$H\sb \infty$ and mu controller design for an actuator

H_∞ AND μ CONTROLLER DESIGN FOR AN ACTUATOR

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Abstract. Robust linear controllers for a rotary electro-mechanical actuator are designed by $\mathcal{H}_\infty$-optimization and μ-synthesis. Meeting the tracking specifications in the presence of input saturation, persistent load disturbances, and Coulomb friction appears difficult: only for the system without saturation, the closed-loop response is satisfactory. The reason for this is believed to be threefold. First, some of the specifications are physically infeasible. Second, accounting for saturation and Coulomb friction in linear controller design is not straightforward. Third, the time domain specifications are not easily translated into the frequency domain. An elaborate trial-and-error procedure may improve the design, but is taxing for the designer.

Keywords. Control system synthesis, actuators, $\mathcal{H}_\infty$ control, μ-synthesis, saturation, Coulomb friction.

1. INTRODUCTION

Robust Performance (RP) is a major issue in control system design. This means that the controller must stabilize the system and meet the performance objectives in the presence of model uncertainties. Several robust controller design methods can be employed. Two options are $\mathcal{H}_\infty$-optimization and μ-synthesis, provided the system model is linear and the uncertainties are $\mathcal{H}_\infty$-norm-bounded. Linear systems hardly occur in practice and it is of interest to know how far one can go in achieving RP for nonlinear systems using linear design techniques. These are in general easier to understand and to apply than nonlinear techniques.

Control of a rotary electro-mechanical actuator serves to illustrate $\mathcal{H}_\infty$-optimization and μ-synthesis. This was proposed as a benchmark problem by Kiendl and Rüger (1993). The system exhibits two frequently occurring, highly nonlinear phenomena: Coulomb friction and control input saturation. Special measures are taken to account for them in the linear design approaches. The goal of the paper is *not* to search for the best performing controller for the benchmark problem, but rather to illustrate the (im)possibilities of the two robust controller design methods, to compare them, and to illustrate some practical problems. To improve RP, nonlinear control is probably the way to go.

Section 2 summarizes the formulation of robust control problems for linear systems. The benchmark problem is discussed in Section 3. Section 4 considers controller design and closed-loop control system evaluation. Starting with a simplified control problem, design requirements are added step-by-step. Finally, in Section 5 the main findings with respect to the controller design methods are discussed and more specific conclusions for the actuator control problem are drawn. For a detailed treatment, see (Van de Wal, 1995).

2. $\mathcal{H}_\infty$-OPTIMIZATION AND μ-SYNTHESIS

The fundamentals of the two robust controller design methods are discussed. The standard set-up for uncertain, linear, time-invariant control systems is shown in Fig. 1.

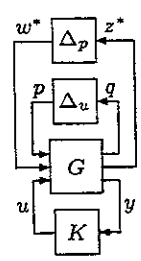

Fig. 1 Standard control system set-up

G represents the generalized plant, including the nominal plant model and weighting functions reflecting performance specifications and uncertainty characterizations. K is the controller. Model uncertainties are represented by Δ_u. A fictitious uncertainty block Δ_p accounts for performance specifications and together with Δ_u it makes up $\Delta := \text{diag}(\Delta_u, \Delta_p)$. It is assumed, that Δ_u and Δ_p are stable and that they are scaled so $\|\Delta_u\|_\infty \le 1$ and $\|\Delta_p\|_\infty \le 1$. The $\mathcal{H}_\infty$-norm of a Transfer Function Matrix (TFM) T is defined as:

$$\|T\|_\infty := \sup_\omega \bar{\sigma}(T(j\omega)). \quad (1)$$

The inputs to G are: the output p of the uncertainty block Δ_u, the exogenous input w^* (disturbances, measurement noise, reference signals), and the control input u. The outputs of G are: the input q to Δ_u, the control objectives z^* (formulated so z^* is ideally zero), and the measurements y. By closing G with K, the generalized closed-loop M is formed, which relates $z := \binom{q}{z^*}$ and $w := \binom{p}{w^*}$ according to $z = Mw$

