Dynamic route planning : finding planning and monitoring algorithms which deal effectively with long range, time dependent traffic information

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Dynamic Route Planning

Finding Planning and Monitoring Algorithms Which Deal Effectively with Long Range, Time Dependent Traffic Information

by
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SIEMENS VDO
Automotive

Preface

It is my pleasure to present to you this Master's thesis as the final written product of my study Applied Mathematics at the Eindhoven University of Technology. This report is the result of my graduation assignment for which the research was done at VDO Car Communication B.V. in Eindhoven within the Route Planner team from December 2001 till September 2002. This report was written in the comfort of my own home in Uden from September 2002 till January 2003.

This report is interesting for people with an interest in shortest path problems, in particular shortest path problems in real road networks. Some familiarity with the notations used in graph theory is assumed, especially for chapters 2, 3 and 4. Chapter 11, which contains the main results, conclusions and recommendations, will be the most interesting for the people who want to use this report as input for further investigation.

At this point, I would like to thank all the people who made my graduation assignment possible. First of all, I wish to thank Jeroen Goes, my supervisor at VDO Car Communication, for all the effort he made guiding me throughout my traineeship and for all the useful comments and ideas. I would also like to thank my supervisors at the University, Judith Keijsper and Cor Hurkens. Judith, thank you for all the discussions which reminded me not to neglect the theoretical aspects. Cor, thank you for supporting me in many ways in writing this report after Judith was called away by motherhood (congratulations with your son). Also, a thank you is in place for Teun Hendriks, who coordinated my assignment at VDO. Of course, to Dot, Ed, Ingrid, Jacques, Jeroen E., Karin and Rob, all my other colleagues from the Route Planner team: Thank you all for making my stay at VDO pleasant, both during working hours (the help with all the trivial and not-so-trivial problems) and outside working hours (second place in the futsal competition).

A sincere thank you is in place for my brothers and my parents, who supported me throughout my study. And last, but most definitely not least, a very special thank you to my girlfriend Monique, for all the moral support and for always trying to push me to be all I can be.

Bart van Eekelen

Uden, January 2003

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Abstract

The route planner of Siemens VDO plans routes from the current car position to destinations specified by the driver. For this planning the actual road network is used with for each road segment the standard driving speed. However, using only one speed for a road segment is not realistic. For example, during rush hours the speed at a certain road segment will be lower than during the rest of the day.

A special form of varying speeds for road segments are caused by traffic congestions. Traffic congestions cause the speed at a road segment to vary with time and most of the time the future development of a congestion is not known. The main objective of this research is to "find planning and monitoring algorithms which can deal effectively with long range, time dependent traffic information within the framework available in the VDO navigation systems".
One of the requirements is that the algorithm should be fast enough, since a driver does not want to wait long before a route is presented.
Finding an algorithm is important since otherwise a seemingly good route planned using the standard speeds can in reality be bad because a traffic congestion is present. Taking the congestion into account could have led to a route only slightly worse than the route in case of no congestions but much better than that same route with the congestion.

First, the different types of minimum cost path problems are investigated. Mathematical models are given and (the lack of) algorithms to solve the minimum cost path problems are discussed. No effective algorithms for minimum cost paths in road networks with congestions are known. Only if the future development of congestions would be known a good route could be planned.

Since no effective algorithms are known, an algorithm is examined by means of simulation. The current way of taking congestions into account is neglecting all congestions more than 150 km. away from the car. This *strategy* is compared with an alternative strategy, which lets the influence of a congestion decrease in a non linear way as the distance between the car and the congestion increases. The influence denotes the weight the expected dynamic cost will have during planning and has a value between 0 and 1. A value of 1 means that the congestion is taken into full account and a value of 0 means that the congestion will be neglected. The influence I of the alternative strategy depends on parameter α and has the following form ($d(car, a)$ is the distance between the car and the congestion):

$$I(a) = \begin{cases} 1 & d(car,a) \leq \alpha; \\ \frac{\alpha}{d(car,a)} & d(car,a) > \alpha. \end{cases}$$

This comparison is done for both the case in which a route is planned only once and the case in which monitoring is applied.
In both cases the alternative strategy with $\alpha = 100$ turns out to perform best. The alternative strategy with $\alpha = 250$ gives the same results (in case monitoring is applied even slightly better) but is not preferred since the planning time is shorter for $\alpha = 100$.

Since most of the times detours are planned locally, it should be investigated what happens if all roads near the congestion are also assumed to be infected by the congestion.
The consequences of the increase in certainty of the course of the congestion as new (updated) information is provided should also be investigated.
In this research, a general model of the expected course of the congestion is used. It is probably beneficial to investigate the courses of certain groups of similar congestions (single car accident, open bridge, tumbled truck etcetera), and to define an expected course for each of these groups.
Based on the fact that most of the detours planned are local, and are only found after a number of replannings, it is interesting to check the quality of the routes planned in case the search level at the area around the congestion is manually set to a greater detail.

Chapter 1

Introduction

In this chapter an introduction is given of what will be discussed throughout the rest of this report. First, in Section 1.1, some background information will be given about the settings of my graduation assignment at Siemens VDO. Also the relevance of my research is explained. Section 1.2 describes the objective of my research. An outline of the rest of this report can be found in the last section of this introductory chapter.

1.1 Background

In the first subsection of this section a short description of the structure of Siemens VDO is given. The product relevance is stated in the second subsection.

1.1.1 Siemens VDO's Organizational Structure

This subsection is used for a description of the organization within which I was allowed to perform my graduation assignment. First we briefly discuss the overall organization Siemens AG, and after that one of its divisions, Siemens VDO Automotive AG. VDO Car Communications Nederland is a subsidiary of Siemens VDO Automotive and has an establishment in Eindhoven. This establishment is where I was allowed to work on my graduation assignment. Finally, an overview of the organizational structure of this site, Development Center Eindhoven, is presented.
This overview is based on [16] and on figures from the Siemens website.

Siemens AG

Sustainability has always been a top priority for Siemens. In the 150 years since the foundation, Siemens developed into an organization with more than 428.000 people spread out over 190 countries and with its headquarters in Munich, Germany. Of this 428.000 people approximately 11% (approximately 53.000) are researchers and developers.
Siemens' goal is to offer life technologies and innovative solutions to its customers to enable them to successfully participate in the digital revolution. In both the areas of electronics and electrical engineering, Siemens is one of the main players. Siemens globally provides products and solutions for the sectors in Figure 1.1.

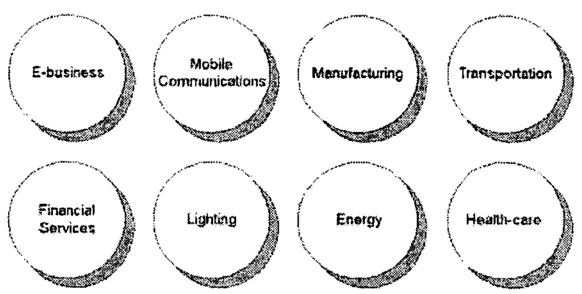

Figure 1.1: Siemens' Active Sectors

The *Transportation* area comprises two groups, namely *Transportation Systems* and *Siemens VDO Automotive*. Siemens VDO Automotive is one of the world's largest suppliers of automotive electronics.

Siemens VDO Automotive AG

In Figure 1.2, the organizational overview is given of the Siemens division *Siemens VDO Automotive*. Siemens VDO Automotive emerged in 2001 from the joining of the former *Siemens Automotive AG* and *Mannesmann VDO AG*.

Its goals are ambitious. A few of them are:

- To become the preferred partner to the automotive industry for innovative solutions and systems integration.
- To grow faster than the overall market and competitors.
- To become the number one supplier for automotive electronics.
- To increase profitability and flexibility.

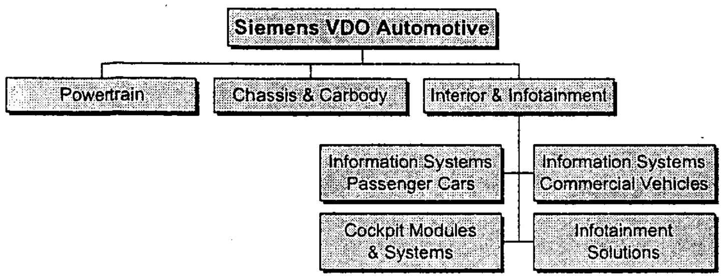

Figure 1.2: Siemens VDO organization scheme

- To expand its position in Asia and North and South America.

Siemens VDO Automotive has interests in over 30 countries worldwide and has 130 different business locations, on five continents. It has over 45.000 employees and its headquarters are in Regensburg and Schwallbach, Germany and in Auburn Hills, Detroit, USA. The Infotainment Solutions group contains among other components *VDO Car Communication Nederland*, which has a development center in Eindhoven.

Development Center Eindhoven

Development Center Eindhoven (DCE) is situated nearby Eindhoven Airport and had about 200 employees in 2002. Its competences are Navigation Key Components and Radio Pre-development. The organization of the site is divided into four disciplines. An overview of the organizational structure is given in Figure 1.3.

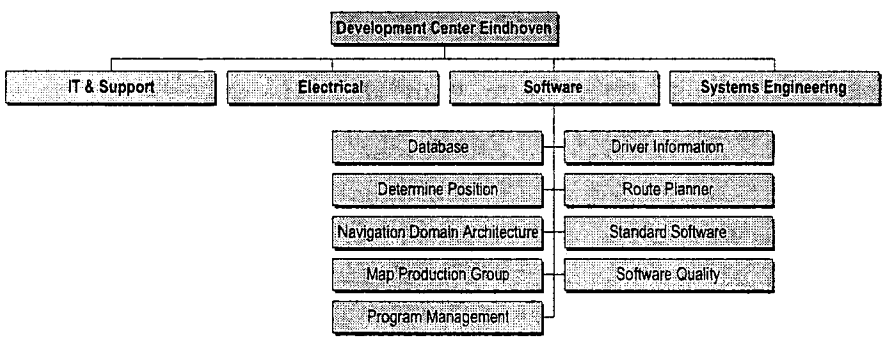

Figure 1.3: DCE organization scheme

The *Route Planner* group belongs to the *Software* discipline. *VDO Dayton* is the brand name for all the Siemens VDO Automotive vehicle navigation systems that are sold in the private sector. Within the Route Planner group, all efforts apply to the Siemens VDO vehicle navigation systems.

