Stuiken en pletten

STUIKEN EN PLETTEN

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Stuiken en Pletten

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W-WPT-Omvormen
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Inhoud.

Literatuur

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[2] B. Avitzur: "Metalforming" The application of the limit analysis
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"Instable Materialflow in Extrusion and Upsetting"
WT 0530

1. Cylinder stuiken , uniforme deformatie

1.1 Energie methode. (σ_v = konst)

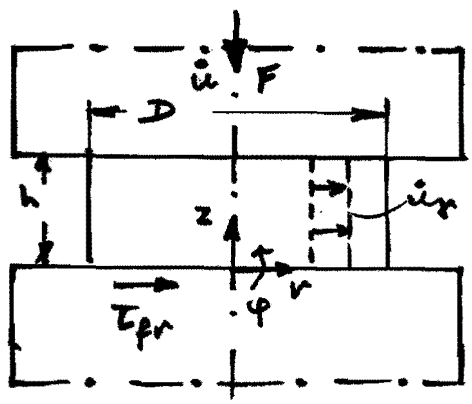


fig 1.1 stuikproef rot. symm.

h - momentane hoogte
D - " " diameter

F - perskracht
$\dot{u}$ - stempelsnelheid

$\tau_{fr} = m \frac{\sigma_v}{\sqrt{3}}$ wrijving

σ_v = konst. eq. $\sigma_v = C\bar{\varepsilon}^n$

snelheidsveld : $\dot{u}_r = \frac{1}{2}\dot{u}\frac{r}{h}$ $\quad \dot{u}_z = -\dot{u}\frac{z}{h}$ $\quad \dot{u}_\varphi = 0$

reksnelheden : $\dot{\varepsilon}_r = \dot{u}_{r,r} = \frac{1}{2}\frac{\dot{u}}{h}$ $\quad \dot{\varepsilon}_\varphi = \frac{\dot{u}_r}{r} = \frac{1}{2}\frac{\dot{u}}{h}$

$\dot{\varepsilon}_z = -\frac{\dot{u}}{h}$ $\quad \dot{\bar{\varepsilon}} = \frac{\dot{u}}{h}$ (uniform!)

def. vermogen: $P_D = \int_V \sigma_v \dot{\bar{\varepsilon}}\, dV = \sigma_v \frac{\dot{u}}{h} \cdot \frac{\pi}{4} D^2 h = \frac{\pi}{4} D^2 \dot{u}\, \sigma_v$

wrijvingsverm: $P_{fr} = \int_S \tau_{fr}\, \dot{u}_r\, dS = \frac{\pi}{4} D^2 \dot{u}\, \sigma_v \frac{m}{3\sqrt{3}} \frac{D}{h}$

energie balans: $F.\dot{u} = \Sigma P$

$$\boxed{F^* = \frac{F}{\sigma_v \frac{\pi}{4} D^2} = 1 + \frac{m}{3\sqrt{3}} \frac{D}{h}} \qquad (1.1)$$

1.2 Energie methode ($\sigma_v = C\bar{\varepsilon}^n$)

eindige rek : $\bar{\varepsilon} = \int \dot{\bar{\varepsilon}}\, dt = \int_{h_0}^{h} -\frac{dh}{h} = \ln \frac{h_0}{h}$

vloeispanning : $\sigma_v = C\left(\ln \frac{h_0}{h} + \bar{\varepsilon}_0\right)^n$

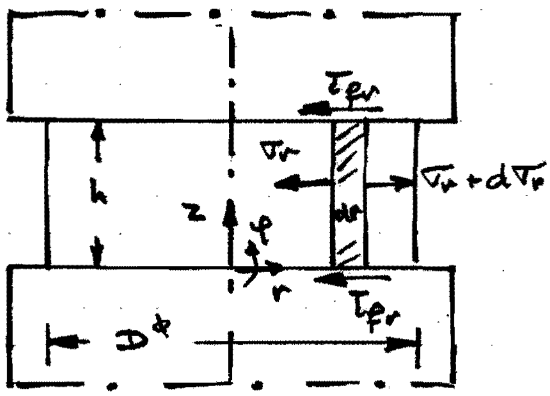
Kracht:

$$\boxed{F^* = \frac{F}{C\frac{\pi}{4}D^2} = \left(\ln\frac{h_0}{h} + \bar{\varepsilon}_0\right)^n \left\{1 + \frac{1}{3}\frac{\tau_{fr}}{\sigma_v}\frac{D}{h}\right\}} \qquad (1.2)$$

NB : $m \neq$ konstant

Met $s = h_0 - h$ = stempelweg kan de kracht-weg kromme uitgerekend worden (zie § 2.2 fig 2.3).

1.3 Schillenmethode (σ_v = konst)



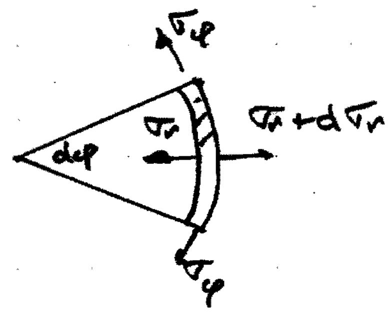


fig 1.2 stuikproef rot. symm. met evenwicht op schil.

evenwicht : $-\sigma_r r d\varphi h + (\sigma_r + d\sigma_r)(r+dr)\, d\varphi\, h +$

$$- 2\, \tau_{fr}\, r d\varphi . dr - 2\, \sigma_\varphi \frac{d\varphi}{2}\, dr\, h = 0$$

$$\Leftrightarrow \quad (\sigma_r - \sigma_\varphi)\, dr\, h + d\sigma_r\, r\, h - 2\, \tau_{fr}\, r\, dr = 0$$

Met $\dot{\varepsilon}_r = \dot{\varepsilon}_\varphi \Rightarrow \sigma_r = \sigma_\varphi \Leftrightarrow \quad d\sigma_r = 2\, \tau_{fr} \frac{dr}{h}$

Met $\sigma_r (r = D/2) = 0$ volgt $\quad \sigma_r = 2\, \tau_{fr} \frac{r - D/2}{h}$

Met v. Mises $\sigma_v = \sigma_r - \sigma_z$ volgt :

"drukberg"
$$\boxed{\frac{\sigma_z}{\sigma_v} = -\left(1 + \frac{m}{\sqrt{3}} \frac{D - 2r}{h}\right)} \qquad (1.3)$$

Met $F = \int_0^{D/2} |\sigma_z|\, 2\pi r\, dr$ en $F^* = \frac{F}{\sigma_v \pi/4\, D^2}$ volgt

$$\boxed{F^* = 1 + \frac{m}{3\sqrt{3}} \frac{D}{h}} \qquad (1.4)$$

zie (1.1) Dit betekent dat binnen deze benadering ∉ schil $\sigma_\varphi \neq \sigma_r(z)$ en $\dot{\varepsilon}_r \neq \dot{\varepsilon}_r(z)$ - boven- en ondergrens oplossing tot hetzelfde resultaat leiden. Dus de "beste" zijn!

