# Particle motion in a traveling wave near cyclotron resonance

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## PARTICLE MOTION IN A TRAVELLING WAVE NEAR CYCLOTRON RESONANCE

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The cyclotron resonance interaction between charged particles and a propagating e.m. wave has been the subject of several papers. In some of these (1, 2) the problem is tackled on a single particle basis, neglecting collective effects. Such a treatment applies for a low-density plasma in a strong e.m. wave. Then the energy gain is such that the influence of the relativistic mass change on the resonance parameter $\Omega/\omega = eB_o/m\omega$ is important. The Doppler effect has then also to be taken into account (3); the $B_{\sim}$ of the wave may not be neglected. Though such exact treatments exist, it is difficult to obtain values for the characteristic quantities, such as attainable energy and specific time for energy gain, from the applied fieldstrengths initial velocity and index of refraction. It is the purpose of this article to satisfy this need for a special case: zero initial transverse velocity. Here as well as in the publications mentioned, the problem is solved in a as simple as possible field configuration: a uniform right-handed circularly polarized TEM wave:

$$E_x = \hat{E}\sin(\omega t-kz) \qquad B_x = \hat{E}/v_p \cdot \cos(\omega t-kz)$$
$$E_y = -\hat{E}\cos(\omega t-kz) \qquad B_y = \hat{E}/v_p \cdot \sin(\omega t-kz)$$

propagating along a homogeneous static magnetic field $B_z = B_o$. Thus non-linearities arising from inhomogeneity of the fields or the density are disregarded. Here the formalism has been extended to include applied or self-generated space charge fields $E_z = E_o(z) = d\varphi/dz$.

A relativistic formulation of the momentum equation is: $\frac{d(\gamma \underline{v})}{dt} = -\frac{e}{m}(\underline{E} + \underline{v} \times \underline{B})$ where $\gamma = m/m_r = 1/(1 - \frac{\underline{v}\cdot\underline{v}}{c^2})^{\frac{1}{2}}$. The assumed fields are inserted in this equation and after some manipulation, including a transformation to a rotating velocity frame, we obtain an extended form of equation 2.3 of ROBERTS & BUCHSBAUM (1):

$$\frac{d\gamma}{d\tau} = \frac{1}{n}\frac{d}{d\tau}\left(\frac{\gamma\dot{z}}{c}\right) - (1 - \dot{z}/v_p)\frac{d\Phi}{d(kz)} \qquad (1)$$

where $\Phi = e\varphi/m_r c^2$, $n$ = index of refraction = $c/v_p$ and $\tau = \omega t$. From eq. (1) it is evident that the relationship between the changes in energy and in axial momentum is influenced by D.C. space charges. Though the influence of this field may have interesting features we will neglect it in the remaining of this paper. The governing differential equation is:

$$-\left\{\frac{d(\gamma/\gamma_o)^2}{d\tau}\right\}^2 = (1-n^2)^2\left(\frac{\gamma}{\gamma_o} - 1\right)^4 + 4\rho(1-n^2)\left(\frac{\gamma}{\gamma_o} - 1\right)^3 +$$
$$+ 4\left(\frac{\gamma}{\gamma_o} - 1\right)^2\left\{\rho^2 - \frac{g^2(1-n^2)}{\gamma_o^2} + \frac{g\dot{x}_o}{\gamma_o c}(1-n^2)\right\} + 8\left(\frac{\gamma}{\gamma_o} - 1\right)\frac{g\dot{x}_o}{\gamma_o c} - \frac{g^2}{\gamma_o^2}\left(\rho + \frac{b}{\gamma_o}\right) - 4\frac{g^2\dot{y}_o^2}{\gamma_o^2 c^2} \qquad (2)$$

and
$$\frac{\dot{z}}{v_p} = n^2 + \left\{(1-n^2) - \left(\rho + \frac{b}{\gamma_o}\right)\right\}\frac{\gamma_o}{\gamma} \; ; \; kz = \int_o^\tau \frac{\dot{z}}{v_p}\,d\tau \qquad (3)$$

Here an electric field parameter $g$ = $g = eE/m_r\omega c$ and a resonance parameter $\rho = 1 - b/\gamma_o - \dot{z}_o/v_p$ where $b = \Omega_r/\omega = eB_o/m_r\omega$ have been introduced. The form of eq. 2 indicates that $\gamma$ oscillates as a function of time (1, 2) and with help of eq. 3 it can be shown that this also happens as function of kz. The quantities to be calculated are the amplitude, time $t_{os}$ and distance $kz_{os}$ of oscillation. The latter two are also important to investigate the applicability of the theory. Because the collisions are neglected, the oscillation time $t_{os}$ has to be small compared to the collision time $t_{coll}$. The gain or loss of the wave must be small over an oscillation distance. Both conditions are supposed to be met.

