MASTER

Achromatic beam focusing with magnetic lenses

Reints, H.

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ACHROMATIC BEAM FOCUSING WITH MAGNETIC LENSES

H.REINTS

Afstudeerdocent:
Prof. Dr. Ir. H.L. Hagedoorn.

Begeleider:
Ir. M. Prins.

Technische Hogeschool Eindhoven,
afdeling Technische Natuurkunde,
vakgroep Deeltjesfysica,
groep Cyclotrontoepassingen.

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SUMMARY

In a magnet lens system that produces a proton microbeam, chromatic aberrations limit the spotsize. A lens system has been designed containing dipoles and quadrupoles to perform focusing in two dimensions. In the dispersive foci of the dipoles, sextupoles are situated to correct the chromatic aberrations. The lens system then contains two horizontal dipoles, two vertical dipoles, six quadrupoles and two sextupoles, and it has an enlargement factor of 0.025. When a diafragma of 1 mm is placed at the entrance of this system, a monochromatic beam will be focused to a spot of $25 \times 25\ \mu m^2$.

The focusing effect of the lens system has been tested numerically. When the system does not contain sextupoles, a beam with an energy spread of 0.3% is focused to a spot of ca. $143 \times 155\ \mu m^2$. With the use of sextupoles, a spot of ca. $40 \times 30\ \mu m^2$ can be achieved with this system. The remaining spot enlargement is mainly due to the geometric aberrations, introduced by the sextupoles. Only chromatic aberrations are corrected, so the minimum spotsize is now determined by the geometric aberrations of the system.

INTRODUCTION

In the group cyclotron applications of the department of physics of Eindhoven University of Technology, element analysis is performed with the PIXE method (PIXE = Proton Induced X-ray Emission).

A 3.5 MeV proton beam, produced by the cyclotron, hits a target which then emits characteristic X-rays. These X-rays are detected with a Si-Li detector, and with a computer the spectrum is analysed. From this spectrum, the concentrations of elements with $Z > 11$ can be determined, with a detection limit of ca. 1 ppm in the element range $20 < Z < 35$ [lit.1].

When the proton beam is focused to a microbeam, the composition of the target can be determined at a specific location. This is called microPIXE. In order to get as much information as possible from this spot in a certain measuring time, the beam current density must be as high as possible. The beam current is however limited because radiation damage of the sample must be prevented.

By scanning the microbeam over the target, topographic element analysis can be performed. With the help of a computer, pictures can then be produced of the element distrubution in the target [lit.1]. This method is called SPIXE (Scanning PIXE). One of the applications of SPIXE is to determine element concentrations and distributions in biological samples. One is interested in tissue structures, i.e. structures with dimensions of several cells.

When a higher spatial resolution is wanted, the number of picture elements (PIXELS) to be measured per area must be increased. When a certain area of the sample has to be scanned, this leads to an increase of the total measuring time. It is therefore necessary to focus the beam to a spot that is small enough to achieve a good spatial resolution and large enough to get acceptable measuring times.

In order to get the same horizontal and vertical spatial resolution, it is neccesary that the spot has equal horizontal and vertical dimensions. At E.U.T. is chosen for a spot of 25 μm in diameter, which gives sufficient spatial information of the sample. This spotsize is approximately equal to the thickness of the biological samples. The time, needed to scan an area of 1x1 mm is then ca. 1 hour and 30 minutes.

At this moment the proton beam is focused with a lens system of four magnetic quadrupoles. This lens system has a linear enlargement factor of ca. 0.025, so a diafragma with a diameter of 1 mm will be projected on the target as a spot of 25x25 μm^2.

Because the lens power of a quadrupole depends on the energy of the protons, a beam of different energies will not be focused optimally by this system. This effect is called chromatic aberration. The Eindhoven cyclotron produces a beam with an energy spread of ca. 0.3%, which leads to a spot enlargement up to ca. 100x100 μm^2. Although this is a second order effect, it is significant because of the small desired spotsize of 25 µm. In lens systems with magnetic quadrupoles, the minimum spot size is limited by these chromatic aberrations [lit.11].

With a combination of a dipole and a sextupole it is possible to correct chromatic aberrations in one dimension [lit.6]. The dipole makes a dispersive focus, i.e. the particles with different energies are separated spatially, normal to the optical axis. Because the vertical lens power of a sextupole is proportional to the horizontal displacement of the particles, and the latter is in a dipole focus proportional to the momentum deflection of the particles, the sextupole focuses the protons with different energies selectively.

Because the dipole introduces dispersion in the lens system, at least one other dipole has to be used to correct for this dispersion. It is possible to perform non-dispersive focusing in one dimension with a system of three dipoles [lit.3], and this is also possible with a small enlargement factor [lit.4].

In order to correct both horizontal and vertical chromatic aberrations, dipole-sextupole combinations have to be used in both directions, so a lens system must be designed containing both horizontal and vertical dipoles and sextupoles.

The purpose of the graduation work was to correct second order chromatic effects in two dimensions using dipoles and sextupoles. Therefore, it is necessary to design a new horizontal and vertical focusing lens system, with dipoles and sextupoles, and if necessary quadrupoles. This lens system has to produce a microspot on the target of ca. 25x25 μm^2.

CHAPTER 1. BEAM OPTICS

This chapter contains some theory, that is used in beam transport and focusing problems. Because of the correspondance between optics and beam transport, it is often referred to as "beam optics", and many terms of optics are used in this theory. Some general theory will be given, and magnetic dipoles, quadrupoles, and sextupoles will be described.

1.1. Phase space of Liouville

Describing a beam of particles is effectively done using the phase space of Liouville. This means that a particle is described by giving the position and momentum at any time. These parameters can be described as one point in a six-dimensional phase space, with dimensions: x, y, z, p_x, p_y, and p_z. A beam of particles is thus described as a collection of such points. Liouville's theorem says, that the local density of such points is constant, when all forces can be derived from a Hamiltonian [lit.5]. This is true for magnetic and electrical forces on a charged particle.

Since a beam of particles has a mean direction, the Z-coordinate will be chosen in this direction, in such a way, that the Z-axis corresponds to a central trajectory. This trajectory will be called the optical axis. Furthermore, one can describe the transversal path of a particle in a linear approximation by using the two-dimensional subspaces with dimensions (x,p_x) and (y,p_y). This means, that the horizontal and vertical motion are treated separately.

Since all particles in a beam have nearly the same direction, that is, $p_x/p_z \ll 1$ and $p_y/p_z \ll 1$ for all particles, and the beam diameter is much smaller than the total distance the particles have to travel, it is possible to use the paraxial approximation, thus eliminating the (z,p_z) subspace from the equations of motion. In this paraxial approximation, p_z is assumed to be equal to the value of the momentum p. Since time independent magnetic forces do not change the value of p, p_z is constant and z may be used as parameter instead of the time-coordinate, which leads to the following paraxial equations of motion:

$$F_x = mv^2.d^2x/dz^2 \tag{1.1a}$$

$$F_y = mv^2.d^2y/dz^2 \tag{1.1b}$$

With equations (1.1) it is possible to describe the motion of a particle by giving both x and y as a function of z, when F_x and F_y are known.

Because p_z is constant, it is allowed to use p_x/p_z and p_y/p_z instead of p_x an p_y respectively, without getting in conflict with Liouville's theorem. By denoting p_x/p_z as x' and p_y/p_z as y', one can describe the motion of a particle by giving (x,x') and (y,y') as vector functions of z. This has the advantage that both x and x' can be interpreted geometrically, because x' is the tangent of the angle between the velocity and the optical axis, which is the derivative of x with respect to z. In the paraxial approximation, this is equal to the angle itself.

In first order it is then possible to describe the motion of a particle in matrix notation [lit.2]. For example, a forcefree motion along a distance L, measured in the Z direction, (hereafter called "driftlength" or "drift") has the form:

$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} \tag{1.2}$$

The indices refer to the vector before and after the transformation. Another simple matrix can be derived for a direction change, proportional to the distance to the optical axis (hereafter called "thin lens" or "lens"):

$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} \tag{1.3}$$

The constant P is called the lens power, which is the reciprocal of the focal length.

With these two transformations, it is possible to describe all first order paraxial particle motions, by multiplying the matrices of all elements of a magnet system. The resultant matrix product is called the system matrix:

$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} \tag{1.4}$$

Because of Liouville's theorem, the determinant of the system matrix must always be equal to 1.

In order to describe the effect of momentum spread of a beam, it is useful to add $\Delta p/p$ as a third component to the vector (x,x') and describe the system in terms of 3x3 matrices:

$$\begin{pmatrix} x_1 \\ x_1' \\ \frac{\Delta p}{p} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \\ \frac{\Delta p}{p} \end{pmatrix} \tag{1.5}$$

Describing both horizontal and vertical effects at a time is possible using 5x5 matrix notation:

$$\begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \\ \frac{\Delta p}{p} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 & a_{15} \\ a_{21} & a_{22} & 0 & 0 & a_{25} \\ 0 & 0 & a_{33} & a_{34} & a_{35} \\ 0 & 0 & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \\ \frac{\Delta p}{p} \end{pmatrix} \tag{1.6}$$

The effect of $\Delta p/p$ in this matrix can be omitted using the 4x4 upper left submatrix.

