The global error approach to the convergence of closed-loop identification, self tuning regulators and self-tuning predictors

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The global error approach to the convergence of closed-loop identification, self-tuning regulators and self-tuning predictors

by

A. Niederlinski

E I N D H O V E N   U N I V E R S I T Y   O F   T E C H N O L O G Y
Department of Electrical Engineering
Eindhoven   The Netherlands

THE GLOBAL ERROR APPROACH TO THE CONVERGENCE OF CLOSED-LOOP IDENTIFICATION, SELF-TUNING REGULATORS AND SELF-TUNING PREDICTORS

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A. Niederlinski

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THE GLOBAL ERROR APPROACH TO THE CONVERGENCE OF CLOSED-LOOP IDENTIFICATION, SELF-TUNING REGULATORS AND SELF-TUNING PREDICTORS

ABSTRACT

The global error is the identification error expressed as a function of unknown system parameters, model parameters and external system inputs. It provides a convenient tool to test mean-square convergence for a number of important identification problems.

Author:

Doc. dr hab. inż. Antoni Niederlinski,
Instytut Automatyki,
Wydzial Automatyki i Informatyki,
Politechnika Sląska,
Gliwice,
Poland

1. The closed-loop identification problem

Fig. 1 presents a block-diagram for direct closed-loop identification of a ML system. The system structure is assumed to be known and given by

$$y(i) = z^{-k} \frac{\mathcal{B}(z^{-1})}{\mathcal{A}(z^{-1})} u(i) + \frac{\mathcal{C}(z^{-1})}{\mathcal{A}(z^{-1})} e(i) \tag{1}$$

where u(i) is the sum of an external testing signal s(i) and the regulator output signal.

$$u(i) = s(i) - R(z^{-1})\, y(i) \tag{2}$$

It is assumed that

$$\mathcal{A}(z^{-1}) = 1 + \alpha_1 z^{-1} + \dots + \alpha_n z^{-n} \tag{3}$$

$$\mathcal{B}(z^{-1}) = \beta_0 + \beta_1 z^{-1} + \dots + \beta_n z^{-n} \tag{4}$$

$$\mathcal{C}(z^{-1}) = 1 + \gamma_1 z^{-1} + \dots + \gamma_n z^{-n} \tag{5}$$

with unknown parameters α_i, β_i and γ_i. It is assumed further that

$$E\{e(i)\} = 0$$

$$E\{e(k)e(j)\} = \begin{cases} 0 & \text{for } i \neq j \\ \lambda^2 & \text{for } i = j \end{cases} \tag{6}$$

$$E\{s(i)\} = 0 \tag{7}$$

$$E\{s(i)e(j)\} = 0 \text{ , all } i \text{ and } j \tag{8}$$

The controller transmittance

$$R(z^{-1}) = \frac{P(z^{-1})}{Q(z^{-1})} = \frac{p_0 + p_1 z^{-1} + \dots + p_p z^{-p}}{1 + q_1 z^{-1} + \dots + q_q z^{-q}} \tag{9}$$

The model is assumed to have the same structure as the system with

$$A(z^{-1}) = 1 + a_1 z^{-1} + \dots + a_n z^{-n} \tag{10}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + \dots + b_n z^{-n} \tag{11}$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + \dots + c_n z^{-n} \tag{12}$$

The identification error is given by

$$\varepsilon(i) = \frac{1}{C(z^{-1})} \left[A(z^{-1})y(i) + z^{-k} B(z^{-1})u(i) \right] \tag{13}$$

Introducing (1), (2) and (9) into (13) gives the <u>global error equation</u>

$$\varepsilon(i) = K(z^{-1})e(i) + L(z^{-1})s(i) \tag{14}$$

with

$$K(z^{-1}) = \frac{\mathcal{C}(z^{-1})}{C(z^{-1})} \; \frac{A(z^{-1})Q(z^{-1}) + z^{-k} B(z^{-1})P(z^{-1})}{\mathcal{A}(z^{-1})Q(z^{-1}) + z^{-k}\mathcal{B}(z^{-1})P(z^{-1})} \tag{15}$$

$$L(z^{-1}) = \frac{Q(z^{-1})}{C(z^{-1})} \; \frac{A(z^{-1})\mathcal{B}(z^{-1}) - \mathcal{A}(z^{-1})B(z^{-1})}{\mathcal{A}(z^{-1})Q(z^{-1}) + z^{-k}\mathcal{B}(z^{-1})P(z^{-1})} \tag{16}$$

Since s(i) and e(i) are uncorrelated,

$$E\{[\varepsilon(i)]^2\} = E\{[K(z^{-1})e(i)]^2\} + E\{[L(z^{-1})s(i)]^2\} \tag{17}$$

Assuming that the global error variance (17) is minimized in the model parameter space a_i, b_i, c_i, the second right-hand term of (17) reaches at minimum the value 0 for

$$A(z^{-1})\mathcal{B}(z^{-1}) = \mathcal{A}(z^{-1}) \, B(z^{-1}) \tag{18}$$

but the first right-hand term of (17) reaches at minimum a value different from zero. In order to determine the value, $K)z^{-1})$ is expanded into a power series, finite or infinite, with the first term equal 1

$$K(z^{-1}) = 1 + k_1 z^{-2} + \dots + k_m z^{-m} + \dots \quad (19)$$

Hence

$$\begin{aligned} \text{Min } E\{[K(z^{-1})]^2\} &= \text{Min } \lambda^2(1 + k_1^2 + k_2^2 + \dots + k_m^2 + \dots) = \\ &= \lambda^2 \end{aligned} \quad (20)$$

for

$$k_1 = k_2 = \dots = k_m = \dots = 0 \quad (21)$$

i.e. for

$$K(z^{-1}) = 1 \quad (22)$$

The following question need to be answered: Are the model parameters which minimize the mean-square error (i.e. which are the solution of (18) and (22) unique, and if so, are they equal to the corresponding system parameters?
In order to answer this question, two special cases are considered:

1. Active identification ($s(i) \neq 0$), noisless system ($e(i) = 0$).
   For this case the model parameters which minimize the mean-square error are given by the solution of (18). It is demonstrated in Appendix 1, that (18) has a unique solution $a_i = \alpha_i$, $b_i = \beta_i$ for all i, if and only if for each i exists at least one such j, that

$$\frac{\alpha_i}{\alpha_j} \neq \frac{\beta_i}{\beta_j}, \quad j = 0, 1, \dots i-1 \quad (23)$$

2. Passive identification ($s(i) = 0$, $e(i) \neq 0$).
   For this case the model parameters which minimize the mean-square error are given by the solution of (22)

$$\begin{aligned} &\mathcal{C}(z^{-1})\, A(z^{-1})\, Q(z^{-1}) + z^{-k}\mathcal{C}(z^{-1})\, B(z^{-1})\, P(z^{-1}) = \\ &C(z^{-1})\mathcal{A}(z^{-1})\, Q(z^{-1}) + z^{-k}\, C(z^{-1})\mathcal{B}(z^{-1})\, P(z^{-1}) \end{aligned} \quad (24)$$

If $2n + q < k$, it is possible to write (24) in the form of two independent polynomial equations

$$\mathcal{C}(z^{-1})\ A(z^{-1}) = C(z^{-1})\mathcal{A}(z^{-1}) \tag{25}$$

$$\mathcal{C}(z^{-1})\ B(z^{-1}) = C(z^{-1})\mathcal{B}(z^{-1}) \tag{26}$$

having the same structure as eq. (18), and so guaranteeing a unique solution $a_i = \alpha_i$, $b_i = \beta_i$ and $c_i = \gamma_i$ under similar, easy to fulfill conditions as for active indentification.