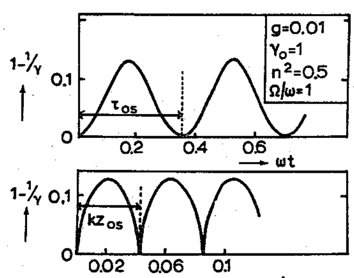


fig. 1

In order to get easily readable results it is felt necessary to limit the number of parameters of the problem; up to now we have six parameters to vary: electric field strength (expressed in g), magnetic field strength ($\rho$), index of refraction n, and three initial velocity components ($\dot{x}_o$, $\dot{y}_o$, $\dot{z}_o$) giving $\gamma_o$. Therefore we will restrict ourselves to a plasma in which thermal velocities, except for a drift velocity along the magnetic field $B_o$, are small compared to the velocities after acceleration: $\dot{x}_o = \dot{y}_o = 0$ (a cold plasma drifting along the $B_o$ field in a strong e.m. wave).

Also a transformation has been used:

$$\delta = \frac{1-n^2}{1-\dot{z}_o/v_p}\left(\frac{\gamma}{\gamma_o} - 1\right) \; ; \; G = \frac{g^2(1-n^2)}{\gamma_o^2(1-\dot{z}_o/v_p)^2} \; ; \; \beta = \frac{\rho}{\gamma_o}\left(\rho + \frac{b}{\gamma_o}\right) = \frac{b}{\gamma_o}\left(1 - \frac{\dot{z}_o}{v_p}\right) - 1 \qquad (4)$$

respectively the to the energy related variable, the new electric field parameter and the new resonance parameter. The equations become (in integral form):

$$\tau_{os} = \int_o^{\delta_{extreme}} 4\frac{\left[\frac{\delta}{1-n^2} + \frac{1}{1-\dot{z}_o/v_p}\right]d\delta}{(P_4(\delta))^{\frac{1}{2}}} \equiv \frac{2I_1}{1-n^2} + \frac{2I_o}{1-\dot{z}_o/v_p} \qquad (5) \quad \text{and}$$

$(\tau - kz)_{os} = 2I_1 + 2I_o$ (6) ; here $P_4(\delta) = -\delta^4 + 4\beta\delta^3 - 4(\beta^2 - G)\delta^2 + 8G\delta$.

The integration takes place between two of the roots of the polynomial $P_4(\delta)$: $\delta = 0$ and $\delta = \delta_{extr.}$, the absolute smallest other root. The sign of the roots depends on the sign of G. If $G < 0$ (slow wave) $\delta_{extr.} < 0$, thus if $\dot{z}_o > v_p$ the initial energy is maximum and the particle looses energy (a Cerenkov-like condition for energy loss). In all other cases ($G < 0$ but $\dot{z}_o < v_p$ and $G > 0$ (fast wave)) the electron always gains energy. $\delta_{extr.}$ is shown in Figs. 2 and 3. With help of these figs. and eq. 4 the attainable energy may be calculated.

The oscillation time is expressed in a sum of two integrals $I_1$ and $I_o$, eq. 5, see Figs. 4 and 5. $(\omega t - kz)_{os}$ is the sum of those integrals. Even at resonance it appears

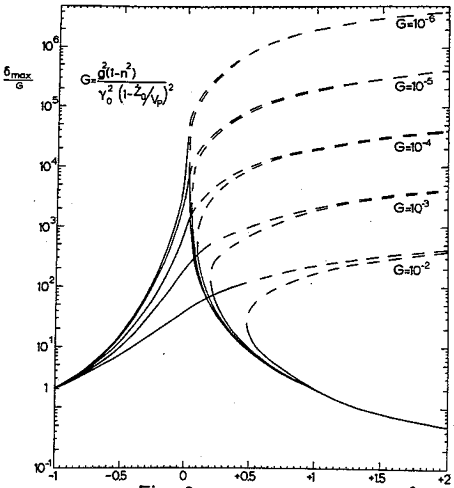


Fig. 2

that $I_1 \ll I_o$ thus $z_{os} \simeq t_{os}\dot{z}_o$. The exact values can be derived from eqs. 4,5, and 6 and the Figs. 4 and 5.