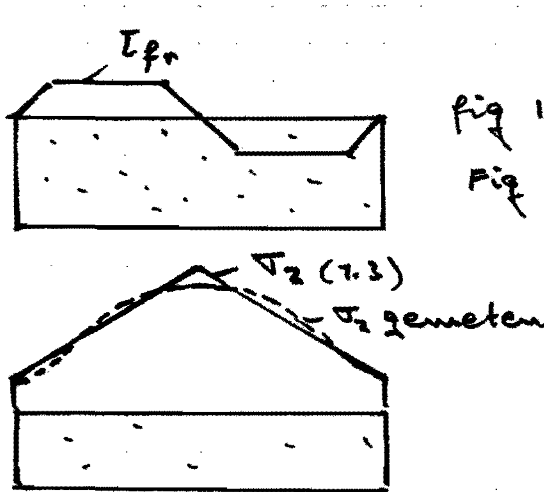


fig 1.3 $\tau_{fr} = \tau_{fr}(r)$ en drukberg

Fig 1.3 geeft juistere τ_{fr} verdeling, echter vaak is dit een technisch niet zo relevante verfijning mbt. de belasting op het gereedschap

2. Strip pletten (vlakke def. $\dot\varepsilon_y = 0$)

2.1 Energiemethode (σ_v = konst.)

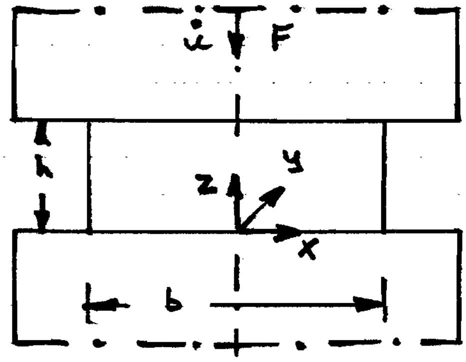


fig 2.1 strip pletten
h – momentane hoogte
b – " " breedte

De berekeningen worden uitgevoerd per eenheid van lengte L.

snelheidsveld: $\dot u_x = \dot u \frac{x}{h}$ $\quad \dot u_y = 0$ $\quad \dot u_z = -\dot u \frac{z}{h}$

reksnelheden: $\dot\varepsilon_x = \dot u_{x,x} = \frac{\dot u}{h}$ $\quad \dot\varepsilon_y = 0$ $\quad \dot\varepsilon_z = \dot u_{z,z} = -\frac{\dot u}{h}$

$$\dot{\bar\varepsilon} = \frac{2}{\sqrt3}\frac{\dot u}{h}$$

Def. vermogen: $P_D = \frac{2}{\sqrt3}\, b L \dot u \sigma_v$

wrijvings verm.: $P_{fr} = b L \dot u \sigma_v \frac{m}{2\sqrt3}\frac{b}{h}$

$$\Leftrightarrow \quad \boxed{F^* = \frac{F}{b \cdot L \cdot \sigma_v} = \frac{2}{\sqrt3}\left(1 + \frac{m}{4}\frac{b}{h}\right)} \qquad (2.1)$$

2.1.1. Rot. symm. ↔ vlak. verg (1.1) ↔ (2.1)

Als we uitgaan van 'n rechthoekige strip ($b \times h \times qb$) met hetzelfde oppervlak als 'n ronde schijf – dus $\frac{\pi}{4}D^2 = q b^2$ cq $b = \frac{D}{2}\sqrt{\frac{\pi}{q}}$ – volgt uit (1.1) en (2.1)

$$\psi = \frac{F^*_{(2.1)}}{F^*_{(1.1)}} = \frac{2}{\sqrt3}\,\frac{1 + \frac{m}{8}\frac{D}{h}\sqrt{\frac{\pi}{q}}}{1 + \frac{m}{3\sqrt3}\frac{D}{h}} = \frac{2}{\sqrt3}\,\frac{1 + \frac{m}{4}\frac{b}{h}}{1 + \frac{2m}{3\sqrt3}\sqrt{\frac{q}{\pi}}\frac{b}{h}}$$

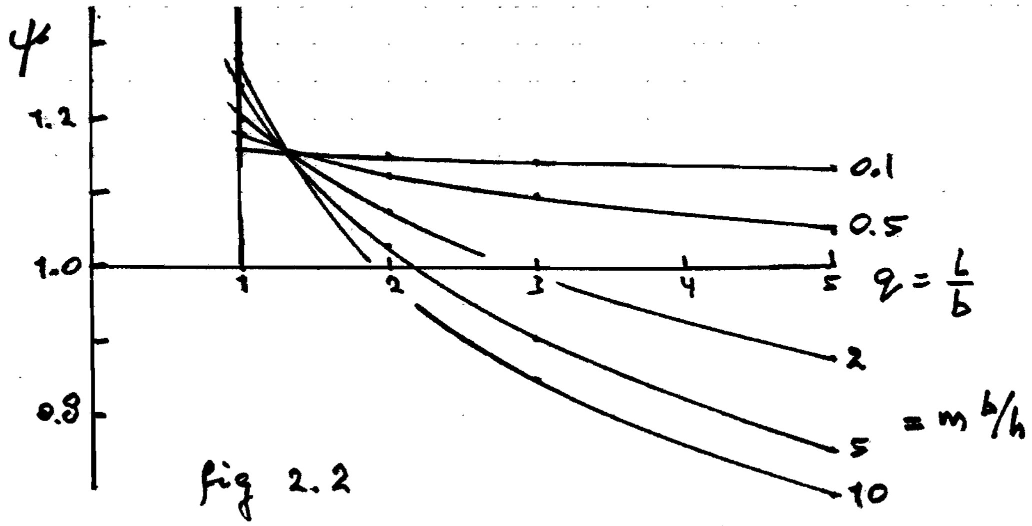


fig 2.2

dus vlak is vaak goede benadering

NB. zie blz 4.3

2.2 Energiemethode $\sigma_v = C\bar{\varepsilon}^n$

eindige rek : $\bar{\varepsilon} = \frac{2}{\sqrt{3}} \ln \frac{h_0}{h}$

vloeispanning: $\sigma_v = C\left(\frac{2}{\sqrt{3}} \ln \frac{h_0}{h} + \bar{\varepsilon}_0\right)^n$

kracht:

$$\boxed{F^* = \left(\frac{2}{\sqrt{3}} \ln \frac{h_0}{h} + \bar{\varepsilon}_0\right)^n \left\{1 + \frac{\sqrt{3}}{4} \frac{\tau_{fr}}{\sigma_v} \frac{b}{h}\right\}} \quad (2.2)$$