The structured singular value $\mu_{\mathbf{\Delta}}(T)$ of a complex-valued constant matrix T with respect to a block structure $\mathbf{\Delta}$ is defined as:

$$\mu_{\mathbf{\Delta}}(T) := \frac{1}{\min\{\bar{\sigma}(\Delta) : \Delta \in \mathbf{\Delta},\ \det(I - T\Delta) = 0\}}, \quad (2)$$

unless no $\Delta \in \mathbf{\Delta}$ makes $(I - T\Delta)$ singular, in which case $\mu_{\mathbf{\Delta}}(T) := 0$. For the frequency-dependent $T(j\omega)$, $\sup_\omega \mu_{\mathbf{\Delta}}(T(j\omega))$ is denoted $\|T\|_{\mathbf{\Delta}}$

The goal of $\mathcal{H}_\infty$-optimization and μ-synthesis is to compute a stabilizing K minimizing $\|M\|_\infty$ and $\|M\|_{\mathbf{\Delta}}$ respectively. The MATLAB μ-Analysis and Synthesis Toolbox (Balas *et al.*, 1991) is employed, relying on a state-space representation of G:

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ y &= C_2 x + D_{21} w + D_{22} u, \end{aligned} \quad (3)$$

where x is the state. G must be proper and fulfill several standard $\mathcal{H}_\infty$ design assumptions (Glover and Doyle, 1988). For more information on $\mathcal{H}_\infty$-optimization and μ-synthesis, see, *e.g.*, (Doyle, 1982; Doyle *et al.*, 1992; Zhou *et al.*, 1996).

3 ACTUATOR CONTROL PROBLEM

The system to be controlled and the performance specifications are described, see also (Kiendl and Rüger, 1993). A model of the rotary electro-mechanical actuator is presented in Fig. 2. The signals have the following meaning: u is the commanded input, x_1 the position, x_2 the angular velocity, M_l a persistent load disturbance, and M_f a Coulomb friction force

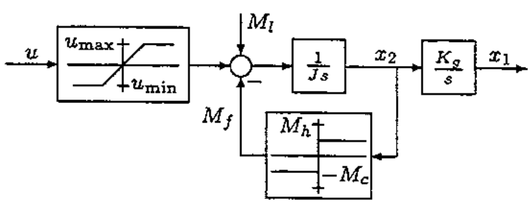

Fig 2. Model of a rotary electro-mechanical actuator

The model neglects the dynamics of the electrical motor, ignores the load dynamics, and includes no viscous friction. The Coulomb friction model accounts for both "slip" (M_c) and "stick" (M_h). The model parameters are partially uncertain and presented in Table 1

Table 1. Actuator model parameters

Parameter	Symbol	Value	Unit
moment of inertia	J	400	kg mm^2
conversion factor	K_g	7.04	mm/2πrad
static load	M_l	1.5	Nm
slip friction	M_c	0.3, . . , 0.8	Nm
stick friction	M_h	$2M_c \leq M_h \leq 3M_c$	Nm
maximal input	$u_{\max}$	4.0	Nm
minimal input	$u_{\min}$	-4.0	Nm

The main task is to make x_1 follow a desired step-wise change in position of 3.5 [mm], for which the requirements are as follows:

(1) the settling time T_{98} (for a 2% band) must be less then 50 [ms],
(2) the final position accuracy must be better than 0.04 [mm],
(3) overshoot is not permitted.

The system should also fulfill requirements (1) and (2) for step-wise position changes of small magnitude, *e.g.*, 0.1 [mm], and of large magnitude, *e.g.*, from -7.5 [mm] to 7.5 [mm]. The time between two desired position changes is at least 60 [ms]

Comment: The following illustrates that fulfilling the requirements for step-wise changes from -7.5 [mm] to 7.5 [mm] is impossible. If, from standstill, the maximum input $u_{\max}$ is applied to the system with $M_l = M_f = 0$, the maximum displacement within T_{98} equals $\frac{K_g}{2J} u_{\max} (T_{98})^2 \approx 14$ [mm]. For this reason, the original specification is replaced by the following one: the system should fulfill requirements (1) and (2) for step-wise changes from -6 [mm] to 6 [mm].

It remains doubtful if the specifications for large step-wise changes can be met, as will be shown next. The fastest way to move x_1 from 0 to the desired position w^* within T_{98} (without overshoot) is to use bang-bang control. Suppose $M_l = 1.5$, $M_f = M_c \operatorname{sign}(x_2)$, $M_c = 0.8$, and $w^* > 0$. Now, w^* is maximally 6.8 [mm]. In case $w^* < 0$, $|w^*|$ is maximally 4.7 [mm]. The difference is due to M_l. So, meeting the requirements for step-wise changes larger than 4.7 [mm] may be hard, even if overshoot is allowed.

