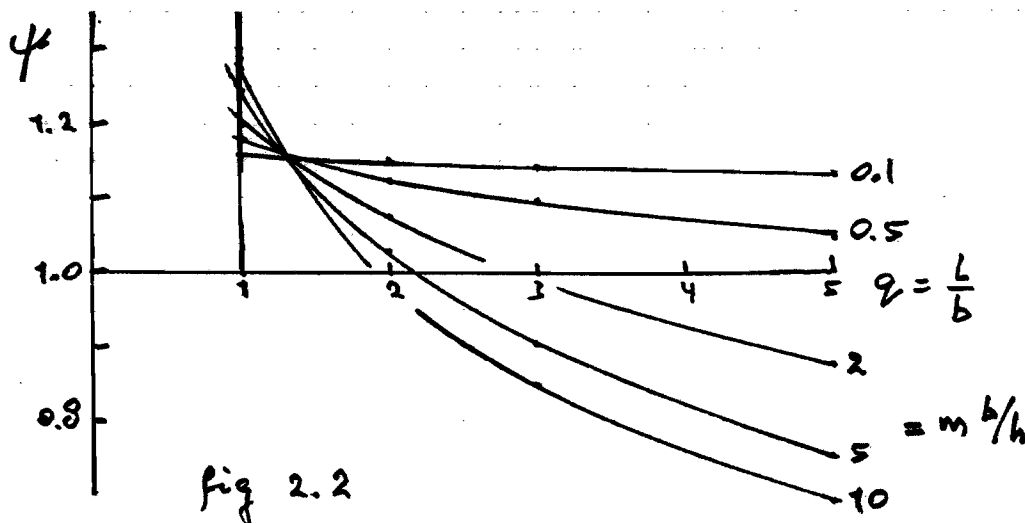
2.1.1. Rot. symm. \leftrightarrow vlak. verg (7.1) \leftrightarrow (2.1)

Als we uitgaan van 'n rechthoekige strip ($b \times h \times 2b$) met hetzelfde oppervlak als 'n ronde schijf

- dus $\frac{\pi}{4} D^2 = 2 b^2$ of $b = \frac{D}{2} \sqrt{\frac{\pi}{2}}$ -

volgt uit (1.1) en (2.1)

$$\psi = \frac{F^*(2.1)}{F^*(2.7)} = \frac{2}{\sqrt{2}} \frac{1 + \frac{\mu}{8} \frac{D}{h} \sqrt{\frac{\pi}{2}}}{1 + \frac{\mu}{3\sqrt{2}} \sqrt{\frac{\pi}{2}} \frac{b}{h}} = \frac{2}{\sqrt{2}} \frac{1 + \frac{\mu}{4} \frac{b}{h}}{1 + \frac{2\mu}{3\sqrt{2}} \sqrt{\frac{\pi}{2}} \frac{b}{h}}$$



dus vlak is vaak goede benadering

NB.
zie
blz 4.3

fig 2.2

2.2 Energiemethode $\sigma_v = C \bar{\epsilon}^n$

2-2

eindige rek: $\bar{\epsilon} = \frac{2}{\sqrt{3}} \ln \frac{h_0}{h}$

vloei spanning: $\sigma_v = C \left(\frac{2}{\sqrt{3}} \ln \frac{h_0}{h} + \bar{\epsilon}_0 \right)^n$

kracht:

$$F^* = \left(\frac{2}{\sqrt{3}} \ln \frac{h_0}{h} + \bar{\epsilon}_0 \right)^n \left\{ 1 + \frac{\sqrt{3}}{4} \frac{\tau_{fr}}{\sigma_v} \frac{b}{h} \right\} \quad (2.2)$$

(fig 2.3)

2.3 Schillenmethode ($\sigma_v = \text{konst}$)

evenwicht analoog § 1.3:

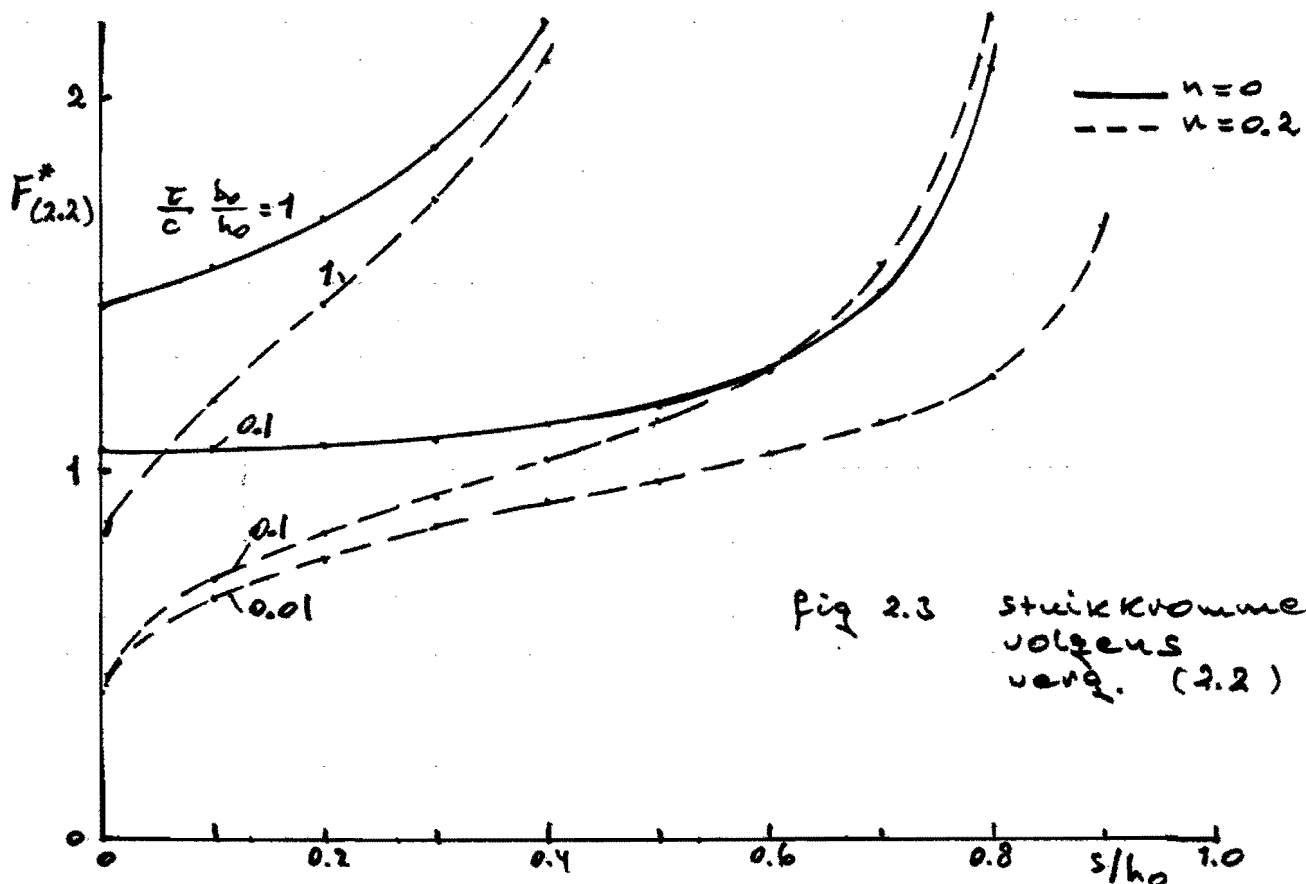
$$-\sigma_x h L + (\sigma_x + d\sigma_x) h L - 2 \tau_{fr} dx L = 0$$

$$d\sigma_x = 2 \tau_{fr} \frac{dx}{h} \quad \text{met } \sigma_x (x=b/2) = 0 \rightarrow \sigma_x = +2 \tau_{fr} \frac{x-b/2}{h}$$

$$\dot{\epsilon}_y = 0 \rightarrow \sigma_y = \frac{\sigma_x + \sigma_z}{2} \rightarrow \sigma_v = \frac{\sqrt{3}}{2} (\sigma_x - \sigma_z)$$

$$\frac{\sigma_z}{\sigma_v} = - \frac{2}{\sqrt{3}} \left(1 + m \frac{b/2 - x}{h} \right) \quad (2.3)$$

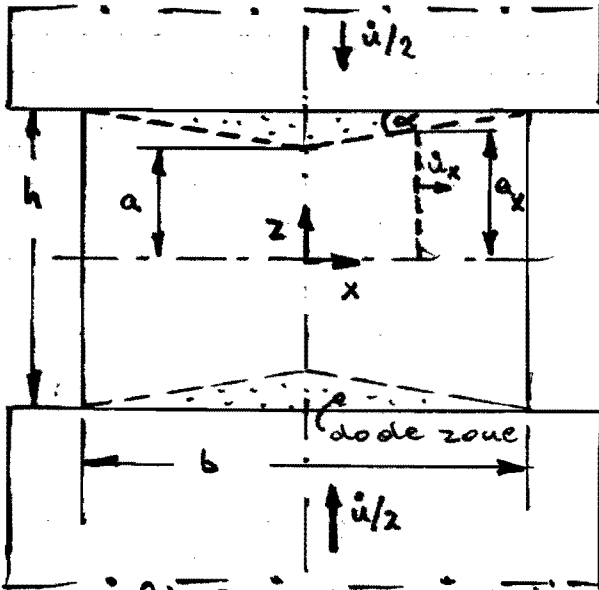
$$\Rightarrow F^* = \frac{2}{\sqrt{3}} \left(1 + \frac{m}{4} \frac{b}{h} \right) \quad (2.4)$$



3. strip pletten met dode zone

3.1 Energie methode ($\sigma_v = \text{konst}$)

$\dot{\epsilon}_y = 0; \dot{\epsilon}_y = 0$



$a_x = a + x \operatorname{tg} \alpha$

$a = \frac{h}{2} - \frac{b}{2} \operatorname{tg} \alpha$

snelheden.