1.1.2 Product Relevance

Everybody knows that the problem when driving towards an unknown destination with the car is how to get there. Before departure the route is planned and with the road map in the car, the driver departs. Usually, the passenger sitting next to the driver will be given the task to read the map, knowing where exactly on the map the car is and give directions to the driver.
In general, this task is not done perfectly by the passenger and several marriages have been tested in the car. Often a failure is made and the actual route driven differs from the one planned. Unnecessary distance and time is the result. A lot of this annoyance, extra distance and time can be avoided by using a navigation system which takes over the task from the passenger. Such a system leads to economic route planning and good car guidance which can give substantial savings and benefits, also for professional road users such as police, ambulances and taxis. One of the main navigation systems are the VDO car navigation and guidance systems of Siemens VDO Automotive. This is an advanced system for both planning a route to a user specified destination from the current car position (which is updated continuously) and guiding the driver along this planned route. The guidance can be given by spoken words or by directions on screen (or both).
The system of Siemens VDO Automotive is not just a plain route planner. The system has a lot of extra features. For example, suppose the car is running low on gasoline, the system can give a list of gas stations nearby. After specifying one of these station as destination, and the system will guide the car right to it. Hotels and restaurants are some of the other things the system has stored.
Suppose the driver knows the optimal route from its current position to a destination but he wants to try a different route for once (for any reason). For this, *alternative planning* can be used, which plans an alternative route. Alternative planning can also be used if the driver gets information about a traffic congestion on the optimal route. With *enhanced local detour* getting information about traffic congestions and planning alternative route if necessary is automated. Information about traffic congestions is provided by the RDS-TMC traffic information system, and the driver is given the option to plan an alternative route or to drive through a congestion. Note that if the driver wants a different route, an even better feature is *via point planning*. With this feature, the driver can specify multiple destinations and with via point planning a route is planned which visits all these destination. Therefore, a route to a destination via a certain point in the middle of the route can be forced by specifying this point as second destination.
Another important feature is *deviation planning*. Deviation planning is used if the driver makes a mistake and leaves the planned route. With deviation planning this deviation is detected and a route is planned either back to the first route planned (not necessary at the point the car deviated) or an entirely different route is planned. These features are described in [14].
One of the problems the route planner encounters is the occurrence of traffic congestions. Recall

that traffic congestions are already taking into account by using *enhanced local detour*, which warns the driver of a congestion and gives the option to replan a route around the congestion. The first step towards automatically taking congestions into account is taken by using *dynamic route planning* [15]. Dynamic route planning automatically takes into account the current information about the traffic congestions and continuously updates the route according to the current traffic situation. However, the dynamic route planner is not perfect. The main problem is that the current traffic situation is assumed to last forever. The congestions are assumed to keep their current lengths. This assumption is not realistic. A congestion develops and the length of the congestion varies with time.
Furthermore, it would take too much processor time to investigate each and every congestion. Therefore, the dynamic route planner only investigates congestions which lie inside an imaginary circle around the car with a diameter of 150 kilometer. However, this means that a congestion at 151 km. from the car is not taking into account.

The assumption that the current traffic situation stays constant can lead to a route that avoids a certain congestion even though afterwards the congestion appeared to be gone by the time the driver should have been at the location of the congestion. On the other side, it can also happen that a route is planned through a congestion which was assumed to have a certain small length but turned out to have a much greater length. A certain part of these problems will be solved when new information about the congestion is provided. For example, a route wrongly planned through a congestion can be replanned when the congestion turns out to be bigger than first expected. But it is also possible that once new information is provided, the driver has passed the junction which separates the current route and a good alternative route avoiding the congestion. It can be concluded that the current way of taking congestions into account can lead to routes with unnecessary delay, either driving through a congestion where a detour should be preferred or taking an unnecessary detour.

1.2 Purpose of Research

In the future, with dynamic navigation, traffic information will be taken into consideration while the "optimal" route is determined. While the driver is guided along the currently "optimal" route new information about the traffic situation may be provided. This information may cause the current driven route to become suboptimal. Therefore, the quality of the planned route will have to be monitored. The question is to which extent it is possible to determine an algorithm which factors in the traffic situation while determining the "best" possible route and which monitors the route while guiding the driver along that route.

The discussion above can be translated into the objective of this report ([7]):

Find planning and monitoring algorithms which can deal effectively with long range, time dependent traffic information within the framework available in the VDO navigation systems.

The algorithms must take the following issues into account:

- In general, the duration of traffic events are given only with approximate bounds. Most of the times, these bounds are given with relative times rather than with absolute times.
- The course of the congestion (the development of the length of the congestion over time) is in general not given.
- The probability that a congestion occurs is never given. The same goes for information about how the congestion will develop in the future.

All the above mentioned issues are consequences of imperfect traffic information.
Another constraint is that the current route planning algorithms can not plan detailed detours for traffic problems far away from the current car position.
Summarizing, the above issues lead to the following objective and constraints:

Objective: Find planning and monitoring algorithms which can deal effectively with long range, time dependent traffic information within the framework available in the VDO navigation systems.

Subject to: Imperfect traffic information.

Subject to: Resource constraints.

In order to get some insight on what kind of algorithm can deal with the dynamic information, some strategies will discussed. A strategy is a certain way of taking congestions (far away) into account during planning. One alternative strategy will be chosen to be compared with the current strategy, which is to take all congestions within a circle of 150 km. around the car into full account and to neglect all congestions outside that circle.
This comparison will be done by means of a simulation. For each of the strategies, routes will be planned and the results will be compared.

1.3 Report Outline

Throughout chapters 2, 3 and 4, several kinds of minimum cost path problems will be described. Each of the chapters starts with a mathematical model of the problem described in that chapter, followed by algorithms known in literature to solve that problem. Chapters 2, 3 and 4 describe the static version, the dynamic version and the stochastic dynamic version of the minimum cost path problem respectively. In Chapter 2 also the route planner used by VDO is discussed.
Chapter 5 focusses on dynamic traffic information. First the types of traffic information and the problems with the provision of that traffic information is described and then a general model of congestions is given.
To check whether the model found in Chapter 5 gives realistic results, in Chapter 6 a comparison is made between the expected cost caused by a congestion and the average cost caused by a congestion.
The possible ways to take congestions (far away) into account are described in Chapter 7. In that chapter, also a choice is made of which strategy will be compared with the current strategy. We are interested in comparing the strategies in both the case in which planning is done only once and the case in which monitoring is applied. The simulations for the case in which planning is done only once is discussed in Chapter 8. In Chapter 9 is explained why monitoring is preferred and how it can be done. The comparison for the case in which monitoring is applied is done in Chapter 10. All the important conclusions are summarized in Chapter 11. In that chapter also recommendations for future work are given.

Chapter 2

Static Route Planning

The most basic form of route planning is static route planning. Static route planning deals with finding a route in a road network in which everything about the roads is known and in which the situation never changes. In this chapter, we first give the mathematical model of the static road network (which is based on [4]) and the problem of finding an optimal route in that network (Section 2.1). After that, in Section 2.2, a description is given of the general ways in which static route planning problems are dealt with, with an emphasis on the two best known algorithms, Dijkstra's *algorithm and algorithm* A* . *In the last section the general (static) route planner used by VDO is explained together with the modifications that are made to that route planner for my research.*

2.1 Mathematical Model

A road network consists of road segments that intersect each other. It can be represented by a directed graph $G = (N, A, c)$.
In this graph, the set of nodes, N, denotes the intersections in the road network, with $|N| = n$ the total number of nodes. The directed edges, set A, represent the road segments, with $|A| = m$ the total number of edges. Note that it is necessary to have *directed* edges because one-way roads exist. An edge $a \in A$ connects two nodes $u \in N$ and $v \in N$ with each other. We say if an edge a is from node u to node v, then $tail(a) = u$ and $head(a) = v$. The function $c : A \rightarrow \mathbf{N}$ assigns cost $c(a)$ to each $a \in A$.
A route from one position in the road network to another will be represented by a path from one node to another. A *path* is a sequence of connected nodes and edges.

Definition 2.1. *A* path p *in a static graph* $G = (N, A, c)$ *is a sequence of nodes and edges* $p = \langle u_1, a_1, u_2, a_2, \ldots, u_k \rangle$ *where* $u_i \in N$, *for* $i = 1, \ldots, k$ *and* $a_i \in A$, *with* $tail(a_i) = u_i$ *and* $head(a_i) = u_{i+1}$, *for* $i = 1, \ldots, k-1$. *We say path* p *is a path* from *(begin node)* u_1 to *(end node)* u_k. *Define*

$A(p)$ as the edge-set *of path p: $A(p) = \{a_1, \dots, a_{k-1}\}$. Define $N(p)$ as the* node-set *of path p: $N(p) = \{u_1, \dots, u_k\}$. The* length *of path p is equal to the number of* edges *in path p.*

Define $P(u,v)$ as the collection of all possible paths in G from u to v. Define P as the collection of all possible paths in G.
The cost of a path $p \in P$ is the sum of the cost of all the edges of that path.

Definition 2.2. *The cost $c(p)$ of a path p in G is given by:*

$$c(p) = \sum_{a \in A(p)} c(a). \tag{2.1}$$

A minimum cost path $p^*(u,v) \in P(u,v)$ is a path from node u to node v with cost less than or equal to the cost of any other path in $P(u,v)$.

Definition 2.3. *A path $p^*(u,v) \in P(u,v)$ is a minimum cost path in $G = (N,A,c)$ from u to v if:*

$$c(p^*) \leq c(p), \qquad \forall p \in P(u,v). \tag{2.2}$$

Given a starting point $s \in N$ and a destination $d \in N$, the problem of finding an optimal route from s to d amounts to finding a minimum cost path $p^*(s,d)$ from s to d in G.
We will denote this problem by *MCP (Minimum Cost Path)*. Now we can formally define *MCP*.

Definition 2.4 (MCP). *Given a directed graph $G = (N,A,c)$, starting node s and destination d, find a minimum cost path $p^*(s,d) \in P(s,d)$.*

2.2 General Route Planner

One of the best-known algorithms for finding a minimum cost path between two nodes in a static directed graph (*MCP*, see definition 2.4) is Dijkstra's uniform cost algorithm (see also [3]). The basic idea of Dijkstra's algorithm is that it searches from the start node s in ever increasing circles around this node until the destination node d is found.
One of the disadvantages of Dijkstra's algorithm is that it does not take into consideration the geographical positions of s and d. An algorithm that does take into account geographical positions is algorithm A* (see also [11]).
Both algorithms will be reviewed in the next two subsections.

