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Citation for published version (APA):

Document license:
TAVERNE

DOI:
10.1016/j.trc.2016.05.011

Document status and date:
Published: 01/01/2016

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
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Modeling duration choice in space–time multi-state supernetworks for individual activity-travel scheduling

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Abstract

Multi-state supernetworks have been advanced recently for modeling individual activity-travel scheduling decisions. The main advantage is that multi-dimensional choice facets are modeled simultaneously within an integral framework, supporting systematic assessments of a large spectrum of policies and emerging modalities. However, duration choice of activities and home-stay has not been incorporated in this formalism yet. This study models duration choice in the state-of-the-art multi-state supernetworks. An activity link with flexible duration is transformed into a time-expanded bipartite network; a home location is transformed into multiple time-expanded locations. Along with these extensions, multi-state supernetworks can also be coherently expanded in space–time. The derived properties are that any path through a space–time supernetwork still represents a consistent activity-travel pattern, duration choice are explicitly associated with activity timing, duration and chain, and home-based tours are generated endogenously. A forward recursive formulation is proposed to find the optimal patterns with the optimal worst-case run-time complexity. Consequently, the trade-off between travel and time allocation to activities and home-stay can be systematically captured.

1. Introduction

It has been widely recognized that observed activity-travel patterns (ATPs) are the results of an underlying scheduling process that involves the planning and execution of activities in time and space. Activity-travel scheduling has become the core of many activity-based travel demand models. Given the high choice dimensionalities and demand–supply complexities in the scheduling process, a one-fits-all scheduling approach cannot be found in the literature. Various scheduling models have been proposed from different perspectives (e.g., Recker, 1995; Lam and Yin, 2001; Bowman and Ben-Akiva, 2001; Balmer et al., 2006; Li et al., 2010; Arentze et al., 2010; Kang et al., 2013; Auld et al., 2016). The multi-state supernetwork approach has been advanced recently for individual activity-travel scheduling based on an integrated and detailed representation of the land-use transport system. Introduced by Arentze and Timmermans (2004) and substantially improved by Liao et al. (2013), a multi-state supernetwork is constructed as an individual's choice space of activity sequences, locations, routes, modes and parking locations for implementing a daily activity program (AP). (Unless stated otherwise, activity alone refers to out-of-home activity). By predicting individuals' daily ATPs, the outcomes offer great potential for systematic assessments of a large range of policy interventions and emerging modalities (e.g., virtual mobility (Kenyon, 2010), shared bike scheme (Lathia et al., 2012), electric vehicles (He et al., 2015), park and ride (Chen et al., 2015)).
However, duration choice of activities and home-stay has not been fully captured in the current state-of-the-art multi-state supernetwork models. First, activity duration only depends on the static characteristics of and arrival times at the activity locations. In other words, activity durations are fixed if arrival times are given. This is a non-issue for certain fixed activities, but it may be problematic for discretionary activities such as leisure and shopping. Second, the choice of home-return and home-stay duration in-between two activities has not been addressed when there are two or more activities in the APs. As evidenced by empirical studies, travel and time allocation to home-stay and activities are interrelated under a constrained time frame. Thus, insufficient treatments of duration choice in the scheduling models would lead to inaccurate and less relevant ATPs along the temporal dimension.

Few fundamental scheduling approaches are capable of simultaneously modeling duration choice of activities and home-stay in the multi-modal multi-activity trip chains. Ettema et al. (2007) concluded that three factors, i.e., activity chain, timing and duration, should be taken into account to obtain an adequate sensitivity to policy scenarios when modeling activity participation. Despite the recognition, these factors have often been modeled separately. Even when timing and duration have been modeled jointly, duration choice is decided using an external model and then fed into an activity-travel scheduler. In such a way, duration choice is not linked to mode and route choice, resulting in weak linkage between travel and activity duration. Likewise, choice of home-stay duration tends to be ignored or given lower priority. In sequential models that build ATPs by blocks, start time and activity duration (or end time) are usually decided first, and the remainder continuous timeslot is automatically reserved for home-stay. In simultaneous models, activity duration choice has recently been considered by allowing random duration variations in MATSim (Feil et al., 2010) and as decision variables in the family of HAPPS model (Yuan, 2014; Chow and Nurumbetova, 2015; Chow and Djavadian, 2015); at the aggregate level, several activity-based traffic assignment models (Li et al., 2010; Ramadurai and Ukkusuri, 2010; Fu and Lam, 2014) have also included activity duration choice, but the ATPs have been limited to either car or transit networks. In these models, multi-modal travel and home-stay duration choice are basically omitted due to the lack of an overall representation of ATPs. In addition, computation complexity is not addressed in such models.

The purpose of this study, therefore, is to incorporate duration choice of activities and home-stay in multi-state supernetworks for individual activity-travel scheduling. To that effect, an activity link with flexible duration is transformed into a time-expanded bipartite network to represent activity duration choice. To represent home-stay choice, it can be considered as an optional activity (a new concept in ATPs); or in an implicit format, a home location is transformed into multiple time-expanded locations. With these extensions, multi-state supernetworks can be coherently expanded in space–time. Any path through a space–time supernetwork still represents a consistent ATP and the (dis)utility of duration choice can be explicitly defined in terms of activity chain, timing and duration. Two further advantages can be derived. First, home-based tours are generated endogenously since the choice of home-return and home-stay are included. Second, space–time supernetworks support a more efficient routing algorithm with reduced run-time complexity than the label correcting algorithm proposed in Liao et al. (2013). Hence, the contribution of the current study is not limited to incorporating duration choice in multi-state supernetworks but also enhancing the behavioral realism and applicability of the approach. To highlight the contributions of the current study, Table 1 gives a short summary of recent simultaneous models on activity-travel scheduling.

The remainder of this paper is organized as follows. The next section introduces the concept of multi-state supernetworks and discusses the previous treatments of duration choice. Section 3 discusses the suggested extensions of duration choice at activity links and home locations, and proposes recursive formulations for finding the optimal ATPs. Section 4 illustrates the proposed extensions and discusses the applicability of the model. Finally, the paper is completed with conclusions and plans of future work.

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Y and N: with and without the referred element respectively.

<sup>a</sup> Trip chains of private vehicle and public transport.

<sup>b</sup> Possible when “staying at home” is an explicit activity.

<sup>c</sup> Convergence analysis instead.

<sup>d</sup> Need to link other components of MATSim.

<sup>e</sup> Referring to activity duration profiles.
2. Multi-state supernetwork

2.1. Preliminaries

The concept of supernetwork was originally defined as network of networks to model route and mode choice simultaneously (Sheffi, 1985). Networks of different travel modes are interconnected by virtual links at locations where individuals can switch modes. The seminal concept has been extended to model multi-modal trip chains (e.g. Lozano and Storchi, 2002). Nagurney (2004) first incorporated one episode of activity in the supernetwork representation. Arentze and Timmermans (2004) proposed the multi-state supernetwork framework for modeling full ATPs. Their representation was substantially improved to activity-travel scheduling by Liao et al. (2010, 2011, 2013).

The essence of multi-state supernetworks is that activity-travel scheduling process is decomposed into path choice through different states of conducting the AP. Three states are distinguished:

1. Activity state: which activities have already been conducted.
2. Vehicle state: where the private vehicles (PVs) are (being in use or parked somewhere).
3. Activity-vehicle state: the combination of activity and vehicle states. All PVs must be parked when conducting activities. (Unless stated otherwise, the term state alone represents an activity-vehicle state.)

In the supernetworks, all nodes denote real locations in space. Three types of links are defined:

1. Travel links: connecting different nodes representing the movement of an individual, staying in the same state.
2. Transition links: connecting the same nodes of different transport modes (i.e., parking/picking-up a PV, boarding/alighting public transport).
3. Transaction links: connecting the same nodes of different activity states representing the implementation of activities.