1.2. Microbeam focusing

In order to obtain a small spot on a target, it is necessary to focus the beam using a lens system with a small enlargement factor. Since there are four conditions to fulfil, i.e. both horizontal and vertical focusing, and in both directions a certain (small) enlargement factor, a system must be designed with four lenses when the drifts are chosen fixed.

In order to obtain focusing, a_{12} and a_{34} must be zero, and then a_{11} and a_{33} are equal to the horizontal and vertical enlargement factors, M_h and M_v. In order to obtain a spot with equal horizontal and vertical dimensions, it is desirable that M_h and M_v have the same absolute value [lit.1], so:

$$M_h = M_v \quad \text{(stigmatic focusing)} \tag{1.7a}$$

Or:

$$M_h = -M_v \quad \text{("antistigmatic" focusing)} \tag{1.7b}$$

It is possible to build such lens systems: an antistigmatic system of four magnetic quadrupole lenses is in use at Eindhoven University of Technology [lit.1].

1.3. Magnetic dipoles

When a charged particle moves through a homogeneous magnetic field, it feels the Lorentz force, normal to its velocity, yielding a circular motion with a radius:

$$R = mv/qB \tag{1.8}$$

When the magnetic field exists only between two planes at an angle ϕ, both normal to the motion of the particle, the particle will be deflected by an angle equal to ϕ (see figure 1.1).

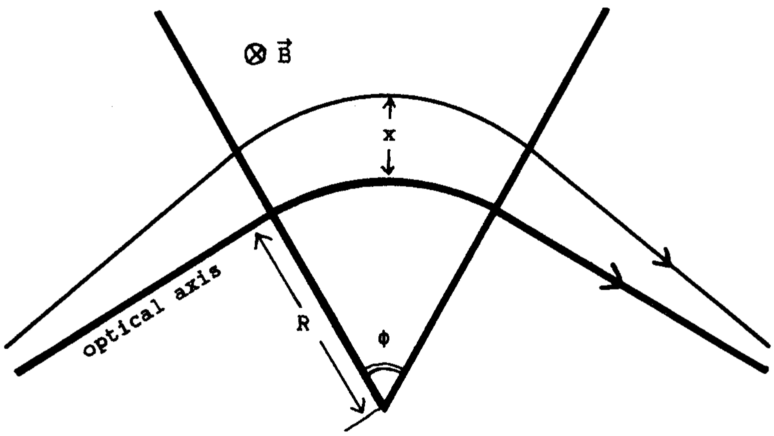


Figure 1.1
Trajectories inside a magnetic dipole.

This trajectory is defined as the optical axis inside the dipole. A particle with the same momentum as the particle for which this axis is defined, traveling at a distance x from the optical axis, will be focused towards it. Because the particle has the same momentum as the particle following the optical axis, it feels the same radial acceleration:

$$a = v^2/R \tag{1.9}$$

Using the paraxial approximation, the acceleration can also be written as [lit.5]:

$$a = v^2/(R+x) - d^2x/dt^2 \tag{1.10}$$

Using the paraxial approximation for d^2x/dt^2 and deviding by v^2, this yields:

$$1/R = 1/(R+x) - x'' \tag{1.11}$$

where the prime means differentiation with respect to z. Because x is small compared to R, this is approximately equal to:

$$1/R = (1-x/R)/R - x'' \qquad (1.12)$$

This leads to:

$$x'' + x/R^2 = 0 \qquad (1.13)$$

which is a harmonic equation. The solution is, using index 0 at z=0 and index 1 at z = L = R.ϕ:

$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} \cos(\phi) & R.\sin(\phi) \\ -\frac{1}{R}\sin(\phi) & \cos(\phi) \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix} \qquad (1.14)$$

Which is the description in matrix notation of the first order paraxial motion through a magnetic dipole.

When a particle has a slightly different energy, the momentum of that particle will not be p, but p+Δp, resulting in a radius inside the dipole of R(1+Δp/p) instead of R. This means that eq.(1.11) changes to:

$$\frac{1}{R(1+\Delta p/p)} = \frac{1}{R+x} - x'' \qquad (1.15)$$

This is approximately equal to:

$$\frac{1}{R}(1-\Delta p/p) = \frac{1}{R}(1-x/R) - x'' \qquad (1.16)$$

Which leads to the differential equation:

$$x'' + x/R^2 - (\Delta p/p)/R = 0 \qquad (1.17)$$

The solution of this equation is in matrix notation:

$$\begin{pmatrix} x_1 \\ x_1' \\ \frac{\Delta p}{p} \end{pmatrix} = \begin{pmatrix} \cos(\phi) & R.\sin(\phi) & R(1-\cos(\phi)) \\ -\frac{1}{R}\sin(\phi) & \cos(\phi) & \sin(\phi) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \\ \frac{\Delta p}{p} \end{pmatrix} \tag{1.18}$$

This matrix can be written as the product of a drift matrix, a lens matrix, and another drift matrix [lit.2]:

$$\begin{pmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -P & 1 & D \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1.19}$$

With:

$$P = \sin(\phi)/R \tag{1.20a}$$

$$D = \sin(\phi) \qquad \text{b)}$$

$$L = R.\tan(\phi/2) \tag{1.20c}$$

D is called the dispersion of the dipole, and leads to first order chromatic effects. This can be used to produce dispersive foci, which means that the foci of beams with different energies are separated spatially, normal to the optical axis.

1.4. Magnetic quadrupoles

An often used device in beam transport systems is the magnetic quadrupole, which exists of two northpoles and two southpoles, that are placed alternating around the optical axis (see figure 1.2). The magnetic field in a quadrupole has the form:

$$B_x = B_0.y \tag{1.21a}$$

$$B_y = B_0.x \tag{1.21b}$$

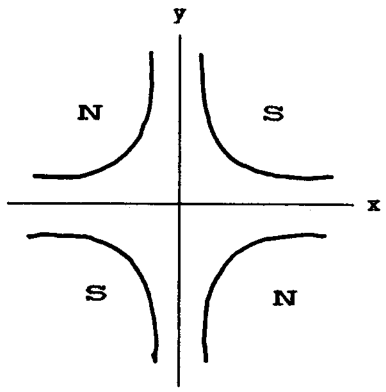


Figure 1.2
Pole shoes and coordinates in a magnetic quadrupole.

Where B_0 is the radial derivative of the absolute value of the magnetic field. The paraxial equations of motion are then, with $qB_0/mv = \omega^2$:

$$x'' - \omega^2.x = 0 \tag{1.22a}$$

$$y'' + \omega^2.y = 0 \tag{1.22b}$$

The solution of these equations is familiar to (1.14) [lit.8]:

$$\begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \end{pmatrix} = \begin{pmatrix} \cosh(\omega L) & \sinh(\omega L)/\omega & 0 & 0 \\ \omega.\sinh(\omega L) & \cosh(\omega L) & 0 & 0 \\ 0 & 0 & \cos(\omega L) & \sin(\omega L)/\omega \\ 0 & 0 & -\omega.\sin(\omega L) & \cos(\omega L) \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \end{pmatrix} \tag{1.23}$$

Where L is the length of the quadrupole. As in (1.19), this matrix can be written as a drift-lens-drift combination:

$$\begin{pmatrix} 1 & L_d & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_d \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ Q & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -Q & 1 \end{pmatrix} \begin{pmatrix} 1 & L_d & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_d \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1.24}$$

With, in second order:

$$Q = \omega^2 L \tag{1.25a}$$

$$L_d = L/2 \tag{1.25b}$$

Because ω^2 is proportional to 1/p, the lens power of a quadrupole depends on the momentum of the particle being focused. This effect is called chromatic aberration, and it is another effect as the dispersion of a dipole. Chromatic aberrations cannot be described as a matrix element on its own, but only change the value of other matrix elements.

1.5. Magnetic sextupoles

A sextupole exists of three northpoles and three southpoles, which are, like in a quadrupole, placed alternating around the optical axis (see figure 1.3). The magnetic field has the form [lit.5,6]:

$$B_x = 2.B_0.x.y \tag{1.26a}$$

$$B_y = B_0.(x^2 - y^2) \tag{1.26b}$$

Where $2B_0$ is the second radial derivative of the absolute value of the magnetic field. The paraxial equations of motion in a sextupole are then, with $\omega^2 = qB_0/mv$:

$$x'' = \omega^2.(x^2 - y^2) \tag{1.27a}$$

$$y'' = -2\omega^2 xy \tag{1.27b}$$

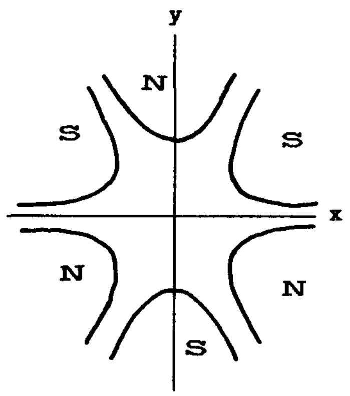


Figure 1.3
Pole shoes and coordinates in a magnetic sextupole.