Vol. Inv. $\dot{u}_x \cdot a_x = \frac{\dot{u}}{2} x$

$\dot{u}_x = \frac{1}{2} \dot{u} \frac{x}{a + x \operatorname{tg} \alpha}$ (3.1)

$\dot{\epsilon}_x = -\dot{\epsilon}_z = \frac{a}{a_x^2} \cdot \frac{\dot{u}}{2}$

$\dot{u}_z = -\frac{1}{2} \dot{u} z \frac{a}{a_x^2}$ (3.2)

Fig 3.1 pletten (dode zone)

$\dot{\epsilon}_{zx} = \frac{1}{2} (\dot{u}_{x,z} + \dot{u}_{z,x}) = \frac{1}{2} (-\frac{1}{2} \dot{u} z a) (-2) \frac{1}{a_x^2} \operatorname{tg} \alpha$

$\dot{\epsilon}_{zx} = \frac{1}{2} \dot{u} z \operatorname{tg} \alpha \frac{a}{a_x^2}$

$\dot{\epsilon} = \sqrt{\frac{4}{3} (\dot{\epsilon}_x^2 + \dot{\epsilon}_{zx}^2)} = \frac{\dot{u}}{\sqrt{3}} \frac{a}{a_x^2} \sqrt{1 + \left(\frac{z \operatorname{tg} \alpha}{a_x}\right)^2}$ (3.3)

Def. vermogen $P_D = 4 \int_V \sigma_v \dot{\epsilon} dV = 4 \sigma_v \int_0^{b/2} \frac{\dot{u}}{\sqrt{3}} \frac{a}{a_x^2} dx \int_0^{a_x} \sqrt{1 + \left(\frac{z \operatorname{tg} \alpha}{a_x}\right)^2} dz$

$P_D = \frac{2}{\sqrt{3}} \sigma_v \dot{u} a \int_0^{b/2} \frac{dx}{a_x} \left\{ \sqrt{\operatorname{tg}^2 \alpha x^2 + 1} + \frac{1}{\operatorname{tg} \alpha} \ln (\operatorname{tg} \alpha + \sqrt{1 + \operatorname{tg}^2 \alpha}) \right\}$

$P_D = \frac{2}{\sqrt{3}} \sigma_v \dot{u} \frac{(h - b \operatorname{tg} \alpha)}{\operatorname{tg}^2 \alpha} \left\{ \operatorname{tg} \alpha \sqrt{1 + \operatorname{tg}^2 \alpha} + \ln (\operatorname{tg} \alpha + \sqrt{1 + \operatorname{tg}^2 \alpha}) \right\} x$

afschuifvermogen.

$P_r = 4 \int_S \frac{\sigma_v}{\sqrt{3}} |\dot{u}_x| dS$

$P_r = \frac{4}{\sqrt{3}} \sigma_v \int_0^{b/2} \frac{\dot{u}_x}{\cos \alpha} \cdot \frac{dx}{\cos \alpha} = \frac{4}{\sqrt{3}} \sigma_v \frac{1}{2} \dot{u} \frac{1}{\cos^2 \alpha} \int_0^{b/2} \frac{x dx}{a + x \operatorname{tg} \alpha}$

$P_r = \frac{2}{\sqrt{3}} \sigma_v \dot{u} \frac{1}{\cos^2 \alpha} \cdot \frac{1}{\operatorname{tg}^2 \alpha} \left\{ \frac{b}{2} \operatorname{tg} \alpha - \frac{1}{2} (h - b \operatorname{tg} \alpha) \ln \frac{h}{h - b \operatorname{tg} \alpha} \right\}$

$P_r = \frac{1}{\sqrt{3}} \sigma_v \dot{u} \frac{1}{\sin^2 \alpha} \left\{ b \operatorname{tg} \alpha - (h - b \operatorname{tg} \alpha) \ln \frac{h}{h - b \operatorname{tg} \alpha} \right\}$ (3.5)

$\left(* \ln \frac{h}{h - b \operatorname{tg} \alpha} \right)$ (3.4)

$$\frac{\Sigma P}{\sigma_c b} = P^* =$$

$$= \frac{1}{\sqrt{3}} \left[\frac{h/b - \tan \alpha}{\tan^2 \alpha} \left\{ \tan \alpha \sqrt{1 + \tan^2 \alpha} + \ln(\tan \alpha + \sqrt{1 + \tan^2 \alpha}) \right\} \ln \frac{h/b}{h/b - \tan \alpha} \right.$$