To express the state transfers efficiently, the land-use transport system is split into private vehicle networks (PVNs) for every type of PV and a single public transport network (PTN) for walking and PT (Liao et al., 2010). A multi-state supernetwork is constructed in two steps for each individual’s AP specifically. First, a copy of PVN or PTN is created for every possible state. Second, PVNs and PTNs are interconnected by transition links (between PVNs and PTNs) and transaction links (between PTNs and PTNs). Fig. 1 is a general supernetwork representation exclusive of the time dimension for an AP including two activities (A₁ and A₂) and two PVs (car and bike). P₁ & P₂ and P₃ & P₄ are the parking locations for car and bike.
respectively. Each column denotes a particular vehicle state. Columns $P_0$ and $P_5$ denote car and bike in use respectively. Other columns denote vehicle states where car and bike are parked at the corresponding locations. Each row denotes an activity state. $s_1s_2$ represents the activity states of $A_1$ & $A_2$ ($0$: not conducted and $1$: conducted). $H^0$ and $H^1$ denote home at the start and end of the AP respectively. Pentagons and hexagons denote PVNs and PTNs respectively; vertices of the PVNs and PTNs denote locations (home, activity and parking locations); undirected links are bi-directed. It can be proven that any path from $H^0$ to $H^1$ expresses a consistent full ATP potentially involving multi-modal multi-activity trip chaining. For instance, the path formed by the bold links represents an ATP that the individual leaves home by car with parking at $P_2$ and travels through PTN to conduct $A_1$, then returns home and rides bike with parking at $P_4$ and goes through PTN to conduct $A_2$, and finally returns home. The individual switches modes between PV and PT at $P_2$ and $P_4$.

The study of Liao et al. (2013) represents the state-of-the-art for individual activity-travel scheduling. For a given individual’s AP, relevant activity/parking locations are selected based on space–time constraints to construct the personalized supernetwork. Travel links in PVNs and PTNs refer to time-dependent road network and time-expanded PT network respectively. Activity participation and parking search look up time-dependent disutility profiles. Thus, link disutility can be generally defined in a state- and time-dependent and personalized way as:

$$\text{disU}_{ilm}(t) = \beta_{ilm} \times X_{ilm}(t) + \epsilon_{ilm}$$

where $\text{disU}_{ilm}(t)$ denotes the disutility on link $l$ for individual $i$ at state $s$ with transport mode $m$ at arrival time $t$, $X_{ilm}(t)$ is a vector of attributes including static and time-dependent components, $\beta_{ilm}$ is an attribute-vector of weights, and $\epsilon_{ilm}$ is an error term. The error term has been considered for estimating the disutility function, but has been ignored in finding optimal path choice. Formally, individual activity–travel scheduling is to find an ATP with the minimal disutility:

$$\min \{ \text{disU}_{i(l|\theta_{-A^t})}, \quad p_{\theta_{-A^t}} \in \text{PATH}_i \}$$

where $p_{\theta_{-A^t}}$ and $\text{PATH}_i$ denote a path from $H^0$ to $H^1$ and the path space respectively.

### 2.2. Treatment of activity duration choice

Liao et al. (2013) assumed that activity duration and disutility follow time-dependent profiles. Fig. 2 is an example, in which $\Gamma_{ij|z}(t)$ and $Z_{ij|z}(t)$ denote the duration and disutility of conducting activity $z$ at the $j$-th activity location $j(z)$ with start time $t$ respectively. As shown, $\Gamma_{ij|z}(t)$ and $Z_{ij|z}(t)$ are known once $t$ is given (e.g., $t = t_1$ or $t_2$). To accommodate the profiles in the scheduling algorithm, the first-in-first-out (FIFO) property of travel (Pyrga et al., 2008; Dehne et al., 2012) has been assumed for activity duration and disutility.

**Assumption 1 (A1).** An individual arriving earlier at an activity location finishes the activity no later than when he/she would arrive at a later time,

$$\Gamma_{ij|z}(t_1) + t_1 \leq t_2 \quad \text{if} \quad t_1 \leq t_2$$

**Assumption 2 (A2).** Once arriving at an activity location, the individual does not wait to conduct the activity at a later time,

$$Z_{ij|z}(t_1) \leq \beta_{iw} \times (t_2 - t_1) + Z_{ij|z}(t_2), \quad \text{if} \quad t_1 \leq t_2$$

where $\beta_{iw}$ is the disutility weight of waiting time in the linear form.

![Fig. 2. Example of duration and disutility profile.](image-url)
The FIFO properties only hold for activity participation that exactly follows the queue phenomenon. They generally prohibit the choice of duration and waiting at activity locations. This restriction oversimplifies activity-travel behavior and thus should be relaxed.

2.3. Treatment of home-stay duration

Choice of home-return in-between two activities has been represented in the general multi-state supernetwork, allowing for several home-based tours in an ATP. In Fig. 1, the individual leaves home (H0) at the first activity state (00), has the possibility to return home at the intermediate states (10 or 01), and must return home (H1) at the last activity state (11). However, if home-return is not an explicit activity specified in the AP, it occurs only when the individual switches to another departing-home mode if and only if the change of modes reduces disutility for conducting the remaining activities according to Eq. (2). Moreover, the individual would immediately depart home without any home-stay even if he/she has to wait outside. For instance, after dropping-off his/her child to school by car, the individual leaves the car at home and walks directly to a nearby supermarket for shopping where the parking fee is high. This modeling feature does not reflect reality. It is more likely that individuals would stay at home for certain duration because people generally gain utility while staying at home. They can opt to conduct in-home activities or enjoy interactions with other household members. In addition, waiting at home is normally preferred to waiting outside (e.g., waiting for public transport services or the opening times of facilities). Thus, home-stay duration choice is also an important facet and should be incorporated in the daily ATPs.

3. Duration choice of activities and home-stay

As discussed in the introduction, it is important to jointly model travel and duration choice of activities and home-stay for accurately capturing the temporal dynamics. Travel dynamics by PVs and PT basically obey the queue phenomenon and have been incorporated in the supernetwork approach. This section first discusses the incorporation of duration choice of activities and home-stay, and then proposes recursive formulations to find the optimal ATPs.

3.1. Duration choice of activities

To incorporate duration choice of activities, A1 and A2 (Section 2.2) are relaxed in two cases, i.e., without and with waiting choice at the activity locations. Suppose individual i arrives at activity location J(\(x\)) at time t and state s, and \([u(\(x\)), v(\(x\))]\) is the time window of J(\(x\)).

3.1.1. Without waiting choice

This subsection considers the case that i does not have the choice of waiting. Thus, if \(u(\(x\)) < v(\(x\)), the activity start time is t; if \(t < u(\(x\)), the start time is u(\(x\)) and the waiting time has to be v(\(x\)) − t; and if \(t ≥ v(\(x\)), there is no duration choice and thus no transaction link through t at J(\(x\)).

We first deal with the situation \(u(\(x\)) < t < v(\(x\)). Given a realization of duration \(\tau\), the activity disutility expressed as a function of \(t\) and \(\tau\), \(Z_{ij}(t, \tau)\), is a time-dependent profile. Fig. 3 is an example of three duration alternatives \(\{\tau_1, \tau_2, \tau_3\}\). Obviously, A1 and A2 tend to be broken at times \(t_1\) and \(t_2\) with different duration choices.

The incorporation of activity duration choice with limited alternatives is relatively straightforward. To associate \(\tau\) with \(t\) and \(s\), a natural extension is to represent duration alternatives explicitly by transaction links. Any transaction link with flexible duration is expanded to as many links as there are duration alternatives. With this expansion, any transaction link disutility can be derived by \(Z_{ij}(t, \tau)\). Fig. 4 shows an example, in which the network on the left-hand-side is a fragment of Fig. 1 with fixed duration if \(t\) is given, while the right-hand-side represents three duration choices.

