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A Note on the Practical Definition of the Parameter of Plastic Anisotropy

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1. Introduction

Many research workers have been trying to establish an experimental relationship between the plastic behaviour, i.e. formability and earing, of sheet in deep-drawing and its plastic anisotropy observed by making use of the tensile test. For that purpose the anisotropic properties are usually characterized by a coefficient R, which is defined as the ratio between the natural contraction on width and the natural contraction on thickness of a rectangular section test-piece under tensile stress.

$$R = \left| \delta_w / \delta_n \right| = \ln \frac{w_o}{w} / \ln \frac{s_o}{s} \qquad (1)$$

An R-value equal to unity indicates that the width- and thickness-strains are equal. In this case the sheet is isotropic, at least in the direction of the tensile test. In general the value of R varies with the angle between the direction from which the test-piece was taken and the rolling direction. The maximum fluctuation can be observed between the rolling direction and under 45° with the rolling direction.

This fluctuation ΔR is called the planar anisotropy of the sheet and is responsible for earing. Since earing is generally regarded as a serious disadvantage in deep-drawing operations, it is necessary to know how to predict the earing height.

The extreme values of R are ∞ and zero. The fact that the definition of R, according to eq. (1), is strongly asymmetric with regard to $R = 0$ must count against it. This implies that the earing height will be dependent not only on ΔR but also on R. In principle this fact is a handicap for the interpretation of test results.

Apparently this problem can be removed by using a quantity $\Delta R/\bar{R}$, where $\bar{R}$ is the average value of the anisotropy coefficient |1|. Defining a symmetric parameter might be another way.

2. Definition of a Symmetric Parameter

According to Hill |2| and, strictly speaking, only for $\Delta R = 0$, we may write the Lévy-von Mises equations for the transverse directions as follows:

$$d\delta_w = \frac{d\lambda}{2} \left\{ G(\sigma_w - \sigma_t) + H(\sigma_w - \sigma_n) \right\}$$

$$d\delta_n = \frac{d\lambda}{2} \left\{ F(\sigma_n - \sigma_t) + H(\sigma_n - \sigma_w) \right\} \qquad (2)$$

where G, F and H are assumed to be constant. In the case of a uniaxial stress state these equations transform to the integrated form

$$\delta_w = -\frac{\lambda}{2} G \sigma_t$$

$$\delta_n = -\frac{\lambda}{2} F \sigma_t \qquad (3)$$

In order to characterize anisotropic behaviour, write as usual

$$R = \delta_w/\delta_n = G/F = \text{constant} \qquad (4)$$

where $R = G = F = 1$ for an isotropic sheet, or choose the difference between the transverse contractions:

$$\delta_w - \delta_n = -\frac{\lambda}{2}(G - F)\sigma_t \qquad (5)$$

With the aid of the third stress-strain relation

$$\delta_t = \frac{\lambda}{2}(F + G)\sigma_t \qquad (6)$$

the hyperbolic expression

$$K = \frac{\delta_w - \delta_n}{\delta_t} = \frac{F - G}{F + G} = \frac{1 - R}{1 + R} = \text{constant} \qquad (7)$$

is obtained. This is independent of δ_t and symmetric with regard to the value $K = 0$ for isotropic sheet, shown clearly by the extreme values $K = +1$ and $K = -1$. R can be calculated from K with the aid of the same function

$$R = \frac{1 - K}{1 + K} \qquad (8)$$

Now, changing the sign of K implies only a rotation $\frac{\pi}{2}$ around the tensile axis, as it should do.

3. Accuracy Problems

In comparison with R the coefficient K offers no advantage with regard to the measuring accuracy.

Using the general formula

$$\Delta f \cong \frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 + \ldots\ldots + \frac{\partial f}{\partial x_n}\Delta x_n$$

the following expression is obtained for the absolute error in K and R:

$$\Delta K = -2\Delta R \cong \frac{2}{\delta_t}\left\{ R\left(\frac{\Delta s}{s} - \frac{\Delta s_o}{s_o}\right) - \left(\frac{\Delta w}{w} - \frac{\Delta w_o}{w_o}\right)\right\} \qquad (9)$$

Analysis of this formula shows that, especially in the case of thin sheet or for little values of the tensile strain δ_t, the error can grow very large. For this reason the experiments have been carried out using rather thick sheet materials.

