Converses for Write-Unidirectional Memories

Research Report
ISSN 0167-9708
Coden: TEUEDE

Eindhoven
University of Technology
Netherlands

Faculty of Electrical Engineering

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EUT Report 89-E-220
ISBN 90-6144-220-6

May 1989

Eindhoven University of Technogy Research Reports
EINDHOVEN UNIVERSITY OF TECHNOLOGY

Faculty of Electrical Engineering
Eindhoven The Netherlands

ISSN 0167-9708 Coden : TEUEDE

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CIP-GEGEVENS KONINKLIJKE BIBLIOTHEEK, DEN HAAG

Willems, F.M.J.

Converses for write-unidirectional memories / by F.M.J. Willems. - Eindhoven : Eindhoven University of Technology, Faculty of Electrical Engineering. - Tab. - (EUT report, ISSN 0167-9708; 89-E-220)
Met lit. opg., reg.
ISBN 90-6144-220-6
SISO 520.6 UDC 681.327.68:519.72 NUGI 832
Trefw.: optische geheugens; informatietheorie.

Contents

CONVERSES FOR WRITE-UNIDIRECTIONAL MEMORIES

Frans M.J. Willems*

ABSTRACT : First we show that we can transform a WUM channel into a channel that behaves identical in even and odd cycles. For this derived channel we then prove that the average-error capacity cannot exceed $\log_2((1+\sqrt{5})/2) = 0.69424\cdots$. For the average-error capacity of the derived channel in the situation where both encoder and the decoder are uninformed, we find an even better upper bound (0.54588··). In this case however we assume that in all cycles the same code is used.

1. Introduction

In 1985, researchers at Philips Research Laboratories were interested in the capacity aspects of a magneto-optical recording system [1]. In this system a laser beam is used to heat up a spot on an optical disc. Depending on the orientation of the magnetic field, either a zero or a one will be written on this spot. The magnetic field is generated by an electromagnet. An optical effect makes it possible to retrieve the information stored in a spot.

A problem arises when we want to record information at high speed. The inductivity of the electromagnet will prevents us from reversing the current too often. Therefore the following strategy is proposed. Suppose a new disc contains only zeros. During the first cycle we can store information on the disc by changing some of the zeros into ones by switching the laser on and off, without having to change the polarity of the current through the electromagnet. In the second cycle we reverse the polarity and we restrict ourselves to writing only zeros, again by switching the laser on and off, and keeping the remaining components unchanged. In the third cycle we write only ones, etc.

An additional feature of the described recording system is that before each cycle the writer knows the state of the disc. This side information may be used to the increase efficiency of the coding process. The reader however is assumed not to be aware of the previous state of the disc.

Philips researchers developed simple (time-sharing) codes that achieved a rate of 0.5 bit per spot and were interested in codes with higher rates. In the "Applications"-

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session at the Seventh Symposium on Information Theory in the Benelux in 1986 Willems and Vinck [2] presented the first article in this field. They found a simple code with a rate of 0.51699 bit/spot, thereby beating the 0.5 barrier. Willems and Vinck noted that there was a connection between the magneto–optical recording channel and the Blackwell broadcast channel (see e.g. Gel'fand [3] and Pinsker [4]), and recognizing this, they stated (however without proof) that $\log_2((1+\sqrt{5})/2) = 0.69424$ is the average–error capacity of the magneto–optical recording channel.

In 1986 Borden [5] investigated a slightly different system which he named "Write–Unidirectional Memory (WUM)". Instead of zeros in even cycles and ones in odd cycles, in Borden's model the encoder is allowed in each cycle, after having inspected the (previous) state of the disc, to choose to write either zeros or ones. Borden showed that 0.69424 bit/spot is the capacity of the WUM–channel in the zero–error case.

Subsequently Simonyi [6] generalized the WUM–model by observing that both the writer and the reader could be either "informed" or "uninformed" of the previous state of the channel (disc). In this way he obtained four different models. He assumed that the current through the electromagnet is reversed at the beginning of each new cycle as in Willems and Vinck [1]. For the "classical" WUM–channel (encoder informed, decoder uninformed) Simonyi generalized the code of Willems and Vinck and found a code with a rate of 0.53254 bit/spot.