2.2.1 Dijkstra's Algorithm

Given a directed graph $G = (N, A, c)$, $s, d \in N$. We want to plan a route from s (start node) to d (destination node).
In the following, let $g(u)$ denote the best known upper bound estimate of $c(p^*(s,u))$, the cost of a minimum cost path from s to u for every $u \in N$.
Let S be the set of nodes for which the minimum cost paths from s have been determined. The basic form of Dijkstra's algorithm is as follows (we assume that a path from s to d exists):

1. Initialization: $S = \{s\}$, $g(s) = 0$, $g(u) = \infty\ \forall_{u \in N \setminus S}$;
2. For all $u \in N \setminus S$ with $(s,u) \in A$, set $g(u) \leftarrow c(s,u)$;
3. While $d \in N \setminus S$:
    (a) Sort the nodes in $N \setminus S$ according to increasing g value;
    (b) Find $v \in N \setminus S$ for which $\forall_{u \in N \setminus S}\, g(u) \geq g(v)$ (ties may be broken arbitrarily). Set $S \leftarrow S \cup \{v\}$;
    (c) For all $u \in N \setminus S$ for which $(v,u) \in A$ set $g(u) \leftarrow \min[c(v,u) + g(v), g(u)]$;

It can easily be seen that once $d \in S$, the minimum cost path from s to d is found. This minimum cost path has cost $c(p^*(s,d))$. Dijkstra's algorithm runs in $O(|N|^2)$ time. Note that Dijkstra's algorithm can be speeded up by using heaps. The running time of the algorithm will then be reduced to $O(|A| + |N| \log |N|)$.
At each iteration, Dijkstra's algorithm uses an evaluation function g to determine the most promising node out of all the nodes in $N \setminus S$. Like stated in the basic form, the value of g of a node u equals the minimum cost path from s to u only passing through nodes from S.

Note that Dijkstra's algorithm can also be used to find shortest paths from s to all other nodes. For this, the algorithm has to run until $N \setminus S$ is empty.

The area searched by Dijkstra's algorithm has the shape of a circle, with s as center and with d on the perimeter. The area that will be searched by the algorithm before the shortest path from s to d is found is displayed in Figure 2.1.

One of the disadvantages of Dijkstra's algorithm is that it does not take into consideration the geographical positions of s and d. The evaluation function $g(u)$ of node u only considers the cost to reach u from s, so it does not distinguish between a node u_1 that geographically lies between node s and d (so leading towards d) and a node u_2 that geographically lies further away from d then s (so leading away from d), see Figure 2.2.

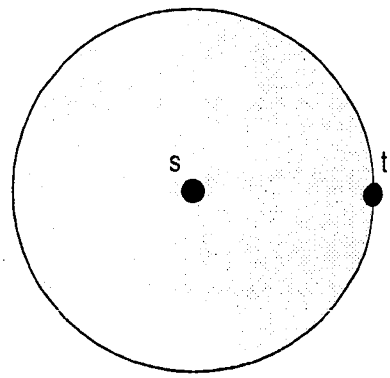

Figure 2.1: Search Area of Dijkstra

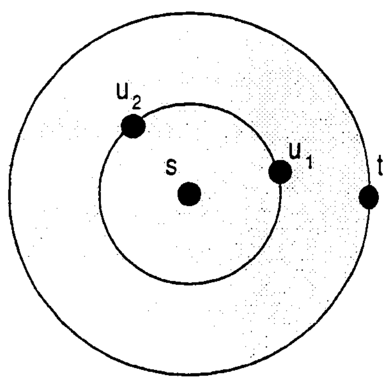

Figure 2.2: Ignored Geography by Dijkstra

In general it will be more likely that the minimum cost route will go via u_1 instead of via u_2. A consequence of neglecting geographical positions is that more nodes will be evaluated, and therefore the search for an optimal route takes much longer than necessary.

2.2.2 Algorithm A*

Let a directed graph $G = (N, A, c)$, s and d be given. We want to find the minimum cost route from s to d in G.
Let S be the set of nodes for which the minimum cost paths from s have been determined. Dijkstra's algorithm does not use the geographical positions in determining the next node to be moved from $N \setminus S$ to S. Algorithm A* is a generalization of Dijkstra's algorithm, which uses an evaluation function f composed of two elements. For evaluation of node u, besides the cost of the minimum cost path from s to u only passing through nodes from S, the g value of Dijkstra's

algorithm, algorithm A* also uses the h value. The h value of a node u is an estimation of the remaining cost from u until the goal node d based on the geographical positions of u and d. Because in general the h value decreases as nodes nearer to the goal node are considered, the search is geographically more directed toward the goal node. If two nodes a_1 and a_2 have the same g value, the node with the smallest h value (closest to the goal node) has the lowest f, and will therefore be chosen as best candidate (provided that all f values of the other nodes are higher). The area searched by the algorithm A* is therefore smaller as can be seen in Figure 2.3. Another observation is that with algorithm A*, in contrast with Dijkstra's algorithm, there is a distinction made between u_1 and u_2 as talked about in the description of Dijkstra's algorithm. Node u_1 will be evaluated by algorithm A* and node u_2 will not.

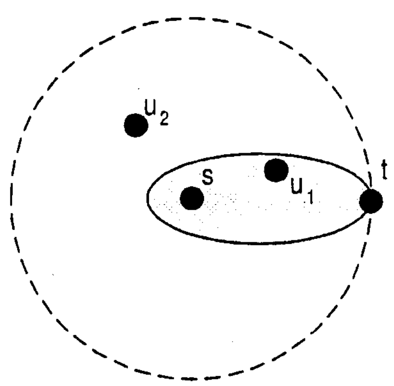

Figure 2.3: Search Area of A*

The basic form of algorithm A* is as follows. We assume that a path from s to d exists, and that $h(u) \leq c(u,d)\ \forall u \in N$ with $c(u,d)$ the remaining cost from u until the goal node d (i.e. the remaining cost is underestimated):

1. Initialization: $S = \{s\}$, $g(s) = 0$, $\forall_{u \in N \setminus S}\, g(u) = \infty$;
2. For all $u \in N \setminus S$ with $(s,u) \in A$, set $g(u) \leftarrow c(s,u)$;
3. While $d \in N \setminus S$:
   (a) Sort the nodes in $N \setminus S$ according to increasing $f = g + h$;
   (b) Find $v \in N \setminus S$ for which $\forall_{u \in N \setminus S} f(u) \geq f(v)$ (ties may be broken arbitrarily). Set $S \leftarrow S \cup \{v\}$;
   (c) For all $u \in N \setminus S$ for which $(v,u) \in A$ set $g(u) \leftarrow \min[c(v,u) + g(v), g(u)]$;

It is easy to see that Dijkstra's algorithm is a special case of algorithm A*. If $h(u,d)$ is set to zero for all $u \in N$, f reduces to g, and thus algorithm A* reduces to by Dijkstra's algorithm.

Theorem 2.5. *If for all $u \in N$ $h(u)$ is at most equal to the cost of a minimum cost path from u to d, algorithm A* finds a minimum cost path from s to d.*

Proof. See [11] □

The running time of algorithm A* depends on the accuracy of the function that is used to estimate the remaining cost.
If the accuracy of the estimation increases (decreases), the running time of algorithm A* decreases (increases). Note that the h value is assumed to remain underestimated. The lowest accuracy is reached if we totally neglect the h value, that is if we set $h(u,d) = 0\ \forall u \in N$. Hence, the running time of algorithm A* is at most equal to that of Dijkstra's algorithm ($O(|N|^2)$). The best estimation is the actual remaining cost. If those are known for all nodes, at each iteration the next node on the minimum cost path is moved from $N \setminus S$ to S, so the running time is $O(|N|)$. The worst instance is the instance in which the minimum cost path goes past all nodes.
It can be concluded that as the estimation of the remaining cost improves from 0 to actual remaining cost, the running time of algorithm A* improves from $O(|N|^2)$ to $O(|N|)$.

2.3 Route Planner of VDO

VDO uses a modified version of algorithm A*. One of the main reasons that the basic algorithm A* is not used (even though it guarantees an optimal route as long an underestimated h value is used) is the long time it takes the algorithm to find the optimal route. It is not acceptable for a driver to have to wait a long time before the navigation system begins to guide him along an optimal route. Therefore, a compromise must be found between the planning time and the quality of the route.
In the first subsection it is explained what causes the long planning times. After that, in the second subsection, it is described which modifications are made to speed up the planning. The third subsection describes an other modification done by VDO to the general route planner. In the last subsection we explain which modifications are made to the route planner of VDO for the benefit of my research.

2.3.1 Causes of Long Planning Time

The route planner of VDO makes use of road maps, which are stored in a database. In order to use this database efficiently, each map is divided into rectangular sections, so-called *buckets*, which contain a maximum of 16K of data. The map is divided alternatively vertically and horizontally into smaller sections until each section contains at most 16K of data and each section is not too large. After the division, the sections are stored on DVD as buckets. Note that the size of a bucket varies with the density of the road information in the bucket.

The relatively long access times of the DVD is a main cause of the long planning times. For 50% of the time, the search process used to be busy retrieving buckets. To reduce the planning time, a limited amount of memory (RAM) is available for temporary storage of bucket information. This buffer pool, with much smaller access times, can contain a fixed amount of data. The route planner can use the buffer to temporarily store buckets. However, also other components of the VDO navigation system use the buffer pool.

2.3.2 Speed Measures

The basic form of the algorithm A* takes too long before a minimum cost route is presented to the driver. Therefore, some modifications have been made to speed up the search process. There are four types of modification. All modifications could cause a decrease in the quality of the route found. Next, we list the types of modifications. For each modification, the impact on both the speed of the search process and the quality of the resulting route will be discussed.

Transition boxes and Leveling

The intuitive approach for planning a route is to find a main road that connects small areas around both the starting point s and the destination d. After finding such a main road, we want to know how to get from s to that main road and from that main road to d. We want to drive along minor roads as little as possible. Therefore, for a major part of the route, the route planner does not search all roads. For this the route planner uses different search levels.
The roads on the map are divided into 7 different road classes. From road class 0 (major roads, for example highways) up to road class 5 (minor roads). Road class 6 is given to roads with a special restriction property like walkways and private areas.