If $2n + q > k$, (24) is equivalent to a set of $2n + q$ or $2n + k + p$ linear equations with $3n + 1$ unknown, with a unique solution being a lucky chance rather than a rule.

For the general case of active identification of a noisy system, the conditions (23) are guaranteeing a unique solution for a_i and b_i, and this in conjunction with (22) automatically leads to the unique solution for c_i.

The global error approach is thus capable of giving a simple explanation of some of the well-known results of closed-loop identification practice.

2. The self-tuning controller problem

The self-tuning controller is analysed for the general case of a ML difference equation model

$$A(z^{-1})y(i) = z^{-k} B(z^{-1})u(i) + C(z^{-1})e(i) \tag{27}$$

all symbols being defined by (6), (10), (11) and (12). A more suitable form than (27) is the predictive model

$$\begin{aligned} & y(i+k) - G(z^{-1})y(i) - B(z^{-1}) F(z^{-1})u(i) = \\ & F(z^{-1})e(i+k) + [C(z^{-1})-1][F(z^{-1})e(i+k) - y(i+k)] \end{aligned} \tag{28}$$

derived in Appendix 2, with

$$F(z^{-1}) = 1 + f_1 z^{-1} + \ldots f_{k-1} z^{-(k-1)} \tag{29}$$

$$G(z^{-1}) = g_0 + g_1 z^{-1} + \ldots g_{n-1} z^{-(n-1)} \tag{30}$$

$$C(z^{-1}) = A(z^{-1}) F(z^{-1}) + z^{-k} G(z^{-1}) \tag{31}$$

Both models are presented by block-diagramms in Fig. 2.

2.1. Some interesting properties of the predictive model

It should be noticed that:

a. The predictive model may be used in an easy way to determine the k-step minimum variance prediction $\hat{y}(i+k|i)_{min}$. Because at time i the disturbances $F(z^{-1})e(i+k)$ are unknown, they are considered to be equal to their expected value i.e. equal zero. Hence (28) takes the form.

$$C(z^{-1})\hat{y}(i+k|i)_{min} - G(z^{-1})y(i) - B(z^{-1}) F(z^{-1})u(i) = 0 \tag{32}$$

b. The predictive model may be used in an easy way to determine the minimum-variance control law for y(i). For a setpoint equal zero it is sufficient to put into (32) $\hat{y}(i+k\ i)_{min} = 0$ to get the well-known control law

$$u(i) = -\frac{G(z^{-1})}{B(z^{-1}(F(z^{-1})} y(i) \tag{33}$$

c. The control-law parameters g_i, f_i, b_i, can be estimated using the LS method and the error equation given by the left-hand side of the predictive model

$$\varepsilon(i) = y(i+k) - G(z^{-1})y(i) - B(z^{-1})\ F(z^{-1})u(i) \tag{34}$$

In the special case of a LS plant $(C(z^{-1}) = 1)$ the estimates are unbiased because

$$\varepsilon(i) = F(z^{-1})e(i+k) \tag{35}$$

are uncorrelated neither with $G(z^{-1})y(i)$ nor with $B(z^{-1})\ F(z^{-1})u(i)$, and $E\{\varepsilon(i)\} = 0$. For the general case of a ML plant $(C(z^{-1}) \neq 1)$ the estimation is biased, the offending term being

$$[C(z^{-1}) - 1]\ F(z^{-1})e(i+k) - y(i+k)$$

on the right-hand side of (28). But when the LS estimation is conducted on the plant (29) regulated with the optimum controller (33), <u>the offending term</u> disappears because for the minimum variance control

$$y(i+k)_{min} = F(z^{-1})e(i+k) \tag{36}$$

This plausible argument will be made precise using the global error approach.

2.2. <u>The global error approach to the self-tuning regulator</u>

It is assumed, that the

1. The plant is described by the following predictor-type relation:

$$y(i+k) - \mathcal{G}(z^{-1})y(i) - \mathcal{B}(z^{-1})\mathcal{F}(z^{-1})u(i) = \mathcal{F}(z^{-1})e(i+k) + [\mathcal{C}(z^{-1})-1]\left[\mathcal{F}(z^{-1})e(i+k) - y(i+k)\right] \quad (37)$$

where

$$\mathcal{G}(z^{-1}) = \eta_0 + \eta_1 z^{-1} + \dots + \eta_{n-1} z^{-(n-1)} \quad (38)$$

$$\mathcal{B}(z^{-1}) = \beta_0 + \beta_1 z^{-1} + \dots + \beta_n z^{-n} \quad (39)$$

$$\mathcal{F}(z^{-1}) = 1 + \rho_1 z^{-1} + \dots + \rho_{k-1} z^{-(k-1)} \quad (40)$$

$$\mathcal{C}(z^{-1}) = 1 + \gamma_1 z^{-1} + \dots + \gamma_n z^{-n} \quad (41)$$

are polynomials of known structure but unknown parameters.

2. The plant is controlled by the regulator

$$u(i) = - \frac{\hat{G}_{i-1}(z^{-1})}{\hat{B}_{i-1}(z^{-1})\hat{F}_{i-1}(z^{-1})} \; y(i) \quad (42)$$

with the polynomials $\hat{G}_{i-1}(z^{-1})$, $\hat{B}_{i-1}(z^{-1})$ and $\hat{F}_{i-1}(z^{-1})$ having the same structure as the corresponding polynomials (38), (39) and (40), but their parameters are determined at the (i-1)-step of the LS recursive algorithm for the system (37).

3. The LS estimation minimizes the sum of squares for the error

$$\varepsilon(i) = y(i+k) - G_i(z^{-1})y(i) - B_i(z^{-1}) \, F_i(z^{-1})u(i) \quad (43)$$

with the polynomials $G_i(z^{-1})$, $B_i(z^{-1})$ and $F_i(z^{-1})$ having the same structure as the corresponding polynomials (38), (39) and (40).