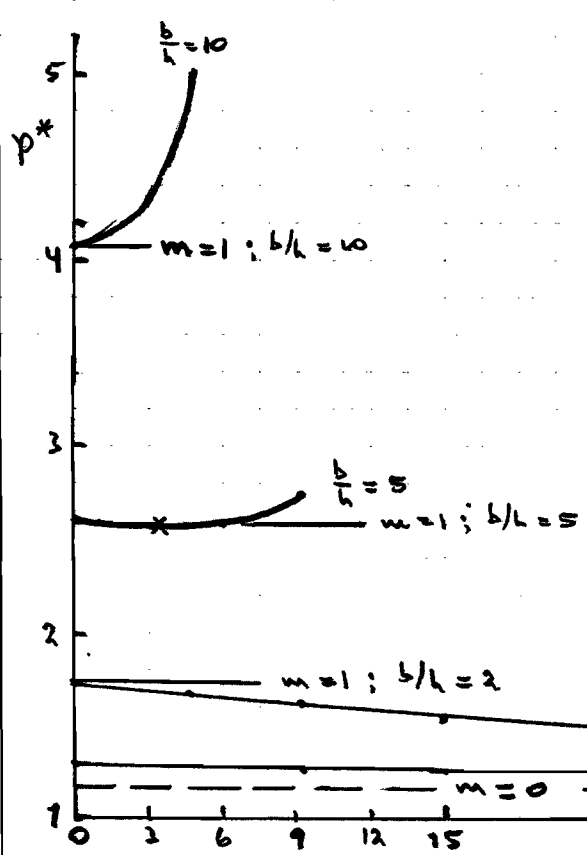
$$\left. + \frac{\gamma}{\sin^2 \alpha} \left\{ \tan \alpha - \left(\frac{h}{b} - \tan \alpha \right) \ln \frac{h/b}{h/b - \tan \alpha} \right\} \right] \quad (3.6)$$

$\frac{b}{h}$	1/2	1	2	5	10
0.01					4.042
0.1	1.299	1.443	1.731	2.597	4.048
1	1.295	1.435		2.597	4.120
2				2.585	4.226
3				2.582	4.337
4					4.607
5	1.278	1.404	1.668	2.586	5.017
6	1.261	1.369	1.608	2.741	
15	1.249	1.337	1.557	6	
20	1.240	1.309	1.496	2.594	
25	1.2349	1.283	1.447		
30	1.2348	1.259	26		
35	1.2402	1.236	1.441		
40	1.252	1.200			
45	1.273	1.175			
50	1.305				

$$P^*_{(m)} = \frac{2}{\sqrt{3}} \left(1 + \frac{m}{4} \frac{b}{h} \right)$$

$\frac{m}{b/h}$	0	0.2	0.4	1
1/2	1.755	1.184	1.213	1.999
1	-	1.213	1.271	1.444
2	-	1.271	1.386	1.733
5	-	1.444	1.733	2.599
10	-	1.733	2.31	4.043

NB. $\tan \alpha < \frac{b}{h}$
 met smering m
 dan $\alpha = 0$



Conclusie:
 als gecsmmeerd wordt $m < 0.4$
 is het ontstaan van 'n
 dode zone zeer onwaar-
 schijnlijk.
 Alleen voor $m \approx 1$ en $b/h < 2$
 kan 'n dode zone ontstaan.

fig 3.2. $P^*(\alpha, b/h, m)$
 verg. (3.6)

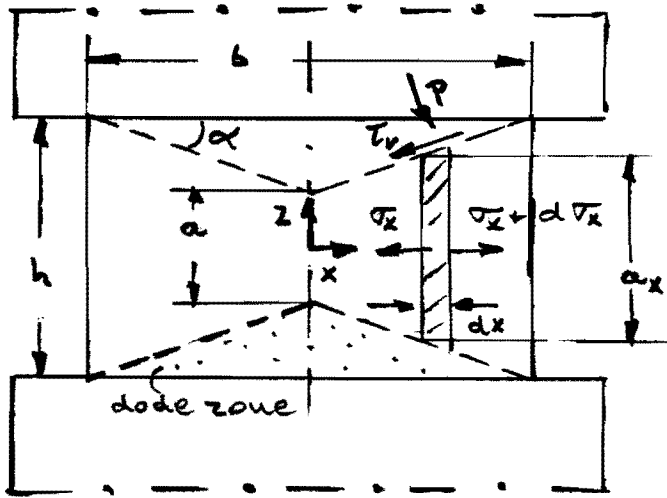


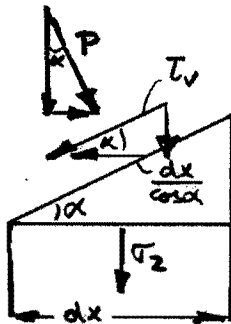
fig 3.3 pletten met dode zone (α) schillenmethode

$$a_x = a + 2x \operatorname{tg} \alpha$$

$$da_x = 2 \operatorname{tg} \alpha \, dx$$

$$\dot{\epsilon}_y = 0 \quad \sigma_y = \frac{\sigma_x + \sigma_z}{2}$$

$$\sigma_x - \sigma_z = \frac{2}{\sqrt{3}} \tau_v$$



horizontaal evenwicht:

$$\begin{aligned}
 & -\sigma_x a_x + (\sigma_x + d\sigma_x) (a_x + da_x) - 2\tau_v \frac{dx}{\cos \alpha} \cos \alpha + 2P \frac{dx}{\cos \alpha} \sin \alpha = 0 \\
 & \downarrow \\
 & \sigma_x da_x + d\sigma_x a_x - \tau_v \frac{da_x}{\operatorname{tg} \alpha} + P da_x = 0
 \end{aligned}$$

vertikaal evenwicht: $\sigma_z dx + P \frac{dx}{\cos \alpha} \cos \alpha + \tau_v \frac{dx}{\cos \alpha} \sin \alpha = 0$

met de vloeivoorwaarde volgt: $P = \frac{2}{\sqrt{3}} \tau_v - \tau_v - \tau_v \operatorname{tg} \alpha$

subst.: $d\sigma_x a_x - \tau_v \frac{da_x}{\operatorname{tg} \alpha} + (\frac{2}{\sqrt{3}} \tau_v - \tau_v \operatorname{tg} \alpha) da_x = 0$

$\tau_v = \tau_v / \sqrt{3} \quad (m=1)$

$$d\sigma_x = \frac{\tau_v}{\sqrt{3}} \left(\operatorname{tg} \alpha + \frac{1}{\operatorname{tg} \alpha} - 2 \right) \frac{da_x}{a_x}$$

met $\sigma_x(a_x=h) = 0$ volgt

$$\boxed{\sigma_x / \tau_v = \frac{1}{\sqrt{3}} \left(\operatorname{tg} \alpha + \frac{1}{\operatorname{tg} \alpha} - 2 \right) \ln \frac{a_x}{h}} \quad (3.7)$$

$$\frac{\sigma_z}{\tau_v} = - \frac{1}{\sqrt{3}} \left[2 + \left(\operatorname{tg} \alpha + \frac{1}{\operatorname{tg} \alpha} - 2 \right) \ln \frac{h}{a + 2x \operatorname{tg} \alpha} \right] \quad (3.8)$$

Met $P^* = F^* = \frac{F}{\sigma_v b}$ en $F = 2 \int_0^{b/2} |\sigma_z| dx$ volgt

$$\boxed{F^* = \frac{2}{\sqrt{3}} \left[1 + \frac{1}{2} \left(\operatorname{tg} \alpha + \frac{1}{\operatorname{tg} \alpha} - 2 \right) \left\{ 1 + \left(\frac{h}{b \operatorname{tg} \alpha} - 1 \right) \ln \left(1 - \frac{b}{h \operatorname{tg} \alpha} \right) \right\} \right]} \quad (3.9)$$

$\frac{b}{h}$ \ α	1/2	1	2	5	10
0.01	1.299	1.444	1.732	2.599	4.043
1	1.295	1.425	1.719	2.591	4.12
2	1.286	1.419	1.693	2.582	4.378
6	1.273	1.395	1.654	2.573	4.607
9	1.260	1.371	1.616	2.672	
15	1.236	1.326	1.542		
21	1.214	1.282	1.474		
30	1.184				
$\frac{m}{0.4}$ →	1.213	1.271	1.386	1.733	2.31