The DVD contains four different versions of the same map, each with its own level of detail. The most detailed map (DTM) contains all road classes. The less detailed map HL2 contains all roads of road classes 0, 1 and 2. The HL1 map contains all roads of road classes 0 and 1 and finally the least detailed map (HL0) contains only the roads of road class 0. Note that in the USA this partition is slightly different. Because of the low network density, each map contains one more road class. So, for example, HL0 also contains road class 1.
For the middle (largest) part of the route only the higher level maps are used. Because the minor roads are left out of the higher level maps, the maps contain less information. Therefore, the buckets will be bigger and fewer buckets will be needed to divide the map. Less chains will be investigated. Thus the planning time will decrease.
To determine when to switch to a higher level map (or lower level on the side of d), so-called *transition boxes* are used. Transition boxes are imaginary squares around s and d. The size of the innermost box (DTM box, in which is searched at the most detailed level) depends on the size of

the bucket in which s lies on the DTM map and the location of the nearest edge on HL2 level. The HL2 box is constructed in the same way and should be at least a certain factor larger than the DTM box. The last transition box, the HL1 box, is determined the same way. The area outside the HL1 box is considered HL0 area.
The same procedure is done for the area around d, although currently the transition boxes around the destination are not used. Note that in general the size of the transition boxes on both sides are different, see Figure 2.4.
When s and d are so close together that the transition boxes of a certain level cross each other, the planning is done with at least the detail of the least detailed level of the crossing boxes.

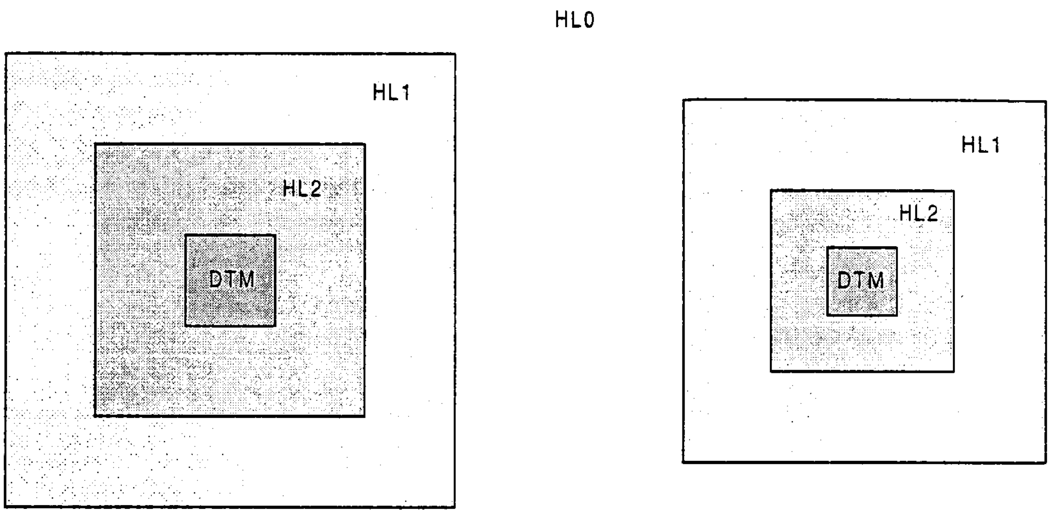

Figure 2.4: Transition Boxes and Leveling

Now the planning is done as follows:
The search starts at d on DTM level. Note that the route planner searches from the destination towards the car, as will be explained further on in this subsection. Once the route planner finds a road on the next search level, it terminates the search along this particular path. The road found becomes a so-called *target*. The search continues on DTM level along other paths until certain stop criteria are satisfied (these stop criteria also will be discussed further on in this subsection). Note that indeed the transition boxes are not used around the destination. The targets that are found will become so-called *sources* when we switch to HL2 map level. The search continues at HL2 level from these sources. Again, we search for targets. This process continues until HL0 is reached. From here on the planning is done on HL0 level until the path enters the transition boxes near s.
When approaching s, as soon as a search path intersects the box of a more detailed level, the search along this path is terminated and the intersecting road is marked as target. Again the search on a level is continued until the stop criteria are satisfied. Search is continued from these targets (now sources) on a more detailed level.

It is easily seen that using the principle of leveling makes the planning a lot faster, since much fewer edges are evaluated.
However, leveling is also a cause of deterioration of the quality of the route found by the route planner. That the minimum cost route will be found can not be guaranteed anymore, because certain roads are not considered. For example, not considering minor roads for the major part of the route may result in not considering some possible shortcuts.

Stop criteria

To decide when the search process has to switch to another level, certain so-called stop criteria are given. When all these criteria are satisfied, the switch is made to a less (near s) or more (near d) detailed level.
There are three stop criteria which have to be satisfied before a switch to the next level is made.

1. At least one target has to be found. The search cannot be continued on the next level if no edge on the next search level has been found yet. It would mean that there are no sources to continu from on the next search level.

2. The minimum number of expansions at the current search level has to be reached. It may well be that the first target found is one that will not lead to a (near) optimal route. To decrease the chance the target corresponding with an optimal route is missed, there is a minimum number of expansions that has to be done at each search level.

3. The maximum number of expansions since the current best target was found has to be reached or there should be no more 'good' candidates. To avoid searching for targets endlessly a maximum is introduced on the number of expansions after finding the current best target. Furthermore, there is no use in evaluating candidates whose f value is too big. The f value of the current best candidate is considered too big if the ratio between the f value of this candidate and that of the current best target is greater than a certain factor.

With the setting of the parameters concerning the stop criteria a balance has to be found between the speed of the search process and the quality of the route found. For example, if the minimum number of expansions and the required factor of difference between current best candidate and current best target are both set low, the switches between levels are done as fast as possible. The search process will then be very fast. However, there is a significant chance that an optimal route will not be found. This can happen if the switch to the next level is made before a target corresponding with an optimal route is found. If both parameters are set high, most likely an optimal route will be found. But the high settings of the parameters will cause the search process to be very slow.

Pruning

During the search process information about evaluated edges must be stored. The amount of information that should be stored can be very large. A feature that is designed to deal with the problem of preventing an overload of information that has to be stored is *pruning*.
Pruning can be described as throwing away data which is considered to be of no importance for the search process.
There are two kinds of data that should be stored during the search. The first is the *candidate list*. This is a list with nodes that can be expanded next. Furthermore, there is the *search tree*, which is information about edges that already have been evaluated. Next, we describe how pruning is done for these two kinds of data.

- Candidate list
  With every expansion that is done during the search process the list of candidates grows. For all candidates information needs to be stored. When the search process progresses it may become clear that certain nodes in the candidate list probably will not be used anymore in the search. These candidates only consume valuable memory. Therefore, the route planner decides to throw away (prune) the least promising candidates in the candidate list. Buckets are deleted when the best candidate from that bucket is more than a certain distance further away from d than the current best candidate. That gives space for a more promising bucket to be stored, and decreases the number of so-called *bucket-requests*, which is the main cause of the long planning time.

- Search tree
  The second kind of data is the search tree. This contains information about the actual cost of partial routes. Only a limited amount of memory can be used for the search tree. Therefore, also the search tree has to be pruned. The route planner throws away buckets of the search tree based on the FIFO principle (First In First Out). So, the oldest bucket is deleted first.

With respect to the speed of the planning, we can say that pruning the candidate list makes the search process proceed faster. After each expansion the candidate list has to be sorted. It is easy to see that if the number of nodes in the candidate list decreases, the time required for sorting that list also decreases. Unfortunately, there are also some disadvantages of pruning. One of the disadvantages is that in certain situations the minimum cost route does not have a 'logical' form. For example, the minimum cost route may not be more or less a straight line if a river is present. In such situations it may occur that a candidate that might seem not so promising may actually be part of the minimum cost route. If this candidate is pruned, the minimum cost route will not be found.
Pruning the search tree has also some disadvantages. For example, certain areas might be evaluated twice because the route planner does not remember having searched those areas before (the information about the first time the area was searched is thrown away).

Overestimating h values

The h value (see Section 2.2.2) is used to prefer nodes in the direction of d over nodes leading in the opposite direction. It will draw the search process in the direction of d.
As long as the h value is underestimated, its value does not influence the quality of the routes found. The route found will always be an optimal route. However, the h value does have an effect on the speed of the route planner. A small h value causes a great number of expansions, because more candidates could lead to an optimal route. The speed of the search can be increased by using overestimated h values. However, if the h value is overestimated, the optimality of the route cannot be guaranteed any more. A candidate that leads to an optimal route may be ignored because the estimated remaining cost is larger than the actual cost.

2.3.3 Search Direction

Besides the modifications made to the general route planner for more speed, there is another difference. In general, an algorithm starts at the CCP (Current Car Position) and searches for DES (Destination). This is called *forward planning*. However, VDO has chosen to use *backward planning*, which means that the search is started at DES and is directed towards the CCP. The reason for this modification is that the car may be moving. Backward planning minimizes the chance that once a route has been found, the CCP is not accurate any more. This is called the *car catching problem*. Furthermore, with backward planning the driver can be given guidance sooner than with forward planning.

2.3.4 Route Planner for Research

For my research, the quality of the route found is more important than the speed of the search process. Therefore, most of the speed measures explained in Section 2.3.2 have been removed in the route planner used during my research. Only the leveling structure is kept. The stop criteria are removed, and nothing will be pruned.

Furthermore, since I have not been testing in a real car, I assumed that the car was stationary. Therefore, forward planning would not cause a car catching problem. With backward planning the route between the edge that is being evaluated and the CCP is not known. Since knowing the arrival time at an edge is important for my search, forward planning is preferred for my research.

Chapter 3

Dynamic Route Planning

In the previous chapter the basic minimum cost path has been described. In this chapter we will investigate minimum cost path problems further and describe the dynamic version. The dynamic minimum cost path problem is concerned with finding a minimum cost path in a road network in which the cost of the roads may vary with time. In the first section, the mathematical model is described (which is partly based on again [4]). This description is followed in the second section by a description of an algorithm which solves the dynamic minimum cost path problem. In the third and last section the search direction is considered in relation with the objective of a search.

3.1 Mathematical Model

The dynamic minimum cost path problem is concerned with finding a minimum cost path in a graph in which the cost of an edge varies with time. Such a graph will be denoted by a *dynamic graph.*

Certain properties of a road network change with time. For example, the average speed on a road is lower during rush hours than during the rest of the day. A consequence is that the cost of an edge of a dynamic graph depends on time. Define the total dynamic cost of an edge of such a graph as the sum of a static part and a variable part.