Fig. 3. shows as block diagram of the system with the controller and identifier. For this diagramm the global error equation can be determined as

$$\varepsilon(i) = H(z^{-1})\,\mathcal{F}(z^{-1})\; e(i+k) \tag{44}$$

with

$$H(z^{-1}) = \frac{\mathcal{C}\hat{B}_{i-1}\hat{F}_{i-1} - \mathcal{C}\hat{B}_{i-1}\hat{F}_{i-1}G_i z^{-k} + \mathcal{C}B_i F_i \hat{G}_{i-1} z^{-k}}{\mathcal{C}\hat{B}_{i-1}\hat{F}_{i-1} - \hat{B}_{i-1}\hat{F}_{i-1}\; z^{-k} + \mathcal{B}\mathcal{F} G_{i-1} z^{-k}} \tag{45}$$

Since the free terms of the $H(z^{-1})$ nominator and denominator are equal, it may be written as a finite or infinite power series

$$H(z^{-1}) = 1 + h_1 z^{-1} + \ldots. + h_n z^{-n} + \ldots \tag{46}$$

The LS algorithm minimizes in the limit the expected value $E\{[\varepsilon(i)]^2\}$ by a proper choice of the regulator parameters of the $H(z^{-1})$ term in (44). In order to establish whether the estimated parameters correspond to the true minimum-variance regulator parameters, use is made of the linear nature of (44). It is obvious that $\varepsilon(i)$ is independent of the order in which the $\mathcal{F}(z^{-1})$ and $H(z^{-1})$ filtrations are performed /fig. 4/. Hence $H(z^{-1})$ chosen to minimize the output variance of the system from fig. 4a minimizes the output variance of the system from fig. 4b, minimizing at the same time the variance of $H(z^{-1})\; e(i+k)$.
This last variance is given by

$$E\{[H(z^{-1})e(i+k)]^2\} = \lambda^2\; (1 + h_1^2 + \ldots + h_n^2 + \ldots) \tag{47}$$

and reaches the minimum value λ^2 for

$$h_1 = h_2 = \ldots = h_n = .. = 0 \tag{48}$$

i.e. for

$$H(z^{-1}) = 1 \tag{49}$$

It is easy to see, that the regulator parameters which fulfil (49) are solutions of the polynomial equation

$$\hat{B}_{i-1}(z^{-1})\hat{F}_{i-1}(z^{-1})\,\mathcal{C}(z^{-1}) = \mathcal{B}(z^{-1})\,\mathcal{F}(z^{-1})\,\hat{G}_{i-1}(z^{-1}) \tag{49}$$

which is equivalent to a system of 2n+k-1 linear equations for the parameters of $(\hat{B}_{i-1}\hat{F}_{i-1})$ and $\hat{G}_{i-1}$, with 2n+k unknown. Hence for a unique solution of (49) one regulator parameter (e.g. b_0) must be known in advance. Because the structure of (49) is the same as the structure of (18), the unique solution of (49) for the optimum regulator is guaranteed under the mild conditions discussed in p.1.

3. The self-tuning predictor problem

3.1. Models for stationary time series

The stationary time series to be predicted k-steps ahead can be described by four types of models:

- the filter model

$$y(i) = \frac{C(z^{-1})}{A(z^{-1})} e(i) \tag{50}$$

with symbols defined by (6),(10) and (12), being the most popular although not the most convenient description.

- the first type predictive model

$$y(i+k) = F(z^{-1})e(i+k) + \frac{G(z^{-1})}{C(z^{-1})} y(i) \tag{51}$$

with symbols defined by (29), (30) and (31).

- the second type predictive model

$$\varepsilon_p(i+k) - G(z^{-1})\ \varepsilon_p(i) + A(z^{-1})\ F(z^{-1})\ \hat{y}(i+k|i) = F(z^{-1})e(i+k) +$$

$$+ \left[1 - \frac{1}{C(z^{-1})}\right]\left[A(z^{-1})F(z^{-1})\hat{y}(i+k|i) - G(z^{-1})\ \varepsilon_p(i)\right] \tag{52}$$

- the third type predictive model

$$y(i+k) - G(z^{-1})\ \varepsilon_p(i) + E(z^{-1})\hat{y}(i+k|i) = F(z^{-1})\ e(i+k) +$$

$$+ \left[1 - \frac{1}{C(z^{-1})}\right]\{[1 + E(z^{-1})]y(i+k|i) - G(z^{-1})\ \varepsilon_p(i)\} \tag{53}$$

with

$$A(z^{-1})F(z^{-1}) = 1 + E(z^{-1}) \tag{54}$$

$$E(z^{-1}) = 1 + e_1 z^{-1} + \ldots + e_{n+k-1} z^{-(n+k-1)} \tag{55}$$

The derivation of those models is presented in Appendix 3. In both (52) and (53) $\hat{y}(i+k|i)$ represents any k-step ahead prediction, not necessarily the optimum, and $\varepsilon_p(i)$ is the prediction error

$$\varepsilon_p(i) = y(i) - \hat{y}(i|i-k) \tag{56}$$

Thus any prediction $\hat{y}(i+k|i)$ is considered to be an input of the system generating the prediction error $\varepsilon_p(i)$ /fig. 5/. It can also be demonstrated that the $\hat{y}(i+k|i\ )$ prediction in (52) and (53) influences only the prediction error and does not influence the real outcome $y(i+k)$.

For the predictive model (51) it is easy to determine the minimum variance prediction

$$\hat{y}(i+k|i)_{opt} = y(i+k) - F(z^{-1})e(i+k) = \frac{G(z^{-1})}{C(z^{-1})}\ y(i) \tag{57}$$

or - introducing the optimum prediction error

$$\varepsilon_p(i)_{opt} = y(i) - \hat{y}(i|i-k)_{opt} \tag{58}$$

and using (31), it is possible to express (57) in the form

$$\hat{y}(i+k|i)_{opt} = \frac{G(z^{-1})}{A(z^{-1})F(z^{-1})}\ \varepsilon_p(i)_{opt} = \frac{G(z^{-1})}{1 + E(z^{-1})}\ \varepsilon_p(i)_{opt} \tag{59}$$

An interesting fact about (59) is that the right-hand side converges to the optimum prediciton $\hat{y}(i+k|i)_{opt}$ if the optimum prediction error $\varepsilon_p(i)_{opt}$ is replaced by any prediction error. To demonstrate this let us define some prediction $\hat{y}(i+k|i)$ by a relation similar to (59)

$$\hat{y}(i+k|i) = \frac{G(z^{-1})}{A(z^{-1})F(z^{-1})}\ [y(i) - \hat{y}(i|i-k)] \tag{60}$$

or

$$\hat{y}(i+k|i) = \left[1 + \frac{z^{-k}\,G(z^{-1})}{A(z^{-1})F(z^{-1})}\right] = \frac{G(z^{-1})}{A(z^{-1})F(z^{-1})}\; y(i) \qquad (61)$$

Taking into account (31) gives

$$\hat{y}(i+k|i) = \frac{G(z^{-1})}{C(z^{-1})}\; y(i)$$

which according to (57) represents the optimum prediction.