4 CONTROLLER DESIGN AND EVALUATION

$\mathcal{H}_\infty$- and μ-based controllers are designed and evaluated, augmenting the control problem step-by-step. First, a controller with one measurement y is considered for the system without Coulomb friction M_f, without load M_l, and without saturation (Section 4.1). Second, input saturation is addressed in the controller design and simulation (Section 4.2). This is followed by considering the constant load and Coulomb friction, but no saturation (Section 4.3). Finally, Section 4.4 is devoted to the complete problem with controllers using two and three measurements

4.1 Design and evaluation for nominal tracking

To gain insight into the problem, the simplified version in Fig. 3 is studied first. There is no Δ_u block and $\Delta = \Delta_p$ is a scalar block without structure. Hence, μ-synthesis and $\mathcal{H}_\infty$-optimization boil down to the same. The generalized closed-loop $M = W_y S$, with $S = (1 + PK)^{-1}$ the sensitivity function. Nominal Performance (NP) is said to be satisfactory if $|S| < |S_{spec}| = |W_y^{-1}|$, or, equivalently, if $\|M\|_\infty < 1$

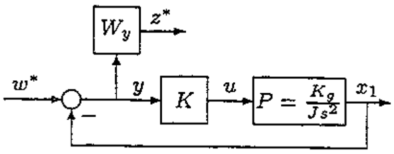

Fig 3 Control system with tracking error weight

To achieve the time-domain specifications, S_{spec} must be chosen suitably. To limit the order of G, and therefore the order of K, first order S_{spec}'s are used:

$$S_{spec,1} = \kappa \frac{s}{s+a}, \qquad S_{spec,2} = \kappa \frac{s+a_2}{s+a_1} \qquad (4)$$

For $S_{spec,1}$ the T_{98} specification is met without overshoot if $a \geq -\ln(0.02/\kappa)/T_{98} \approx 78$ ($a = 100$). Zero steady-state errors are imposed by $S_{spec,1}(0) = 0$ By using $S_{spec,2}$ the requirements at low frequencies are less demanding: a_1 is set to meet the T_{98} specification ($a_1 = 100$), while a_2 is set to obtain a final accuracy of x_1 better then 0.04 [mm] for a step-wise change of 12 [mm] ($a_2 = \frac{0.04 a_1}{12\kappa}$).

Two standard $\mathcal{H}_\infty$ design assumptions are violated: $D_{12} = 0$, so it lacks full column rank and $G(s)$ has at least two poles at $s = 0$, so $\left[\begin{smallmatrix} A-\mathrm{j}\omega I & B_1 \\ C_2 & D_{21} \end{smallmatrix}\right]$ lacks full row rank at $\omega = 0$ To solve the first problem, a small weight on u is imposed: $W_u = 10^{-8}$, yielding $D_{12} = [0\ 10^{-8}]^T$. The second problem is solved by using a "bilinear transformation", see, *e.g.*, (Chiang and Safonov, 1992, Chapter 1) The $\mathrm{j}\omega$-axis eigenvalues of A are shifted to the $(-\alpha + \mathrm{j}\omega)$-axis by modifying A to $A - \alpha I$ The positive real number α is chosen so the plant behavior is not significantly changed in the frequency range of interest between 0.01 and 1000 [rad/s] ($\alpha = 10^{-6}$). An $\mathcal{H}_\infty$ controller can now be designed. Its system matrix A_K is transformed back to $A_K + \alpha I$, giving a sub-optimal solution for $\alpha = 0$.