(fig 2.3)

2.3 Schillenmethode (σ_v = konst)

evenwicht analoog § 1.3 :

$$-\sigma_x h l + (\sigma_x + d\sigma_x) h l - 2 \tau_{fr} dx\, l = 0$$

$$d\sigma_x = 2 \tau_{fr} \frac{dx}{h} \quad \text{met} \quad \sigma_x (x = b/2) = 0 \rightarrow \sigma_x = +2 \tau_{fr} \frac{x - b/2}{h}$$

$$\dot{\varepsilon}_y = 0 \rightarrow \sigma_y = \frac{\sigma_x + \sigma_z}{2} \rightarrow \sigma_v = \frac{\sqrt{3}}{2} (\sigma_x - \sigma_z)$$

$$\boxed{\frac{\sigma_z}{\sigma_v} = -\frac{2}{\sqrt{3}}\left(1 + m \frac{b/2 - x}{h}\right)} \quad (2.3)$$

$$\Rightarrow \quad \boxed{F^* = \frac{2}{\sqrt{3}}\left(1 + \frac{m}{4} \frac{b}{h}\right)} \quad (2.4)$$

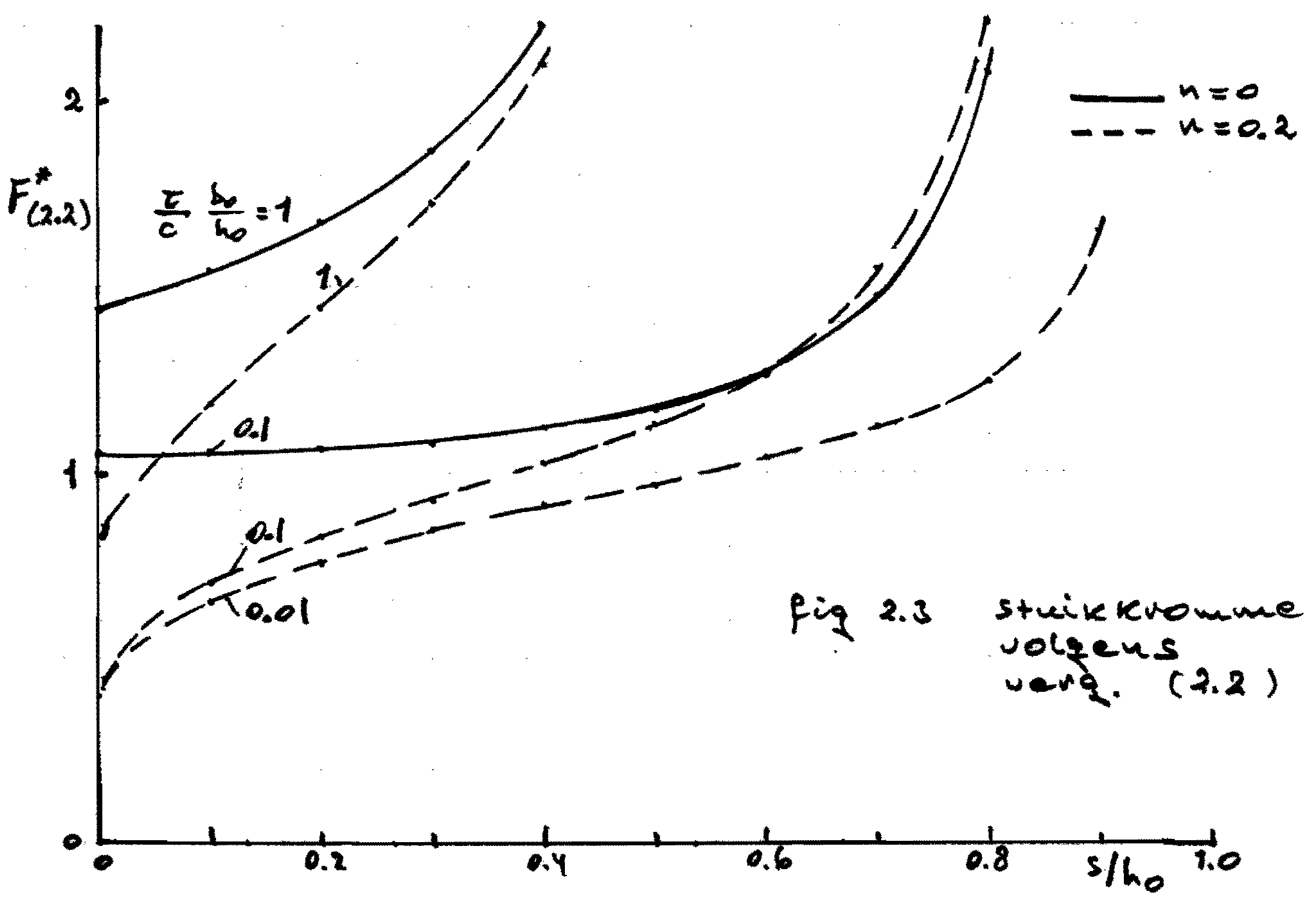


fig 2.3 stuikkromme volgens verg. (2.2)

3. strip pletten met dode zone

3.1 Energie methode (σ_v = konst) $\dot u_y = 0$; $\dot\varepsilon_y = 0$

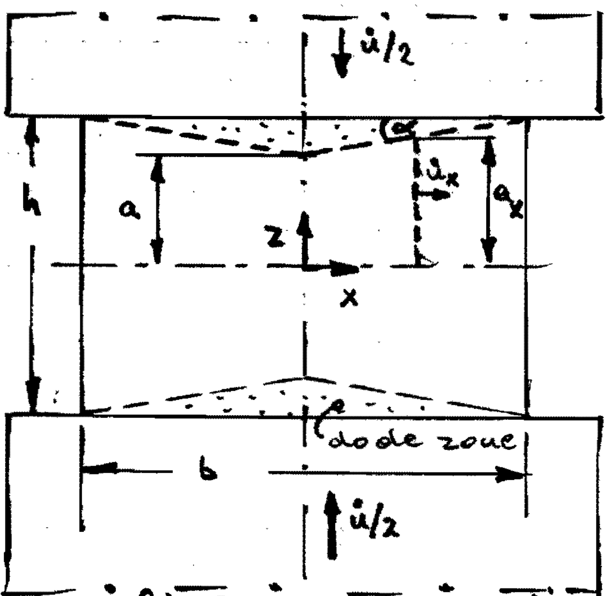


fig 3.1 pletten (dode zone)

$$a_x = a + x\,\mathrm{tg}\alpha$$

$$a = \frac{h}{2} - \frac{b}{2}\,\mathrm{tg}\alpha$$

snelheden.