Because of the coupling between x'' and y'', it is not convenient to solve these equations analytically. It is however possible to describe these equations in a matrix notation [lit.6]. Like the quadrupole, the sextupole can be split into a drift-lens-drift combination, with drifts equal to half of the sextupole length. The direction change, due to the lens power, is:

$$\Delta x' = L.x'' = L.\omega^2(x^2 - y^2) \tag{1.28a}$$

$$\Delta y' = L.y'' = -2L.\omega^2 xy \tag{1.28b}$$

In a thin lens approximation, both x and y are constant and equal to x_0 and y_0 respectively. This leads to the following matrix notation [lit.4,6]:

$$\begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ Q_n x_0 & 1 & -Q_n y_0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2Q_n x_0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \\ y_0 \\ y_0' \end{pmatrix} \tag{1.29}$$

Where $Q_n = \omega^2 L$ is called the normalized lens power (in dioptr/m). Because x_0 an y_0 appear inside this matrix, the effect of a sextupole is of second order, and cannot be described using the normal first order matrix methods such as matrix multiplication.

When a sextupole is placed at a dispersive horizontal focus, produced by a dipole, it can be used as a correction element. It is then necessary that the horizontal displacement is mainly due to dispersive effects [lit.6], or:

$$b_{11}.x_s \ll b_{13}.\Delta p/p \tag{1.30}$$

where b_{ij} is the corresponding matrix element of the horizontal subsystem from source to sextupole, and x_s is the horizontal displacement of the incoming particle. If eq.(1.30) is true for any x_s that may occur, it is allowed to use the following approximation:

$$x_0 = b_{13}.\Delta p/p \tag{1.31}$$

This means that the a_{34} element of (1.29) becomes equal to $-2Q_n.b_{13}.\Delta p/p$, so the vertical lens power of the sextupole is now proportional to $\Delta p/p$. Because the dipole focus is an intermediate focus in the lens system, it is focused once again on the target. This means that the horizontal direction change, due to the sextupole matrix elements a_{21} and a_{23} has no first order effect on the endfocus at the target, so that the combination of a dipole and a sextupole can be used to correct chromatic aberrations in one dimension without having any first order effect in the other dimension.

1.6. Chromatic effects

As shown in section 1.3, dipoles have a first order effect due to momentum spread, called dispersion. In section 1.4 is found, that quadrupoles have chromatic aberrations, i.e. the lens power depends on the momentum of the particles. The lens power of a dipole is given by eq.(1.21a) and substituting eq.(1.8) shows, that a dipole also has chromatic aberrations.

The first order dispersive effects result in a transversal focus displacement, while the second order chromatic aberrations cause a longitudinal focus displacement.

In order to avoid misunderstandings, it is necessary to distinguish between the different types of chromatic effects. A lens system will be called dispersive when there is a transversal spot displacement due to momentum spread and non-dispersive if not. When a system has no longitudinal spot displacement it is called achromatic. The usage of the word "achromatic" will be restricted to lens systems that have no chromatic aberrations, neglecting any dispersion of the system. (In the literature on beam transport and focusing problems however, non-dispersive systems are often called achromatic).

CHAPTER 2. BEAM FOCUSING WITH DIPOLES

2.1. Introduction

As shown in section 1.5, sextupoles can be used to correct chromatic aberrations, when they are placed in a dispersive focus. When this is a horizontal focus, the horizontal action of the sextupole has no first order effect on the endfocus at the target, while the vertical sextupole action corrects chromatic aberrations. In a vertical focus, the sextupole has to be turned 90°.

Since sextupoles have to be placed in a dipole focus, dispersion is introduced in the lens system. This dispersion must be corrected by at least one other dipole, in order to enable non-dispersive focusing on the target. Non-dispersive focusing means that in the system matrix a_{13} has to be zero, as an extra demand to the lens system, so there are now three conditions:

- A certain enlargement factor: $a_{11} = M$ (2.1a)
- Focusing on the target: $a_{12} = 0$ b)
- No dispersion at the target: $a_{13} = 0$ (2.1c)

Because there are three conditions, the system must have three variable parameters, for example three lens powers. In that case the driftlengths are to be chosen fixed. These conditions can be fulfilled by a system of three dipoles [lit.4] or two dipoles and a quadrupole. For the dipoles, either the radius or the deflection angle can be chosen fixed and the other as variable. In order to perform horizontal and vertical focusing, the following six conditions have to be fulfilled:

- A certain horizontal enlargement factor: $a_{11} = M_h$ (2.2a)
- Horizontal focusing on the target: $a_{12} = 0$ b)
- No horizontal dispersion at the target: $a_{15} = 0$ c)
- A certain vertical enlargement factor: $a_{33} = M_v$ d)
- Vertical focusing on the target: $a_{34} = 0$ e)
- No vertical dispersion at the target: $a_{35} = 0$ (2.2f)

In order to fulfil these conditions, a system must have six variables. When the driftlengths are chosen fixed, such a system must have six lenses. It also must have at least two horizontal and two vertical dipoles in order to create dispersive intermediate foci and non-dispersive focusing on the target.

Because the horizontal and vertical focusing can be performed independently, most problems will be solved using (2.1), once for the horizontal and once for vertical subsystem, instead of (2.2).

Because a system, existing of both horizontal and vertical dipoles, may have a rather complex geometric configuration, one must be carefull to use the words "horizontal" and "vertical". In this document "horizontal" will always refer to the local X-direction and "vertical" to the local Y-direction, which are not necessarily the real horizontal or vertical directions.

2.2. Beam focusing with a dipole sextuplet

A system of three horizontal dipoles can be used to perform non-dispersive beam focusing in one dimension with a dispersive intermediate focus [lit.4] (see figure 2.1).

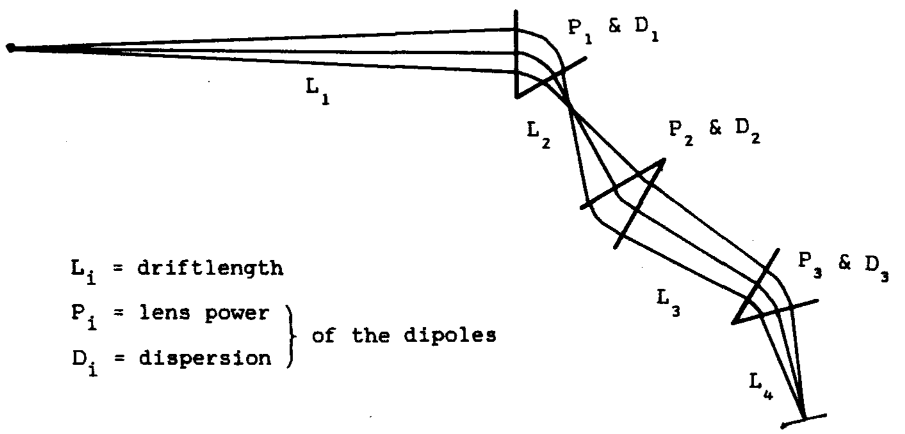


Figure 2.1
Beam focusing with three dipoles. The drifts are measured from and to the dipole centres.

The system matrix elements mentioned in eq.(2.1) are then:

$$\begin{aligned} a_{11} = {} & 1 - P_1(L_2+L_3+L_4) - P_2(L_3+L_4) - P_3L_4 \\ & + P_1P_2L_2(L_3+L_4) + P_1P_3(L_2+L_3)L_4 + P_2P_3L_3L_4 \\ & - P_1P_2P_3L_2L_3L_4 \end{aligned} \qquad (2.3a)$$

$$\begin{aligned} a_{12} = {} & L_1+L_2+L_3+L_4 \\ & - P_1L_1(L_2+L_3+L_4) - P_2(L_1+L_2)(L_3+L_4) - P_3(L_1+L_2+L_3)L_4 \\ & + P_1P_2L_1L_2(L_3+L_4) + P_1P_3L_1(L_2+L_3)L_4 + P_2P_3(L_1+L_2)L_3L_4 \\ & - P_1P_2P_3L_1L_2L_3L_4 \end{aligned} \qquad (2.3b)$$

$$\begin{aligned} a_{13} = {} & D_1\left\{L_2+L_3+L_4 - P_2L_2(L_3+L_4) - P_3(L_2+L_3)L_4 + P_2P_3L_2L_3L_4\right\} \\ & + D_2\left\{L_3+L_4 - P_3L_3L_4\right\} \\ & + D_3L_4 \end{aligned} \qquad (2.3c)$$

Where P_i, D_i and L_i are the lens powers, dispersions and drift-lengths respectively. Substituting the values of a_{ij} from eq.(2.1) in (2.3) three parameters can be solved. Because a dipole has two parameters of its own, i.e. the radius and the deflection angle, one of them may be chosen fixed for each dipole, and then the other one can be solved using eq.(1.20). This leads to two main solutions, i.e. one where all dipoles have a fixed radius and one where they have fixed deflection angles. These solutions are:

With fixed radii:

$$P_3 = \frac{(ML_1+L_2+L_3+L_4)(R_1-R_2) + (M-1)L_1R_1}{(L_2+L_3)L_4(R_1-R_2) + L_2L_4(R_3-R_1)} \qquad (2.4a)$$

$$P_2 = \frac{(1-M)R_1 + L_4P_3(R_3-R_1)}{(R_1-R_2)(L_3+L_4 - P_3L_3L_4)} \qquad (2.4b)$$

$$P_1 = \frac{M-1 + P_2(L_3+L_4 - P_3L_3L_4) + P_3L_4}{ML_1} \qquad (2.4c)$$

And with fixed angles:

$$P_3 = \frac{L_3 + L_4}{L_3 L_4} - \frac{ML_1 D_1 - L_4 D_3}{L_3 L_4 D_2} \tag{2.5a}$$

$$P_2 = \frac{ML_1 + L_2 + L_3 + L_4 - P_3 (L_2 + L_3) L_4}{L_2 (L_3 + L_4 - P_3 L_3 L_4)} \tag{2.5b}$$

$$P_1 = \frac{M-1 + P_2 (L_3 + L_4 - P_3 L_3 L_4) + P_3 L_4}{ML_1} \tag{2.5c}$$

For the solution of eq.(2.4) R_i are known, and D_i can be calculated using eq.(1.20). From the solution of eq.(2.5), where D_i are known, R_i can be calculated using eq.(1.20).