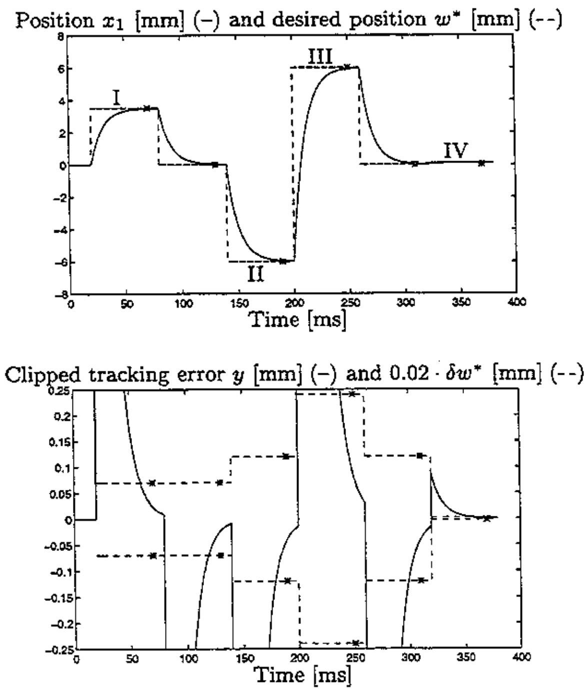

Fig. 4 Closed-loop behavior for the design with Fig. 3

For $\kappa = 2$, NP is easy to achieve The controllers make $\bar{\sigma}(M)$ flat in the frequency range of interest and $\|M\|_\infty = 0.51$ So, κ could be decreased to make the S_{spec}'s tighter. For the strictly proper third order controller with $S_{spec,1}$, Fig 4 displays closed-loop simulation results The desired trajectory w^* incorporates all step-wise changes of interest, so all time domain specifications in Section 3 can be checked. w^* takes four values other than zero: w^*=3 5 [mm] (I), w^*=−6 [mm] (II), w^*=6 [mm] (III), and w^*=0 1 [mm] (IV) The time between two changes in w^* is always 60 [ms]. Within the settling time, x_1 must be and remain within 2% of the step size of interest The "target zone" for the tracking error y is indicated by the dashed line in the lower plot of Fig. 4 ($0.02 \cdot \delta w^*$). The times at which x_1 must be and remain inside this zone are indicated by "*"

The three time domain goals are achieved. Due to $S(0) = 0$, y asymptotically approaches zero Large control inputs u are required: when w^* changes, peaks of u with an order of magnitude of 10^3 [Nm] occur. If the plant input saturates for u larger than 4 [Nm], the control goals are not achieved anymore.

4.2 Design and evaluation for saturation

Two distinct ways to account for plant input saturation are studied In Section 4 2.1, the weighted commanded input u is included in z^*. In Section 4.2.2, the nonlinear saturation element is modeled via Δ_u. The latter approach makes Δ structured and μ-synthesis becomes worthwhile.

4.2.1. Weighting the controller output

Consider Fig. 5 with z_2^* an additional control objective compared to Fig. 3. $M = \left[\begin{smallmatrix} W_y S \\ W_u R \end{smallmatrix}\right]$, with the input sensitivity function $R = K(1+PK)^{-1}$. This is an NP control problem with a 1×2 unstructured Δ_p.

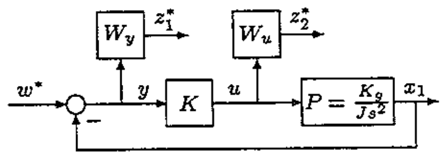

Fig. 5. Control system with tracking error weight and controller output weight

For the scalar S and R considered here, minimizing $\|M\|_\infty$ amounts to minimizing:

$$\sup_\omega (|W_y(\mathrm{j}\omega)S(\mathrm{j}\omega)|^2 + |W_u(\mathrm{j}\omega)R(\mathrm{j}\omega)|^2) \qquad (5)$$

The solution often has the equalizing property, *i.e.*, the frequency-dependent function whose peak value is minimized is a constant γ:

$$|W_y(\mathrm{j}\omega)S(\mathrm{j}\omega)|^2 + |W_u(\mathrm{j}\omega)R(\mathrm{j}\omega)|^2 = \gamma^2 \qquad (6)$$

So, for the optimal solution:

$$|S(\mathrm{j}\omega)| \leq \frac{\gamma}{|W_y(\mathrm{j}\omega)|}, \ |R(\mathrm{j}\omega)| \leq \frac{\gamma}{|W_u(\mathrm{j}\omega)|} \qquad (7)$$

By suitable W_y and W_u, S and R can be made small in appropriate frequency regions. Performance is satisfactory if $|S| < |S_{spec}| = |W_y^{-1}|$ and $|R| < |R_{spec}| = |W_u^{-1}|$. S is specified by $S_{spec,1}$ in (4) To avoid high frequencies in u, occurring for step-wise changes in w^* and causing plant input saturation, the following inverse weight is used:

$$R_{spec} = \zeta \frac{s+b}{s} \qquad (8)$$

Parameters ζ and b must be chosen so the time domain specifications are achieved under saturation. The frequency above which u is penalized most severely is indicated by b Because of the required settling time of 50 [ms], control signals of 20 [Hz] must be allowed and it seems reasonable to use $b = 2\pi \quad 100$ [rad/s]. Finding a suitable ζ is not straightforward, but based on trial-and-error

It is attempted to achieve performance by iteratively changing the parameters in $S_{spec,1}$ and R_{spec} With

$\zeta = 1100$, $a = 100$, and $\kappa = 3$ (instead of $\kappa = 2$), $\|M\|_\infty = 0.71 < 1$ and so the frequency domain specifications are met $|W_y(j\omega)S(j\omega)|$ dominates for $\omega < 100$ [rad/s] where W_y is large, while $|W_u(j\omega)R(j\omega)|$ dominates for $\omega > 100$ [rad/s] where W_u is large.