Another interesting property of the second and third type predictive models are the similarities with the predictive model for the self-tuning regulator:

1. Their left-hand side is a linear function of those polynomials which are necessary to build the optimum prediction algorithm (59). This suggests the possibility of LS recursive estimation of the predictor parameters.

2. The bias-causing right-hand side term of the models (52) and (53) disappear when the prediction is done accordingly to the optimum prediction algorithm (59).

This plausible argument in favour of an adaptive prediction algorithm based on the LS estimation can once again be made precise with the help of the global error concept.

3.2. The global error approach to the self-tuning prediction

It is assumed that

1. The time series is generated by a system described with a third-type predictive relation

$$y(i+k) - \mathcal{G}(z^{-1})\ \varepsilon_p(i) + \mathcal{E}(z^{-1})\hat{y}(i+k|i) = \mathcal{F}(z^{-1})e(i+k) +$$

$$+ \left[1 - \frac{1}{\mathcal{C}(z^{-1})}\right]\left\{\left[1 + \mathcal{E}(z^{-1})\right]\ \hat{y}(i+k|i) - \mathcal{G}(z^{-1})\ \varepsilon_p(i)\right\} \qquad (62)$$

where $\mathcal{G}(z^{-1})$, $\mathcal{F}(z^{-1})$ and $\mathcal{C}(z^{-1})$ are polynomials defined by (38), (40) and (41), and

$$\mathcal{E}(z^{-1}) = 1 + \varepsilon_1 z^{-1} + \dots + \varepsilon_{n+k-1} z^{-(n+k-1)} \qquad (63)$$

The structure of all those polynomials is known, but their parameters are unknown.

2. The prediction is performed according to the algorithm

$$\hat{y}(i+k|i) = \frac{\hat{G}_{i-1}(z^{-1})}{1 + \hat{E}_{i-1}(z^{-1})}\quad \varepsilon_p(i+k) \qquad (64)$$

where $\hat{G}_i(z^{-1})$ and $\hat{E}_i(z^{-1})$ have the same structure as correspondingly $\mathcal{G}(z^{-1})$ and $\mathcal{E}(z^{-1})$, and their parameters are determined at the i-step of the LS recursive algorithm for system (62).

3. The LS estimation minimizes the sum of squares for the error

$$\varepsilon(i) = y(i+k) - G_i(z^{-1})\ \varepsilon_p(i) + E_i(z^{-1})\hat{y}(i+k|i) \qquad (65)$$

Fig. 6. represents the block diagram of the system with adaptive predictor. From this diagram the global error can be expressed as

$$\varepsilon(i) = M(z^{-1})\ \mathcal{F}(z^{-1})e(i+k) \qquad (66)$$

where

$$M(z^{-1}) = \frac{(1 + \hat{E}_{i-1}) + \mathcal{C}\hat{G}_{i-1}(1 + E_i)z^{-k} - \mathcal{C}G_i(1 + \hat{E}_{i-1})z^{-k}}{\mathcal{C}(1 + \hat{E}_{i-1}) + (1 + \mathcal{E})\hat{G}_{i-1}z^{-k} - (1 + \hat{E}_{i-1})\mathcal{G}z^{-k}} \qquad (67)$$

which can be expanded as

$$M(z^{-1}) = 1 + m_1 z^{-1} + \dots + m_i z^{-i} + \dots \tag{68}$$

By a similar argument as for the self-tuning regulator, minimizing $E\{[\varepsilon(i)]^2\}$ in the predictor parameters space gives

$$M(z^{-1}) = 1 \tag{69}$$

which is equivalent to /see (67) /

$$[1 + \quad (z^{-1})]\, \hat{G}_{i-1}(z^{-1}) = [1 + \hat{E}_{i-1}(z^{-1})]\, \mathcal{G}(z^{-1}) \tag{70}$$

This polynomial equation represents 2n+k-1 linear equations for 2n+k-1 unknown, which are the predictor parameters. Its structure is the same as for eq. (18). Hence it gives under similar mild conditions a unique solution for these parameters, which correspond to the optimum prediction parameters.

4. Conclusions

It has been demonstrated that the global error approach presents a straightforward way to determine the mean-square convergence conditions for a number of complicated estimation problems.