Vol. Inv. $\dot u_x \cdot a_x = \frac{\dot u}{2}\, x$

$$\dot u_x = \frac{1}{2}\dot u\,\frac{x}{a + x\,\mathrm{tg}\alpha} \qquad (3.1)$$

$$\dot\varepsilon_x = -\dot\varepsilon_z = \frac{a}{a_x^2}\cdot\frac{\dot u}{2}$$

$$\dot u_z = -\frac{1}{2}\dot u\, z\,\frac{a}{a_x^2} \qquad (3.2)$$

$$\dot\varepsilon_{zx} = \frac{1}{2}(\dot u_{x,z} + \dot u_{z,x}) = \frac{1}{2}\left(-\frac{1}{2}\dot u\, z\, a\right)(-2)\,\frac{1}{a_x^3}\,\mathrm{tg}\alpha$$

$$\dot\varepsilon_{zx} = \frac{1}{2}\dot u\, z\,\mathrm{tg}\alpha\,\frac{a}{a_x^3}$$

$$\dot{\bar\varepsilon} = \sqrt{\frac{4}{3}\left(\dot\varepsilon_x^2 + \dot\varepsilon_{zx}^2\right)} = \frac{\dot u}{\sqrt3}\,\frac{a}{a_x^2}\sqrt{1+\left(\frac{z\,\mathrm{tg}\alpha}{a_x}\right)^2} \qquad (3.3)$$

Def. vermogen $P_D = 4\int_V \sigma_v\,\dot{\bar\varepsilon}\,dV = 4\sigma_v\int_0^{b/2}\frac{\dot u}{\sqrt3}\,\frac{a}{a_x^2}\,dx\int_0^{a_x}\sqrt{1+\left(\frac{z\,\mathrm{tg}\alpha}{a_x}\right)^2}\,dz$

$$P_D = \frac{2}{\sqrt3}\sigma_v\,\dot u\, a\int_0^{b/2}\frac{dx}{a_x}\left\{\sqrt{\mathrm{tg}^2\alpha+1} + \frac{1}{\mathrm{tg}\alpha}\ln\left(\mathrm{tg}\alpha + \sqrt{1+\mathrm{tg}^2\alpha}\right)\right\}$$

$$P_D = \frac{1}{\sqrt3}\sigma_v\,\dot u\,\frac{(h - b\,\mathrm{tg}\alpha)}{\mathrm{tg}^2\alpha}\left\{\mathrm{tg}\alpha\sqrt{1+\mathrm{tg}^2\alpha} + \ln\left(\mathrm{tg}\alpha+\sqrt{1+\mathrm{tg}^2\alpha}\right)\right\} * \ln\frac{h}{h - b\,\mathrm{tg}\alpha} \qquad (3.4)$$

afschuifvermogen.

$$P_f = 4\int_S \frac{\sigma_v}{\sqrt3}\,|\dot u_\alpha|\,dS$$

$$P_f = \frac{4}{\sqrt3}\sigma_v\int_0^{b/2}\frac{\dot u_x}{\cos\alpha}\cdot\frac{dx}{\cos\alpha} = \frac{4}{\sqrt3}\sigma_v\,\frac{1}{2}\dot u\,\frac{1}{\cos^2\alpha}\int_0^{b/2}\frac{x\,dx}{a + x\,\mathrm{tg}\alpha}$$

$$P_f = \frac{2}{\sqrt3}\sigma_v\,\dot u\,\frac{1}{\cos^2\alpha}\cdot\frac{1}{\mathrm{tg}^2\alpha}\left\{\frac{b}{2}\mathrm{tg}\alpha - \frac{1}{2}(h - b\,\mathrm{tg}\alpha)\ln\frac{h}{h-b\,\mathrm{tg}\alpha}\right\}$$

$$P_f = \frac{1}{\sqrt3}\sigma_v\,\dot u\,\frac{1}{\sin^2\alpha}\left\{b\,\mathrm{tg}\alpha - (h - b\,\mathrm{tg}\alpha)\ln\frac{h}{h-b\,\mathrm{tg}\alpha}\right\} \qquad (3.5)$$

$$\frac{\Sigma P}{\sigma_0 \cdot u \cdot b} = P^* =$$

$$= \frac{1}{\sqrt{3}}\left[\frac{h/b - tg\,\alpha}{tg^2\alpha}\left\{ tg\,\alpha\sqrt{1+tg^2\alpha} + \ln\left(tg\,\alpha + \sqrt{1+tg^2\alpha}\right)\right\} \ln\frac{h/b}{h/b - tg\,\alpha} + \frac{1}{\sin^2\alpha}\left\{ tg\,\alpha - \left(\frac{h}{b} - tg\,\alpha\right)\ln\frac{h/b}{h/b - tg\,\alpha}\right\}\right] \tag{3.6}$$