Simulation results for the main task $w^* = \pm 3.5$ [mm] including saturation are depicted in Fig. 6. The time domain specifications are met, except for a slight overshoot when returning from $x_1 = 3.5$ [mm] to $x_1 = 0$ [mm]. For step-wise changes of ± 0.1 [mm] all specifications are met, but for ± 6 and ± 12 [mm] the specifications are *not* met. Attempts to improve the response to step-wise changes of 6 [mm] (*e.g.*, by raising ζ in R_{spec}) cause deterioration of the response to smaller step-wise changes, in the sense that overshoot occurs

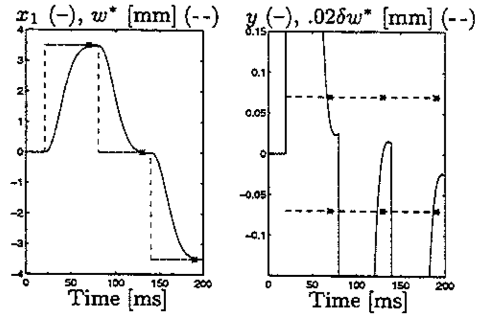

Fig. 6 Closed-loop behavior for the design with Fig. 5

4.2.2. Saturation as sector bounded uncertainty

In Fig. 7, the saturation element is modeled as the parallel connection of a gain 0.5 and a weighted ($W_u = 0.5$) uncertainty Δ_u (Chiang and Safonov, 1992, Chapter 1). Δ_u is a nonlinear operating mapping q into p, for which $\|\Delta_u q\|_2 \le \|q\|_2$. Because of this, Δ_u is called sector bounded with an $\mathcal{L}_2$-gain equal to one. The RP control problem in Fig. 7 has a 2×2 structured $\Delta = \mathrm{diag}(\Delta_u, \Delta_p)$. $S^{-1}_{spec,1}$ is used as W_y

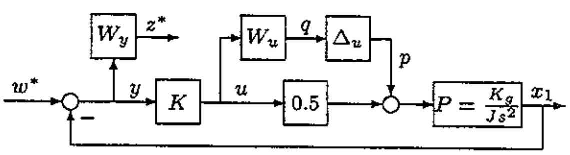

Fig. 7. Control system with tracking error weight and sector bounded plant input uncertainty

The D-K iteration approach to μ-synthesis is applied. Initially, a stabilizing K is computed by $\mathcal{H}_\infty$ optimization. With $\kappa = 10$ and $a = 150$, $\|M\|_\infty = 9.54$. For a good fit and a fast convergence, the order of the diagonal D-scale approximation should be high during D-K iteration. However, for implementation reasons the controller order is desirably low, and so the order of the fit should be low. Therefore, first order D-scale approximations are used for the first iteration steps (for higher orders, numerical problems occur during D-K iteration), while zeroth order approximations are used for the final (third) step. Like G, the controller K from the third iteration has order three.

The left plot of Fig. 8 shows the magnitude of this K. The right plot shows, that for successive designs $\mu_\Delta(M)$ is flattened across the frequency range. $\|M\|_\Delta$ decreases, while it shifts to higher frequencies. RP is not guaranteed for this controller, since $\|M\|_\Delta = 2.07$. Continuing the D-K iteration, it seems that $\|M\|_\Delta$ cannot be reduced further than about 1.33. Ultimately, $\mu_\Delta(M) = 1$ for the whole frequency range, except for a small bump for frequencies above 100 [rad/s]: $\|M\|_\Delta = 1.33$. However, for this continued D-K iteration numerical problems occur and the results cannot be relied upon.