According to eq.(1.20), D_i is equal to $\sin(\phi_i)$, while R_i and ϕ_i have the same sign. When ϕ_i is in the interval $[-\pi/2,+\pi/2]$ the following relations must be true:

$$-1 \leq D_i \leq +1 \tag{2.6a}$$

$$P_i \geq 0 \tag{2.6b}$$

All solutions of eq.(2.4) or (2.5) have to be tested with eq.(2.6) for validity.

In order to perform horizontal and vertical focusing, two of such systems can be combined to a system of three horizontal and three vertical dipoles. Such a system is called a dipole sextuplet. For the sequence of the horizontal and vertical dipoles exist ten independent possibilities:

- H,H,H,V,V,V (2.7a)
- H,H,V,H,V,V b)
- H,H,V,V,H,V c)
- H,H,V,V,V,H d)
- H,V,H,H,V,V e)
- H,V,H,V,H,V f)
- H,V,H,V,V,H g)

- H,V,V,H,H,V h)
- H,V,V,H,V,H i)
- H,V,V,V,H,H (2.7j)

Where "H" and "V" stand for a horizontal and a vertical dipole respectively. Systems, beginning with a vertical dipole give the same set of lens configurations, rotated over 90°.

2.3. Beam focusing with dipoles and quadrupoles

The conditions (2.1) can also be fulfilled by a system of two dipoles and one quadrupole (see figure 2.2).

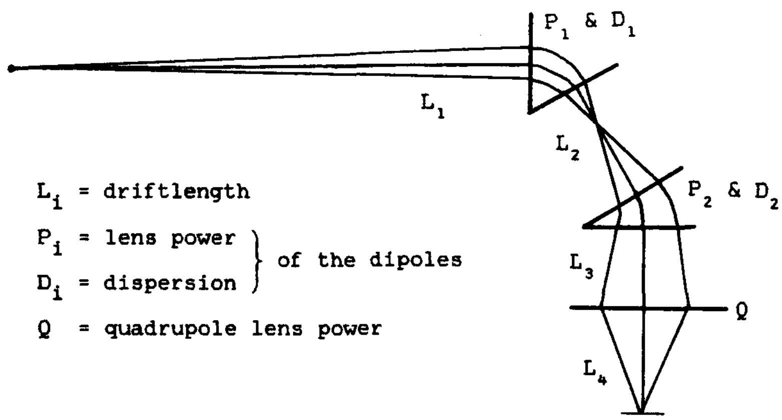


Figure 2.2
Beam focusing with two dipoles and one quadrupole. The drifts are measured from and to the dipole and quadrupole centres.

In order to focus in two dimensions, this system has to be combined with an equivalent vertical focusing system. Then the two quadrupole lens actions will interfere because a quadrupole has both horizontal and vertical action. This means, that the horizontal and vertical subsystem both exist of two dipoles and two quadrupoles. The matrix elements of eq.(2.1) are then:

$$a_{11} = 1 - P_1(L_2+L_3+L_4+L_5) - P_2(L_3+L_4+L_5) - Q_1(L_4+L_5) - Q_2L_5$$
$$+ P_1P_2L_2(L_3+L_4+L_5) + P_1Q_1(L_2+L_3)(L_4+L_5)$$
$$+ P_1Q_2(L_2+L_3+L_4)L_5 + P_2Q_1L_3(L_4+L_5)$$
$$+ P_2Q_2(L_3+L_4)L_5 + Q_1Q_2L_4L_5$$
$$- P_1P_2Q_1L_2L_3(L_4+L_5) - P_1P_2Q_2L_2(L_3+L_4)L_5$$
$$- P_1Q_1Q_2(L_2+L_3)L_4L_5 - P_2Q_1Q_2L_3L_4L_5 \qquad (2.8a)$$
$$+ P_1P_2Q_1Q_2L_2L_3L_4L_5$$

$$a_{12} = L_1+L_2+L_3+L_4+L_5$$
$$- P_1L_1(L_2+L_3+L_4+L_5) - P_2(L_1+L_2)(L_3+L_4+L_5)$$
$$- Q_1(L_1+L_2+L_3)(L_4+L_5) - Q_2(L_1+L_2+L_3+L_4)L_5$$
$$+ P_1P_2L_1L_2(L_3+L_4+L_5) + P_1Q_1L_1(L_2+L_3)(L_4+L_5)$$
$$+ P_1Q_2L_1(L_2+L_3+L_4)L_5 + P_2Q_1(L_1+L_2)L_3(L_4+L_5)$$
$$+ P_2Q_2(L_1+L_2)(L_3+L_4)L_5 + Q_1Q_2(L_1+L_2+L_3)L_4L_5$$
$$- P_1P_2Q_1L_1L_2L_3(L_4+L_5) - P_1P_2Q_2L_1L_2(L_3+L_4)L_5$$
$$- P_1Q_1Q_2L_1(L_2+L_3)L_4L_5 - P_2Q_1Q_2(L_1+L_2)L_3L_4L_5$$
$$+ P_1P_2Q_1Q_2L_1L_2L_3L_4L_5 \qquad (2.8b)$$

$$a_{13} = D_1\Big\{L_2+L_3+L_4+L_5 - P_2L_2(L_3+L_4+L_5)$$
$$- Q_1(L_2+L_3)(L_4+L_5) - Q_2(L_2+L_3+L_4)L_5$$
$$+ P_2Q_1L_2L_3(L_4+L_5) + P_2Q_2L_2(L_3+L_4)L_5$$
$$+ Q_1Q_2(L_2+L_3)L_4L_5 - P_2Q_1Q_2L_2L_3L_4L_5\Big\}$$
$$+ D_2\Big\{L_3+L_4+L_5 - Q_1L_3(L_4+L_5) - Q_2(L_3+L_4)L_5$$
$$+ Q_1Q_2L_3L_4L_5\Big\} \qquad (2.8c)$$

As in (2.4) and (2.5), solutions can be found with fixed radii or with fixed angles. It is also possible to use R and ϕ of the second dipole as parameter and choose the first dipole with fixed radius and angle. Then the location of the dipole focus is known a priori. This yields:

$$Q_1 = \frac{ML_1+ML_2+L_3+L_4+L_5 - P_1ML_1L_2 - Q_2(L_3+L_4)L_5}{L_3(L_4+L_5) - Q_2L_3L_4L_5} \qquad (2.9a)$$

$$R_2 = \frac{ML_1D_1}{1-M + P_1ML_1 - Q_1(L_4+L_5) - Q_2L_5 + Q_1Q_2L_4L_5} \qquad (2.9b)$$

$$D_2 = \frac{ML_1D_1}{L_3+L_4+L_5 - Q_1L_3(L_4+L_5) - Q_2(L_3+L_4)L_5 + Q_1Q_2L_3L_4L_5} \qquad (2.9c)$$

Now, both parameters of the second dipole are defined and eq.(2.9a) gives a relation between the two quadrupole lens powers. For the vertical subsystem, an equivalent relation is found for the quadrupoles. Both relations are of the general form:

$$Q_1 = \frac{X_1+ X_2Q_2}{X_3+ X_4Q_2} \qquad (2.10)$$

With:

$$X_1 = ML_1+ML_2+L_3+L_4+L_5 - P_1ML_1L_2 \qquad (2.11a)$$

$$X_2 = -(L_3+L_4)L_5 \qquad b)$$

$$X_3 = L_3(L_4+L_5) \qquad c)$$

$$X_4 = -L_3L_4L_5 \qquad (2.11d)$$

With $Q_{1h} = -Q_{1v}$ and $Q_{2h} = -Q_{2v}$, this yields:

$$-(H_2V_4+V_2H_4)Q_2^2 + (H_2V_3-V_2H_3+V_1H_4-H_1V_4)Q_2 + H_1V_3+V_1H_3 = 0 \qquad (2.12)$$

Where H_i and V_i represent the values of X_i for the horizontal and vertical subsystem respectively. Eq.(2.12) can have two independent solutions. By substituting these solutions in eq.(2.9a), all parameters are defined.