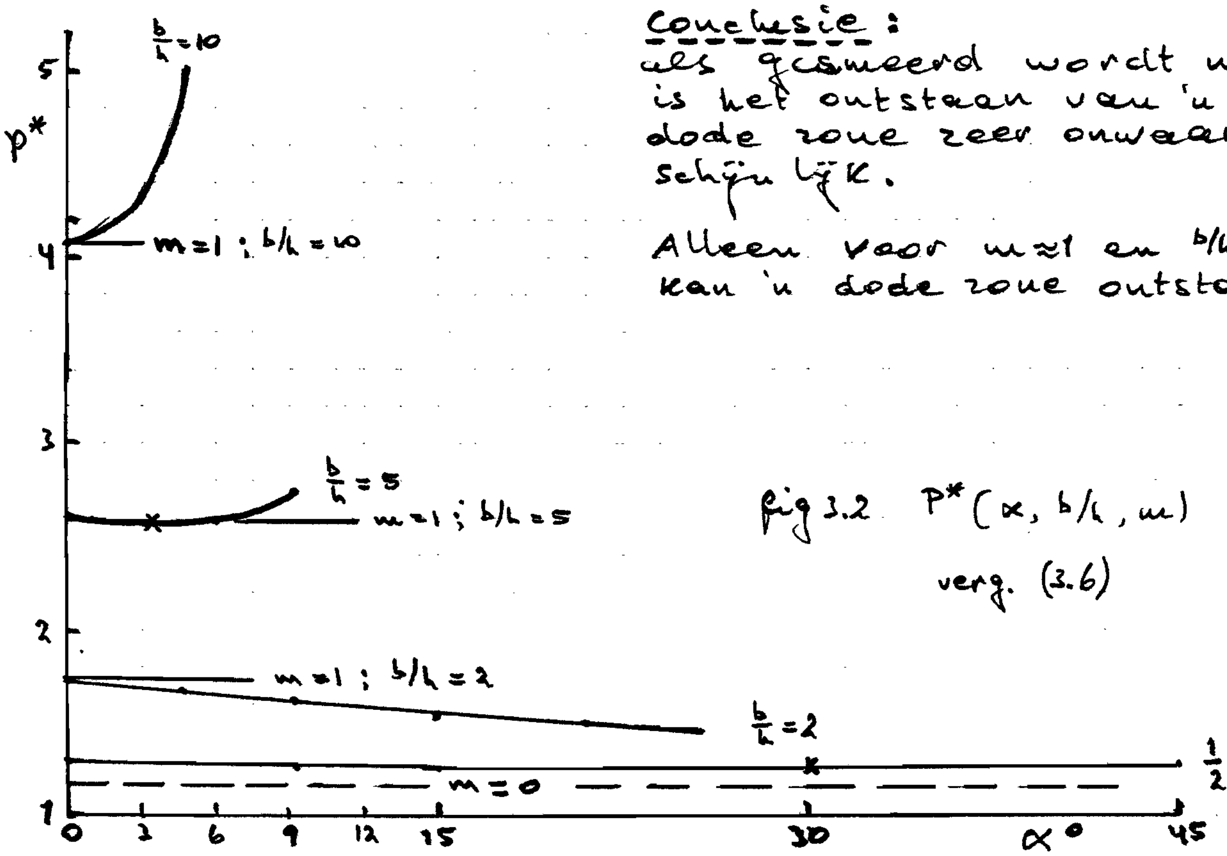
α \ b/h	1/2	1	2	5	10
0.01					4.042
0.1	1.299	1.443	1.731	2.597	4.048
1	1.295	1.435		2.591	4.120
2				2.585	4.226
3				2.582	4.337
4					4.607
5	1.278	1.404	1.668	2.586	5.017
10	1.261	1.369	1.608	2.741	
15	1.249	1.337	1.551	6	
20	1.240	1.309	1.496	2.594	
25	1.2349	1.283	1.447		
30	1.2348	1.259	26		
35	1.2402	1.236	1.441		
40	1.252	1.240			
45	1.273	44 1.175			
50	1.305				

$$P^*_{(m)} = \frac{2}{\sqrt{3}}\left(1 + \frac{m}{4}\frac{b}{h}\right)$$

b/h \ m	0	0.2	0.4	1
1/2	1.155	1.184	1.213	1.299
1	"	1.213	1.271	1.444
2	"	1.271	1.386	1.733
5	"	1.444	1.733	2.599
10	"	1.733	2.31	4.043

NB. $tg\,\alpha < \frac{h}{b}$

met smering m
dan $\alpha = 0$



<u>Conclusie</u>:
als gesmeerd wordt m≤0.4 is het ontstaan van 'n dode zone zeer onwaarschijnlijk.

Alleen voor m≈1 en b/h $\tilde{<}$ 2 kan 'n dode zone ontstaan.

fig 3.2 $P^*(\alpha, b/h, m)$

verg. (3.6)

3.2 Schillenmethode

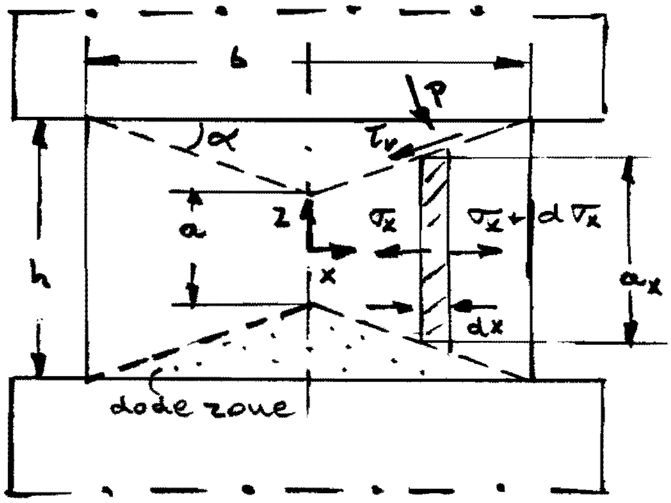
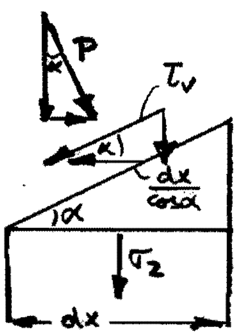


fig 3.3 pletten met dode zone (α) schillenmethode

$$a_x = a + 2x\,\mathrm{tg}\,\alpha$$
$$da_x = 2\,\mathrm{tg}\,\alpha\,dx$$

$$\dot{\varepsilon}_y = 0 \qquad \sigma_y = \frac{\sigma_x + \sigma_z}{2}$$

$$\sigma_x - \sigma_z = \frac{2}{\sqrt{3}}\sigma_v$$



horizontaal evenwicht:

$$-\sigma_x a_x + (\sigma_x + d\sigma_x)(a_x + da_x) +$$
$$- 2\tau_v \frac{dx}{\cos\alpha}\cos\alpha + 2P\frac{dx}{\cos\alpha}\sin\alpha = 0$$