In this report we will give the weak converses for the four models that are described by Simonyi. We assume that the error probability concept is the average–error concept. The current trough the electromagnet is assumed to be reversed each cycle.

2. Definitions

Let $N \in 1,2,\cdots$. A WUM of block length N consists of N components $y^N := (y_1,y_2,\cdots,y_N)$. Each component y_n, $n \in \{1,2,\cdots,N\}$ may assume a value from $\{0,1\}$.

A decoder can inspect the components of the memory. An encoder is a device that can alter them.

A cycle (indexed by $k \in 1,2,\cdots$) is a time interval that starts when the k–th "message" is stored in the WUM and that ends when message k+1 is about to be stored. During odd cycles the encoder can decide to leave a component unchanged, we say the encoder writes a "?" or to write a "1". In an even cycle a component can remain unaltered, a "?" is written, or can be set equal to "0". The tables below give the updated value $y_n(k)$ of a WUM-component given its previous value $y_n(k-1)$ and the "input" $f_n(k)$.

k odd:

f(k)	y(k-1)	y(k)
?	0	0
?	1	1
1	0	1
1	1	1

k even:

f(k)	y(k-1)	y(k)
0	0	0
0	1	0
?	0	0
?	1	1

Table 1.
y(k) as a function of y(k-1) and the input f(k) for both odd and even k (n fixed).

At the beginning of cycle k we denote the state of the WUM by $y^N(k) = (y_1(k),y_2(k),\cdots,y_N(k))$. An information source generates the message $w(k) \in \{0,1,\cdots,M-1\}$. We assume that $\Pr\{W(k)=m\} = 1/M$ for $m \in \{0,1,\cdots,M-1\}$. The encoder maps the message w(k) into a "prescription" $f^N(k) := (f_1(k),f_2(k),\cdots,f_N(k))$. When k is odd $f_n(k) \in \{?,1\}$, when k is even $f_n(k) \in \{0,?\}$. Yet we will see that the distinction between odd and even cycles is artificial. Therefore we introduce the random variables $G_n(k)$ and $Z_n(k)$ for $k = (0,)1,2,\cdots$. When we define

$$G_n(k) := \begin{cases} i & \text{if } F_n(k) = ?, \\ r & \text{else}, \end{cases} \qquad \text{for } k = 1,2,\cdots \text{ and} \tag{1a}$$

$$Z_n(k) := \begin{cases} 1-Y_n(k) & \text{for k odd}, \\ Y_n(k) & \text{for k even}, \end{cases} \qquad \text{for } k = 0,1,2,\cdots , \tag{1b}$$

we see that the mapping that determines a component $z_n(k)$ from the $z_n(k-1)$ and $g_n(k)$ does not depend on k anymore (see table 2).

g(k)	z(k-1)	z(k)
i	0	1
i	1	0
r	0	0
r	1	0

Table 2.
z(k) as a function of z(k-1) and the input g(k) (n fixed).

As can be seen in the table, the combination $(z_n(k),z_n(k-1)) = (1,1)$ never occurs. From now on we consider a WUM as a channel, where for each component there is an input $g(k) \in \{i,r\}$, state $z(k-1) \in \{0,1\}$ and an output $z(k) \in \{0,1\}$ that depends on $g(k)$ and $z(k-1)$ as in table 2.

We say that an encoder is "uninformed" when the previous state of the channel is not used in the determination of $g^N(k)$, hence

$$g^N(k) := E(w(k),k). \tag{2a}$$

The encoder is "informed" when

$$g^N(k) := E(w(k),z^N(k-1),k). \tag{2b}$$

Likewise a decoder is said to be "uninformed" when

$$\hat{w}(k) := D(z^N(k),k), \tag{3a}$$

and "informed" when

$$\hat{w}(k) := D(z^N(k),z^N(k-1),k). \tag{3b}$$

Note that both the encoder and decoder are allowed to use codes that depend on the cycle index k. For practical reasons however, we assume that there exists a "period" $T \in \{1,2,\cdots\}$ such that $E(\cdot,k) = E(\cdot,k \bmod T)$ or $E(\cdot,\cdot,k) = E(\cdot,\cdot,k \bmod T)$ and $D(\cdot,k) = D(\cdot,k \bmod T)$ or $D(\cdot,\cdot,k) = D(\cdot,\cdot,k \bmod T)$. The entire coding scheme is now referred to as a "period–T" code. E.g. if we use one odd–cycle code and one even–cycle code we have a period–2 code.