In a lens system of two horizontal and two vertical dipoles three independent dipole sequences exist:

- H,H,V,V (2.13a)
- H,V,H,V b)
- H,V,V,H (2.13c)

For each sequence there may be two solutions of eq.(2.12), so there are six possible lens configurations for the system of four dipoles and two quadrupoles, with both quadrupoles placed behind all dipoles.

2.4. Driftlength corrections

When combining the horizontal and vertical subsystem to a double focusing system, it is necessary to describe dipoles as 5x5 matrices. A horizontal dipole is a driftlength for the vertical subsystem and vice versa. This driftlength is equal to (see figure 2.3):

$$L_v = R.\phi \qquad (2.14)$$

This driftlength depends on the parameters of the dipole, so if the horizontal dipoles are not yet known exactly, the driftlengths are not correct, and neither are the parameters of the vertical dipoles, because these depend on the driftlengths.

The solutions given so far are derived using the thin lens model, with eq.(1.20c) as an approximation for the vertical driftlength in a horizontal dipole and vice versa. Using the calculated values of the dipole parameters, the driftlengths can be corrected using eq.(1.20c) and (2.14), and then the dipole settings have to be recalculated. Then the driftlengths have to be corrected again etc. This leads to a successive substitution iteration proces that can be performed by a computerprogram.

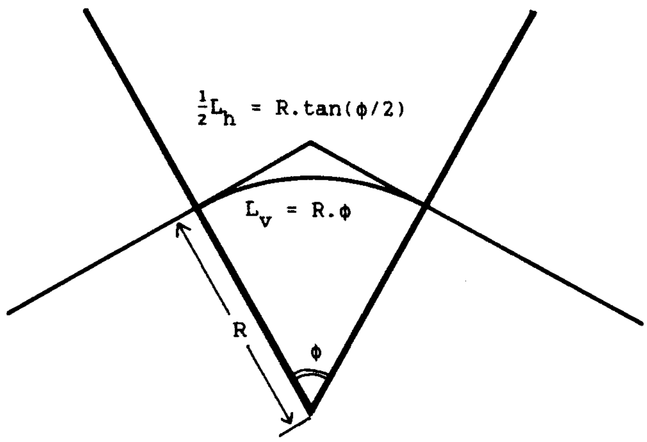


Figure 2.3
Horizontal and vertical driftlengths inside a dipole.

For technical reasons, it is preferrable that different dipoles have equal deflection angles and radii. When changing the solution so that some dipoles become equal, other parameters of the lens system (e.g. the driftlengths) have to be corrected.

This is done using a Newton iteration method. This Newton method uses eq.(2.2) as a six-dimensional vector function of six variables (six different driftlengths). The Newton method needs the Jacobian, which can be calculated from the matrix model of the lens system as follows:

The system matrix can be written as:

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L_i & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ 0 & 0 & 1 \end{pmatrix} \qquad (2.15)$$

Where C is the matrix of the subsystem that is physically after the drift L_i and B is the subsystem matrix before the drift. The upper row of the system matrix is then:

$$a_{1j} = c_{11}.b_{1j} + c_{12}.b_{2j} + c_{11}.b_{2j}.L_i \qquad (2.16)$$

Yielding:

$$\partial a_{1j}/\partial L_i = c_{11}.b_{2j} \qquad (2.17)$$

With eq.(2.17) all elements of the Jacobian can be calculated, when the subsystem matrices are known.

A computerprogram, called "DQ" (= Dipoles & Quadrupoles), is written to perform both iteration methods described above on lens systems with all possible configurations previously described. When checking the result of the succesive substitution method by calculating the system matrix, it may be necessary to correct the solution. This is also done with the Newton method. The Jacobian with respect to the lens powers is then calculated similar to eq.(2.16) and (2.17). The Newton method gives the correct solution because it uses eq.(2.2) as a stopcriterium for the iteration process.

CHAPTER 3. FIRST ORDER RESULTS

All possible lens configurations described in chapter 2 are checked with the computerprogram "DQ", and some results are given in this chapter.

3.1. Possible lens configurations

Because a lens system of six dipoles and seven drifts has nineteen independent parameters (six radii, six angles and seven drifts), while only six parameters are calculated with eq.(2.4), (2.5) or (2.9) and (2.12), the other thirteen must still be chosen. The driftlengths are chosen as 1.00 m between two horizontal or two vertical dipoles, and as 0.50 m otherwise. The drift before the first lens is chosen as 6.00 m, in order to obtain a small enlargement factor.

For practical reasons the total system length must not be significantly larger than ca. 10 m, so the driftlengths must not be too long. Because of the size of the lenses, the drifts cannot be much smaller than 0.50 m.

When the driftlengths decrease, the lens powers must increase. The maximum lens power of the quadrupoles at E.U.T. is ca. 10 m^{-1} for 3.5 MeV protons. The maximum lens power of a dipole is, according to eq.(1.20a), equal to 1/R and following eq.(1.8) this is equal to qB/mv. For a proton beam of 3.5 MeV this maximum is ca. 5 m^{-1} with a magnetic field of ca. 1.35 T.

The drifts of 0.50 m are practically useful, because the total system length and the maximum lens power do not exceed the limits given above. For example: the quadrupole lens system currently in use at E.U.T. has driftlengths of 0.50 m and the maximum lens power used is ca. 7.6 m^{-1} [lit.1]. Because the maximum lens power of a dipole is ca. a factor 2 smaller than that of a quadrupole, the drifts between dipoles are chosen as 1.00 m.

The ten possibilities of eq.(2.7) plus the six of eq.(2.13) yield sixteen possible lens configurations to be tested. For each absolute value of the enlargement factor exist four possibilities, i.e. positive or negative and stigmatic or antistigmatic. Thus, sixty four lens configurations are to be tested.

For an enlargement factor with an absolute value of 0.025 there is only one dipole sextuplet as a valid solution of eq.(2.4) for dipoles with radii of 0.25 m. This solution is listed in table 3.1 and shown in figure 3.1. With radii of 0.20 m three valid solutions exist. This is mainly due to the fact that with larger radii, the same lens power can only be achieved with a larger dispersion so that eq.(2.6a) is violated.

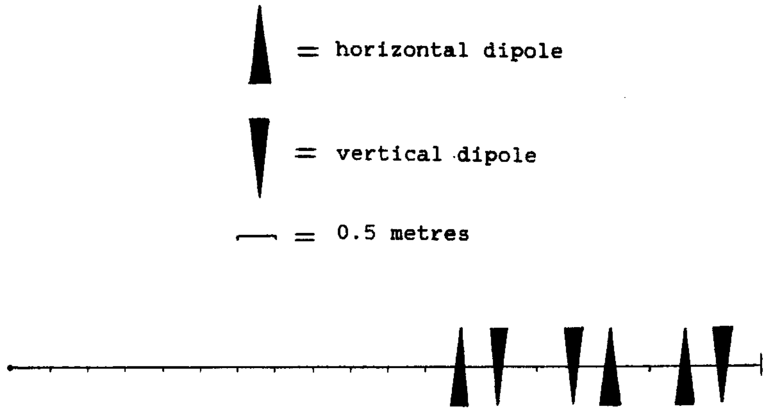


Figure 3.1
The dipole sextuplet of table 3.1. In this table the values of the driftlengths and dipole parameters are given. The system has a stigmatic enlargement factor of 0.025.

Trying to make some dipoles equal and correct the driftlengths appears not to be possible for these solutions. The convergence area of the Newton method is too small. The lens system shown in table 3.1 is a mathematically possible system, but all dipoles are different. If a system with some equal dipoles can be found that would be preferrable.

The lens configurations with a dipole quadruplet plus a quadrupole doublet have many solutions for the four types of enlargement factors. For all of these configurations however, at least one dipole radius is less then 0.15 m, which would require a magnetic field of ca. 1.8 T. This can be made but it is a rather strong field. Changing the value of that radius and trying to correct the driftlengths leads to a significant increase of these lengths, or negative driftlengths occur which is physically not possible.

3.2. Lens trimming

In any existing lens system, it is necessary to be able to adjust the settings of the lenses. This is called trimming. For a dipole this is difficult, because the lens effect of the dipole is an entirely geometric effect. The deflection angle and the radius determine the lens parameters of the dipole. When a dipole is to be trimmed, either the radius or the deflection angle should be adjustable. Changing the radius needs a mechanical device to shift the pole shoes in or out the system (see figure 3.2) while changing the deflection angle would change the geometry of the entire lens system.

Quadrupole lens powers can be adjusted without changing the lens geometry by changing the current in the coils. When a quadrupole is placed near a dipole, it can be used to trim that dipole because the quadrupole and the dipole can be considered as one lens. This means that any adjustable non-dispersive lens system must have six quadrupoles since there are six conditions to be fulfilled according to eq.(2.2).

The computer program is written to enable insertion of quadrupoles in the lens system, and with the Newton method, the system can be trimmed numerically. It appears then possible to change the dipole parameters, so that the system contains some equal dipoles with radii of 0.20 - 0.25 m, and correct the system by adjusting the quadrupoles. Table 3.2 contains the listing of such a system, which is also shown in figure 3.3. In this system the drifts are longer than previously described, in order to achieve acceptable dipole radii. The total length of this system is ca. 15 m. This system has two pairs of equal dipoles with "standard" deflection angles of 15° and 60°.