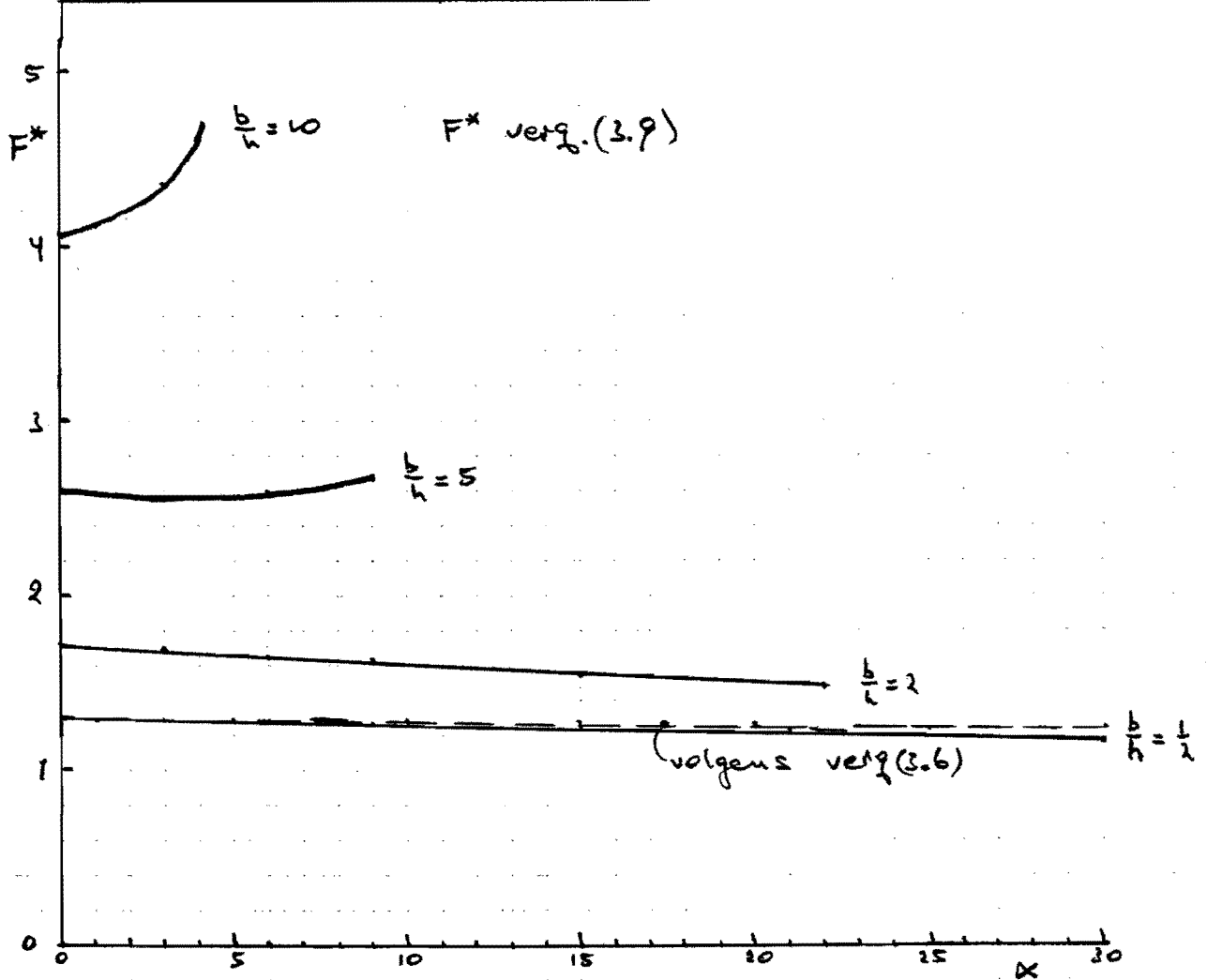


fig 3.3 F^* (3.9)

conclusie: bovengrens (3.6) en schillen (3.9) leveren praktisch dezelfde uitkomst

- schillen is dus hier geen echte ondergrensmethode
- conclusie bij fig 3.2 wordt hier bevestigd.

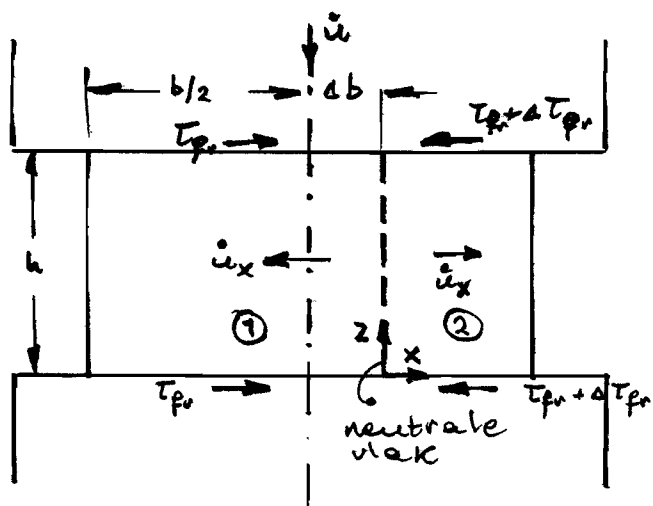
4. strip pletten met verstoring4.1 verstoring Δm , energiemethode ($\dot{u}_y = 0$)

fig 4.1

strip pletten met verstoring $\Delta T_{fr}(\Delta m)$ en verschuiving van het neutrale vlak $-\Delta b$.

snelheden: $\dot{u}_x = \dot{u} \frac{x}{h}$

$$\dot{\epsilon}_x = \frac{\dot{u}}{h} \quad \dot{\epsilon}_z = -\frac{\dot{u}}{h}$$

$$\dot{\epsilon} = \frac{2}{\sqrt{3}} \frac{\dot{u}}{h}$$

deformatie vermogen in ①: $P_{D1} = \sigma_v \frac{2}{\sqrt{3}} \frac{\dot{u}}{h} \cdot \left(\frac{b}{2} + \Delta b\right) h$.

deformatie vermogen in ②: $P_{D2} = \sigma_v \frac{2}{\sqrt{3}} \frac{\dot{u}}{h} \left(\frac{b}{2} - \Delta b\right) h$.