In principle, i has infinite duration alternatives in the continuous interval of \((0, v(\(x\)) − t]\). However, in reality, i is expected to choose duration only from a limited set of discrete choice alternatives. For instance, people may plan activity durations in the unit of 5 min. In that sense, duration choice can be incorporated in the supernetwork in a discretized scheme. Let \(T_0\) be the smallest time unit for activity-travel scheduling. Then, there are \(\frac{T_0 - t}{T_0}\) time steps from \(t\) to \(v(\(x\))\), which defines the maximum number of duration choice alternatives.

A time-expanded bipartite network is adopted to describe the expansion of a transaction link. Given \(t\) and \(s\) at J(\(x\)), the bipartite network is depicted in Fig. 5(a) with \(T_0\) as the unit of time. All nodes refer to the same location J(\(x\)). Nodes at the same rows have the same time labels on the left. Columns s and s’ represent two different activity states, i.e., \(x\) being unconducted and conducted respectively. Only those nodes at activity state s’ whose time labels are greater than \(t\) can be the end nodes of transaction links. Those unfilled nodes cannot be parts of transaction links. The duration of each expanded transaction link equals to the time differences of the two nodes. The duration choice space can be represented as an arithmetic sequence from \(t\) to \(v(\(x\))\) with \(T_0\) as the common difference. Fig. 5(b) is a reduced bipartite network with \(T_k\) (\(T_k > T_0\)) as the time step of duration choice. In Fig. 5, any transaction link can be defined by \(Z_{ij}(t, \tau)\).

In the situation \(t < u(\(x\)), i has to wait until \(u(\(x\)). The start time is updated as \(u(\(x\))\) and all transaction links of the bipartite network start from the node with time label \(u(\(x\)). The numbers of duration choice alternatives are \(\frac{v(\(x\)) - u(\(x\))}{T_0}\) and \(\frac{v(\(x\)) - u(\(x\))}{T_k}\) in...
The disutility of waiting in the linear form and conducting the activity are $\beta_{iw} \times (u_{j(t)} - t)$ and $Z_{iuJ(t)}(u_{j(t)}, \tau)$, respectively. Fig. 5 represents the duration choice space for conducting $\alpha$. If $i$ only concerns a small set of duration alternatives, the subset of the choices can also be represented in the bipartite network.

### 3.1.2. With waiting choice

This subsection concerns the case that $i$ is allowed to wait until a better time to start activity $\alpha$ even if arrival time $t$ is inside $[u_{j(t)} - \tau_j]$. Although waiting at $f(\alpha)$ cause disutility, it may decrease the disutility of conducting $\alpha$ due to a better start time and lead to less overall disutility. Choice of waiting can also be incorporated into the bipartite network.

We first deal with the situation $u_{j(t)} < t < v_{j(t)}$. At activity state $s$ in Fig. 5, the links originating from a node to any other nodes with bigger time labels can be used to represent waiting. Once $i$ has the freedom to wait, the activity start time can be later than $t$. Then, with the choice of waiting time $t_w$, the new start time $t + t_w$ has the duration choice again. Fig. 6 illustrates the choice of waiting. In Fig. 6(a), the bold links indicate that $i$ arrives at $f(\alpha)$ at $t$, chooses to wait $T_0$, and conducts $\alpha$ with a duration $v_{j(t)} - t - T_0$. In total, there are $v_{j(t)} - T_0$ and $\frac{v_{j(t)} - t - t_w}{T_0}$ links of waiting and conducting the activity respectively. Fig. 6(b) shows a reduced bipartite network with time step of $T_0$ for waiting choice and $T_0$ for duration choice. In this way, activity timing, chain, and duration are still associated. Let $t_w$ denote the waiting time, then the disutility of waiting and conducting the activity are $\beta_{iw} \times t_w$ and $Z_{iuJ(t)}(i + t_w, \tau)$ respectively.

Possibly, $t$ can be outside of $[u_{j(t)} - \tau_j]$. If $t > v_{j(t)}$, there is neither choice of waiting nor duration. If $t < u_{j(t)}$, $i$ has to wait until $u_{j(t)}$ and is allowed to wait longer for a better start time. This situation can be captured by adding a waiting link from $t$ to $u_{j(t)}$ in Fig. 6.

The above treatment of duration choice applies to any activities with flexible durations in the AP. An original transaction link is transformed into a bipartite network that represents the duration choice space contingent on the arrival time. Just like PVN and PTN connections referring to time-dependent networks, it is reasonable to let transaction links refer to on-the-fly bipartite networks. The multi-state supernetwork representation discussed in Section 2.1 is still the choice space for conducting the AP. A personalized multi-state supernetwork is a highly sparse networks considering choice (see Appendix A for a description of the pseudo-code). The algorithm keeps a non-dominated label set for every node in the supernetwork based on a strong dominance relationship that one time instance keeps one label of disutility. A label is corrected if traversing a link results in a smaller disutility at the same time instance. If waiting is involved at a location, the non-dominated set is self-corrected. Then, the optimal ATP can be backtracked from node $H^1$ to $H^0$. It is noteworthy that there are at most $N_i$ non-dominated labels for each node ($N_i$ is bounded by 1440 if $T_0 = 1$ min). The algorithm terminates when no label of a node can be further updated, which is bounded by finite steps because the traverse times on the links are positive.

**Fig. 3.** Example of duration choice.

**Fig. 4.** Example of incorporation duration choice.
3.2. Duration choice of home-stay

This section discusses two formats of representing home-stay for an AP of individual \( i \) (\( AP_i \)) with at least two activities. The focus is on the home-stay episodes during the period of implementing \( AP_i \). The home-stay choice refers to deciding the numbers and durations of home-stay episodes. Delineated by the departure and arrival time at \( H_0 \) and \( H_1 \) respectively, the remainder timeslot of a day is reserved for other in-home activities.

3.2.1. Explicit representation

Explicitly representing choice facets as different types of links is the main mechanism underlying multi-state supernet-works. Following the same logic, a straightforward format to accommodate home-stay is to represent possible home-stay episodes explicitly as links. Home-stay cannot be represented as travel link or transition link because it does not involve travel and not necessarily involve a change of transport mode. Instead, it can be represented as a transaction link by considering home-stay as an activity. Nonetheless, home-return and home-stay during the implementation period are only optional, since \( i \) may go for the next activity directly after finishing one activity. Thus, it is reasonable to regard home-stay as an optional activity, which is new concept in the activity-travel scheduling models. To incorporate home-stay choice, all episodes of optional home-stay activities need to be appended to \( AP_i \). The content of \( AP \), activity state and their reachability should also be extended.

Let \( \tilde{AP}_i \) denote the \( AP \) after appending home-stay episodes to \( AP_i \). Given \( |A| \) activities in \( AP_i \), there are \( |A| - 1 \) optional home-stay activities in \( \tilde{AP}_i \). With respect to a certain sequencing of activities in \( AP_i \), let \( |S| \) be the number of activity states. \( |S| \) equals to \( |A| + 1 \) if the sequence of activities is completely fixed; and \( |S| \) equals to \( 2^{|A|} \) if sequences are all free to choose. Each episode of home-stay can be implemented immediately after finishing one activity except the last one in the \( ATP \). Therefore, there is one extra activity state accommodating home-stay for each intermediate activity state, which renders \( 2 \times |S| - 2 \) activity states for \( \tilde{AP}_i \).

Let \( S_p = \ldots, s_l, \ldots, s_j \in \{0, 1\} \) be an activity state and \( V_q \) a vehicle state for \( AP_i \), and \( \tilde{S}_p \) an activity state for \( \tilde{AP}_i \). \( \tilde{S}_p \) can be defined as \( S_p|h \), where the part before delimiter “|” represents the activity state of \( AP_i \), and \( h \) represents whether \( i \) has stayed at home or not: \( h = 1 \) means “yes” and \( h = 0 \) “no”. Suppose \( \tilde{S}_p \) is an intermediate activity state and can reach another activity state \( \tilde{S}_p \) by conducting an activity of \( AP_i \). Thus, only \( \tilde{S}_p|0 \) can reach \( \tilde{S}_p|1 \) by conducting one episode of home-stay; both \( \tilde{S}_p|0 \) and \( \tilde{S}_p|1 \) can reach \( \tilde{S}_p|0 \) by conducting the activity. The extension of activity state reachability enables the choice of optional home-stay activities. This incorporation increases the number of activity states but does not affect vehicle states. Fig. 7 is an
example of supernetwork representations for APi and APj with one vehicle state only. As shown, a PTN at activity state “10|1” is created in the right-hand-side to explicitly represent home-stay choice. After the explicit representation of home-stay, the features of the multi-state supernetworks remain. Following Section 3.1.1, time-expanded bipartite networks can be used to represent the duration choice of home-stay.