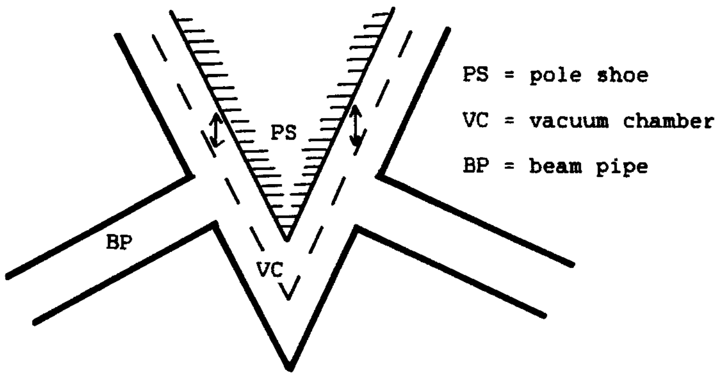
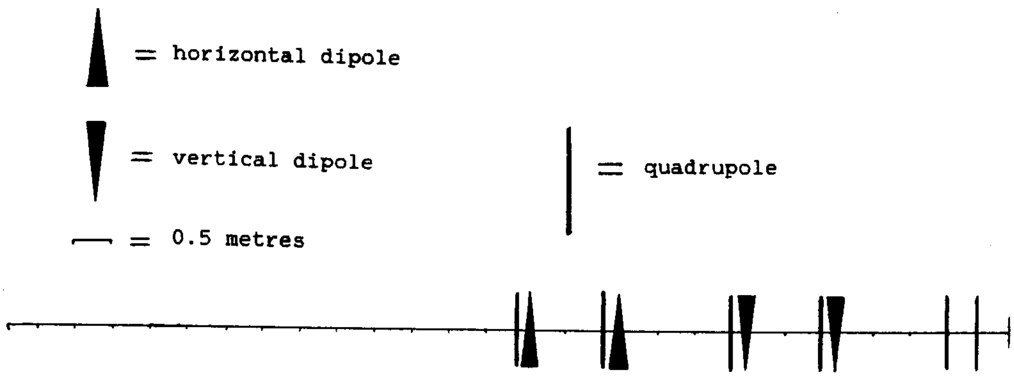


Figure 3.2
Dipole with adjustable radius. When the pole shoe shifts in or out, the magnetic field strength must be changed to achieve the desired radius. When the radius is changed, the changes of the driftlengths inside and outside the dipole compensate each other.

This system has another advantage, i.e. the capability of nearly independent horizontal and vertical focusing. This can be seen in the Jacobian with respect to the quadrupole lens powers, which gives the sensitivity of the system for the various lens powers. Each column in this Jacobian is the derivative of the vector function, mentioned in section 2.4, with respect to the corresponding quadrupole lens power. This Jacobian is listed in table 3.2a. The horizontal focusing (second row of the Jacobian) is trimmed mainly by the 5th quadrupole (5th column) and the vertical focusing (5th row) by the 4th quadrupole.



<u>Figure 3.3</u>
Dipole quadruplet with trim lenses plus quadrupole doublet. The values of the driftlengths, quadrupole lens powers and dipole parameters are listed in table 3.2. The lens system has a stigmatic enlargement factor of -0.025.

A lens system is now designed that performs first order focusing in two dimensions. It has dispersive intermediate foci in which sextupoles can be placed to correct chromatic aberrations, and it is now necessary to check the effect of these sextupoles.

Table 3.1: Dipole sextuplet

Lens system:

1:	drift,	L = 6.00 m	
2:	horizontal dipole,	R = 0.25 m,	ϕ = 61.604°
3:	drift,	L = 0.50 m	
4:	vertical dipole,	R = 0.25 m,	ϕ = 54.637°
5:	drift,	L = 1.00 m	
6:	vertical dipole,	R = -0.25 m,	ϕ = -15.610°
7:	drift,	L = 0.50 m	
8:	horizontal dipole,	R = -0.25 m,	ϕ = -6.246°
9:	drift,	L = 1.00 m	
10:	horizontal dipole,	R = 0.25 m,	ϕ = 12.921°
11:	drift,	L = 0.50 m	
12:	vertical dipole,	R = 0.25 m,	ϕ = 27.874°
13:	drift,	L = 0.50 m	
14:	target.		

Enlargement factor = 0.025 stigmatic

Intermediate horizontal focus: 14.9 cm after element # 2
Intermediate vertical focus: 19.2 cm after element # 4

System matrix:

$$\begin{pmatrix} 0.025 & 0.000 & 0.000 & 0.000 & 0.000 \\ 6.865 & 40.000 & 0.000 & 0.000 & -1.731 \\ 0.000 & 0.000 & 0.025 & 0.000 & 0.000 \\ 0.000 & 0.000 & 6.002 & 40.000 & 0.153 \\ 0.000 & 0.000 & 0.000 & 0.000 & 1.000 \end{pmatrix}$$

Table 3.2: Dipole quadruplet with trim lenses plus quadrupole doublet

Lens system:

1:	drift,	L =	6.90	m
2:	trim quadrupole,	Q =	0.252	m^{-1}
3:	drift,	L =	0.10	m
4:	horizontal dipole,	R =	0.25	m, ϕ = 60°
5:	drift,	L =	1.00	m
6:	trim quadrupole,	Q =	0.501	m^{-1}
7:	drift,	L =	0.10	m
8:	horizontal dipole,	R =	0.20	m, ϕ = 15°
9:	drift,	L =	1.50	m
10:	trim quadrupole,	Q =	-0.426	m^{-1}
11:	drift,	L =	0.10	m
12:	vertical dipole,	R =	0.25	m, ϕ = 60°
13:	drift,	L =	1.00	m
14:	trim quadrupole,	Q =	-0.443	m^{-1}
15:	drift,	L =	0.10	m
16:	vertical dipole,	R =	0.20	m, ϕ = 15°
17:	drift,	L =	1.50	m
18:	quadrupole,	Q =	2.290	m^{-1}
19:	drift,	L =	0.40	m
20:	quadrupole,	Q =	-5.327	m^{-1}
21:	drift,	L =	0.40	m
22:	target.			

Enlargement factor = -0.025 stigmatic

Intermediate horizontal focus: 13.5 cm after element # 4
Intermediate vertical focus: 13.7 cm after element # 12

Table 3.2a: Continuation of table 3.2

System matrix:

-0.025	0.000	0.000	0.000	0.000
-6.200	-40.000	0.000	0.000	1.846
0.000	0.000	-0.025	0.000	0.000
0.000	0.000	-4.973	-40.000	0.533
0.000	0.000	0.000	0.000	1.000

Jacobian with respect to the quadrupole lens powers:

-0.173	-1.59	-0.007	-2.03	-16.8	-0.982
-1.19	-10.8	-0.033	-13.1	-109	-6.40
0.00	0.514	0.002	0.606	5.04	0.295
0.173	0.357	1.72	15.5	0.012	0.786
1.19	2.83	14.4	129	0.109	6.40
0.00	0.00	0.00	-1.78	-0.001	-0.085

CHAPTER 4. USEAGE OF SEXTUPOLES

In chapter 3, lens systems are described that perform two-dimensional focusing. These lens systems have horizontal and vertical dispersive intermediate foci, in order to use sextupoles to correct chromatic aberrations. Because sextupoles are second order lenses, their matrix elements depend on the position of the protons in the lens. This means that the usual method of matrix multiplication cannot be used for a sextupole. To check the effect of sextupoles, the spotsize is calculated for beams of different energies, by "shooting" a large number of protons. This method is called "ray tracing".

4.1. Ray tracing

A computer program was written that performs ray tracing by integrating the equations of motion for each proton. In this way, both chromatic and geometric aberrations can be calculated. The program has to compute with an accuracy of ca. 1 μm on the target. To achieve this accuracy, a 4th order Runge-Kutta integration method was used to calculate the proton trajectories.

A proton that starts following the optical axis has to hit the target on the optical axis. This is used to check the accuracy of the program. It appeared necessary to use an integration step of ca. 0.25 ps, which for a 3 MeV proton corresponds to a distance of ca. 6 μm. The program was written on a PDP 11/23 computer, and on this machine, the time needed to calculate one proton trajectory with this integration step appeared to be ca. 10 hours.

In order to simulate a beam of protons, several trajectories for protons with different starting conditions have to be calculated. The source has four dimensions in phase space, and in each dimension three particles are needed. This means that a monochromatic beam exists of $3^4 = 81$ protons. A beam with energy spread is simulated by shooting three beams with different energies, so it exists of 243 particles. The time to calculate the spotsize for such a beam is then 243 x 10 = 2430 hours, which is nearly 15 weeks.

One reason for this large computing time is probably the fact that the integration of the equations of motion is done using the time as integration variable. This means that the number of steps between the boundaries of a magnetic field is not the same for different trajectories, and it is also not an integer number. It is then necessary to use the small integration step of ca. 0.25 ps.