↓

$$\sigma_x da_x + d\sigma_x a_x - \tau_v \frac{da_x}{\mathrm{tg}\,\alpha} + P\,da_x = 0$$

vertikaal evenwicht: $\sigma_z dx + P\frac{dx}{\cos\alpha}\cos\alpha + \tau_v\frac{dx}{\cos\alpha}\sin\alpha = 0$

met de vloeivoorwaarde volgt: $P = \frac{2}{\sqrt{3}}\sigma_v - \sigma_x - \tau_v\,\mathrm{tg}\,\alpha$

subst. : $d\sigma_x a_x - \tau_v \frac{da_x}{\mathrm{tg}\,\alpha} + \left(\frac{2}{\sqrt{3}}\sigma_v - \tau_v\,\mathrm{tg}\,\alpha\right)da_x = 0$

$\tau_v = \sigma_v/\sqrt{3}$ (m=1)

$$d\sigma_x = \frac{\sigma_v}{\sqrt{3}}\left(\mathrm{tg}\,\alpha + 1/\mathrm{tg}\,\alpha - 2\right) da_x/a_x$$

met $\sigma_{x\,(a_x=h)} = 0$ volgt

$$\boxed{\sigma_x/\sigma_v = \frac{1}{\sqrt{3}}\left(\mathrm{tg}\,\alpha + \frac{1}{\mathrm{tg}\,\alpha} - 2\right)\ln\frac{a_x}{h}} \qquad (3.7)$$

$$\frac{\sigma_z}{\sigma_v} = -\frac{1}{\sqrt{3}}\left[2 + \left(\mathrm{tg}\,\alpha + \frac{1}{\mathrm{tg}\,\alpha} - 2\right)\ln\frac{h}{a + 2x\,\mathrm{tg}\,\alpha}\right] \qquad (3.8)$$

Met $P^* = F^* = \frac{F}{\sigma_v b}$ en $F = 2\int_0^{b/2}|\sigma_z|\,dx$ volgt

$$\boxed{F^* = \frac{2}{\sqrt{3}}\left[1 + \frac{1}{2}\left(\mathrm{tg}\,\alpha + \frac{1}{\mathrm{tg}\,\alpha} - 2\right)\left\{1 + \left(\frac{h}{b\,\mathrm{tg}\,\alpha} - 1\right)\ln\left(1 - \frac{b}{h\,\mathrm{tg}\,\alpha}\right)\right\}\right]} \qquad (3.9)$$

α \ b/h	1/2	1	2	5	10
0.01	1.299	1.444	1.732	2.599	4.043
1	1.295	1.435	1.719	2.591	4.12
3	1.286	1.419	1.693	2.582	4.378
6	1.273	1.395	1.654	2.593	4→ 4.607
9	1.260	1.371	1.616	2.672	
15	1.236	1.326	1.542		
21	1.214	1.282	1.474		
30	1.184				
$m = 0.4$ →	1.213	1.271	1.386	1.733	2.31

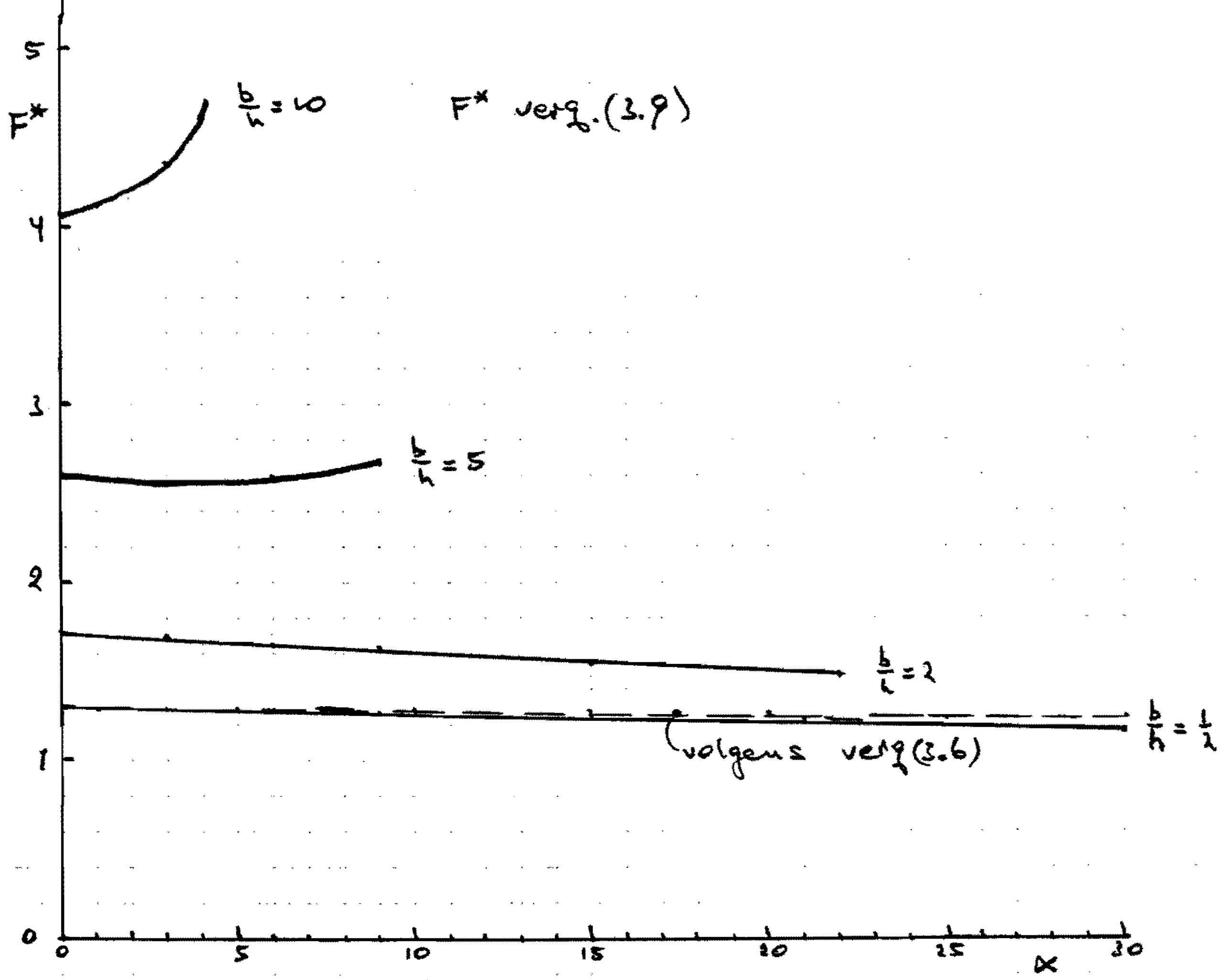


fig 3.3 F* (3.9)

conclusie: bovengrens (3.6) en schillen (3.9) leveren practisch dezelfde uitkomst

- schillen is dus hier geen echte ondergrens methode

- conclusie bij fig 3.2 wordt hier bevestigd.