wrijvingsvermogen in ①: $P_{f1} = \frac{m}{\sqrt{3}} \sigma_v \frac{\dot{u}}{h} \left(\frac{b}{2} + \Delta b\right)^2$

wrijvingsvermogen in ②: $P_{f2} = \frac{m + \Delta m}{\sqrt{3}} \sigma_v \frac{\dot{u}}{h} \left(\frac{b}{2} - \Delta b\right)^2$

$$P^* = \frac{\Sigma P}{\sigma_v \dot{u} b} = \frac{2}{\sqrt{3}} \left\{ \frac{b/2 + \Delta b}{b} + \frac{b/2 - \Delta b}{b} + \frac{m}{2} \frac{1}{bh} \left(\frac{b}{2} + \Delta b\right)^2 + \frac{m + \Delta m}{2} \frac{(b/2 - \Delta b)^2}{bh} \right\}$$

$$P^* = \frac{2}{\sqrt{3}} \left\{ 1 + \frac{m}{2} \frac{b}{h} \left(\frac{1}{2} + 2\left(\frac{\Delta b}{b}\right)^2\right) + \frac{\Delta m}{2} \frac{b}{h} \left(\frac{1}{2} - \frac{\Delta b}{b}\right)^2 \right\}$$

Met $\frac{\partial P^*}{\partial \Delta b/b} = 0$ volgt:
$$\boxed{\frac{\Delta b}{b} = \frac{1}{2} \frac{\Delta m}{2m + \Delta m}} \quad (4.1)$$

4.2 verstoring Δm , schillenmethode

Analoog § 1.1 wordt voor $x \geq 0$ afgeleid:

$$d\sigma_x = 2 (T_{fr} + \Delta T_{fr}) \frac{dx}{h} = \frac{2}{\sqrt{3}} \sigma_v (m + \Delta m) \frac{dx}{h}$$

met $\sigma_x(x = \frac{b}{2} - \Delta b) = 0$ volgt: $\sigma_x = \frac{2}{\sqrt{3}} \sigma_v (m + \Delta m) \frac{x - (\frac{b}{2} - \Delta b)}{h}$

analoog voor $x \leq 0$: $\sigma_x(x \leq 0) = -\frac{2}{\sqrt{3}} \sigma_v m \frac{x + (\frac{b}{2} + \Delta b)}{h}$

uit de evenwichtsvoorwaarde voor $x=0$ volgt:

$$\boxed{\frac{\Delta b}{b} = \frac{1}{2} \frac{\Delta m}{2m + \Delta m}}$$

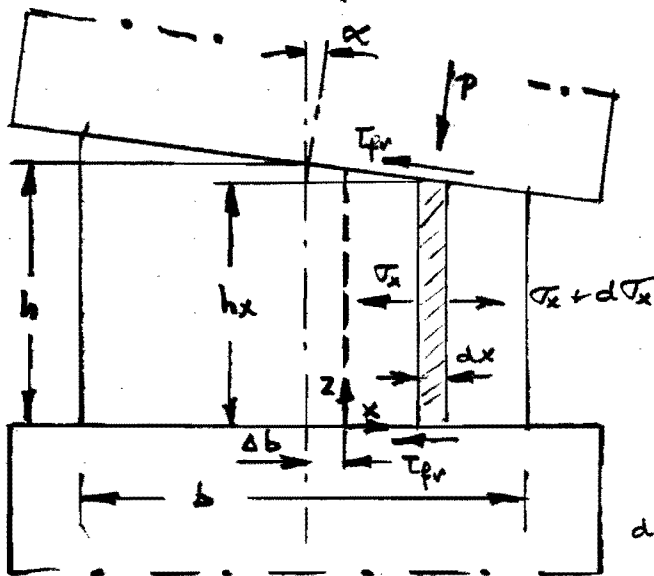
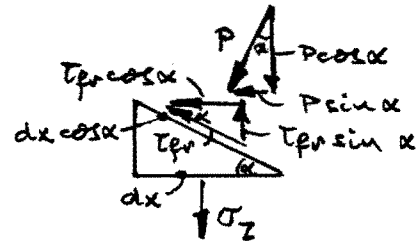


fig 4.2 Pletten met verstoring α (Δh) en verschuiving v.d. neutrale laag Δb .

met:
 $h_x = h - (x + \Delta b) \operatorname{tg} \alpha$
 $dh_x = -\operatorname{tg} \alpha \, dx$



horizontaal evenwicht ($x \geq 0$):

$$-\sigma_x h_x + (\sigma_x + d\sigma_x)(h_x + dh_x) - T_{\perp} dx - T_{\perp} \cos \alpha \frac{dx}{\cos \alpha} + P \sin \alpha \frac{dx}{\cos \alpha} = 0$$

$$d\sigma_x h_x + \sigma_x dh_x + 2T_{\perp} \frac{dh_x}{\operatorname{tg} \alpha} + P dh_x = 0 \quad (4.1)$$

$$i_y = 0 \rightarrow \dot{i}_y = 0 \rightarrow \sigma_y = \frac{\sigma_x + \sigma_z}{2} \rightarrow \sigma_x - \sigma_z = \frac{2}{3} \sigma_v \quad (4.2)$$

vertikaal evenwicht ($x \geq 0$):