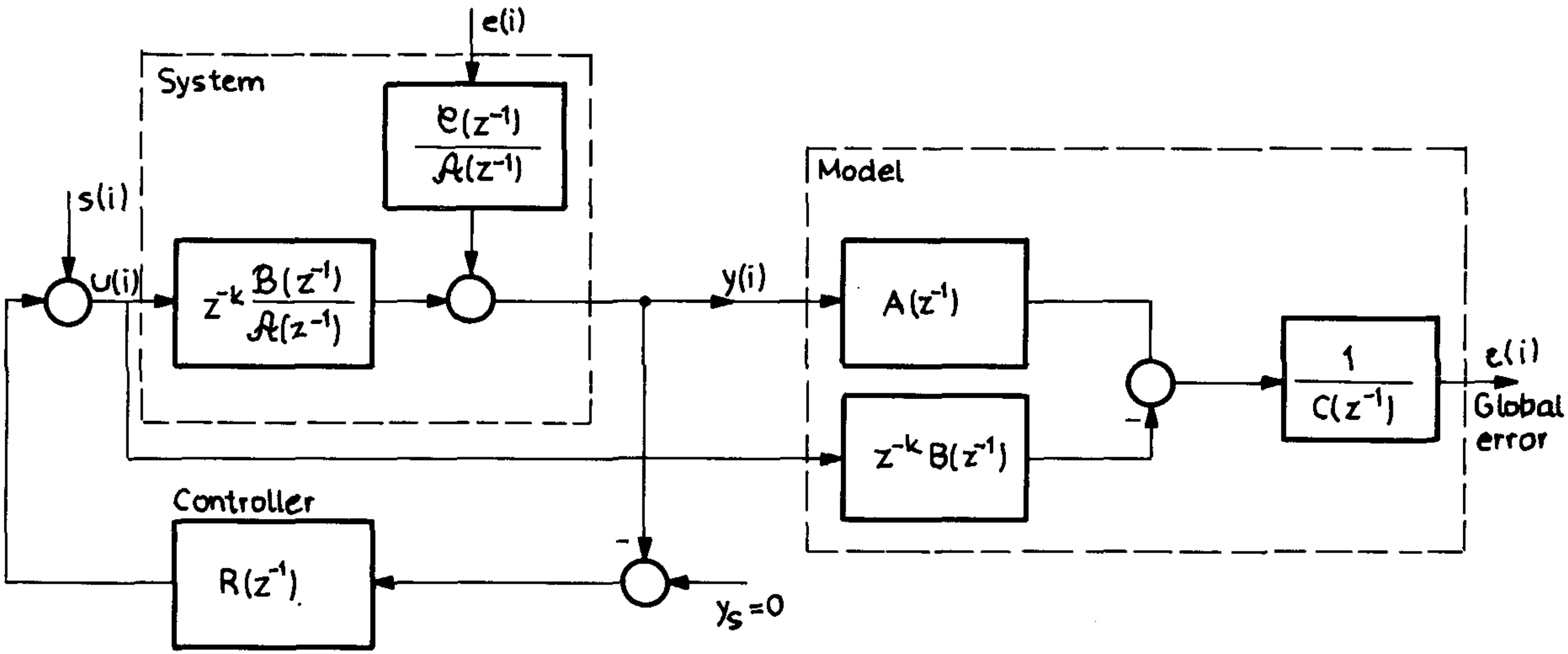

Fig. 1. Block diagram of closed loop identification of a ML plant

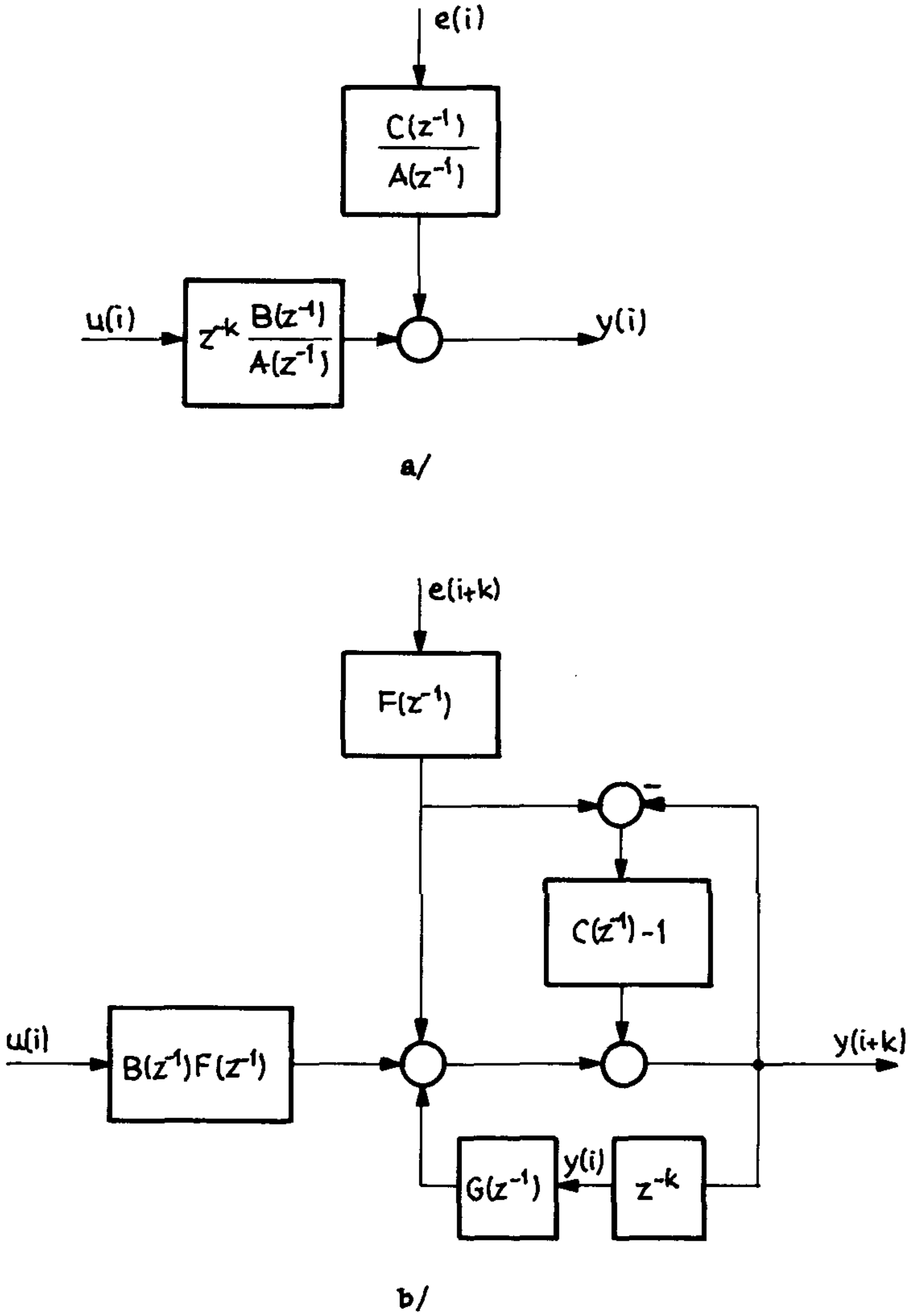

Fig. 2. Difference equation model a/ and predictive model b/ for a ML plant

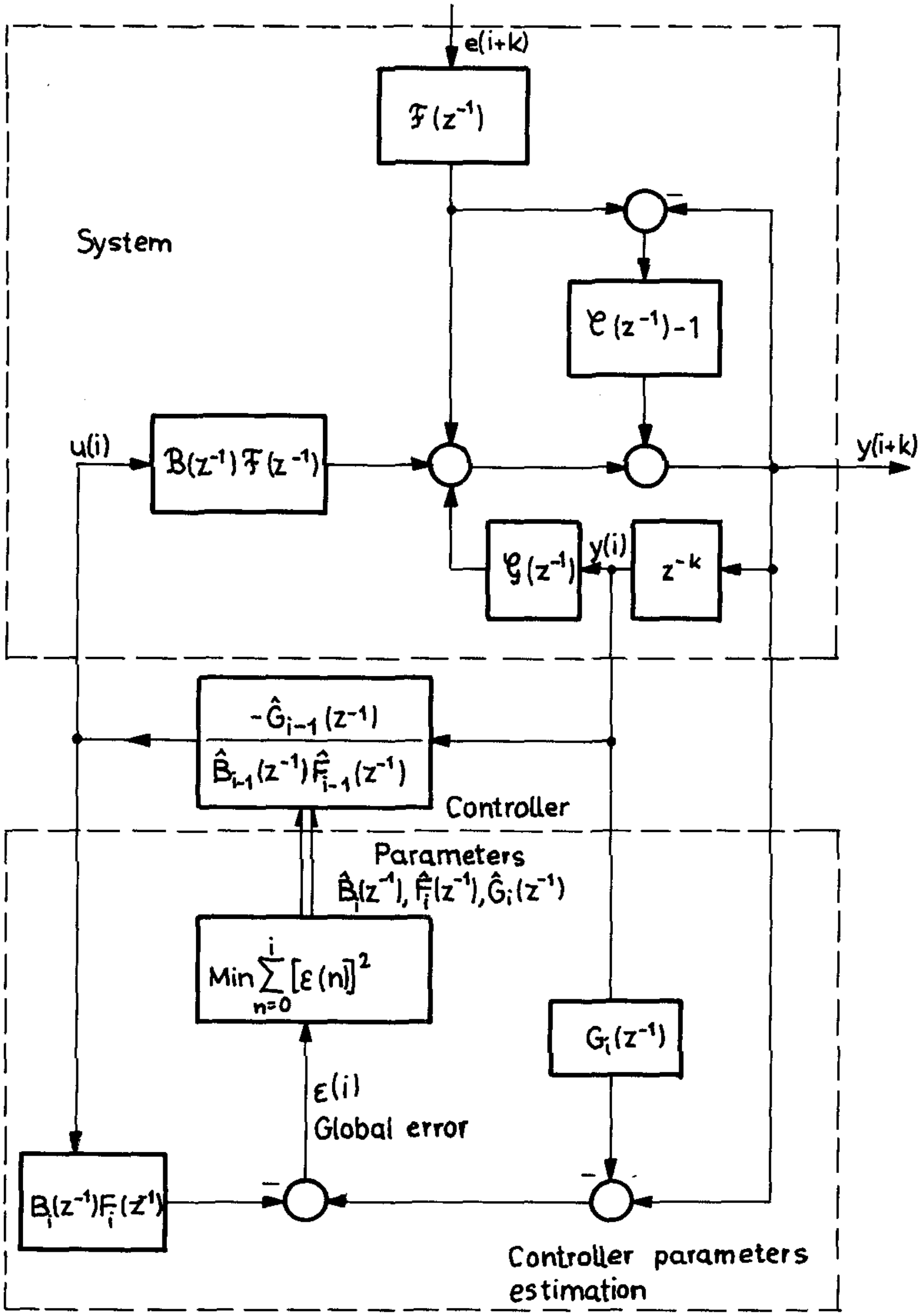

Fig. 3. Block diagram of the self-tuning control system

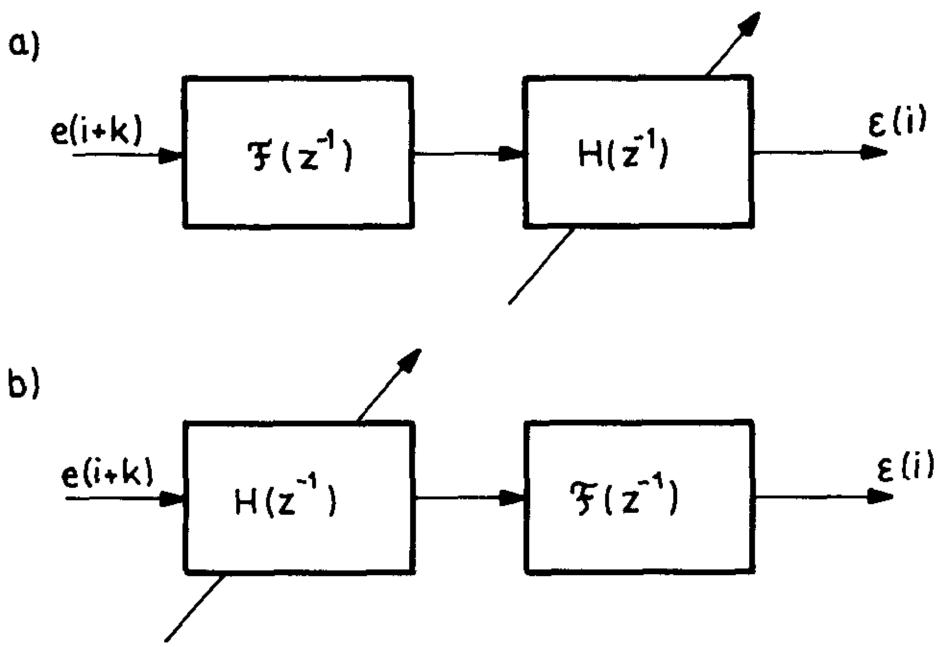

Fig. 4. Two equivalent representations of the global error

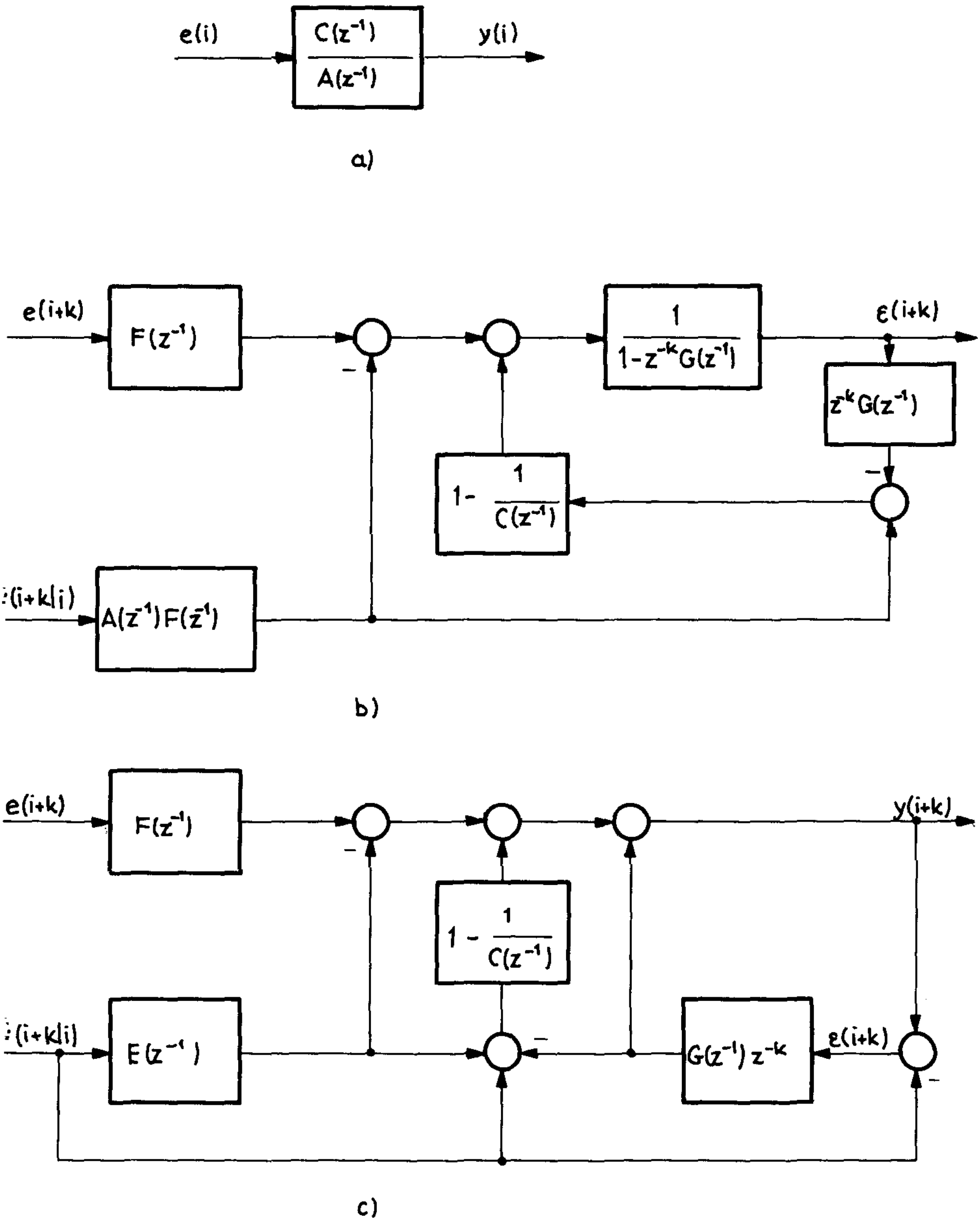

Fig. 5. Block diagrams for time series models: a/filter model, b/second type predictive model, c/third type predictive model

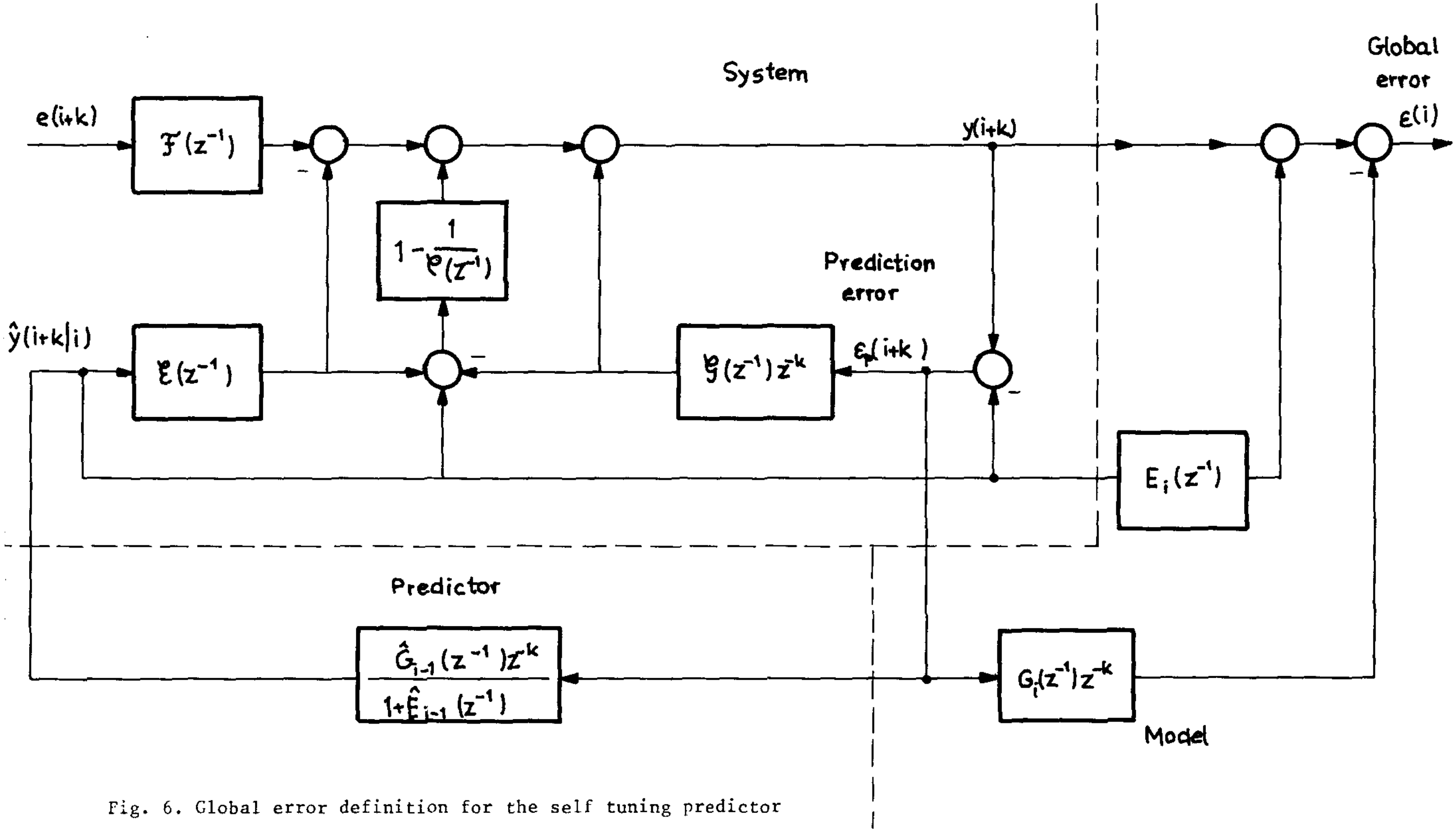