4. strip pletten met verstoring

4.1 verstoring Δm, energiemethode ($\dot{u}_y = 0$)

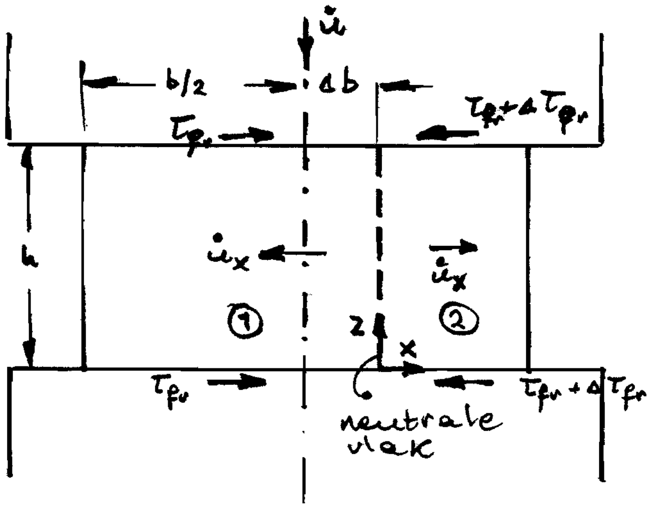


fig 4.1 strip pletten met verstoring $\Delta\tau_{fr}$ (Δm) en verschuiving van het neutrale vlak $-\Delta b-$.

snelheden: $\dot{u}_x = \dot{u}\,\frac{x}{h}$

$\dot{\varepsilon}_x = \frac{\dot{u}}{h} \qquad \dot{\varepsilon}_z = -\frac{\dot{u}}{h}$

$\dot{\bar{\varepsilon}} = \frac{2}{\sqrt{3}}\,\frac{\dot{u}}{h}$

deformatie vermogen in ①: $P_{D1} = \sigma_v\,\frac{2}{\sqrt{3}}\,\frac{\dot{u}}{h}\cdot\left(\frac{b}{2}+\Delta b\right)h$.

deformatie vermogen in ②: $P_{D2} = \sigma_v\,\frac{2}{\sqrt{3}}\,\frac{\dot{u}}{h}\left(\frac{b}{2}-\Delta b\right)h$.

wrijvingsvermogen in ①: $P_{f1} = \frac{m}{\sqrt{3}}\,\sigma_v\,\frac{\dot{u}}{h}\left(\frac{b}{2}+\Delta b\right)^2$

wrijvingsvermogen in ②: $P_{f2} = \frac{m+\Delta m}{\sqrt{3}}\,\sigma_v\,\frac{\dot{u}}{h}\left(\frac{b}{2}-\Delta b\right)^2$

$$P^* = \frac{\Sigma P}{\sigma_v\,\dot{u}\,b} = \frac{2}{\sqrt{3}}\left\{\frac{b/2+\Delta b}{b} + \frac{b/2-\Delta b}{b} + \frac{m}{2}\,\frac{1}{bh}\left(\frac{b}{2}+\Delta b\right)^2 + \frac{m+\Delta m}{2}\,\frac{(b/2-\Delta b)^2}{b\,h}\right\}$$

$$P^* = \frac{2}{\sqrt{3}}\left\{1 + \frac{m}{2}\,\frac{b}{h}\left(\frac{1}{2}+2\left(\frac{\Delta b}{b}\right)^2\right) + \frac{\Delta m}{2}\,\frac{b}{h}\left(\frac{1}{2}-\frac{\Delta b}{b}\right)^2\right\}$$

Met $\frac{\partial P^*}{\partial\,\Delta b/b} = 0$ volgt: $\boxed{\frac{\Delta b}{b} = \frac{1}{2}\,\frac{\Delta m}{2m+\Delta m}}$ (4.1)

4.2 Verstoring Δm, schillenmethode

Analoog § 1.1 wordt voor $x \geq 0$ afgeleid:

$$d\sigma_x = 2\left(\tau_{fr}+\Delta\tau_{fr}\right)\frac{dx}{h} = \frac{2}{\sqrt{3}}\,\sigma_v\,(m+\Delta m)\,\frac{dx}{h}$$

met $\sigma_x\left(x=\frac{b}{2}-\Delta b\right) = 0$ volgt: $\sigma_{x\,(x\geq 0)} = \frac{2}{\sqrt{3}}\,\sigma_v\,(m+\Delta m)\,\frac{x-(b/2-\Delta b)}{h}$

analoog voor $x \leq 0$: $\sigma_{x\,(x\leq 0)} = -\frac{2}{\sqrt{3}}\,\sigma_v\,m\,\frac{x+(b/2+\Delta b)}{h}$

uit de evenwichtsvoorwaarde voor $x=0$ volgt:

$$\boxed{\frac{\Delta b}{b} = \frac{1}{2}\,\frac{\Delta m}{2m+\Delta m}}$$

4.3 verstoring α (Δh), schillenmethode

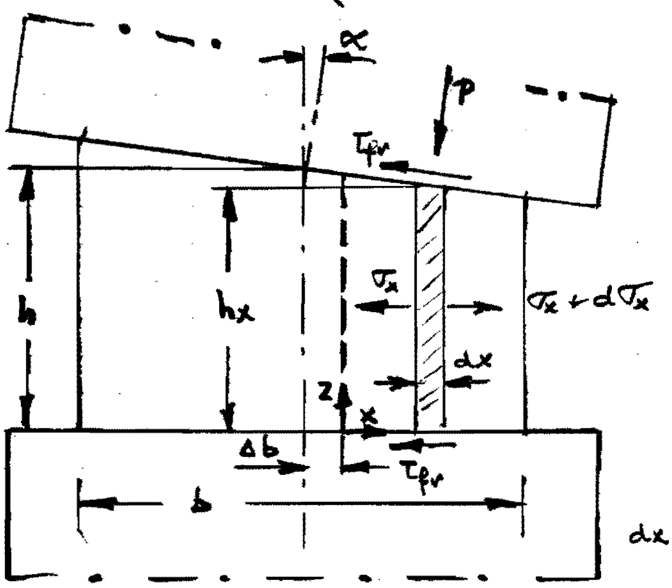
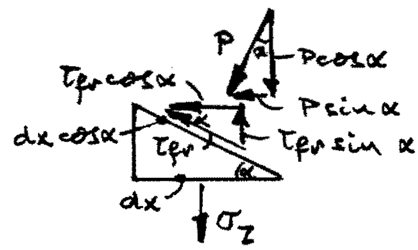


fig 4.2 Pletten met verstoring $\alpha(\Delta h)$ en verschuiving v.d neutrale laag Δb.