By combining (2a) or (2b) with (3a) or (3b) we find four different cases. For each of these cases we can design codes. The (average–) error probability of a code is defined as

$$P_e := \lim_{K\to\infty} \frac{1}{K}\cdot \sum_{k=1,K} P_e(k), \text{ with } P_e(k) := \Pr\{\hat{W}(k) \neq W(k)\}, \tag{4}$$

where we assume that $z^N(0) = 0^N$.

A rate R is said to be T–achievable if for every $\delta > 0$ there exist for all N large enough period–T codes with $\frac{1}{N}\cdot\log(M) \geq R - \delta$ and error probability $P_e \leq \delta$. The capacity C(T) is defined as the maximum of all T–achievable rates.

It will be clear now that there are four different WUM–capacities, possibly depending on the period T, if we consider the average–error concept.

The binary entropy function $h(\cdot)$ is defined as

$$h(\alpha) := -\alpha\cdot\log(\alpha) - (1-\alpha)\cdot\log(1-\alpha), \text{ for } 0 \leq \alpha \leq 1. \tag{5}$$

Throughout this report we assume that the basis of the logarithm is 2.

3. Results

In this report we prove two converses :

Lemma 1 : For the WUM with informed encoder and informed decoder for arbitrary period T

$$C_{\text{informed,informed}}(T) \leq \log((1+\sqrt{5})/2). \qquad \square$$

Lemma 2 : For the WUM with uninformed encoder and uninformed decoder for period T = 1

$$C_{\text{uninformed,uninformed}}(T=1) \leq \max_{\beta}\, (1-\beta)\cdot h(\beta/(1+\beta)). \qquad \square$$

From Lemma 1 we immediately obtain

Corollary 1 : For the WUM with only the encoder or only the decoder informed and arbitrary period T

$$C_{\text{uninformed,informed}}(T) \le \log((1+\sqrt{5})/2), \text{ and} \qquad \square$$
$$C_{\text{informed,uninformed}}(T) \le \log((1+\sqrt{5})/2). \qquad \square$$

4. Proof of Lemma 1

For $k = 1,2,\cdots$ the Fano–inequality yields

$$H(W(k)|\hat{W}(k)) \le \phi(P_e(k)), \text{ where} \tag{6a}$$
$$\phi(p) := h(p) + p\cdot\log(M-1), \text{ for } 0 \le p \le 1. \tag{6b}$$

Note that $\phi(p)$ is convex–$\cap$ in p and that $\phi(p)/N \downarrow 0$ when $p \downarrow 0$ (M constant). Now for $k = 1,2,\cdots$

$$\begin{aligned}\log(M) &= H(W(k)) = H(W(k)|Z^N(k-1)) \\ &= I(W(k);Z^N(k)|Z^N(k-1)) + H(W(k)|Z^N(k),Z^N(k-1)) \\ &\overset{(a)}{=} I(W(k);Z^N(k)|Z^N(k-1)) + H(W(k)|Z^N(k),Z^N(k-1),\hat{W}(k)) \\ &\le H(Z^N(k)|Z^N(k-1)) + H(W(k)|\hat{W}(k)) \\ &\overset{(b)}{\le} H(Z^N(k)|Z^N(k-1)) + \phi(P_e(k)).\end{aligned} \tag{7}$$

Here (a) follows from the fact that $\hat{w}(k)$ is determined by $z^N(k)$ and $z^N(k-1)$ as stated in (3b), and (b) is Fano's inequality. Next we find

$$\begin{aligned}\frac{1}{N}\cdot\log(M) &\le \frac{1}{K\cdot N}\cdot\sum_{k=1,K}\sum_{n=1,N} H(Z_n(k)|Z^N(k-1),Z^{n-1}(k)) + \frac{1}{K\cdot N}\cdot\sum_{k=1,K}\phi(P_e(k)) \\ &\le \frac{1}{K\cdot N}\cdot\sum_{k=1,K}\sum_{n=1,N} H(Z_n(k)|Z_n(k-1)) + \frac{1}{K\cdot N}\cdot\sum_{k=1,K}\phi(P_e(k)) \\ &\overset{(c)}{\le} \frac{1}{K\cdot N}\cdot\sum_{k=1,K}\sum_{n=1,N} H(Z_n(k)|Z_n(k-1)) + \frac{1}{N}\cdot\phi\Big[\frac{1}{K}\cdot\sum_{k=1,K}P_e(k)\Big]\end{aligned}$$