Definition 3.1. *Define $c^s(a) : A \rightarrow \mathbb{N}$ as the static cost of an edge $a \in A$ and define $c^v(a,t) : A \times \mathbb{R}^+ \rightarrow \mathbb{N}$ as the variable cost of an edge a at time t. The total dynamic cost of an edge a at time t, $\hat{c}(a,t) : A \times \mathbb{R}^+ \rightarrow \mathbb{N}$, is now:*

$$\hat{c}(a,t) = c^s(a) + c^v(a,t), \qquad c^s(a), c^v(a,t) \geq 0, t \in \mathbb{R}^+ \tag{3.1}$$

A dynamic directed graph $\hat{G}$ is a graph $\hat{G} = (N, A, \hat{c})$ with dynamic cost function $\hat{c}$.
Because the cost is dynamic, the definition of a path has to be extended to a time-path.
But before we do this, first we define a time-function.

Definition 3.2. *Let $a \in A$ and let $t \in \mathbb{R}^+$. The time-function $\hat{T} : A \times \mathbb{R}^+ \to \mathbb{R}^+$ gives for each $(a,t) \in A \times R^+$ the time that is needed to traverse $tail(a)$ plus the time needed to traverse a, starting the traversal of $tail(a)$ at time t. Thus, $\hat{T}(a,t)$ is the time needed to get to $head(a)$ when departing at time t at $tail(a)$.*

Now, a dynamic directed graph can be denoted by $\hat{G} = (N, A, \hat{c}, \hat{T})$. Next, the definition of time-path is given.

Definition 3.3. *Given a dynamic directed graph $\hat{G} = (N, A, \hat{c}, T)$, a time-path is a tuple $\pi = (p_\pi, \tau_\pi)$, with $p_\pi = \langle u_1, a_1, u_2, a_2, \ldots, u_k \rangle$ a path in $\hat{G}$ and $\tau_\pi = \langle t_1, \ldots, t_k \rangle$ a time vector with $t_i \in \mathbb{R}^+$, $\forall i$, such that $t_{i+1} \geq t_i + T(a_i, t_i)$ for $i = 1, \ldots, k-1$. In this time-path, t_i is the departure time from node u_i, so the arrival time at the end of the time-path equals $t_{k-1} + T(a_k, t_{k-1})$.*

Note that from the fact that $\hat{T}(a,t) \in \mathbb{R}^+$ for all $a \in A$ and $t \in \mathbb{R}^+$, it follows that $t_1 \leq t_2 \leq \ldots \leq t_k$.

Let $\Pi(u,v)$ be the collection of all time-paths from u to v ($u_1 = u$ and $u_k = v$). Define Π as the collections of all time-paths in $\hat{G}$.

As was stated in definition 3.3, the departure time from a node is not necessarily equal to its arrival time (i.e. if $t_{i+1} > t_i + \hat{T}(a_i, t_i)$ for a certain $i \in \{1, \ldots, k-1\}$. Waiting at nodes is allowed. However, waiting at nodes costs time and therefore a penalty function is given for the time waited at a node.

Definition 3.4. *Define $p(n, t_1, t_2) : N \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ as the penalty cost for staying at node n from time t_1 to t_2. Of course, this function is only defined for times t_1 and t_2 with $t_1 \leq t_2$.*

This penalty function gives that a dynamic directed graph is now denoted by $\hat{G} = (N, A, \hat{c}, \hat{T}, p)$. Now we can define the total cost of a time-path.

Definition 3.5. *Given a dynamic directed graph $\hat{G} = (N, A, \hat{c}, \hat{T}, p)$ and time-path $\pi = (p_\pi, \tau_\pi) \in \Pi$, with $p_\pi = \langle u_1, a_1, \ldots, u_k \rangle$ and $\tau_\pi = \langle t_1, \ldots, t_k \rangle$. The total dynamic cost of time-path π is given by the total dynamic cost function $\hat{c}(\pi)$:*

$$\hat{c}(\pi) = \sum_{i=1}^{k-1} [c^s(a_i) + c^v(a_i, t_i)] + \sum_{i=0}^{k-2} [p(\mu_{i+1}, t_i + \hat{T}(a_i, t_i), t_{i+1})] \tag{3.2}$$

Similar to the 'normal' minimum cost path, we can now define the minimum cost time-path from node u to node v, $\pi^*(u,v)$.

Definition 3.6. *A time-path $\pi^*(u,v) \in \Pi(u,v)$ is a minimum cost time-path in $\hat{G} = (N, A, \hat{c}, \hat{T}, p)$ from u to v if:*

$$\hat{c}(\pi^*) \leq \hat{c}(\pi), \qquad \forall \pi \in \Pi(u,v). \tag{3.3}$$

Given a starting point $s \in N$ and a destination $d \in N$, the problem of finding an dynamic optimal route from s to d amounts to finding a minimum cost time-path $\pi^*(s,d)$ from s to d in $\hat{G}$. This problem is denoted by *DMCP* (*Dynamic Minimum Cost Path*).

Definition 3.7 (DMCP). *Given a dynamic directed graph $\hat{G} = (N, A, \hat{c}, \hat{T}, p)$, starting node s and destination d. Find a minimum cost time-path $\pi^*(s,d) \in \Pi(s,d)$.*

3.2 Search Algorithms

Now that the mathematical model of the dynamic route planner is known, in this section a description is given of an algorithm that solves the *DMCP* problem (Dynamic Minimum Cost Path).

In the literature, the minimum cost path problem in a dynamic graph has been addressed in several different forms and with different constraints, see for example [9], [10], [6] and [1].

The standard way of finding a minimum cost path in a dynamic graph is by using an algorithm that uses a certain type of graph, which will be described further on in this section. This algorithm is for example mentioned in [1].

The algorithm that will be described in this section is based on three assumptions. Next, these assumptions will be listed. In this list is also explained why these assumptions are made.

Assumption 3.8. *Time can be considered discrete. The unit of time will be set at seconds.*

The discreteness of the time is needed to be able to use a finite graph. The unit of time is set at seconds because a small unit of time leads to a more realistic graph. However, making the unit of time even smaller would not make that much difference to make up for the increase in the size of the graph (and thus the planning time).

Assumption 3.9. *There is a maximum D to the duration of the route found.*

This assumption is also needed to be sure that the graph used during the search is finite. If the route found takes longer than D seconds (D can be large) then the route is assumed to be not interesting.

Assumption 3.10. *The cost function of the edges has such a form that all minimum cost routes are concatenated.*

A minimum cost route is concatenated when all sub routes are also minimum cost routes. This assumption is needed to validate the claim used in the description of Dijkstra's algorithm (see Section **??**) that once the destination is selected from the list of candidates the minimum cost path is found.

Now that these three assumptions are given, the algorithm can be explained.
The algorithm uses a modified graph. This modified graph is called a *time-expanded graph*. The time-expanded graph is denoted with $G^D = (N^D, A^D, c^D, T^D, p^D)$. For each node $n \in N$ from the original graph $\hat{G} = (N, A, \hat{c}, \hat{T}, p)$ there are $D+1$ copies $n_0, \ldots, n_D$ in N^D, where node n_i represents node n in $\hat{G}$ at time i. For each edge $(i, j) \in A$, there are multiple edges (at most D) corresponding to different starting times. There is an edge (n_i, l_j) in the time-expanded graph whenever $(n, l) \in A$, $j - i = \hat{T}((n, l), i)$, and $j \leq D$. This edge represents travelling from node n to l starting the traversal at time i and therefore arriving at node l at time $j = i + \hat{T}((n, l), i)$. The cost of edge (n_i, l_j) is set to be equal to $\hat{c}((n, l), i)$. The cost of the edge $(n_i, n_{i'})$ with $i \leq i'$ is set to be the penalty for waiting in node n from time i to i'.
The above results in a static graph. Finding a minimum cost route from s to d in $\hat{G}$ amounts to finding a minimum cost route from s_0 to d_i, for each i.
A consequence is that Dijkstra's algorithm can be used for finding a minimum cost route in G^D. The algorithm terminates once d_i, for some i, shifts from set $N \setminus S$ to S.

3.3 Search Direction

During a search in a dynamic graph it is important to know the time at which the car is at a certain node. In this section it will be explained what kind of consequences this observation has on the search direction.

One of the main consequences is that backward planning is no longer possible. Like was said, the time at which the car is at a certain node n is important to know. With backward planning this is not known, since the path from s to n is not known ($s \in N$ the starting point of the search). If backward planning is used by an algorithm, such an algorithm can find a good minimum route just to find out that the car should already have departed.
Therefore, given a starting time, forward planning should be used.

Sometimes, a search is conducted with an other objective. It may be that a desired arrival time at the destination is given. The objective can be to find the latest departure time at the starting point for which the car will arrive at the destination before the desired arrival time.
In this case, backward planning should be used. Forward planning can be used, only a recursive

algorithm would be needed. Let a arbitrary starting time be given. If the resulting minimum cost route arrives in time, the starting time is increased. If not, the starting time is decreased. After some subsequential modifications of the starting time, the latest departure time for which the car arrives in time will be known.

Note that, opposed to *latest departure time* with backward planning, the objective of the search for which backward planning can not be used (also only with a recursive algorithm) can be denoted with *earliest arrival time*.

Chapter 4

Stochastic Dynamic Route Planning

In the previous two chapters the (dynamic) minimum cost path problems have been discussed. This chapter contains a final extension. The stochastic dynamic minimum cost path problem will be discussed. The stochastic dynamic cost path problem is concerned with finding a minimum cost path in a road network in which the costs of the roads do not only vary with time, but are also stochastic. This is the situation which represents finding routes in a real road network with traffic congestions.
The first section describes the mathematical model. The second section discusses the (lack of) algorithms to solve the stochastic minimum cost path problem.

4.1 Mathematical Model

The stochastic dynamic minimum cost path problem is concerned with finding a minimum cost path in a graph in which the costs of edges vary with time and are stochastic. Such a graph will be denoted by a *stochastic dynamic graph.*

Recall that congestions cause the cost of the edges of a dynamic graph to depend on time. Since the development of a congestion is not always known, the state of the congestion at the time of arrival can only be estimated. The dynamic part of the cost of the congested edge is a stochastic variable.
Also the time needed to travel a certain congested edge is stochastic (also depends on the state of the congestion). This leads to the following definition.