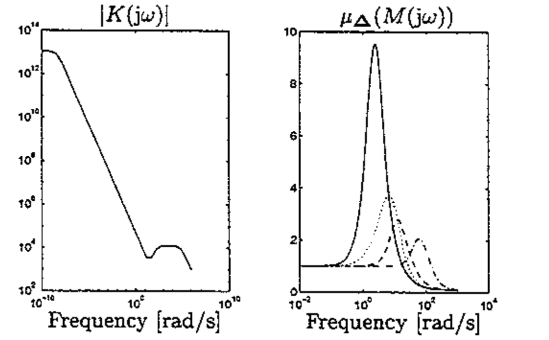

Fig. 8. *left:* magnitude of K; *right:* $\mu_\Delta(M)$: initial design (–), first (··), second (--), third iteration (–·)

In Fig. 9, some simulation results are depicted. The time domain specifications are *not* met, not even for small step-wise changes. It is remarkable, that for $w^* = \pm 3.5$ [mm] input saturation occurs for a relatively large time interval after a change in w^*

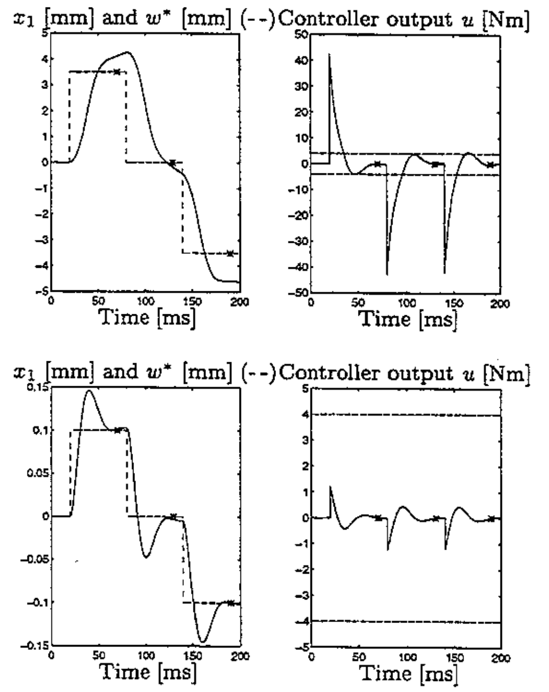

Fig. 9. Closed-loop behavior for the design with Fig. 7

4.3 Design and evaluation for load disturbance and Coulomb friction

The attention is focussed on achieving set-point tracking under load disturbance M_l and Coulomb friction M_f (see Fig. 2). In that case, the controller from Section 4.1 does *not* achieve the time domain requirements anymore, indicating the need to explicitly account for these phenomena. Saturation is assumed not to play a role in this section.

For the purpose of controller design, M_l and M_f have to be incorporated in the standard plant setting of Fig. 1. The static load disturbance is represented in the exogenous input w^*, while a filter V_1 is included in G to account for the nature of M_l. For a constant disturbance like M_l, this is done by choosing $V_1 = \rho_1 M_l/s$, with ρ_1 a design parameter

Incorporating the highly nonlinear Coulomb friction in the linear standard control system set-up is less straightforward: the friction characterization should be linear, possibly accompanied by a bounded nonlinear uncertainty However, Van der Linden and Lambrechts (1993) show, that this is impossible due to the discontinuity for $x_2 = 0$ The Coulomb friction is now modeled as an external disturbance by including it in w^* and including a filter V_2 in G. Unfortunately, the knowledge of the friction's feedback nature is lost in this way. To account for both slip ("step") and stick ("impulse"), V_2 is chosen as:

$$V_2 = \rho_2 \frac{M_c}{s} + \rho_3 M_h = \frac{\rho_2 M_c + \rho_3 M_h s}{s} \qquad (9)$$

The largest friction values $M_c = 0.8$ and $M_h = 3M_c$ are used. V_1 and V_2 are added to give V in Fig. 10. The corresponding NP control problem has a 2×1 unstructured block Δ_p

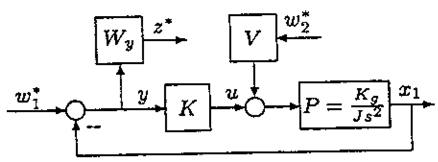

Fig. 10. Tracking error weight and load/friction weight

For controller design, $W_y = S^{-1}_{spec,1}$ is used with $\kappa = 2$ and $a = 100$. To meet the rank condition on D_{12}, a small weight on u is added ($W_u = 10^{-8}$). Finding good settings for the design parameters in V relies on trial-and-error To start with, ρ_1, ρ_2, and ρ_3 are set to guarantee a fast rejection of M_l, while limiting the displacement x_1. Studying various simulations, a compromise between tracking on the one hand and rejection of Coulomb friction and load disturbance on the other hand is made with $\rho_1 = \rho_2 = 10^7$ and $\rho_3 = 0$ A non-zero ρ_3 does not improve performance.