To avoid unnecessarily extending the activity states, an admissible heuristic rule can be applied: there is no optional home-stay at an intermediate state if the total lower bound travel time of returning home and going to the next locations is too long to meet the time window constraints. As aforementioned, individuals normally gains utility from staying home and thus ZiSp(t, τ) should be negative in terms of disutility. Despite this fact, the label correcting algorithm can still find the optimal ATP because transaction links in the supernetworks never cause cycles with negative disutilities.

3.2.2. Implicit representation

Another format accommodating home-stay is to represent it at nodes (home locations) because the general representation has already included links of returning home at intermediate activity states (Fig. 1). Given Nf time instances of a node, a time instance can always be associated with a later one through waiting. Hence, duration choice of home-stay is supported by allowing the individual to stay some time at home at the intermediate activity states.

Let H(Sp − Vq) denote the home location at an intermediate activity state Sp and vehicle state Vq, and (t, d) be a two-tuple label of arriving at H(Sp − Vq) at time t with sub-total disutility d following a sub-ATP starting from H0. By staying time τ (τ > 0), the disutility is updated as d + ZiSp(t, τ). It will be compared with the disutility of time instance t + τ, which is initially set as positive infinity. If d + ZiSp(t, τ) is larger, it means home-stay with duration τ does not benefit; otherwise, the label (t + τ, d + ZiSp(t, τ)) will be temporarily kept at time instance t + τ. This self-correction is applied for every possible τ at H(Sp − Vq). Thus, we have an updated non-dominated label set at H(Sp − Vq) that implicitly incorporates home-stay duration choice. The representation can be realized by connecting the time labels of H(Sp − Vq) from t and any other labels greater than t.

In case that ZiSp(t, τ) is an additive function of τ, i.e., ZiSp(t, τ) = ZiSp(t, τ1) + ZiSp(t + τ1, τ2) if τ = τ1 + τ2, home-stay can be represented in an efficient way. Given t at H(Sp − Vq), there is only one link directed to the next time label t + T0 with added link disutility. It only takes NT − 1 links to represent all possible arrival times, while it needs \( \frac{NT(NT-1)}{2} \) links in the general (non-additive) case. A simple additive structure is often adopted in activity-based modeling. For instance, βiahl · τ (βiahl < 0) is used as the opportunity cost of conducting an activity for duration τ, which i might otherwise gain by staying at home. Fig. 8(a) and (b) show an example of home-stay with additive and non-additive disutility functions respectively. With this representation, the disutilities of conducting activities are disentangled from the opportunity costs of staying at home. Similarly, a lower time resolution of home-stay duration may apply.

The implicit format does not change the general supernetwork representation. A small twist on the label correcting algorithm can still find the optimal ATP. The step for correcting the labels of H(Sp − Vq) is inserted between lines 5 and 6 of Appendix A. This step takes run-time complexity O(Nf) in the same order of dealing with time-expanded transaction links. No matter what forms ZiSp(t, τ) may exude (e.g., negative or positive, additive or non-additive), the algorithm terminates within finite steps as the traverse time on all the links is positive.

With either explicit or implicit format, home-based tours are generated endogenously as the choice of home-return is incorporated. Home-stay durations and home-based tours are wrapped in the predicted ATP, resulting from considering all relevant choice facets simultaneously. Each format has its own advantage. While the implicit format does not extend states, the explicit provides an avenue for including specific in-home activities. Practically, the explicit format involves more computation time, which is bounded by two times since the supernetwork scale for APj doubles approximately.
and it takes run-time complexity of $O$ space–time supernetwork representation including state transfers. Hence, it takes run-time complexity of $O$ to traverse the duration space for either an episode of activity or home-stay at one time instance. Hence, it takes run-time complexity of $PASS = O(P \cdot M \cdot \log M + Q \cdot N \cdot \log N)$ to make a pass (traversing all links once) in $SNK_i$. Given there are $N_t$ time instances at a node and at most $N_t$ links in an ATP, we get a worst-case run-time complexity of $O(N^2 \cdot \text{PASS})$ by the label correcting algorithm, which is reduced to $O(V_{SNK} \cdot N_t \cdot \text{PASS})$ as $V_{SNK} < N_t$ holds in most cases.

$O(P)$ and $O(Q)$ can be further derived based on the principle that complexity analysis excludes coefficients and lower order factors. In addition to the notations used above, let $|C|$ and $|V|$ be the numbers of private vehicles and vehicle states of $AP_i$, respectively, and $N^i_C (N^i_A ≥ 1)$ the number of alternative locations selected for each flexible activity if any. Then, we have $O(|S|) = O(2^{|A|})$ and $O(|V|) = O(|C| \cdot |A| \cdot N^i_C)$ as $|V|$ is determined by the number of parking locations, which is in the same order as the number of activity locations. The numbers of nodes and links in a PTN or PVN are equal to $O(|A| \cdot N^i_A)$ and $O((|A| \cdot N^i_A)^2)$ respectively. Thus, we can obtain $O(P) = O(|S| \cdot |V| \cdot (|A| \cdot N^i_A)^2)$, $O(Q) = O(|S| \cdot |C| \cdot (|A| \cdot N^i_C)^2)$.

In the context of individual activity-travel scheduling, $|A|$ and $|C|$ are quite small; so is $|S|$; after reduction, $O(P) = O(N^i_A)$ and $O(Q) = O(N^i_C)$; and if there is no flexible activity, $O(P) = O(Q) = O(1)$.

The label correcting process involves a large number of useless correcting trials. This section discusses a more efficient algorithm based on recursive formulations from $H_0$ to $H_1$ and start time $v_{HI}$ to end time $v_{HI}$ in a non-FIFO ordinary network $G$; finding the minimum-cost paths is a NP-hard problem. If time is considered in the discrete domain, a pseudo-polynomial algorithm exists to find the optimal with the worst-case run-time complexity $O(m \cdot N_t)$, where $m$ and $N_t$ are the numbers of links in $G$ and discrete time instances respectively. $G$ is restructured as a space–time network (Chabini, 1999; Dean, 2004; George, 2008; Tong et al., 2015), which is a topological ordering and acyclic graph of $m \cdot N_t$ links. As a result, one pass of the space–time network is sufficient to find the optimal paths. Similar to discretizing duration of activities and home-stay, we can also restructure $SNK_i$ in space–time. Although space–time networks have been applied in unimodal networks (e.g., Dean, 2004) and public transit networks (e.g., Li et al., 2010), individual activity-travel scheduling of multi-modal multi-activity ATPs is inherently a problem of a different magnitude involving a high level of detail and spatial–temporal resolution. Thus, space–time multi-state supernetwork is an innovative application of this network technique.

A space–time multi-state supernetwork is constructed by extending the links of $SNK_i$. For travel and transition links, one time instance at the entry node corresponds to one time instance at the exit node. For transition links, time instance $t$ leads to at most $\frac{v_{HI}}{v_{HI}}$ time instances at the exit nodes. If waiting choice is not allowed and home-stay episodes are explicitly represented, we can construct the space–time supernetwork by linking all reachable time instances. Fig. 9 shows an example, in which $H^0 \rightarrow n, n \rightarrow w$ and $w \rightarrow H^1$ are links of $SNK_i$ and particularly link $n \rightarrow w$ is a transaction link. (See Appendix B for a space–time supernetwork representation including state transfers.)