Definition 4.1. *Define $\{C_a(t), t \in \mathbb{R}^+\}$ and $\{T_a(t), t \in \mathbb{R}^+\}$ as two stochastic processes of the dynamic cost of edge $a \in A$ and the travel time of edge $a \in A$ respectively. For each $t \in \mathbb{R}^+$, $C_a(t)$ ($T_a(t)$) is considered a continuous random variable with its* PDF *(Probability Distribution Function) denoted by $f_{C_a}(C_a, t)$ ($f_{T_a}(T_a, t)$).*

This definition of the stochastic processes of the cost and travel time of edges leads to a stochastic dynamic graph that can be denoted by $\tilde{G} = (N, A, \tilde{c}, \tilde{T}, p)$, with $\tilde{c}$ and $\tilde{T}$ containing the two stochastic processes for the cost and the time respectively.

Now that the shape of a stochastic dynamic graph is defined, what remains is stating the problem of finding a stochastic dynamic minimum cost path in that graph. However, what is considered a stochastic dynamic minimum cost path is not easily determined. There are more than one possible objectives. For example, an objective can be to find the path that is expected to be of minimum cost. Another objective can be to minimize the variance in the arrival time. This is particular interesting for a person that wants to reach the destination on time rather than in time. Yet another objective can be to find an optimal route according to the *worst case scenario*, in which the cost of the road segments are set according the most negative case.
In this research the emphasis will be on the expected minimum cost path.

Note that when a expected minimum cost path is found, it will be denoted by a simple path, containing only the sequential nodes. It will not be a time-path, since the actual travel time is stochastic. One can speak of a expected time-path.

Given a starting point $s \in N$ and a destination $d \in N$, the problem of finding an stochastic dynamic optimal route from s to d amounts to finding a expected minimum cost path $p(s,d)$ from s to d in $\hat{G}$. This problem is denoted by *SDEMCP* (*Stochastic Dynamic Expected Minimum Cost Path*).

Definition 4.2 (SDEMCP). *Given a stochastic dynamic graph* $\tilde{G} = (N, A, \tilde{c}, \tilde{T}, p)$*, starting node* s *and destination* d*. Find an expected minimum cost path* $p(s,d) \in \Pi(s,d)$*.*

4.2 Search Algorithms

That computing the probability distribution function of the arrival time at the destination is not a trivial problem, even for small networks will be shown in an example. This example is based on an example given in [5]. Suppose that the route from start node s to destination d is found via only one other node i. Suppose that the travel time from s to i is normally distributed ($T_{si}(t) \sim N(\mu, \sigma) \forall t \in \mathbb{R}^+$) and the travel time from i to d is only time dependent ($T_{id}(t) = 5 + \frac{1}{2}(t-5)^2$). Even with the travel time from s to d being a rather simple function of a normally distributed variable, its probability distribution function is not easily computed.
Since the cost of a path is dependent of the travel time of that path, it is clear that computing the probability distribution function of the cost of a path will become intractable quickly.

The problem of finding an optimal path in dynamic and stochastic graph has been addressed in several articles. For example, Fu and Rilett ([5]) studied "... the problem of finding the expected shortest path in a traffic network where the dynamic and stochastic nature of link travel times is modelled explicitly ..." and tried to "... develop an algorithm which can provide improved solutions without significantly adding to the overall computation time." Another example is [12], in which Polychronopoulos and Tsitsiklis assume that "... information on arc cost values is accumulated as the graph is being traversed."

In none of these articles an efficient algorithm is given to solve the minimum path problem in case of a stochastic dynamic graph. Furthermore, due to the processor time constraints it is not possible to use excessive calculations before finding a route. Therefore, in this research the stochastic dynamic graph will be reduced to a solely dynamic graph by setting the stochastic variables to a static values. The best values that can be chosen is the expectation. It is assumed that the minimum dynamic cost path in the resulting graph is also on average the expected minimum cost path of the stochastic dynamic graph. Since this is not obviously true, in Chapter 6 a comparison is made between the expected cost and the average cost of the routes found.

Chapter 5

Traffic Congestions: a Model

In the dynamic minimum cost path problem as well as in the stochastic dynamic minimum cost path problem congestions play an important role. In this chapter a description will be given of those congestions and will be shown how they can be modelled in such a way that they can be inserted in a stochastic dynamic graph.
In the first section will be shown what is usually known about a congestion and will be explained why the state of the congestion at the time of arrival is a stochastic variable. In the second section a mathematical model of a congestion will be given.

5.1 Traffic Information

A essential part of (stochastic) dynamic route planning is traffic information. This section is concerned with the kinds of traffic information (Section 5.1.1) and with the problems with the provision of that traffic information (Section 5.1.2).

5.1.1 Kinds of Traffic Information

There are several providers of traffic information. Each of these providers has its own opinion about which information is important to the driver. A consequence is that each provider gives a different kind of information about congestions. Of course, all the providers give information about the location of the congestion. But that information is about all that the information of the providers have in common.
There are providers which suffice with the length of the congestion (apart from the location). Sometimes the actual speed at the congestion is given. A combination of the length and the actual speed is the delay that will be experienced by the driver. This can be calculated if the *free flow speed* (speed at the congested road in the situation without congestions) is known. There

are providers that provide the delay of rather than the speed at the congestion.
So far, only the current status of the congestion is considered. Some providers try to predict the development of a congestion.
They provide information about whether the congestion just started or whether it is almost cleared. Some providers even give an estimation of the future development of the congestion. These estimations can be based on information about the road which is congested or on information about the type of congestion (a congestion can for example be caused by an accident or a congestion can appear when the traffic on a road exceeds its capacity).

5.1.2 Imperfect Traffic Information

The assumption that traffic information is perfect is not realistic. One of the main problems with the provision of traffic information is caused by the fact that messages about the state of a congestion are only sent at a certain resolution (for example, every three minutes). Therefore, it can happen that a congestion is reported not earlier than a few minutes after its start. Another problem is that most of the time the report about a congestion is done by a human, so inherently errors can occur. Also, the observations are not done on a continuous scale, so traffic information is rounded. Last but not least, not every report of a congestion contains all the information you want to know about the congestion, like was stated in Section 5.1.1.

In summary, this results in the following reasons why traffic information is not perfect:

- Traffic information is not 100% timely.
- There is uncertainty about the traffic information.
- Traffic information is incomplete.

5.2 General Model of Congestions

To be able to insert a congestion into a stochastic dynamic graph, a mathematical model of the development of the congestion should be described.
For this purpose, this section describes a general model.

The description that will be given is a simplification of reality. For example, the length of a congestion will be assumed to be constant during the time the congestion is on its top. In reality, the length will fluctuate around its assumed maximum length.

In general, the course of a congestion is always the same. At first, the road is not congested. The starting time of the congestion will be denoted by t_s. Once the congestion starts, the length

of the congestion increases with a constant rate (Δ^+ m/s) during an interval with length $\delta_{s,ts}$. The time at which the congestion reaches its maximum length will be denoted by t_{ts}. Note that $t_{ts} = t_s + \delta_{s,ts}$. The maximum length of a congestion will be denoted by l (the unit of this length is kilometers).
After the congestion has reached its maximum length, it will keep this length for a certain period of time. The length of the period in which the congestion has its maximal length is equal to $\delta_{ts,te}$. At time $t_{te} = t_{ts} + \delta_{ts,te}$ the length of the congestion starts to decrease. The decrease will have a constant rate of Δ^- m/s. After $\delta_{te,e}$ seconds, at time t_e ($= t_{te} + \delta_{te,e}$), the congestion will have disappeared totally.
A car arriving after time t_e will not experience a delay anymore.
Summarizing, a congestion can be split up in three schematic phases:

I. The first phase is the phase in which the length of the congestion increases. During $\delta_{s,ts}$ seconds, from t_s until t_{ts}, the congestion increases with a constant rate of Δ^+ meters per seconde. At time t_{ts}, the length of the congestion will be equal to l kilometer.

II. During the second phase, the length of the congestion stays constant. The length of this phase is $\delta_{ts,te}$ seconds, from t_{ts} until t_{te}.

III. During the third phase, the length of the congestion decreases. This takes $\delta_{te,e}$ seconds, from t_{te} until t_e. The length of the congestion decreases with a constant rate of Δ^- meters per second.

A schematic overview of the course of a congestion is displayed in Figure 5.1.

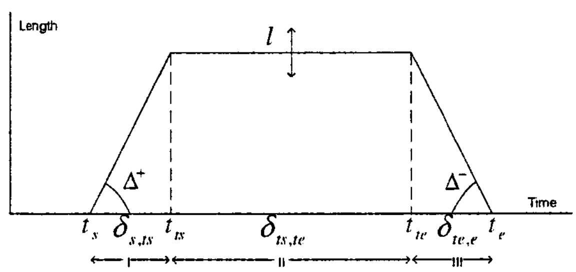

Figure 5.1: Schematic Overview

This simplification of the real course of a congestion uses a number of important assumptions:

- The length of the congestion stays constant during the period of time in which the congestion is at its maximum.
  Like stated before, in reality the length of the congestion will fluctuate.

- The length of the congestion increases with a constant rate. In reality, it can happen that the length increases x kilometer in the first 10 minutes and y kilometers in the next 10 minutes (with $x < y$). Of course, the opposite can also happen.

- The length of the congestion decreases with a constant rate. This assumption uses the same reasoning as the previous one.

Looking at the mathematical model of the congestion, it is clear that the following five stochastic variables are important. Each of these variables has its own distribution function:

T_s
: The starting time of the congestion is independent of the rest of the stochastic variables. A realization from the distribution of this stochastic variable will be denoted by t_s.

L
: The maximum length of the congestion can be dependent of $\Delta_{s,ts}$ and Δ^+, but it can also be the variable on which the variable $\Delta_{s,ts}$ depends. The same goes for the variables $\Delta_{te,e}$ and Δ^-. These variables can also determine L. Therefore, $\Delta_{s,ts}$, $\Delta_{te,e}$, Δ^+ and Δ^- are not pairwise independent. A realization of distribution of L will be denoted by l.

$\Delta_{s,ts}$
: Like said, the length of the period in which the length of the congestion increases can be one of the factors on which the maximum length of the congestion depends. A realization of the distribution of $\Delta_{s,ts}$ will be denoted by $\delta_{s,ts}$.

$\Delta_{ts,te}$
: The length of the period in which the congestion is assumed to have its maximum length is independent to the rest of the stochastic variables. A realization will be denoted by $\delta_{ts,te}$.