met :
$h_x = h - (x+\Delta b)\, tg\alpha$
$dh_x = tg\alpha\, dx$



horizontaal evenwicht ($x \geq 0$) :

$$-\sigma_x h_x + (\sigma_x + d\sigma_x)(h_x + dh_x) - \tau_{fr}\, dx - \tau_{fr}\cos\alpha \frac{dx}{\cos\alpha} + $$
$$- p \sin\alpha \frac{dx}{\cos\alpha} = 0$$

$$d\sigma_x h_x + \sigma_x dh_x + 2\tau_{fr}\frac{dh_x}{tg\alpha} + p\, dh_x = 0 \qquad (4.1)$$

$$\dot{u}_y = 0 \rightarrow \dot{\varepsilon}_y = 0 \rightarrow \sigma_y = \frac{\sigma_x + \sigma_z}{2} \longrightarrow \sigma_x - \sigma_z = \frac{2}{\sqrt{3}}\sigma_v \qquad (4.2)$$

vertikaal evenwicht ($x \geq 0$) :

$$\sigma_z dx + p\cos\alpha \frac{dx}{\cos\alpha} - \tau_{fr}\sin\alpha \frac{dx}{\cos\alpha} = 0 \rightarrow p = -\sigma_z + \tau_{fr}\, tg\alpha$$

$$p = \frac{2}{\sqrt{3}}\sigma_v - \sigma_x + \tau_{fr}\, tg\alpha \qquad (4.3)$$

subst. in (4.1) geeft:

$$d\sigma_x h_x + \sigma_x dh_x + 2\tau_{fr}\frac{dh_x}{tg\alpha} + \left(\frac{2}{\sqrt{3}}\sigma_v - \sigma_x + \tau_{fr}\, tg\alpha\right) dh_x = 0$$

$$d\sigma_x = -\frac{2}{\sqrt{3}}\sigma_v\left(\frac{m}{tg\alpha} + 1 + \frac{m}{2}tg\alpha\right)\frac{dh_x}{h_x} \qquad (4.4)$$

Met $\sigma_x(x = b/2 - \Delta b) = 0$ volgt :

$$\boxed{\sigma_{x\,(x \geq 0)} = -\frac{2}{\sqrt{3}}\sigma_v\left(1 + \frac{m}{tg\alpha} + \frac{m}{2}tg\alpha\right) \ln\frac{h_x}{h - b/2\, tg\alpha}} \qquad (4.5)$$

voor $x \leq 0$ kan analoog afgeleid worden:

$$\sigma_{x\,(x \leq 0)} = -\frac{2}{\sqrt{3}}\sigma_v\left(1 - \frac{m}{tg\alpha} - \frac{m}{2}tg\alpha\right) \ln\frac{h_x}{h + b/2\, tg\alpha} \qquad (4.6)$$

uit evenwicht voor $x=0$ volgt met (4.5) en (4.6) :

$$\left(1+\frac{m}{tg\alpha}+\frac{m}{2}tg\alpha\right)\ln\frac{h-\Delta b\, tg\alpha}{h-b/2\, tg\alpha} = \left(1-\frac{m}{tg\alpha}-\frac{m}{2}tg\alpha\right)\ln\frac{h-\Delta b\, tg\alpha}{h+b/2\, tg\alpha}$$

voor $\alpha \ll 1$ volgt

$$\boxed{\frac{\Delta b}{b} = \frac{1}{2}\,\frac{\alpha}{m}} \qquad (4.7)$$

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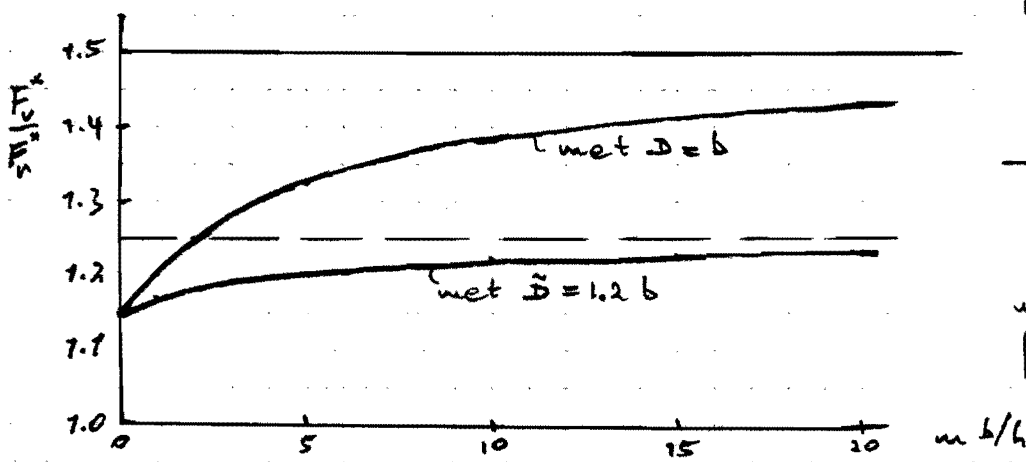
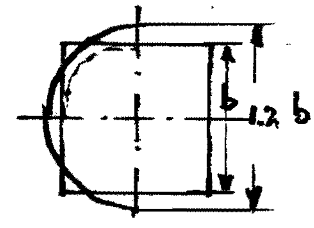
vervolg § 2.1.1 blz 2-1

Een vierkant produkt kunnen we als rot sym. of als vlak benaderen. Met als afmetingen $b \times b \times h$ volgt uit (1.1) resp (2.1) :

$$\frac{F^*_{vlak}}{F^*_{rond}} = \frac{2/\sqrt{3}\,(1+ m/4\ b/h)}{1+ m/3\sqrt{3}\ b/h} = 1.155 \div 1.5$$



Met $\tilde{D} = b \cdot 1{,}2$



wordt de max. fout 1.25

Bij een proefstuk met $L \times b \times h$ en $L \gg b$ "loopt" het materiaal langs de kortste weg naar de rand, weg van de minste weerstand. Een goede benadering is dan vlakke def.