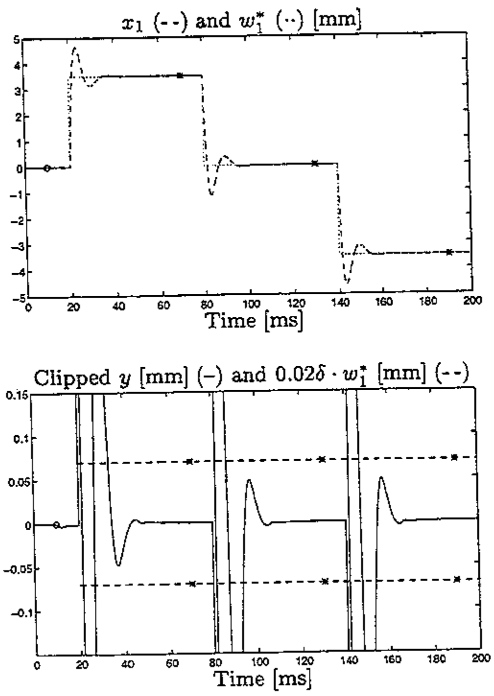

Fig. 11. Simulation for the design with Fig 10

With the computed controller, $\|M\|_\infty = 0.56$ and Fig 11 shows some simulation results. The symbol "o" for $t = 10$ [ms] indicates when M_l starts acting on the system. The third time domain requirement is *not* met, since overshoot occurs. The influence of M_l and M_f on the tracking error appeared to be marginal. Apparently, rejection of load disturbance and Coulomb friction is more restrictive for controller design than tracking specifications, *i.e.*, $|W_y PSV|$ dominates $|W_y S|$ To avoid overshoot, $W_y S$ (tracking) must be emphasized during controller design, which may be easier achieved with a different controller structure.

4.4 *Multi-input controllers*

It is attempted to improve performance by designing multi-input controllers First, a controller with *two* inputs as depicted in Fig. 12 will be studied. While the input for the previously studied controller was the difference between the desired and actual position (tracking error), now the desired position w_1^* and the actual position x_1 are used as separate inputs

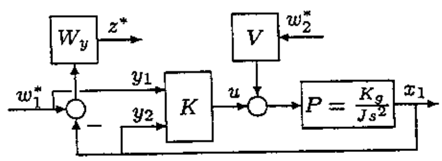

Fig. 12 Control system with two inputs y

To make D_{12} full column rank, a small weight is imposed on u. In addition, a small "measurement error" on x_1 is used to give D_{21} full row rank. The parameters of $S_{spec,1}$ and V are the same as in Section 4.3, except for ρ_1 and ρ_2 which are now set to one (the previous settings meet the frequency domain specifications, but give rise to unnecessarily large controller gains). An $\mathcal{H}_\infty$ optimization yields $\|M\|_\infty = 0.51$ and the time domain specifications are achieved, see Fig. 13. The influence of the static load disturbance and the Coulomb friction again appeared to be negligible

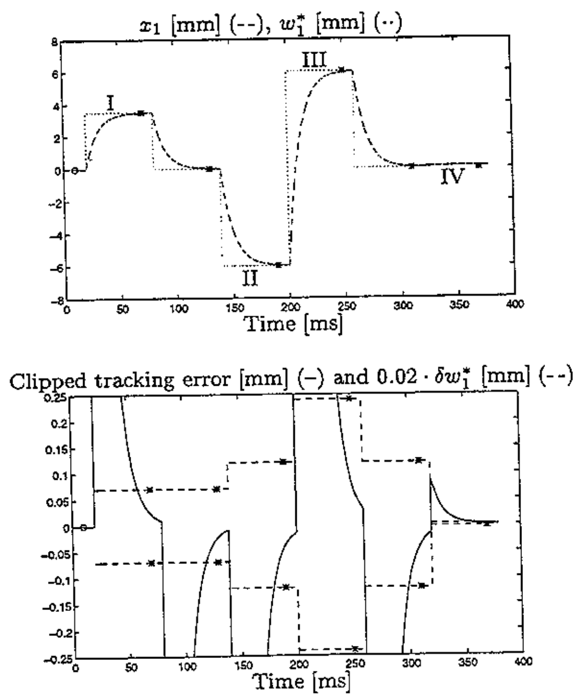

Fig. 13 Simulation for the design with Fig 12

These results are obtained without saturation. If controller design accounts for saturation by weighting u