$\Delta_{te,e}$
: The length of the period in which the length of the congestion decreases will be denoted by $\delta_{te,e}$.

Besides these five stochastic variables, there are two more factors on which the course of a congestion can depend.

Δ^+
: The speed with which the length of the congestion increases can be one of the factors determining either L or $\Delta_{s,ts}$. If both L and $\Delta_{s,ts}$ are not determined by the value of Δ^+, then Δ^+ is determined by the values of L and $\Delta_{s,ts}$. In that case, it is not necessary to use Δ^+ in the model.

Δ^-

The speed with which the length of the congestion decreases can be one of the factors determining either L or $\Delta_{te,e}$. If both L and $\Delta_{te,e}$ are not determined by the value of Δ^-, then Δ^- is determined by the values of L and $\Delta_{te,e}$. In that case, it is not necessary to use Δ^- in the model.

Chapter 6

Traffic Congestions: Expectation vs. Reality

In the previous chapter a mathematical model of a congestion is given. As was discussed in the previous chapter, the course of a congestion depends on several stochastic variables. In the first section will be described in which way this stochastic information will be used during planning. The second section contains a theoretical discussion of the consequences of the way the stochastic information will be used during planning. This discussion emphasizes on whether (the cost of) the route found is realistic. In the third section this comparison of what is the expected cost of the route found and what is the real distribution of this cost will be verified by means of a simulation.

6.1 Using a Stochastic Model During Route Planning

In the previous chapter a mathematical model of a congestion is given. What remains is to define a way to use this model during planning. The current route planner can not cope with stochastic costs. Therefore, the distribution of the dynamic cost of a road segment should be transformed to one value (which can change over time). This value has to be an estimation of the real dynamic cost at a certain road segment at a certain time. The maximum likelihood estimator for the dynamic cost of a road segment is the mean of the underlying distribution. This estimator will be used during planning.

When a road segment a is evaluated during planning, the current time t is compared with the course of the average congestion. The corresponding length of the congestion will be used to determine the dynamic cost at the road segment. In more mathematical words:

$$c^v(a,t) = \mathbb{E}[c(a,t)] - c^s(a) \tag{6.1}$$

with $c^v(a,t)$ the dynamic cost, $c^s(a)$ the static cost and $\mathbb{E}[c(a,t)]$ the expectation of the total cost of the road segment.

After a route is found, the expected cost will be defined as the cost of the route based on an average congestion.

6.2 Theoretical Consequences

The expected cost of a route is based on the assumption that a congestion will develop as expected. However, the chance that the congestion will develop just as expected can be rather small. Take for example a road segment on which in 50% of the time the travel time will be 1 minute, and in 50% of the time the travel time will be 3 minutes. Then, on the average, the travel time will be 2 minutes, but it is obvious that an actual travel time of 2 minutes will never occur.

Since the cost of the route based on the expected congestion is not (always) correct, one can not use the model straight away. It has to be verified that it is meaningful to use the model. This section continues with a theoretical view on the severity of possible errors. The practical verification of the model will be discussed further in the next section.

There are two different kinds of possible problems. The actual congestion can be the average congestion shifted horizontally or vertically. A horizontally shifted congestion means that the congestion starts earlier or later than expected. A vertically shifted congestion means that the maximum length of the congestion is bigger or smaller than expected.
Of course, a combination of a horizontally and vertically shifted congestion is also possible.

In the next subsection the horizontal shift and its consequences will be discussed. After that, the vertical shift will be discussed.

6.2.1 Horizontal Shift

A horizontally shifted congestion occurs when the congestion starts earlier or later than expected or when the congestion is on its top longer or shorter than expected. If the starting time of the congestion changes, all the other phases (see Section 5.2) shift accordingly. If the length of the

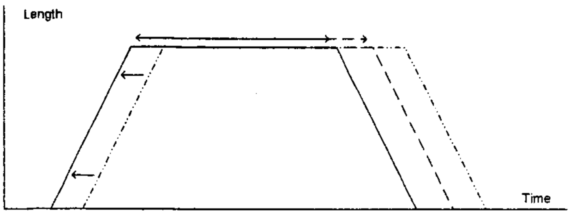

Figure 6.1: Horizontal Shift

period the congestion has its maximum length changes, this effects the starting time for the phase in which the congestion decreases. For an example, see Figure 6.1.

A horizontally shifted congestion can cause the car to arrive at a congested road segment during a unexpected phase of the congestion. For example, if a car is supposed to arrive during the time the length of the congestion is increasing, a horizontal shift can cause the car to arrive after the congestion has reached its maximum length. Most of the time a horizontal shift will have no consequences on the average cost of the congested road segment. That is because most of the time a horizontal shift will not cause the car to arrive in a different phase of the congestion. For example, suppose that the car arrives at road segment a at 6 am. and there is a congestion expected to start at 7 am.. It makes no difference for the car if the congestion starts 15 minutes earlier or later. The car will always pass long before the road segment gets congested.

A closer inspection learns that a horizontal shift is only cause for extra cost if the car is expected to arrive near the beginning or the end of the course of the congestion. The average extra cost does not depend on whether the car arrives when the congestion just started or when the congestion is just about to start.
Other problem areas are near the beginning and the end of the period in which the congestion has its maximum length. These problem areas do not cause extra average cost. As a matter of fact, the average cost will be lower than expected.

The fact that some problem areas are and some problem areas are not cause for extra average cost can be seen by looking at the shape of the graph representing the course of a congestion.
The first two problem areas (where the average cost will exceed the expectation) lie in parts of the graph which are convex (the derivative of the graph does not decrease). Therefore, a part of the graph lies above the straight line (through the point of interest) that would give an average cost equal to the expectation.

In the same way the fact that the graph is *concave* (never increasing derivative) at the other two problem areas is reason for the average costs to be lower than expected.

6.2.2 Vertical Shift

A vertical shift occurs when the realized maximum length of the congestion is not equal to the expected maximum length.
The consequence is that the expected graph will be multiplied with a certain scalar. For example, if the congestion is expected to have a maximum length of 5 kilometer and it turns out to have a maximum length of 7 (3) kilometer, then for every point in time, the value of the graph will be multiplied with $\frac{7}{5}$ ($\frac{3}{5}$). This multiplication principle is shown in Figure 6.2.

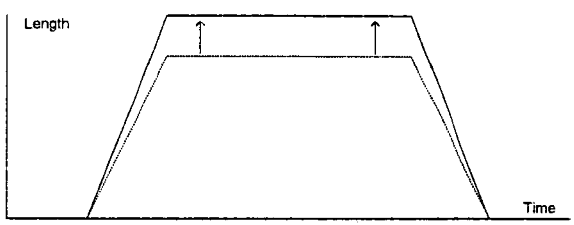

Figure 6.2: Vertical Shift

Average costs which are not equal to the expectation can not be caused by vertical shifts. This follows from the fact that at every point in time the graph of the possible values is a straight line.

However, in the above reasoning the situation with a vertical shift is not accurate for the model that will be used in this research. That reasoning implicitly assumes that the length of the period in which the congestion increases is constant. This is not realistic. It would imply that for extreme large (short) congestions the length of the congestion would increase with an extreme high (low) rate.
Our model assumes that the rates at which the length of the congestion increases and decreases are constant. The consequence of this assumption is that if the length of the congestion is not equal to the expectation, not only that maximum length will differ from the expected congestion, but also the points in time at which the phase of the congestion changes. For example, given a congestion with a expected maximum length of 5 kilometer which should start at 7*am* and reach its maximum length at 7.05*am*. If the maximum length of the congestion turns out to be 7 kilometer, that length will be reached at 7.07*am*. The start time of all the other phases will also shift 2 minutes. As a consequence the length of the period in which the congestion increases/ decreases will vary as the length of the congestion differs from the expectation.
Thus, a vertical shift is in matter of fact also a horizontal shift. This principle is shown in Figure 6.3.

A vertically shifted congestion is cause for two horizontal shifts. First, the length of the period in which the congestion increases changes, which difference continues throughout the rest of

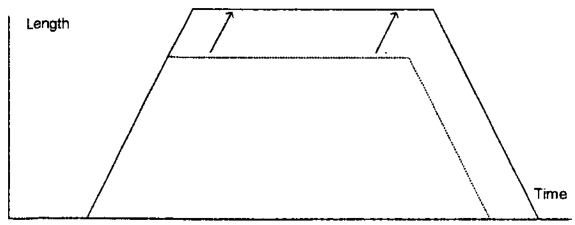

Figure 6.3: Vertical Shift With Constant Increase/Decrease Rate

the course of the congestion. And second, also the length of the period in which the length of the congestion decreases changes. Therefore, the point in time at which the congestion ends is influenced twice by the change in the maximum length of the congestion.
Note that we assume that the length of the period in which the congestion has its maximum length is constant. Another model is a model in which the length of the time window in which the congestion exists is assumed constant. In this case, a vertical shift causes the length of the period in which the congestion has its maximum length to change.

The conclusion is that we expect that if the car is expected to arrive at the congested road segment at the time the congestion either starts or ends, the average cost will be higher than the cost in the situation with the expected congestion. If the car arrives at the time the congestion is expected to either reach or leave its maximum length, the average cost will be lower that the cost in the situation with the expected congestion.
The variance is expected to increase throughout the course of the congestion. Especially there will be a dividing line between either the first part of the congestion (low variance) and the second part (high variance).

6.3 Practical Verification

In the previous section, the theoretical consequences of the fact that a congestion does not always develop as expected are discussed. In this section, these consequences will be verified by means of a simulation.
Throughout the simulations done in this research, 10 different pairs of start and end points in Germany will be used. In Table 6.1, for all those pairs information about the routes that will be found by the route planner in case there are no congestions is listed. This information includes the distance (meters), time (minutes) and cost of the routes. More detailed information about these pairs of start and end points will be given in appendix B.

For the verification in this section, we need information about routes with congestions. However, for the pairs 2, 3 and 9 it is not possible to simulate a route with a congestion (at its full strength). If on those routes a congestion is inserted, the route planner calculates a route avoiding the in-

Route	Distance (m)	Time (min)	Cost
1	311187	161.2	157422
2	310242	160.0	157906
3	281463	157.2	153804
4	282788	154.8	153874
5	360019	188.9	183180
6	352582	186.0	180551
7	328896	179.4	175089
8	321868	177.8	173178
9	334710	173.5	169383
10	330576	171.9	167104

Table 6.1: Information About Routes Without Congestion

Route	t_{d_1}	t_{d_2}	t_{d_3}	t_{d_4}
1	0,13	23,98	24,98	0,00
4	0,33	16,15	15,82	0,00
5	0,05	23,77	25,10	0,00
6	0,00	24,98	24,25	0,00
7	0,00	19,07	25,23	0,00
8	0,00	10,02	11,03	0,13
10	0,03	15,70	17,78	0,00

Table 6.2: Extra Time of Routes With Expected Congestion (minutes)

serted congestion. Therefore, these routes will be omitted in the remainder of this verification.