$$\sigma_z dx + P \cos \alpha \frac{dx}{\cos \alpha} + T_{\perp} \sin \alpha \frac{dx}{\cos \alpha} = 0 \rightarrow P = -\sigma_z + T_{\perp} \operatorname{tg} \alpha$$

$$P = \frac{2}{3} \sigma_v - \sigma_x + T_{\perp} \operatorname{tg} \alpha \quad (4.3)$$

subst. in (4.1) geeft:

$$d\sigma_x h_x + \sigma_x dh_x + 2T_{\perp} \frac{dh_x}{\operatorname{tg} \alpha} + \left(\frac{2}{3} \sigma_v - \sigma_x + T_{\perp} \operatorname{tg} \alpha \right) dh_x = 0$$

$$d\sigma_x = -\frac{2}{3} \sigma_v \left(\frac{m}{\operatorname{tg} \alpha} + 1 + \frac{m}{2} \operatorname{tg} \alpha \right) \frac{dh_x}{h_x} \quad (4.4)$$

Met $\sigma_x(x = b/2 - \Delta b) = 0$ volgt:

$$\sigma_x(x \geq 0) = -\frac{2}{3} \sigma_v \left(1 + \frac{m}{\operatorname{tg} \alpha} + \frac{m}{2} \operatorname{tg} \alpha \right) \ln \frac{h_x}{h - b/2 \operatorname{tg} \alpha} \quad (4.5)$$

voor $x \leq 0$ kan analoog afgeleid worden:

$$\sigma_x(x \leq 0) = -\frac{2}{3} \sigma_v \left(1 - \frac{m}{\operatorname{tg} \alpha} - \frac{m}{2} \operatorname{tg} \alpha \right) \ln \frac{h_x}{h + b/2 \operatorname{tg} \alpha} \quad (4.6)$$

uit evenwicht voor $x=0$ volgt met

4-3

(4.5) en (4.6) :

$$\left(1 + \frac{m}{\tan \alpha} + \frac{m}{2} \tan \alpha\right) \ln \frac{h - \Delta b \tan \alpha}{h - b/2 + \tan \alpha} = \left(1 - \frac{m}{\tan \alpha} - \frac{m}{2} \tan \alpha\right) \ln \frac{h - \Delta b \tan \alpha}{h + b/2 + \tan \alpha}$$

voor $\alpha \ll 1$ volgt

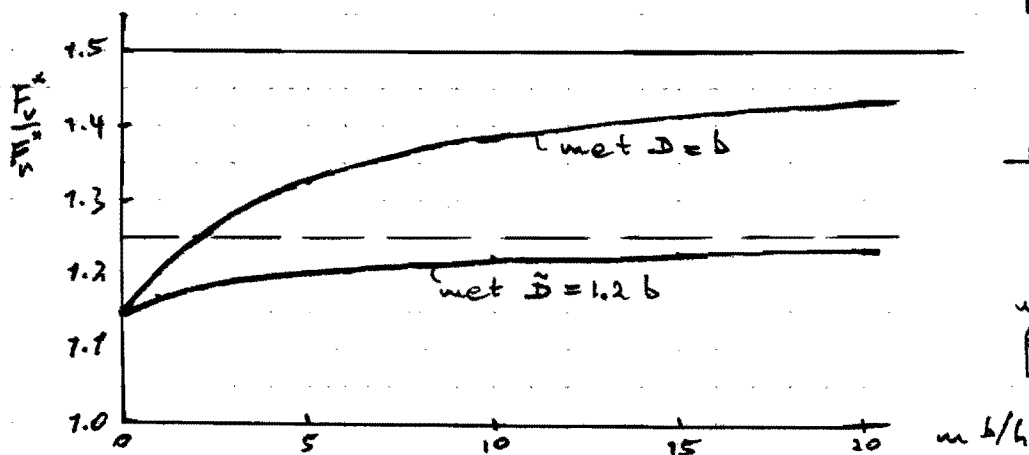
$$\boxed{\frac{\Delta b}{b} = \frac{1}{2} \frac{\alpha}{m}}$$

(4.7)

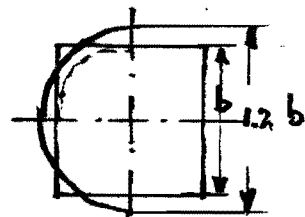
vervolg § 2.1.1 blz 2-1

Een vierkant profiel kunnen we als rot. sym. of als vlak benaderen. Met als afmetingen $b \times b \times h$ volgt uit (1.1) resp (2.1) :

$$\frac{F_{\text{vlak}}^*}{F_{\text{rond}}^*} = \frac{2/\sqrt{3} (1 + m/4 \cdot b/h)}{1 + m/3\sqrt{3} \cdot b/h} = 1.155 \div 1.5$$



Met $\tilde{D} = b \cdot 1,2$



wordt de max. fout 1.25

Bij een proefstuk met $L \times b \times h$ en $L \gg b$ "loopt" het materiaal langs de kortste weg naar de rand, weg van de minste weerstand. Een goede benadering is dan vlakke def.