$$\leq H(Z_I|Z_{II}) + \frac{1}{N}\cdot\phi\Big[\frac{1}{K}\cdot\sum_{k=1,K} P_e(k)\Big], \tag{8}$$

where (c) follows from the convexity of $\phi(\cdot)$, and where Z_I and Z_{II} are random variables with

$$\Pr\Big\{(Z_I,Z_{II})=(z_1,z_2)\Big\} := \frac{1}{K\cdot N}\cdot\sum_{k=1,K}\sum_{n=1,N}\Pr\Big\{(Z_n(k),Z_n(k-1))=(z_1,z_2)\Big\},$$

$$\text{for } (z_1,z_2) \in \{0,1\}^2. \tag{9}$$

Now note that

$$\Pr\Big\{(Z_I,Z_{II})=(1,1)\Big\} = 0, \text{ and that} \tag{10a}$$

$$\lim_{K\to\infty}\frac{1}{K}\Pr\Big\{Z_I=z\Big\} = \lim_{K\to\infty}\frac{1}{K}\Pr\Big\{Z_{II}=z\Big\} \text{ for } z \in \{0,1\}. \tag{10b}$$

Therefore assume that $\lim_{K\to\infty}\frac{1}{K}\Pr\{(Z_I,Z_{II})=(0,1)\} = \lim_{K\to\infty}\frac{1}{K}\Pr\{(Z_I,Z_{II})=(1,0)\} = \alpha$, and $\lim_{K\to\infty}\frac{1}{K}\Pr\{(Z_I,Z_{II})=(0,0)\} = 1-2\alpha$, for some α with $0 \leq \alpha \leq 1/2$. Hence, taking limits for $K \to \infty$, we obtain from (8)

$$\frac{1}{N}\cdot\log(M) \leq (1-\alpha)\cdot h(\alpha/(1-\alpha)) + \frac{1}{N}\cdot\phi\Big[\lim_{K\to\infty}\frac{1}{K}\cdot\sum_{k=1,K}P_e(k)\Big]. \tag{11}$$

Therefore for any T–achievable rate R for all (small enough) $\delta > 0$ there exists an $0 \leq \alpha \leq 1/2$ such that

$$R - \delta \leq \frac{1}{N}\cdot\log(M) \leq (1-\alpha)\cdot h(\alpha/(1-\alpha)) + \frac{1}{N}\cdot\phi(\delta), \tag{12}$$

hence we can conclude that for all T

$$C_{\text{informed,informed}}(T) \le \max_{\alpha} (1-\alpha)\cdot h(\alpha/(1-\alpha)) \overset{(d)}{=} \log((1+\sqrt{5})/2). \tag{13}$$

Equality (d) follows from simple analysis.

5. Proof of Lemma 2

For $k = 1,2,\cdots$ we have that

$$\begin{aligned}
\log(M) &= H(W(k)) = I(W(k);Z^N(k)) + H(W(k)\,|\,Z^N(k)) \\
&\overset{(e)}{=} I(W(k);Z^N(k)) + H(W(k)\,|\,Z^N(k),\hat{W}(k)) \\
&\overset{(f)}{\le} I(W(k);Z^N(k)) + \phi(P_e(k)) \\
&= \sum_{n=1,N} I(W(k);Z_n(k)\,|\,Z^{n-1}(k)) + \phi(P_e(k)) \\
&= \sum_{n=1,N} \left[H(Z_n(k)\,|\,Z^{n-1}(k)) - H(Z_n(k)\,|\,Z^{n-1}(k),W(k))\right] + \phi(P_e(k)),
\end{aligned} \tag{14}$$