The space–time supernetwork is also a topological ordering and acyclic graph. Let $C_n(t)$ denote the least disutility of arriving at node $n$ at time $t$ departing $H^0$ at any time, $a_{nw}(t)$ be the arrival time at node $w$ of departing $n$ at $t$, and $c_{nw}(t)$ is the disutility of traversing $n \rightarrow w$. Note that $a_{nw}(t)$ and $c_{nw}(t)$ are vectors when $n \rightarrow w$ is a transaction link. $C_n(t)$ is initialized...
as \( +\infty \), \( \forall n \) and \( \forall t \). Regarding departure time choice at \( H^0 \), suppose there is an initialized disutility \( d_{i\rho} |t| \) at \( H^0 \). A recursive formulation on \( C_n(t) \) is:

\[
C_n(t) = d_{i\rho} |t|, \quad \text{if } n = H^0, \quad u_{B^1} \leq t \leq v_{B^1}
\]

(5)

\[
C_w(a_{nw}(t)) = \left\{
\begin{array}{ll}
+\infty, & \text{if } a_{nw}(t) > v_{B^1} \\
\min(C_w(a_{nw}(t)), C_n(t) + c_{nw}(t)), & \text{if } a_{nw}(t) \leq v_{B^1}
\end{array}
\right.
\]

(6)

With Eqs. (5) and (6), we can obtain \( C_{i\rho}^H(t), \forall t \). The main computation time is used for traversing \( N_T \cdot P \) PVN links, \( N_T \cdot Q \) PTN links, and \( O(N_T^2) \cdot R \) transaction links. Given \( R \leq \max\{P, Q\} \) and \( N_T \ll \max\{M, N\} \), the run-time complexity of the formulation is \( O(N_T \cdot \text{PASS}) \), which is an order lower than the label correcting algorithm.

A reasonable way to treat home-stay before departing \( H^0 \) is setting \( C_{i\rho}^H(u_{B^1}) = 0 \) as the base disutility. \( C_{i\rho}^H(t) \) at other \( t \) is updated at the first activity state \( s^0 \) as:

\[
C_{i\rho}^H(t) = Z_{w^0H}(u_{B^1}, t - u_{B^1}), \quad t \neq u_{B^1}
\]

(7)

As the opportunity cost of home-stay is also taken into account, arriving at \( H^1 \) earlier should be compensated. Thus, \( C_{i\rho}^H(t) \) is updated at the last activity state \( s^1 \) as:

\[
C_{i\rho}^H(t) = C_{i\rho}^H(t) + Z_{w^0H}(v_{B^1} - t, t), \quad t \leq v_{B^1}
\]

(8)

where \( C_{i\rho}^H(t) \) is the final disutility of arriving \( H^1 \) at \( t \). With \( \text{argmin}_i C_{i\rho}^H(t) \), the detailed optimal ATP can be backtracked from \( H^1 \) to \( H^0 \).

Similarly, we can construct the space–time supernetwork with implicit representation of home-stay and waiting choice. Fig. 10 is an example of this kind, combining the representations of Figs. 6 and 8. Without loss of generality, we first consider the additive structural disutility \( Z_{at}(t, T_0) \) of waiting or staying at location \( L \) at time \( t \) for duration \( T_0 \). Thus, using \( a_{nw}(t') = t \), in addition to Eq. (5), \( C_w(t) \) can be formulated as follows:

\[
C_w(t) = \left\{
\begin{array}{ll}
+\infty, & \text{if } t > v_{B^1} \\
\min(C_w(t), c_n(t') + c_{nw}(t'), C_w(t - T_0) + Z_{at}(t - T_0, T_0)), & \text{if } t \leq v_{B^1}
\end{array}
\right.
\]

(9)

With Eqs. (5), (8) and (9), we can obtain \( C_{i\rho}^H(t), \forall t \). Since Eq. (9) only adds \( O(N_T \cdot \text{ESNK}) \) steps of comparison to Eq. (6), the runtime complexity of Eq. (9) remains \( O(N_T \cdot \text{PASS}) \). In case of non-additive waiting or staying disutility, \( O(N_T^2 \cdot \text{ESNK}) \) steps of comparison at worst are needed, making the final run-time complexity \( O(N_T^2 \cdot \text{ESNK} + N_T \cdot \text{PASS}) \). It comes down to \( O(N_T \cdot \text{PASS}) \) if home-stay is only allowed at vehicle state \( H \) when all PVs are at home. According to \( \text{Dean (2004)} \), \( C_w(t) \) to \( \forall w \) is processed in a chronological order. Every link at every time instance is only traversed once; thus, the worst-case run-time complexity is also the optimal.

Fig. 9. Explicit representation of home-stay and not allowing waiting.

Fig. 10. Implicit representation of home-stay and allowing waiting.
4. Numerical examples

This section exemplifies the incorporation of duration choice in multi-state supernetworks for individual activity-travel scheduling. The approach is executed using C++ on a PC using one core of Intel® CPU Q9400@ 2.67 GHz, 8 G RAM. The study area concerns the Eindhoven–Helmond corridor (The Netherlands), which is about 15 km long between the two city centers. Suppose individual i, living in Helmond, has to conduct an AP on a typical day, which may include: (1) three activities, i.e., dropping-off a child to day-care center, work and shopping. Dropping-off and work are conducted with fixed durations of 3 and 510 min, and time windows of [8:00 am, 8:10 am] and [9:00 am, 5:30 pm], respectively. Shopping can be conducted at one of multiple locations with a flexible duration between 10 and 60 min; (2) availability of a car; (3) free to choose sequence; and (4) time window \([u_{ili}, v_{ili}]\) for activities: [7:00 am, 7:00 pm]. Fig. 11 and other related data are described below:

1. The two big dots are transport hubs (THs). In the time-expanded PT network, there are 176,163 nodes and 309,979 links. Fare for train and bus are 0.15 €/km and 0.25 €/km respectively.
2. Roads are classified into four types: \(<\text{urban}, \text{local}, \text{regional}, \text{national}\>). Walking speeds on them are set \(<5,6,0,0\) km/h, and car fuel costs \(<0.18,0.16,0.12,0.1\) €/km. Car speeds refer to time-dependent profiles in Fig. 12(a). There are 28,734 nodes and 81,360 links in the road network.
3. Potential parking locations are activity locations, \(P + R\), TH/1, TH/2. Parking ticket costs are in linear functions as \(\alpha_p + \beta_p \times t\). \((\alpha_p, \beta_p)\) is in unit of \(\langle t, \epsilon/h\rangle\) as \((0.8,0.18)\) for \(P + R\) and THs. \((\alpha_p, \beta_p)\) in other locations is dependent on the zoning, which is \((1.0,0.6)\) if within 1 km to the city center points of Eindhoven and Helmond, \((0.0)\) if more than 2 km to the city center points, and otherwise, \((0.5,0.3)\). Parking search time refers to time-dependent profiles in Fig. 12(b). Suppose one minute for car picking-up. As discussed in Liao et al. (2013), the fixed disutility including base parking cost and search time is attached on the parking links; the disutility of transfer is dealt on the PTN connections; parking-duration dependent disutility is attached to the time expense in the PTNs and activity durations under the specific vehicle state.
4. Boxed H, D and O denote home, day-care center and office respectively. Small dots represent the alternative locations for shopping, which are classified into three types as \(\{1,2,3\}\) labeled in the right brackets with their IDs. The time windows of the shopping locations are \([8:00 \text{am}, 21:00 \text{pm}], [8:00 \text{am}, 19:00 \text{pm}]\) and \([9:00 \text{am}, 18:00 \text{pm}]\).
5. For the sake of simplicity, suppose activity states do not affect link costs. Time and monetary cost are the two components of travel link costs. Personalized parameters are set in Table 2, estimated from a survey in the Netherlands (Arentze and Molin, 2013). Note that for real-world applications those parameters should be individualized by personal characteristics. The disutility of work and dropping-off are set to zero because they are fixed activities. The disutility of shopping is defined as \(Z_{ij}(t, \tau) = U_B - U_{ij}(t, \tau)\), where \(j\) denotes the type of shopping location, \(U_B\) and \(U_{ij}(t, \tau)\) are the utility of shopping at an ideal and a specific location respectively.