4. Experiments

In order to verify the practical usefulness of K, a series of experiments has been carried out for different 2 mm thick sheet materials and several values of the drawing ratio β_o. The punch radius r_{st} was 40 mm. R and K have been evaluated from tensile tests, where the contraction strains have been measured at the maximum load. The results are given in Table 1. The plane anisotropy values ΔR and ΔK have been calculated according to

$$\Delta R = \left| R_o - R_{45} \right|$$

and

$$\Delta K = \left| K_o - K_{45} \right| \qquad (10)$$

A proportional relation between the earing height ΔH and the square value of the drawing ratio β_o appeared to exist. Some typical curves are shown in Fig. 1. If a constant scale ratio between the earing height and the punch radius may be assumed, then

$$\Delta H / r_{st} \cong f_e(\beta_0^2 - 1.9) \qquad (11)$$

where the earing factor f_e is apparently dependent on the sheet material. Hence, in Fig. 2., f_e has been plotted versus ΔR and ΔK. Although a rather large scatter can be observed in both of the representations, it can be seen clearly that f_e correlates better to ΔK than to ΔR.

Now, the following experimental formula holds on an average:

$$\Delta H \cong 0.21\, r_{st}\, \Delta K (\beta_0^2 - 1.9) \qquad (12)$$

This equation seems to give a true picture of the main conditions affecting the earing height of circular cups.

Finally, the differences between the plotted points and the mean straight lines in Fig. 2 have been plotted in Fig. 3 over the average values

$$\overline{R} = (R_0 + R_{45}) / 2$$

and (13)

$$\overline{K} = (K_0 + K_{45}) / 2$$

In contrast to $\overline{R}$, the parameter $\overline{K}$ appears not to affect systematically the earing height.

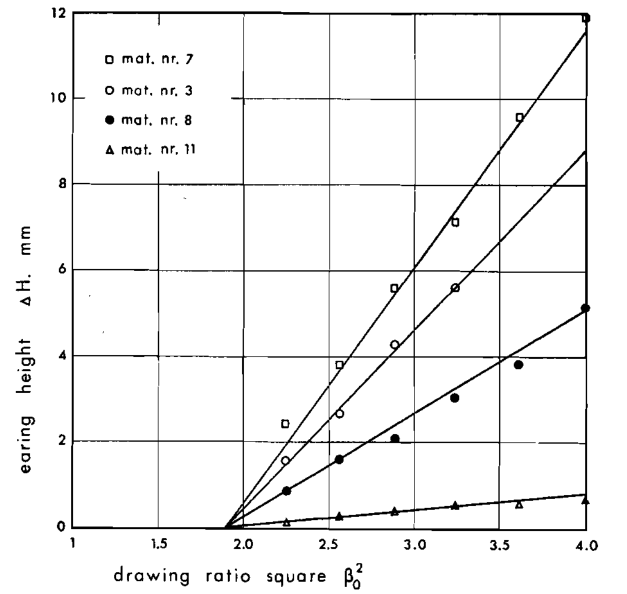

Fig. 1. Experimental relationship between absolute average earing height and the drawing ratio.

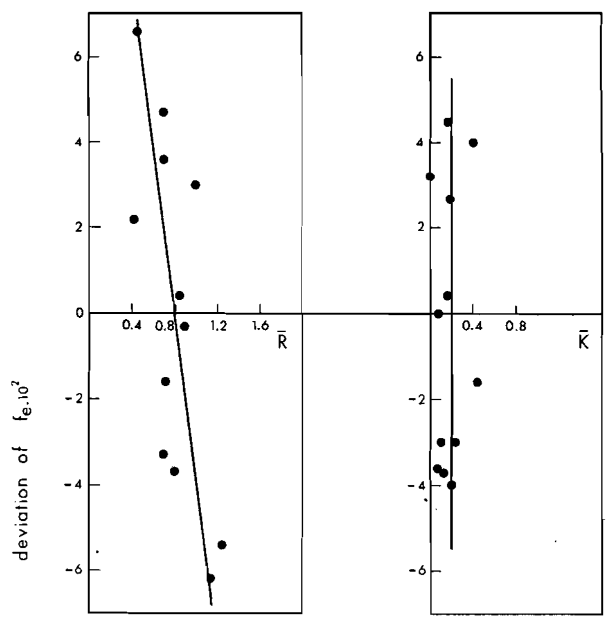

Fig. 3. The systematic effect of the average anisotropy coefficients on the deviation of the earing factor with regard to the straight lines of Fig. 2