where (e) follows from (3a), and where (f) is Fano's inequality. Next consider

$$\begin{aligned}
&H(Z_n(k)\,|\,Z^{n-1}(k),W(k)) \\
&\quad \ge H(Z_n(k)\,|\,Z^{n-1}(k),W(k),G_n(k),Z^{n-1}(k-1)) \\
&\quad = \Pr\{G_n(k)=i\}\cdot H(Z_n(k)\,|\,Z^{n-1}(k),W(k),G_n(k)=i,Z^{n-1}(k-1)) \\
&\quad = \Pr\{G_n(k)=i\}\cdot H(Z_n(k-1)\,|\,Z^{n-1}(k),W(k),G_n(k)=i,Z^{n-1}(k-1)) \\
&\quad \overset{(g)}{=} \Pr\{G_n(k)=i\}\cdot H(Z_n(k-1)\,|\,Z^{n-1}(k-1)),
\end{aligned} \tag{15}$$

where (g) follows from

$$\begin{aligned}
&P\left[z^{n-1}(k-1),z_n(k-1),w(k),g_n(k),z^{n-1}(k)\right] \\
&\quad = P\left[z^{n-1}(k-1),z_n(k-1)\right]\cdot P\left[w(k),g_n(k)\right]\cdot P\left[z^{n-1}(k)\,|\,z^{n-1}(k-1),w(k)\right] \\
&\quad = P\left[z^{n-1}(k-1)\right]\cdot P\left[z_n(k-1)\,|\,z^{n-1}(k-1)\right]\cdot P\left[w(k),g_n(k),z^{n-1}(k)\,|\,z^{n-1}(k-1)\right].
\end{aligned} \tag{16}$$

From (14) and (15) we conclude that

$$\log(M) - \phi(P_e(k)) \leq \sum_{n=1,N} \Big[H(Z_n(k) \mid Z^{n-1}(k)) - \Pr\{G_n(k)=i\} \cdot H(Z_n(k-1) \mid Z^{n-1}(k-1)) \Big]. \quad (17)$$

This implies that (note that (h) follows from the convexity of $\phi(\cdot)$)

$$\begin{aligned}
\tfrac{1}{N} \cdot \log(M) - \tfrac{1}{N} \cdot \phi\Big[\tfrac{1}{K} \sum_{k=1,K} P_e(k)\Big]\Big)
&\overset{(h)}{\leq} \tfrac{1}{N} \cdot \log(M) - \tfrac{1}{K \cdot N} \cdot \sum_{k=1,K} \phi(P_e(k)) \\
&\leq \tfrac{1}{K \cdot N} \cdot \sum_{k=1,K} \sum_{n=1,N} \Big[H(Z_n(k) \mid Z^{n-1}(k)) - \Pr\{G_n(k)=i\} \cdot H(Z_n(k-1) \mid Z^{n-1}(k-1)) \Big] \\
&= \tfrac{1}{N} \cdot \sum_{n=1,N} \tfrac{1}{K} \cdot \sum_{k=1,K} \Big[H(Z_n(k) \mid Z^{n-1}(k)) - \Pr\{G_n(k)=i\} \cdot H(Z_n(k-1) \mid Z^{n-1}(k-1)) \Big] \\
&\leq \max_n \tfrac{1}{K} \cdot \sum_{k=1,K} \Big[H(Z_n(k) \mid Z^{n-1}(k)) - \Pr\{G_n(k)=i\} \cdot H(Z_n(k-1) \mid Z^{n-1}(k-1)) \Big] \\
&\leq \max_n \tfrac{1}{K} \cdot \Big[\sum_{k=2,K} (1 - \Pr\{G_n(k)=i\}) \cdot H(Z_n(k-1) \mid Z^{n-1}(k-1)) + 1 \Big] \\
&\leq \tfrac{1}{K} \cdot \Big[\sum_{k=2,K} (1 - \Pr\{G(k)=i\}) \cdot H(Z(k-1)) + 1 \Big],
\end{aligned} \quad (18)$$

for some set of random variables ($Z(0)$, $G(1)$,) $Z(1)$, $\cdots$, $Z(K-1)$, $G(K)$ with distribution

$$\begin{aligned}
P\Big[z(0),g(1),z(1),\cdots,z(K-1),g(K)\Big] &= P\Big[z(0)\Big] \cdot P\Big[g(1)\Big] \cdot P\Big[z(1) \mid z(0),g(1)\Big] \cdot \ldots \\
&\cdot P\Big[g(k-1)\Big] \cdot P\Big[z(K-1) \mid z(K-2),g(K-1)\Big] \cdot P\Big[g(k)\Big]. \quad (19)
\end{aligned}$$