A timing-duration combined model is adopted for \(U_{ij}(t, \tau)\), \(U_B(t, \tau) = f_j(t) \times \log(1 + \beta_d \times \tau)\). \(f_j(t)\) is the coefficient for time-dependency and the log-shape explains the relationship between duration and utility. \(\beta_d\) is a constant for re-scaling. The parameters are set as \(U_B = 35,\ \beta_d = 1,\ f_j(t) = -0.002(t - 8.5)(t - 9)(t - 18.5)(t - 19) + 1.7,\ f_j(t) = 1.25 \times f_j(t),\ \) and \(f_j(t) = 1.5 \times f_j(t), f_j(t)\) is set in the form of quadratic function in that two time-dependent peaks are obtained. These settings meet the condition that different types of shopping locations exhibit different levels of attractiveness along with timing and duration.

Three individual APs are set accumulatively to evaluate duration choice; and simulations on a small population of three scenarios are run to test the aggregate results and computational efficiency.

4.1. Example 1: two activities with PT only

Suppose \(i\) only uses PT to conduct AP\(_i\) including two activities, i.e. work and shopping. As \(i\) is free to choose the activity sequence, there are four activity states; and using PT makes only one vehicle state. Thus, there are four PTNs in the multi-state supernetwork. In Fig. 11, there are 52 alternative locations for shopping. Without selecting a small choice set, the supernetwork scale becomes very large and the scheduling query cannot be solved in an acceptable time. A heuristic selection method is based on the estimated disutility of shopping and extra travel involving the locations (Liao et al., 2011, 2013). With the space–time constraints conditioned on the minimum shopping duration and a time margin allowing for travel dynamics, only eight alternatives are left in the choice set, including IDs \(#2,7,11,14,18,23,34,41\)\). The travel times are estimated by Euclidean distance and average speed, while random sampling is applied for shopping duration since the actual duration is unknown at the estimation stage. Through the location selection, we obtain a reduced list \(#18,14,23,41,7,11\) ranked in terms of the estimated disutility starting from the least. With these location alternatives, home-stay between the two activities is not possibly by applying the admissible heuristics for avoiding unnecessary representing home-return and home-stay (Section 3.2.1). In this example, we can examine the choice of departure time, shopping duration and activity location together.
We first use a reasonable time resolution to delineate the alternatives for shopping duration and departure time, i.e., $T_R = T_K = 5$ minutes. Furthermore, we assume no waiting choice for shopping and a linear disutility for home-stay, i.e., $b_i w = 1$ and $Z_{H}(t,s) = b_i H / C_s$. The choice set for shopping duration is $\{s = TR_0 / C_s | 10 \leq s \leq 60, X \in N\}$. By setting $b_i H$ as $0.05$, $C_{H}(t)$ is initialized as:

$$C_{H}(t) = \begin{cases} 
\beta_{Bi} \times t, & \tau = TR_0 \times X, t = 7:00 \text{ am}, 0 \leq \tau \leq 120, X \in Z \\
+\infty, & \text{otherwise}
\end{cases}$$

(a) car speed profiles

(b) car parking search time profiles

Fig. 11. Eindhoven-Helmond corridor (scale: 1:100,000).

Fig. 12. Profiles of car use.

Table 2
Personalized parameters.

<table>
<thead>
<tr>
<th>Constant of travel links</th>
<th>Coefficient of time (min)</th>
<th>Transition</th>
<th>Coefficient of cost (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walk</td>
<td>Bus</td>
<td>Train</td>
<td>Car</td>
</tr>
<tr>
<td>$\rho_{Bi}^{W}$</td>
<td>$\rho_{Bi}^{B}$</td>
<td>$\rho_{Bi}^{T}$</td>
<td>$\rho_{Bi}^{C}$</td>
</tr>
<tr>
<td>0.0</td>
<td>0.8</td>
<td>1.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

We first use a reasonable time resolution to delineate the alternatives for shopping duration and departure time, i.e., $T_R = T_K = 5$ minutes. Furthermore, we assume no waiting choice for shopping and a linear disutility for home-stay, i.e., $b_i w = +\infty$ and $Z_{Bi}(t, \tau) = \beta_{Hi} \times \tau$. The choice set for shopping duration is $\{\tau = TR \times X | 10 \leq \tau \leq 60, X \in N\}$. By setting $\beta_{Hi}$ as $-0.05$, $C_{Hi}(t)$ is initialized as:

$$C_{Hi}(t) = \begin{cases} 
\beta_{Hi} \times t, & \tau = TR \times X, t = 7:00 \text{ am}, 0 \leq \tau \leq 120, X \in Z \\
+\infty, & \text{otherwise}
\end{cases}$$

(10)
The label correcting algorithm and recursive formulation are executed with different values of $N_A^d$ ($1 \leq N_A^d \leq 6$). At each time, the first $N_A^d$ shopping locations of the ranked list are included in $SNK_i$. Table 3 displays the scheduling results, indicating that the optimal ATP is found when $N_A^d = 2$ (highlighted in bold) and thus the optimal shopping location is ID-14. While the number of PTN queries and computation time increase as $N_A^d$ gets larger for both algorithms, the recursive formation is much more efficient. Note that the scale of $SNK_i$ refers to the numbers of nodes/links in the supernetwork representation.

By backtracking, the optimal ATP (when $N_A^d = 2$) suggests that $i$ leave home at 8:10 am by PT, arrive at the office at 8:53 am, and leave the office at 5:30 pm; then, $i$ walk 8 min to location ID-14 (in Eindhoven) to shop for 45 min; and finally arrive home at 6:59 pm by PT. The detailed PTN connections can be further backtracked through the time-expand PT network. In the morning, $i$ chooses the best departure time in confrontation with the PT timetable and time constraints at the office. After work, $i$ chooses a shopping duration at the high end minimizing the disutility for waiting PT. However, the predicted ATP of $N_A^d = 1$ suggests that after work $i$ go to location ID-18 (in Helmond) to shop for 10 min. These results indicate that choices of location and duration are interrelated. The space–time paths of the predicted ATPs are shown in Fig. 13.

In addition, if fixing the activity sequence as shopping prior to work, the optimal ATP is also found with $N_A^d = 2$. The optimal label is (6:29 pm, 39.25), and the optimal ATP suggests that $i$ depart home at 7:25 am, do shopping at location ID-14 for 45 min, and arrive at the office at 8:58 am. Although in the end $i$ can arrive home 30 min earlier than when shopping is conducted after work, $i$ needs to depart home 45 min earlier in the morning.

To investigate the effects of duration choice resolution, a sensitivity analysis is conducted on $TR$ and $TR_0$ when $N_A^d$ equals to 4 and other settings remain the same. The results (Table 4) indicate that the predicted ATP approaches the theoretical optimality as the time resolution increases, and it gets more computation-demanding, manifesting the “no-free-lunch” principle. It is also shown that the predicted ATPs are similar if the time resolutions are within a certain range (e.g., [1, 15] min), although they differ slightly in the temporal dimension. It implies a balance of efficiency and accuracy can be achieved by setting an acceptable time resolution. It is noteworthy that a lower resolution may lead to a totally different or worse pattern (e.g., $TR = TR_0 = 20$ min).

Table 3
Comparison with different values of $N_A^d$ of example 1.