The error introduced because the number of integration steps is not integer, is caused by the fact that the last step does not fit to the sharp edge of the magnet field. Using a variable stepsize it must be possible to reduce this effect, but the accuracy did not increase significantly. The reason for this is not known. Reduction of the effect is also tried by inserting fringe fields in the magnet system. These fringe fields are given the length of several integration steps. The accuracy of the program did however not increase significantly, so the large computing time remained. It is necessary to use a much faster computer when this method of ray tracing is to be performed with acceptable computing times.

Since the main purpose of the program is to calculate chromatic aberrations, the detailed informations of the magnetic fields are not important, because these give only geometric aberrations. By using the paraxial equations of motion, the position measured along the optical axis can be used as integration variable. Then the number of integration steps can be integer and equal for all trajectories.

When assuming ideal fields in a dipole, quadrupole or sextupole, the computing time can be reduced significantly. The analytical solution for a quadrupole (eq.1.23)) can be used. For an ideal dipole, i.e. a homogeneous magnetic field with sharp edges, normal to the optical axis, the exact solution can be found analytically. Knowing that the trajectory inside the dipole is a circular motion, it is only necessary to calculate the radius and centre of this circle, and the crossing point of this circle with the second edge. The trajectory in a sextupole has to be integrated numerically.

This method has the disadvantage, that no details of magnetic fields, such as fringe fields, can be included in the calculations, but the computing speed is acceptable.

When the results of the program "DQ" are entered in the ray tracing program just described, the spot does not have the size that it should have according to the first order calculations. This can be caused by the fact that the analytical solutions that were used, differ from the thin lens approximations. The pricipal planes of the lenses are not exactly in the middle of the lenses.

It is necessary that the program can minimize the spotsize. This is done using a negative gradient method [lit.7]. The spotsize is treated as a function of the quadrupole lens powers and by adding a small value to these lens powers the variation of the spotsize is determined. This gives an approximation of the gradient. Then a step is done in the direction of steepest descent.

On the PDP 11/23 computer, the time required to calculate a spotsize with a source of 81 particles appeared to be ca. 1 minute when the lens system does not contain sextupoles, so the time to compute the gradient with respect to the quadrupole lens powers is about 7 minutes when the system has six quadrupoles. This is approximately equal to the time to perform one iteration step. It is necessary that the start values are as good as possible, in order to reduce the number of iteration steps needed to minimize the spotsize.

The minimum spotsize found by the program for the lens system of table 3.2 with a monochromatic beam was ca. 500x500 μm^2, which is still far too large.

The main cause is probably the fact that the exact solution of the motion in an ideal dipole has too large geometric aberrations, which leads to spot enlargement. These aberrations can be corrected with a sextupole field, which can be made inside the dipole by shaping the pole shoes correctly [lit.9]. The program was however not written to contain such fields, because the computing time became too long, as previously described.

A program that can give results with sufficient accuracy needs unacceptable computing times, while a program that uses less time cannot give the desired accuracy. No more time was spent to test or rewrite this program once again.

Since only the chromatic effects of a lens system containing sextupoles are to be checked, a program was written to check the effect of sextupoles with ray tracing using the matrix descriptions given in eq.(1.18), (1.24) and (1.29). In this program, which is called "BF" (Beam Focusing), drifts are measured measured between dipole edges, while quadrupoles and sextupoles are treated as thin lenses, so all lenses are assumed to be corrected for geometric aberrations, especially the dipoles. The chromatic effects can thus be checked without interference with geometric effects caused by non-ideal lenses.

4.2. Results

In order to show the chromatic aberrations, the parameters of the quadrupole lens system currently in use at E.U.T. have been entered in the program "BF" (see table 4.1). A monochromatic beam gives a spot of 26x26 μm^2, while a beam with an energy spread of 0.3% leads to a spot of 166x74 μm^2. This spot enlargement is only due to chromatic aberrations, since the program uses the ideal thin lens approximations for the quadrupoles.

The results of the system listed in table 3.2 are given in table 4.2. This system also has a significant spot enlargement due to chromatic aberrations. In this system sextupoles have been placed at the intermediate foci, which indeed compensates the chromatic aberrations (see table 4.3). Because eq.(1.30) is not exactly fulfilled, the sextupoles introduce geometric aberrations in the system. These are however much smaller than the chromatic aberrations (compare table 4.2 and 4.3 for $\Delta p/p = 0$), so because of the sextupoles, there is a significant reduction of the spotsize for a beam with energy spread. The remaining chromatic spot enlargement is due to the fact that the sextupoles themselves also have chromatic aberrations.

Table 4.1: Chromatic aberrations of the quadrupole lens system

Lens system:

1:	drift,	L = 6.00 m
2:	quadrupole,	Q = 7.455 m^{-1}
3:	drift,	L = 0.50 m
4:	quadrupole,	Q = -2.686 m^{-1}
5:	drift,	L = 0.50 m
6:	quadrupole,	Q = 2.920 m^{-1}
7:	drift,	L = 0.50 m
8:	quadrupole,	Q = -2.879 m^{-1}
9:	drift,	L = 0.40 m
10:	target.	

Source diafragma: 1mm

Spotsize:

Δp/p (%)	Xmin (μm)	Xmax (μm)	Ymin (μm)	Ymax (μm)
-0.075	-69	+69	-37	+37
0.0	-13	+13	-13	+13
+0.075	-83	+83	-30	+30

Table 4.2: Chromatic aberrations in the system of table 3.2

Lens system: see table 3.2

Source diafragma: 1 mm
Spotsize:

Δp/p (%)	Xmin (μm)	Xmax (μm)	Ymin (μm)	Ymax (μm)
-0.075	-71	+72	-73	+82
0.0	-13	+13	-13	+13
+0.075	-58	+47	-72	+69

Table 4.3: Correction of chromatic aberrations with sextupoles

Lens system: see table 3.2, sextupoles at the intermediate foci

Source diafragma: 1 mm
Spotsize:

Δp/p (%)	Xmin (μm)	Xmax (μm)	Ymin (μm)	Ymax (μm)
-0.075	-22	+20	-34	+25
0.0	-15	+15	-19	+19
+0.075	-22	+24	-29	+32

Q_{nh} = -0.340 dioptr/mm

Q_{nv} = -0.222 dioptr/mm

CHAPTER 5. DISCUSSION

5.1. Lens system geometry

A lens sytem that contains both horizontal and vertical dipoles, has a rather peculiar geometric configuration. It contains various curves and goes up and down. Such a system may be diffucult to build with sufficient accuracy, for example because it is hard to place all system elements "in line". It also contains many lenses because with each dipole, a quadrupole is included in the system to trim it. The simplest complete system that corrects chromatic aberration with sextupoles, contains 4 dipoles, 6 quadrupoles and 2 sextupoles, as listed in table 3.2.

Every lens, especially the dipoles, also have geometric aberrations, so that when the chromatic aberrations can be corrected, the minimum spotsize will be determined by geometric aberrations. The geometric aberrations of the dipoles can be corrected by shaping the pole shoes correctly. The geometric aberrations of the sextupoles must be corrected by placing more sextupoles in the lens system at places where the dispersive effect is smaller than the beam diameter, which leads to another expansion of the lens system.

5.2. Acceptance of the system

Because the system contains dipoles, both the horizontal and the vertical drifts are very long without significant lens action inbetween. This is due to the fact that horizontal dipoles have no vertical lens action and vice versa, and the trim quadrupoles have only a small lens power. Because of these large drifts a significant loss of acceptance occurs.

Figure 5.1 shows some trajectories in the system of table 3.2 and it shows that the distance to the optical axis can be rather large. Because the beam pipe has a limited diameter (50 mm, shown by the dashed lines), this leads to a significant loss of acceptance, which reduces the beam current density at the target. This causes a higher detection limit and larger measur-

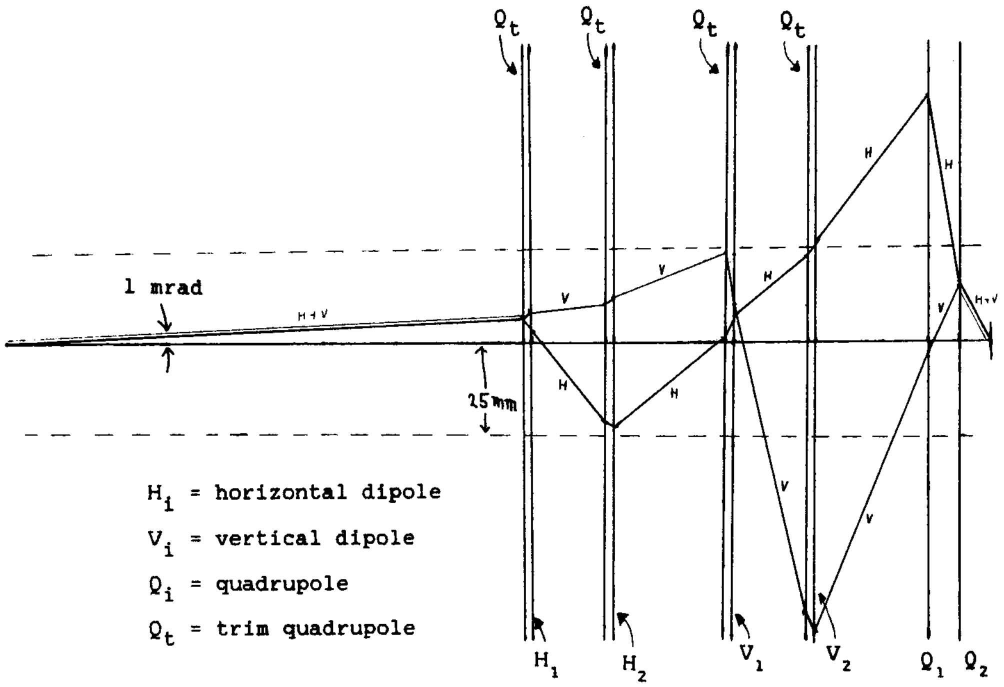


Figure 5.1
First order particle trajectories in the lens system listed in table 3.2. The quadrupoles and dipoles are shown as thin lenses. The dashed line shows the beam pipe.