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1. Wright, J. C., The Phenomenon of Earing in Deep Drawing, Sheet Metal Industries, Nov. 1965, p. 814.
2. Hill, R., A Mathematical Theory of Plasticity, Oxford University Press, 1956.

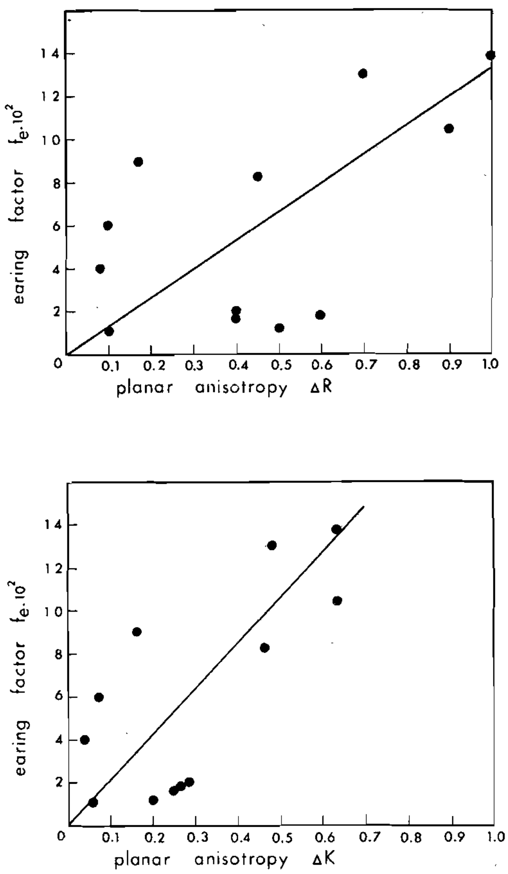

Fig. 2. Comparison of experimental relations between the earing factor and the parameters of planar anisotropy.

Table 1. Results of tensile tests and deep drawing tests.

nr.	material	R_0	R_{45}	ΔR	ΔK	earing height / mm						f_e
						β_0=1.5	β_0=1.6	β_0=1.7	β_0=1.8	β_0=1.9	β_0=2.0	
1	alum. 2S-0	0.75	0.65	0.10	0.07	0.90	1.60	2.05	3.05	3.75	5.12	0.06
2	alum. 2S-$\frac{1}{2}$H	0.50	0.90	0.40	0.28	0.60	0.80	0.90	1.10	1.20	—	0.02
3	alum. 57S-$\frac{1}{2}$H	0.55	0.38	0.17	0.23	1.00	1.60	2.90	3.95	5.20	7.95	0.09
4	alum. 2S-H	0.40	1.10	0.70	0.48	1.90	2.75	4.30	5.55	8.40	11.85	0.13
5	alum. 51S-T	1.00	0.60	0.40	0.25	0.35	0.50	0.55	0.75	0.85	—	0.02
6	alum. 57S-H	0.30	1.20	0.90	0.63	1.60	2.60	4.30	5.60	—	—	0.10
7	alum. N3S-$\frac{1}{2}$H	0.35	1.35	1.00	0.63	2.50	3.80	5.60	7.15	9.60	12.08	0.14
8	steel NP-0	0.20	0.65	0.45	0.46	1.30	2.60	3.40	4.25	—	—	0.08
9	steel SP-0	1.00	1.50	0.50	0.20	0.10	0.20	0.50	0.65	0.90	1.20	0.01
10	stainless steel	0.96	1.04	0.08	0.04	0.50	0.95	1.55	2.00	2.90	3.35	0.04
11	nickel	0.82	0.92	0.10	0.06	0.15	0.30	0.40	0.50	0.55	0.60	0.01
12	copper	0.85	1.45	0.60	0.26	0.15	0.60	0.80	0.90	1.05	1.30	0.02