Fig. 6. Global error definition for the self tuning predictor

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Appendix 1

The polynomial equation (18)

$$A(z^{-1})\mathcal{B}(z^{-1}) = \mathcal{A}(z^{-1})B(z^{-1})$$

or

$$(1 + a_1 z^{-1} + \ldots + a_n z^{-n}) \quad (\beta_0 + \beta_1 z^{-1} + \ldots + \beta_n z^{-n}) =$$

$$= (1 + \alpha_1 z^{-1} + \ldots + \alpha_n z^{-n}) \quad (b_0 + b_1 z^{-1} + \ldots + b_n z^{-n}) \tag{A1}$$

is equivalent to 2n+1 linear equations with 2n+1 unknown a_i and b_i, which may be written in the following form

$$\begin{bmatrix} 1 & 0 & 0 & \ldots 0 & 0 & 0 & \ldots 0 \\ \alpha_1 & 1 & 0 & \ldots 0 & -\beta_0 & 0 & \ldots 0 \\ \alpha_2 & \alpha_1 & 1 & \ldots 0 & -\beta_1 & -\beta_0 & \ldots 0 \\ \vdots & & & & & & \\ \alpha_n & \alpha_{n-1} & \alpha_{n-2} & \ldots 1 & -\beta_{n-1} & -\beta_{n-2} & \ldots -\beta_0 \\ 0 & \alpha_n & \alpha_{n-1} & \ldots \alpha_1 & -\beta_n & -\beta_{n-1} & \ldots -\beta_1 \\ 0 & 0 & \alpha_n & \ldots \alpha_2 & 0 & -\beta_n & \ldots -\beta_2 \\ \vdots & & & & & & \\ 0 & 0 & 0 & \ldots \alpha_n & 0 & 0 & \ldots -\beta_n \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_n \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{A2}$$