<table>
<thead>
<tr>
<th>$N_A^d$</th>
<th>Scale of $SNK_i$</th>
<th>Optimal label ($t$, $C_H^t(t)$)</th>
<th>Label correcting</th>
<th>Space–time supernetwork</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node</td>
<td>All links</td>
<td>PTN</td>
<td>PTN queries</td>
<td>Run-time (s)</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>14</td>
<td>10</td>
<td>123</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>22</td>
<td>16</td>
<td>221</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>30</td>
<td>22</td>
<td>367</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>38</td>
<td>28</td>
<td>490</td>
</tr>
<tr>
<td>5</td>
<td>28</td>
<td>46</td>
<td>34</td>
<td>624</td>
</tr>
<tr>
<td>6</td>
<td>32</td>
<td>54</td>
<td>40</td>
<td>809</td>
</tr>
</tbody>
</table>

Fig. 13. Predicted space–time paths with $N_A^d = 1$ and $N_A^d = 2$ of example 1.
4.2. Example 2: two activities with PT and PV

Based on example 1, one PV-car is appended to $AP_i$. Potential car parking locations are activity locations, $P + R$ and THs in Fig. 11. Given $N_A^d$, there are $N_A^d + 4$ out-of-home parking locations, making $N_A^d + 6$ vehicle states in $SNK_i$ (one extra for car being in use and another for being parked at home). Different from example 1, $i$ is allowed to return home after work or shopping by car. There are 6 activity states with the explicit representation of home-stay and 4 with the implicit format. For fast scheduling, implicit representation is used. Setting $T_R = T_{R0} = 5$ minutes and others the same as example 1, the execution results with different $N_A^d$ are displayed in Table 5.

Compared with example 1, the queries for PTN and PVN connections increase substantially because the use of car scales up the supernetworks. As highlighted in bold in Table 5, location ID-14 ($N_A^d = 2$) is the optimal shopping location. By backtracking, it is found that the activity sequences are both shopping prior to work when $N_A^d = 1$ and $N_A^d = 2$; however, the choices of shopping location and duration are different. The detailed space–time paths are shown in Fig. 14. As specified by the disutility functions, shopping causes more disutility right after work than right before work with the same duration. Despite that car parking is more difficult (7 min search time), $i$ is facilitated to avoid the afternoon-peak by car, whereas it is not in example 1 due to the long travel time by PT. The optimal shopping duration in Helmond is 25 min, while it is 30 min in Eindhoven. Meanwhile, it also takes less travel time by car from home to office in the morning (21 min) than from the office to home in the afternoon (24 min). The detailed PVN connections can be backtracked through the time-dependent road network. According to the predictions, home-return between the two activities does not occur as it does not reduce the disutility. This example illustrates that the interactions between an individual's choices and travel-facility dynamics are captured.

4.3. Example 3: three activities with PT and PV

Based on example 1, one PV-car is appended to $AP_i$. Due to the space–time constraints, the activity sequencing must satisfy dropping-off a child to day-care center, prior to work and shopping, making 5 activity states. Given $N_A^d$, there are $N_A^d + 7$ vehicle states as one more activity location is added. In this example, waiting is allowed at the shopping locations for a better start time. With the same settings as example 2, the scheduling results are shown in Table 6. As highlighted in bold (when $N_A^d = 1$), location ID-18 is the optimal shopping location. By backtracking, the space–time path of the predicted ATP is shown in blue in Fig. 15. After dropping-off, the spare time before the opening time for work is too long, but it is too short to go shopping with a favorable duration. Thus, $i$ would return home first and then go to the office after home-stay for 15 min. Thus, two home-based tours are produced endogenously. Meanwhile, waiting at the shopping location for a better start time does not occur.

**Table 4**
Predicted ATPs of different values of $T_R$ and $T_{R0}$.

<table>
<thead>
<tr>
<th>$T_R = T_{R0}$ (min)</th>
<th>PTN queries</th>
<th>Run-time (s)</th>
<th>Optimal label $(t, C_{pt}(t))$</th>
<th>Departure time (am)</th>
<th>Activity sequence</th>
<th>Shopping location</th>
<th>Shopping duration (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1048</td>
<td>3.72</td>
<td>(6:59 pm, 36.20)</td>
<td>8:12</td>
<td>Work first</td>
<td>ID-14</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>421</td>
<td>1.79</td>
<td>(6:59 pm, 36.37)</td>
<td>8:12</td>
<td>Work first</td>
<td>ID-14</td>
<td>46</td>
</tr>
<tr>
<td>5</td>
<td>193</td>
<td>0.44</td>
<td>(6:59 pm, 36.52)</td>
<td>8:10</td>
<td>Work first</td>
<td>ID-14</td>
<td>45</td>
</tr>
<tr>
<td>10</td>
<td>143</td>
<td>0.28</td>
<td>(6:59 pm, 37.81)</td>
<td>8:10</td>
<td>Work first</td>
<td>ID-14</td>
<td>40</td>
</tr>
<tr>
<td>15</td>
<td>95</td>
<td>0.15</td>
<td>(6:59 pm, 37.95)</td>
<td>8:00</td>
<td>Work first</td>
<td>ID-14</td>
<td>40</td>
</tr>
<tr>
<td>20</td>
<td>65</td>
<td>0.08</td>
<td>(6:29 pm, 40.29)</td>
<td>8:00</td>
<td>Work first</td>
<td>ID-18</td>
<td>10</td>
</tr>
</tbody>
</table>

**Table 5**
Comparison with different values of $N_A^d$ of example 2.

<table>
<thead>
<tr>
<th>$N_A^d$</th>
<th>Scale of $SNK_i$</th>
<th>Optimal label $(t, C_{pt}(t))$</th>
<th>Space–time supernetwork</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Node All links PTN PVN</td>
<td></td>
<td>PTN queries PVN queries Run-time (s)</td>
</tr>
<tr>
<td>1</td>
<td>64</td>
<td>63</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>111</td>
<td>42</td>
</tr>
<tr>
<td>3</td>
<td>144</td>
<td>135</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>196</td>
<td>157</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>256</td>
<td>177</td>
<td>66</td>
</tr>
<tr>
<td>6</td>
<td>324</td>
<td>199</td>
<td>74</td>
</tr>
</tbody>
</table>
Table 6
Comparison with different values of $N^A$ of example 3.

<table>
<thead>
<tr>
<th>$N^A$</th>
<th>Scale of $SNK_i$</th>
<th>Optimal label ($t, C_H(t)$)</th>
<th>Space–time supernetwork</th>
<th>Run-time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Node</td>
<td>All links</td>
<td>PTN</td>
<td>PVN</td>
</tr>
<tr>
<td>1</td>
<td>125</td>
<td>85</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>151</td>
<td>48</td>
<td>39</td>
</tr>
<tr>
<td>3</td>
<td>265</td>
<td>174</td>
<td>60</td>
<td>52</td>
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<tr>
<td>4</td>
<td>320</td>
<td>198</td>
<td>68</td>
<td>60</td>
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<tr>
<td>5</td>
<td>405</td>
<td>226</td>
<td>76</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>500</td>
<td>245</td>
<td>88</td>
<td>72</td>
</tr>
</tbody>
</table>

Fig. 14. Predicted space–time paths with $N^A = 1$ and $N^A = 2$ of example 2.

Fig. 15. Predicted space–time path with $N^A = 1$ of example 3.
To assess the behavior adaptation to change from the supply side, a scenario promoting location ID-7 from type 2 to type 1 is analyzed. The predicted optimal ATP indicates that i would do shopping after work at location ID-7 for 35 min (in green in Fig. 15) while the sub-pattern before work remains the same. This adaptation is also attributed to the fact that no parking cost is involved at location ID-7.

As a side note, the length of the time window for dropping-off is wider than the activity duration, but i still has to wait 2 min for the opening of the day-care center. Although this phenomenon is not rare in reality, the predicted ATP is not the theoretical optimality due to the departure time resolution. If TR and TR0 are reduced to 1 min, departure time would be adapted to 7:53 am and no waiting will be involved at the day-care center, while the rest of the ATP is exactly the same as that of TR = TR0 = 5 min. This result confirms the findings in Table 4 that if the time resolutions are set in a close range, the predictions are approximately the same.