From the fact that $T = 1$ we obtain that there must exist a β, $0 \leq \beta \leq 1$, such that

$$\Pr\{G(k)=i\} = \beta, \text{ for } k = 1,2,\cdots . \quad (20a)$$

Note that the right side of (18) is equal to $1/K$ when $\beta = 1$. Therefore we assume in what follows that $\beta < 1$. We will now determine $\lim_{k\to\infty} \Pr\{Z(k)=1\}$. Define

$$\gamma_k := \Pr\{Z(k)=1\},\ \ k = 0,1,2,\cdots . \tag{20b}$$

then

$$\begin{aligned} \gamma_0 &= 0 \text{ and} \\ \gamma_k &= \Pr\{Z(k)=1\} = \Pr\{Z(k-1)=0\}\cdot\Pr\{G(k)=i\} \\ &= (1-\Pr\{Z(k-1)=1\})\cdot\Pr\{G(k)=i\} \\ &= (1-\gamma_{k-1})\cdot\beta,\ \ k = 1,2,\cdots . \end{aligned} \tag{21}$$

Define $\Gamma := \beta/(1+\beta)$ and $\delta_k := \gamma_k - \Gamma$. Then

$$\delta_{k+1} = \gamma_{k+1} - \Gamma = (1-\gamma_k)\cdot\beta - (1-\Gamma)\cdot\beta = (\Gamma-\gamma_k)\cdot\beta = -\delta_k\cdot\beta. \tag{22}$$

Therefore $|\delta_{k+1}| < |\delta_k|$ and $\lim_{k\to\infty} \gamma_k = \beta/(1+\beta)$. Now taking in (18) the limit for $K\to\infty$ at both sides we find that

$$\begin{aligned} &\tfrac{1}{N}\cdot\log(M) \\ &\le \lim_{K\to\infty} \tfrac{1}{K}\cdot\Big[\sum_{k=2,K}(1-\Pr\{G(k)=i\})\cdot H(Z(k-1)) + 1\Big] + \tfrac{1}{N}\cdot\lim_{K\to\infty} \phi\Big[\tfrac{1}{K}\sum_{k=1,K} P_e(k)\Big] \\ &= (1-\beta)\cdot h(\beta/(1+\beta)) + \tfrac{1}{N}\cdot\phi\Big[\lim_{K\to\infty} \tfrac{1}{K}\sum_{k=1,K} P_e(k)\Big]). \end{aligned} \tag{23}$$

Note that in the limit for $K\to\infty$, (23) holds for both $\beta < 1$ and $\beta = 1$.

Therefore for any 1–achievable rate R for all (small enough) $\delta > 0$ there exists a $0 \le \beta \le 1$ such that

$$R - \delta \le \tfrac{1}{N}\cdot\log(M) \le = (1-\beta)\cdot h(\beta/(1+\beta)) + \tfrac{1}{N}\cdot\phi(\delta). \tag{24}$$

From this we can conclude that

$$C_{\text{uninformed,uniformed}}(T=1) \leq \max_{\beta} (1-\beta)\cdot h(\beta/(1+\beta)). \tag{25}$$

Numerical computation shows that $\max_{\beta} (1-\beta)\cdot h(\beta/(1+\beta)) = 0.54588\cdots$ is achieved for $\beta = 0.2887\cdots$.

6. Conclusion

We have shown that also in the average–error case $\log_2((1+\sqrt{5})/2)$ is an upper bound for the T–capacities in all four (Simonyi–) situations. In addition we found an upper bound for the average–error 1–capacity in the case where both the encoder and the decoder are uninformed.

Wyner and Ozarow [7] independently found an upper bound for the average–error capacity in the uninformed–uninformed case. Their proof is more concise than ours, but not detailed as far as the limiting behavior (for $K\to\infty$) is concerned. Berger [8] informed the author about the existence of this unpublished material.

Van Overveld [9] demonstrated that also the 2–capacity is upper–bounded by $0.54588\cdots$ in the uninformed–uninformed case. It is still unknown however whether this holds for all T–capacities or not.

7. Acknowledgement

Part of the research for this report was performed at Bielefeld University in December 1987. Rudy Ahlswede and Zhen Zhang, thank you for inviting me, and for providing the right atmosphere.

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ISSN 0167-9708
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