The following theorem holds:

Eq. (A2) has a unique solution

$$a_i = \alpha_i \quad , \quad i=1, \ldots n$$
$$b_i = \beta_i \quad , \quad i=0, \ldots n$$

if and only if for each i exists at least one such j, that

$$\frac{\alpha_i}{\alpha_j} \neq \frac{\beta_i}{\beta_j} \quad , \quad j=0,1,\ldots i-1 \tag{A3}$$

The proof is by complete induction:

a. for n=1 (A1) has the form

$$(1 + a_1 z^{-1})\ (\beta_0 + \beta_1 z^{-1}) = (1 + \alpha_1 z^{-1})\ (b_0 + b_1 z^{-1})$$

Hence $b_o = \beta_0$

and $\frac{b_1}{\beta_1}\ (\beta_0\alpha_1 - \beta_1) = \beta_0\ \alpha_1 - \beta_1$

which gives $b_1 = \beta_1$ if and only if

$$\frac{\alpha_1}{1} \neq \frac{\beta_1}{\beta_0}$$

b. Assuming that for n=i the polynomial equation (A1) has the solution

$$a_j = \alpha_j \quad , \quad j=1, \ldots i$$
$$b_j = \beta_j \quad , \quad j=0, \ldots i$$

the equation (A1) for n=i+1

$$(1+a_1 z^{-1} +\ldots+ a_i z^{-i}+a_{i+1} z^{-(i+1)})\ (\beta_o+\beta_1 z^{-1} +\ldots+ \beta_i z^{-i}+\beta_{i+1} z^{-(i+1)}) =$$

$$= (1+\alpha_1 z^{-1} +\ldots+ \alpha_i z^{-i}+\alpha_{i+1} z^{-(i+1)})\ (b_0+b_1 z^{-1} +\ldots+ b_i z^{-i}+b_{i+1} z^{-(i+1)})$$

gives

$$(a_{i+1} - \alpha_{i+1})\ (\beta_0 + \beta_1 z^{-1} + \ldots + \beta_i z^{-i}) +$$

$$+ (\beta_{i+1} + b_{i+1})\ (1 + \alpha_1 z^{-2} + \ldots + \alpha_i z^{-1}) + (a_{i+1}\beta_{i+1}-\alpha_{i+1}b_{i+1})z^{-(i+1)} = 0$$

or

$$(a_{i+1} - \alpha_{i+1})\beta_j = (b_{i+1} - \beta_{i+1})\alpha_j \quad , \quad j=0,\ldots i$$

$$a_{i+1}\beta_{i+1} = \alpha_{i+1}b_{i+1}$$

which results in

$$\frac{b_{i+1}}{\beta_{i+1}} (\alpha_{i+1}\beta_j - \alpha_j\beta_{i+1}) = \alpha_{i+1}\beta_j - \alpha_j\beta_{i+1} \tag{A4}$$

$$j = 0,\ldots i$$

Eq. (A4) has the unique solution $b_{i+1} = \beta_{i+1}$ if and only if at least for one j

$$\frac{\alpha_{i+1}}{\alpha_j} \neq \frac{\beta_{i+1}}{\beta_j}$$

Appendix 2

Multiplying both sides of (27) by $F(z^{-1})$ gives

$$A(z^{-1})F(z^{-1})y(i) = z^{-k} B(z^{-1})F(z^{-1})u(i) + C(z^{-1})F(z^{-1})e(i) \tag{A5}$$

which - considering (31) may be written as

$$[C(z^{-1}) - z^{-k}G(z^{-1})]y(i) = z^{-k} B(z^{-1})F(z^{-1})u(i) + C(z^{-1})F(z^{-1})e(i) \tag{A6}$$

Multiplying both sides of (A6) by z^k, adding and subtracting from the right-hand side of A6 the term $F(z^{-1})\, e(i+k)$ gives the predictive model

$$y(i+k) - G(z^{-1})y(i) - B(z^{-1})F(z^{-1})u(i) = F(z^{-1})e(i+k) +$$

$$+ [C(z^{-1}) - 1]\, [F(z^{-1})\, e(i+k) - y(i+k)] \tag{A7}$$

Appendix 3

1. Introducing (31) into (50) gives

$$y(i) = \frac{C(z^{-1})}{A(z^{-1})} e(i) = F(z^{-1})e(i) + z^{-k} \frac{G(z^{-1})}{A(z^{-1})} e(i) \tag{A8}$$

Hence

$$y(i+k) = F(z^{-1})e(i+k) + \frac{G(z^{-1})}{A(z^{-1})} e(i) \tag{A9}$$

Since from (50)

$$e(i) = \frac{A(z^{-1})}{C(z^{-1})} y(i) \tag{A10}$$

it follows that

$$y(i+k) = F(z^{-1})e(i+k) + \frac{G(z^{-1})}{C(z^{-1})} y(i) \tag{A11}$$

2. From (56) and (A11)

$$\varepsilon_p(i+k) + \hat{y}(i+k|i) = F(z^{-1})e(i+k) + \frac{G(z^{-1})}{C(z^{-1})} [\varepsilon_p(i) + \hat{y}(i|i-k)] \tag{A12}$$

or

$$\varepsilon_p(i+k) + 1 - z^{-k} \frac{G(z^{-1})}{C(z^{-1})} \hat{y}(i+k|i) = F(z^{-1})e(i+k) + \frac{G(z^{-1})}{C(z^{-1})} \varepsilon_p(i) \tag{A13}$$

(A13) and (31) gives

$$\varepsilon_p(i+k) + \frac{A(z^{-1})F(z^{-1})}{C(z^{-1})} \hat{y}(i+k|i) = F(z^{-1})e(i+k) + \frac{G(z^{-1})}{C(z^{-1})} \varepsilon_p(i) \tag{A14}$$

adding to both sides of (A14) the term
$-G(z^{-1}) \varepsilon_p(i) + A(z^{-1})F(z^{-1})\hat{y}(i+k|i)$
gives the second type predictive model (52)

3. Introducing (54) into (52) gives the third type predictive model (53)

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