(Section 4.2 1), a two-input controller is not successful A good response was only achieved for w_1^* with a magnitude of 0.1 [mm]. Therefore, saturation will next be accounted for by the sector bounded uncertainty proposed in Section 4.2.2

Consider Fig 14, where *three* signals are fed back to K: the desired position w_1^*, the real position x_1, and the output of the saturation block, *i.e.*, the plant input The RP problem related to Fig. 14 has a structured Δ made up of a scalar Δ_u and a 3×1 Δ_p. Using the same design parameters as earlier in this section and $W_u = 0.5$, a fourth order K is designed by μ-synthesis After two iterations, $\mu_\Delta(M) = 1$ for all considered frequencies The closed-loop behavior is only satisfactory for $w_1^* = \pm 0.1$ [mm] For $w_1^* = \pm 3.5$ [mm] or $w_1^* = \pm 6$ [mm], all time domain requirements are violated due to input saturation.

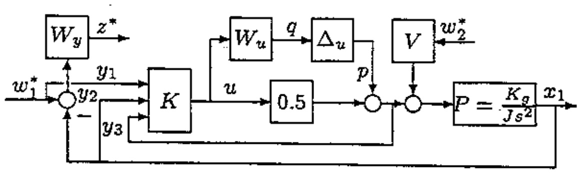

Fig. 14 Control system with three inputs y

To make the design more "robust" to saturation, W_u is raised to two, while the other design parameters are maintained. A three-input controller is computed with $\mu_\Delta(M(j\omega)) = 4$ for all frequencies. The response for the main task $w_1^* = \pm 3.5$ [mm] indeed improves, see Fig. 15, but the requirements are still not met. This response is the best from a number of designs in which both W_u and the parameters in V and $S_{spec,1}$ were changed iteratively. Figure 15 shows, that the controller output still saturates during large time intervals after changes in w_1^* For $w_1^* = \pm 0.1$ [mm] and $w_1^* = \pm 6$ [mm], the response is somewhat worse.

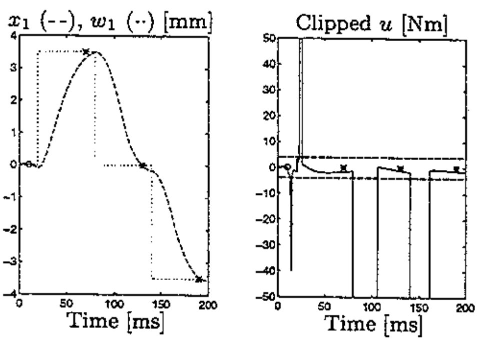

Fig. 15 Simulation for the design with Fig. 14

5. DISCUSSION

$\mathcal{H}_\infty$- and μ-based controller design methods were applied to an electro-mechanical actuator control problem The main findings are discussed below

Performance specifications in the time domain are not trivially translated into frequency domain equivalents. For some attempts, see (Franchek, 1996; Hu *et al.*, 1996). Therefore, it might be difficult to find suitable weighting functions and meeting the frequency domain specifications may not imply that the time domain specifications are also met

For the considered benchmark problem, performing μ-synthesis with μ-Tools is sometimes troublesome First, μ may increase for successive D-K iteration steps Second, numerical problems in solving the Riccati equations may occur during $\mathcal{H}_\infty$ optimization for the (D-)scaled system Moreover, a satisfactory D-scale approximation is not always possible due to numerical conditioning problems and a lower order fit must be used. The controller order was restricted by using a zero order fit in the final iteration, provided that the μ-value did not increase. An alternative is to apply order reduction to the final design

The design of a controller meeting the time domain specifications was not successful. Satisfactory tracking, load disturbance, and Coulomb friction was only achieved without saturation. Moreover, the controller must use the desired position and the measured one as two separate input signals. If the tracking error is the single controller input, tracking is only satisfactory without saturation, load disturbance, and Coulomb friction. Modifying design parameters or weighting filter types might improve closed-loop behavior.

Two nonlinearities played a role in the control problem. First, plant input saturation was handled by introducing a controller output weight, or by modeling the saturation element as an uncertainty Second, the Coulomb friction was modeled as an external disturbance, but this neglects its feedback nature.

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