The cases $\dot{z}_o = v_p$ and n=1 are exactly solvable. If $\dot{z}_o = v_p$ the energy of the particle does not change: $\gamma \equiv \gamma_o$ and $\dot{z} \equiv \dot{z}_o = v_p$. If n=1 the differential equation becomes:

$$-\left\{\frac{\gamma}{\gamma_o}\frac{d(\gamma/\gamma_o)}{d\tau}\right\}^2 = \rho^2\left(\frac{\gamma}{\gamma_o} - 1\right)^2 - 2\left(\frac{\gamma}{\gamma_o} - 1\right)\frac{g^2}{\gamma_o^2}(1-\dot{z}_o/v_p).$$

A resonance occurs if $\rho = 0$; the energy increases indefinitely, so it appears as if the Doppler effect cancels the relativistic effect (1). $\gamma$ may be given in the form

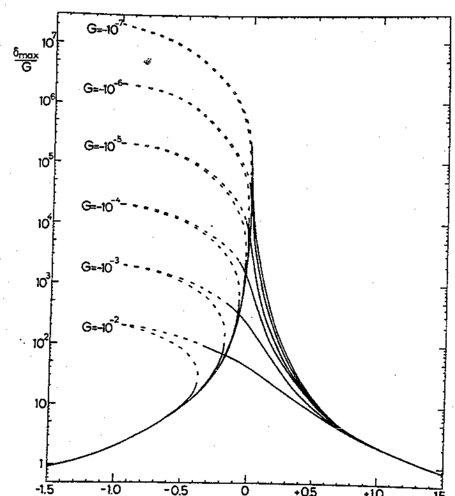


Fig. 3

$\gamma(\tau)$ and it appears that for long times ($\tau \gg \frac{16}{9}\frac{b}{g}$) approximately

$$\frac{\gamma}{\gamma_o} - 1 = \frac{1}{2b/\gamma_o}(3g\tau/\gamma_o)^{1/3}$$

If $\rho \neq 0$ again energy oscillation occurs:

$$\left(\frac{\gamma}{\gamma_o} - 1\right)_{max} = 2\frac{g^2}{\rho^2\gamma_o^2}(1-\dot{z}_o/v_p).$$

The oscillation time $\tau_{os}$ =

$$\frac{2\pi}{\rho}\left\{1 + \frac{g^2(1-\dot{z}_o/v_p)}{\rho^2\gamma_o^2}\right\} \text{ and}$$

$\tau_{os} - kz_{os} = (1-\dot{z}_o/v_p)\,2\pi/\rho$. So for this case too the characteristic quantities have been calculated.

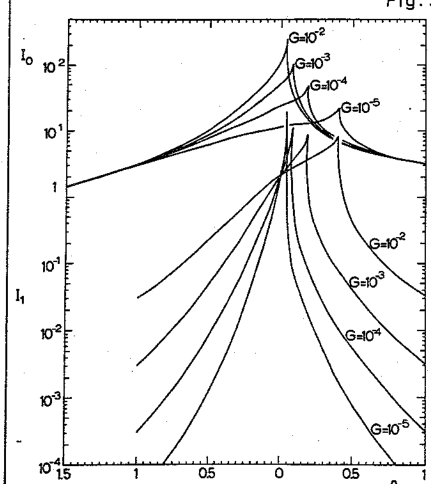


Fig. 4

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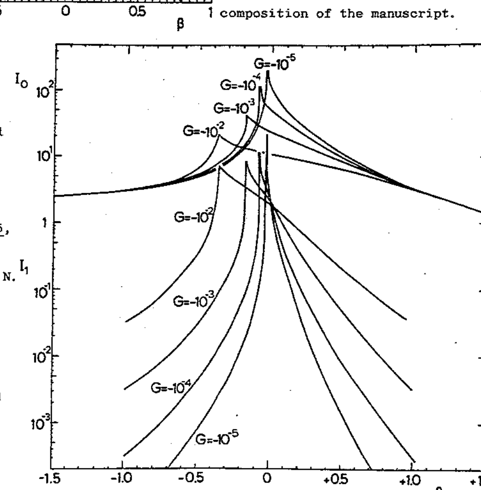


Fig.5