4.4. Example 4: a small population

In this example, the scheduling model is applied to a small population under three scenarios to demonstrate the applicability of obtaining the aggregate choices, particularly activity sequence, timing and duration choice. Suppose there are 200 car travelers whose home locations are randomly distributed in the corridor and offices are randomly distributed inside or close to the ring (gray circle of Fig. 11) of Eindhoven. Among those, half are full-time workers and half are part-time with fixed work agenda from 9:00 am to 5:30 pm and from 10:00 am to 2:00 pm respectively. Moreover, each has a shopping
activity to do with a duration varying from 10 to 60 min. The setup on the supply side above is considered the base scenario (S0), in which ID-7 is upgraded to type 1 (Example 3). Based on S0, two imaginative scenarios are added: S1: the attractive timings for shopping are shifted a little later in the morning and a little earlier in the afternoon, which can be obtained by setting \( f_1(t) = -0.002(t - 10)(t - 10)(t - 16.5)(t - 18) + 1.7 \); S2: the parking costs at locations inside the ring of Eindhoven are doubled except that of TH/1.

The micro-simulations are run by setting \( T_R = T_K = 1 \) min and \( N^A = 4 \). On average, it takes around 4 s for each individual activity-travel scheduling. Fig. 16 shows the aggregate distributions of travelers in space–time under S0 and S1, while those under S0 and S2 are much similar. Compared with S0, as the attractive timings for shopping shift in S1, travelers in general obtain less utility if shopping is conducted too early or too late. As shown, all those full-time workers who do shopping before work under S0 adapt their activity sequences under S1; since part-time workers start working later at 10:00 am, a small part still benefit from doing shopping before work other than from the opposite. In addition, as full-time workers finish work at 5:30 pm (close to attractive timing), returning home and home-stay does not lead to good timing and duration for shopping, hence no home-stay episode is evoked; in contrast, the majority of part-time workers have the freedom to choose the best duration and timing for shopping by means of home-stay duration choice.

Although the difference of the aggregate distributions of S0 and S2 are not identifiable, travelers do adapt their choice of parking and shopping location. As parking costs increase under S2, more travelers use TH/1 (Eindhoven train station) as park and ride service, 42 of S2 opposed to 28 of S0. Besides the mode change from car to car and PT (park and ride), there is also a notable mode change from car to PT, 24 PT users of S0 opposed to 37 of S2. Accordingly, more travelers choose ID-14 for shopping as it is spatially close to TH/1 and of type 1 (Fig. 17). Despite causing disutility, the mode change does not necessarily result in much longer travel times because TH/1 has good access by bus to the locations of offices in the city center. Therefore, attractive timing and duration are not missed, which explains why the aggregate distributions under S0 and S2 are much similar.

4.5. Remarks

The above examples demonstrate that duration choice of activities and home-stay has been well-incorporated in multi-state supernetworks. As indicated, duration choice is closely related to and has effects on all other choice facets. Given the illustration purpose, the examples include all relevant elements of conducting activities at a city scale. In terms of the computation feasibility, a supernetwork application without duration choice and space–time extension to a larger area (The Hague-Rotterdam corridor, The Netherlands) already showed the evidence (Liao et al., 2016). Logically, in a larger study area, individuals may travel further to conduct activities, but it is likely that there are less activities in the daily APs and thus possibly no home-stay episode between activities due to space–time constraints. According to the Dutch national daily travel diary, 87% of the surveyed individuals have no more than three daily activities. Thus, the scales of the examples are realistically set.

The computation time has been mainly spent on the queries for PTN and PVN connections. Although \( N^A \) has a big impact on the scales of the supernetworks, the optimal locations for flexible activities can often be picked out by setting low values of \( N^A \). In addition, the less \( T_R \) and \( T_K \) are, the more queries are needed. Time resolution should be carefully chosen depending on the characteristics of the activities. In rare cases where individuals may travel far and conduct many activities of short durations, it is computationally burdensome for activity-travel scheduling in the multi-modal context. According to the latest progress on point-to-point routing (Bast et al., 2014), speedup factors for PVN and PTN queries are thousands up with data pre-processing. With the speedups, the process time for one individual activity-travel scheduling is comparable to that of modern micro-simulation systems even when high values are set for \( N^A, T_R \) and \( T_K \). Thus, the applicability of the proposed extensions to large-scale simulations is viable.

5. Conclusions and future work

Individual activity-travel scheduling is at the center of activity-based micro-simulation models that inherently focus on high level of detail and spatial–temporal resolution. Multi-state supernetwork approach is an emerging technique of such kind. This study has incorporated duration choice of activities and home-stay in multi-state supernetworks using space–time extension. Consequently, the trade-off between travel and time allocation to home-stay and activities can be captured, home-based tours are generated endogenously, and an efficient algorithm is supported to find the optimal ATPs with a lower order of run-time complexity. Therefore, the study means a substantial improvement of the prediction capability and feasibility of the approach.

However, several issues are worth consideration. First, the accuracy of the predicted ATPs are dependent on consistent disutility measurements of different components. Thus, the parameters of home-stay, activity participation and travel should be estimated in an integrated fashion. Second, as substitution effects among travel, home-stay and activities can be captured,
the study on their effects on externalities such as energy use, emission and budget constraints becomes more approachable. Third, duration-related transport modes, such as electric vehicles and public shared bikes, may have significant influences on activity durations. Their inclusion in the suggested approach has implications to the infrastructure management of these emerging modalities. Fourth, with the ever-enhancing capability of the supernetwork approach, its application to operational travel demand forecasting systems becomes promising. The connection with traffic models aiming for user equilibrium (Liu et al., 2015) and system optimization (Tong et al., 2015) should also be established. These issues will be addressed in future research.

Acknowledgements

This study is supported by the Dutch Science Foundation (NWO). Valuable comments from the three anonymous referees and the editor are greatly appreciated.

Appendix A

Notation:

- $\beta_i$: personalized parameters
- $SNK$: a personalized multi-state supernetwork of $AP$
- $n, w$: nodes of $SNK$
- $n \rightarrow w$: a link of $SNK$
- $B_n$: non-dominated label set of $n$
- $b_n(t, d)$: a label of $B_n$
- $B^n_w(t, d)$: the label set at $w$ resulting from traversing $n \rightarrow w$ with $b_n(t, d)$
- $B^H_0$: non-dominated departure labels at $H^0$

Pseudo-code of label correcting algorithm:

1. input: $<\beta_i, SNK, B^H_0, [u_{li}, v_{li}]>$
2. initialization: $scanList = \{H^0\}$, $B_n = \emptyset$ for $\forall n \in SNK \setminus \{H^0\}$
3. while $scanList \neq \emptyset$
4. choose the first node $n$ from $scanList$, and $scanList = scanList \setminus \{n\}$
5. for each link $n \rightarrow w$
6. for each label $b_n(t, d) \in B_n$ that have not traversed $n \rightarrow w$ earlier
7. update $B^n_w(t, d)$ in terms of link type of $n \rightarrow w$
8. if $t \leq v_{li}$
9. merge $B_w$ and $B^n_w(t, d)$ into a non-dominated set
10. end if
11. end for
12. end for
13. if $B_w$ changes and $w \notin scanList$
14. $scanList = scanList + \{w\}$
15. end if
16. end while
17. backtrack the optimal path from $H^1$ to $H^0$

Appendix B

Fig. 18(a) shows the PVN and PTN for an simpler $AP$ than Fig. 1, including two activities at $A_1$ & $A_2$ with fixed activity sequence and two parking locations ($P_1$ & $P_2$) for a PV (car). Fig. 18(b) shows the multi-state representation with a highlighted $ATP$ of implicit representation of home-stay, which is further extended in space–time multi-state supernetwork in Fig. 18(c).
(a) PVN and PTN

(b) Multi-state supernetwork representation

(c) Space-time multi-state supernetwork

Fig. 18. Illustration of space–time multi-state supernetwork.
References