ing times.
The beam current density on the target, J_t, can be written as follows:

$$J_t = I/S_t \tag{5.1}$$

Where I is the beam current and S_t the area of the spot on the target. The beam current is proportional to the acceptance of the system:

$$I = I_o.A_h.A_v \tag{5.2}$$

Here I_0 is the beam current per $mm^2.mrad^2$ delivered by the cyclotron, and A_h and A_v are the horizontal and vertical acceptance of the lens system in mm.mrad. When the lens system performs stigmatic or antistigmatic focusing, the area of the spot is given by:

$$S_t = \frac{\pi}{4}.D^2.M^2 \tag{5.3}$$

Where D is the diameter of the source diafragma and M the absolute value of the enlargement factor. When D is small enough, the acceptance can be approximated by (see figure 5.2):

$$A = D.A' \tag{5.4}$$

Where A' is the aperture of the system. Combination of eq.(5.1) through (5.4) leads to:

$$J_t = 4I_o.A_h'.A_v'/\pi M^2 \tag{5.5}$$

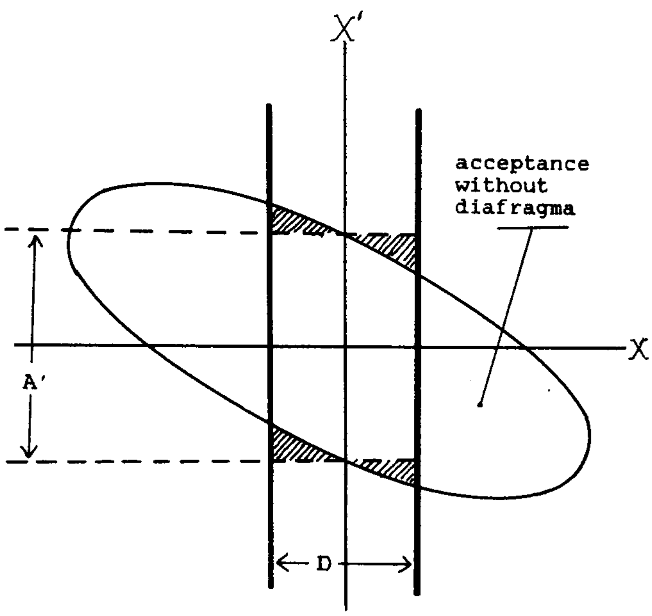


Figure 5.2
Acceptance in phase space of a system with a small diafragma at the entrance. D is the diameter of the diafragma and A' the aperture of the system.

So the beam current density on the target does not depend on the diameter of the source diafragma.

Decreasing the spotsize can be done using a smaller diafragma instead of a smaller enlargement factor. A lens system with a small enlargement factor often has also a small acceptance, so reducing the enlargement factor is in fact not the best method to obtain a small spot with high beam current density.

This means that a lens system should not be designed with a small enlargement factor, but the ratio of the acceptance and the enlargement factor should be optimized. In most cases, this leads to a greater enlargement factor [lit.12]. The program "DQ" was not written to optimize this ratio, but when a solution was computed with a greater enlargement factor (M = 0.1 stigmatic), the acceptance became much better. The ratio $A'_h . A'_v / M^2$ is ca. 3000 mrad^2 for this system, while it was 400 mrad^2 for the system of table 3.2. With a diafragma of 0.25 mm instead of 1 mm, a spot of 25x25 µm^2 can be achieved with this system and it can also be corrected for chromatic aberrations by using sextupoles (see table 5.1).

5.3. Other achromatic lens systems

When magnetic and electric quadrupoles are combined to a single lens, this lens can be made achromatic because the chromatic effects of the electric lens are twice as large as for the magnetic lens. When the lens power of the magnetic lens is twice the opposite lens power of the electric lens, the aberrations compensate each other, while the lens powers do not. Then an achromatic lens is made [lit.10].

This method has a disadvantage, because the lenses have to be placed at the same place, or the electric lens must be inside the magnet. This reduces the diameter inside the lens and thus the acceptance is reduced. Moreover, the electric lens is very sensible to external fields, for example the presence of any conducting material, such as the magnetic pole shoes of the quadrupole. It is preferrable that the electric lens is separated from the magnetic lens.

When the magnetic and electric lenses are located at different places the achromatism cannot always be achieved [lit.12]. Recent calculations show however, that when independent electric and magnetic lenses are combined to a system of three lenses, it is possible to perform achromatic focusing in one dimension [lit.12]. The achromatism is nearly perfect (see table 5.2), which cannot be said of the systems with sextupoles (table 3.2).

This means that it is most probably also possible to focus in two dimensions, when magnetic and electric quadrupoles are combined to a system of six lenses. If this is really possible, such a system must be preferred instead of a complex system with dipoles, quadrupoles and sextupoles. A system of only quadrupoles has less geometric aberrations than a system containing dipoles and sextupoles. The geometry of the lens system is also preferrable, because a system of quadrupoles is an in line lens system, which is much easier to build than a system containing dipoles.

5.4. Conclusion

It is shown that chromatic aberrations of a magnet lens system can be corrected by using dipole-sextupole combinations. A lens system that performs focusing to a microbeam must then contain both horizontal and vertical dipoles and sextupoles. This means that the system will not have a rectilinear geometry. Because the lens power of dipoles is limited to ca. 5 m^{-1}, the driftlengths become rather large, so that the system will have a total length of ca. 15 m. An entire compensation of the chromatic aberrations cannot be achieved using sextupoles, and moreover, geometric aberrations are introduced by the sextupoles. The dipoles must also be corrected for geometric aberrations.

It is possible to create a nearly achromatic lens system with dipoles, quadrupoles and sextupoles, but it is probably not very usefull to really build such a system, because it has practical disadvantages. It still needs corrections because the geometric aberrations now determine the minimum spotsize.

If it is really possible to perform achromatic focusing with only quadrupoles, this must be preferred because the achromatism is nearly perfect and the geometry of such a system is easier than that of a system containing horizontal and vertical dipoles. It might then even be possible to achieve a much smaller spot than 25 μm.

Table 5.1: Lens system with high beam current density

Lens system:

1:	drift,	L =	5.90 m
2:	trim quadrupole,	Q =	-0.074 m^{-1}
3:	drift,	L =	0.10 m
4:	horizontal dipole,	R =	0.25 m, ϕ = 45°
5:	drift,	L =	0.90 m
6:	trim quadrupole,	Q =	-0.380 m^{-1}
7:	drift,	L =	0.10 m
8:	horizontal dipole,	R =	0.25 m, ϕ = 20°
9:	drift,	L =	0.40 m
10:	trim quadrupole,	Q =	0.365 m^{-1}
11:	drift,	L =	0.10 m
12:	vertical dipole,	R =	0.25 m, ϕ = 45°
13:	drift,	L =	0.90 m
14:	trim quadrupole,	Q =	-0.641 m^{-1}
15:	drift,	L =	0.10 m
16:	vertical dipole,	R =	0.25 m, ϕ = 20°
17:	drift,	L =	1.00 m
18:	quadrupole,	Q =	1.618 m^{-1}
19:	drift,	L =	0.50 m
20:	quadrupole,	Q =	-2.183 m^{-1}
21:	drift,	L =	0.50 m
22:	target.		

Enlargement factor = 0.100 stigmatic

Intermediate horizontal focus: 28.2 cm after element # 4
Intermediate vertical focus: 25.1 cm after element # 12

Source diafragma: 0.25 mm
Spotsize with sextupoles of -0.015 and -0.070 dioptr/mm
at intermediate foci: 42 x 38 μm^2
Spotsize without sextupoles: 156 x 98 μm^2

Table 5.2: Achromatic quadrupole lens system in one dimension

Lens system:

1:	drift,	L =	9.00 m
2:	magnetic quadrupole,	Q =	-2.2698 m^{-1}
3:	drift,	L =	0.15 m
4:	electric quadrupole,	Q =	3.9336 m^{-1}
5:	drift,	L =	0.30 m
6:	magnetic quadrupole,	Q =	-2.100 m^{-1}
7:	drift,	L =	0.6695 m
8:	target.		

Enlargement factor = -0.09 horizontal

Source diafragma: 1 mm

Spotsize:

Δp/p (%)	Xmin (μm)	Xmax (μm)
-0.10	-45	+45
0.00	-45	+45
+0.10	-45	+45

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