For the verification and throughout the rest of this research, each route is planned with four different departure times for the car. The first departure time (t_{d_1}) is chosen in a way the car arrives around the time the congestion is expected to start. The second departure time (t_{d_2}) causes an arrival around the time the congestion is expected to reach its maximum length. The third departure time (t_{d_3}) is connected with an arrival around the time the length of the congestion starts to decrease again and the fourth departure time (t_{d_4}) causes the car to arrive around the time the congestion ends.

To be able to compare the results of the simulation with our expectations, in Table 6.2 we list the *dynamic time* (extra time, caused by a congestion) of each of the routes and each of the departure times (with minutes as unit of time).

In our simulation, we used 100 realizations of the congestions (drawn from the according distributions) for each pair of start and end point. In Table 6.3 the results of the simulations are summarized. For each combination of route and departure time, the average and the standard

Route	t_{d_1}		t_{d_2}		t_{d_3}		t_{d_4}	
	Av.	St.dev.	Av.	St.dev.	Av.	St.dev.	Av.	St.dev.
1	3,88	4,89	21,29	5,74	21,06	9,10	6,78	9,48
4	4,03	5,11	13,03	4,13	11,94	6,02	4,78	6,18
5	3,93	5,04	21,49	4,89	20,79	7,34	5,78	7,99
6	3,84	4,96	21,81	5,40	20,79	9,27	6,45	9,28
7	2,50	3,02	18,20	6,67	21,42	8,80	6,83	9,51
8	3,33	4,17	7,68	3,85	7,58	4,58	3,93	4,55
10	3,19	4,05	13,49	4,82	13,49	6,22	5,03	6,54

Table 6.3: Average Extra Time of Routes With Congestion (minutes)

deviation of the results are given (again the unit of time is minutes).

Looking at the results of the simulation, we see that these results are as we had argued in Section 6.2. If the car is expected to arrive at the congested road segment around the time the congestion is either about to start or end, the average extra dynamic cost is higher than expected. On the other side, if the car is expected to arrive around the edge of the period in which the congestion is expected to have its maximum length, the average extra dynamic cost is lower than expected. Also, the variance increases with the course of the congestion.
The fact that the values for t_{d_2} tend to be higher than for t_{d_3} (as can be seen in Table 6.3) is a consequence of *double chains* (for an explanation of this term, its cause and consequences see appendix A). Double chains cause the car to arrive later at the congested road segment than expected. So, if the car is expected to arrive at the time the length of the congestion is about to decrease, double chains cause the car to arrive when the length of the congestion is already decreasing.
On the whole the results are as expected. However, there is one pair of start and end points for which the results differ from what was expected. For pair 7, the average extra dynamic cost for departure time t_{d_2} is fairly lower than for departure time t_{d_3}. The deviating results can be explained by the fact that the departure times for this pair are wrongly chosen. The car should depart later than it does. A consequence is that departure time t_{d_2} does not lead to a car arriving around the time the congestion is about to reach its maximum length, but a car arriving while the length of the congestion is still increasing. Furthermore, t_{d_3} is small enough to nullify the effect of double chains.

In the simulation as described above, the car is always expected to arrive at a time the length of the congestion is changing or is about to change. It would also be interesting to see what would be the difference between the expected extra dynamic cost and the average extra dynamic cost in case the length of the congestion is at its maximum. This would give an indication of the consequence of a vertical shift solely. Horizontal shifts would not infect the extra dynamic cost. Unfortunately, data for this comparison is only available for the pairs 1 and 7. The data is listed in Table 6.4.

Route	Exp.	Av.	St.dev.
1	25,23	24,57	6,00
7	25,23	25,58	4,72

Table 6.4: Extra Time When Car Arrives in Middle of Congestion (minutes)

From these results it can be seen that in these cases indeed the average extra dynamic cost is almost as expected.
However, the variance gives some other insight. It was expected that the variance would increase with the course of the congestion. This is not true for pair 7. It could be that it is another consequence of the wrongly chosen departure times for pair 7. Another explanation could be that indeed only the effect of a vertical shift is present, since the edges of the period in which the congestion has its maximum length are too far away to notice a horizontal shift.

From the results of Table 6.4 an interesting calculation can be made to clarify some of the results in Table 6.3.
Since on average the congestion needs 25 minutes to reach its maximum, and on average the maximum length of the congestion is cause for a delay of 25 minutes (in case the length of the congestion is normally distributed with $\mu = 5$ and $\sigma = 1$), it can be concluded that the delay increases 1 minute per minute. With this observation we can explain that if the car is expected to arrive at the congestion when that congestion reaches its maximum length, a standard deviation of 8 minutes lead to an average delay of 21. Without loss of generality, assume that the maximum length of the congestion is always 5 km.. Since 50% of the time the congestion reaches its maximum length earlier than expected, 50% of the time the delay is 25 minutes. For also 50% of the time, the congestion start later than expected. Since the standard deviation is 8 minutes, the average of the delays experienced when the congestion starts later will also be 8 minutes less than expected and will thus be 17 minutes (recall that the length of the congestion increases with 1 minute per minute). Thus, for 50% of the time, the delay is (on average) 25 minutes and for the other 50% of the time, the delay is (on average) 17 minutes. Therefore, the total average is $(17+25)/2 = 21$ minutes.

From the verification can be concluded that although the model used in this research is not entirely accurate, the errors made are only minor. The model is good enough to be used during the research. Due to lack of time, improvements can not be made.

Chapter 7

Planning: Ways to Factor in Congestions

One of the objectives of my research is to find an efficient way to factor in congestions during planning. The first section of this chapter contains a list of several different strategies, *i.e. ways to factor in congestions during planning. In the second section a strategy is chosen to be compared with the current strategy.*

7.1 List of Possible Strategies

Apart from the method used by the current route planner, there are many other ways to factor in congestions during planning, so called strategies. Next, a list is made of the different strategies with their advantages and disadvantages.
But first, the method used by the current route planner will be explained.
In the current route planner the way congestions are factored in is based on the airline distance between the car and the edges infected by the congestion. The expected dynamic cost of a congested road segment a is only factored in if the airline distance between the car and a is less than or equal to 150 km. The airline distance between the car and a will be denoted by $d(CCP, a)$, with CCP the car. This means that the dynamic cost will be factored in if $d(CCP, a) \leq 150$ km and will not be factored if $d(CCP, a) > 150$ km. Note that this strategy can be generalized by using a general boundary L instead of the current 150 km.

Let the function $I : a \rightarrow [0 \ldots 1]$ denote the influence of a road segment a, with a being congested by a certain congestion $j \in J$, with J the set of all congestions. The static cost of the road segment a infected by a congestion j will be incremented with the expected dynamic cost of the congestion at a times the value of $I(j)$.

With the current strategy, the following influence function is used:

$$I(j) = \begin{cases} 1 & d(CPP, j) \leq 150; \\ 0 & d(CCP, j) > 150. \end{cases} \tag{7.1}$$

+ This strategy is simple. The one thing that has to be calculated is the distance between the car and the congested road segments.

+ A large amount of all the congestions will fall outside the circle with diameter 150 km, and can therefore be ignored. So, no time is wasted to evaluate congestions which are expected to be of no importance.

– A disadvantage of the strategy is the fixed boundary distance. A congestion 149 km from the car will be factored in, but a congestion at 151 km will not.

– During planning, no distinction is made between a congestion nearby and a congestion far away, as long as both congestions are not more than 150 km away. For example, a congestion at 149 km is considered equally important as a congestion at 2 km.

One of the disadvantages (see above) is that no distinction is made between a congestion nearby and a congestion far away. The expected dynamic costs of both congestions will be factored in equally during planning, i.e. the congestions both have the same *influence*. The influence of a congestion is the extent in which the dynamic cost of that congestion will be added to the static cost of the road segment during planning.
The next list contains strategies that are based on the assumption that the influence of a congestion should decrease gradually as the distance between the car and the congestion increases. This assumption is based on the fact that it will take the driver a longer time to reach the congested road segment. As the time needed to reach the congestion increases, the state of the congestion at the time of arrival will become less certain.

1. The first new strategy uses an influence function that decreases linearly as the distance increases. Again, there is a sort of a boundary. However, this boundary does not cause a 'jump' in the influence of the congestions. The decrease of influence goes gradually. From a certain boundary distance congestions are ignored. Next an example to clarify the principle.

   Consider a boundary distance of L km. Then, the influence of a congested road segment a can be defined as follows:

$$I(a) = \begin{cases} 1 - \frac{d(CCP,a)}{L} & d(CPP, a) \leq L; \\ 0 & d(CCP, a) > L. \end{cases} \tag{7.2}$$

   + Congestions near the car have more influence than congestions far away.

+ There is no hard boundary for which the influence of a congestion makes a 'jump'.

2. The second new strategy does not have a boundary distance. The influence of a congestion decreases with distance, and goes to 0 as the distance goes to infinity. For example, the following influence function I can be used:

$$I(a) = \begin{cases} 1 & d(CCP,a) \leq \alpha; \\ \frac{\alpha}{d(CCP,a)} & d(CPP,a) > \alpha. \end{cases} \tag{7.3}$$

   The parameter α indicates how fast the influence of a congestion decreases. The influence of a congestion less than α km from the car will be equal to 1.

   + Congestions near the car have more influence than congestions far away.
   + No congestion will be ignored totally. Note that this can also be considered a disadvantage.

   The difference between this strategy and the previous strategy is that with this strategy, in contrast with the previous strategy, the influence does not decrease in a linear way.

3. The previous two strategies are based on the assumption that the influence of a congestion should decrease as the distance between the car and the congestion increases. This assumption does not hold for every congestion. Take for example a blockade. No matter how far away the blockade is, there is no uncertainty about the congestion at the time of arrival. The line of reasoning that the congestion might be disappeared before arrival does not apply here.
   That is why the third new strategy uses an influence function that depends on the variance of the dynamic cost at the time of arrival. This means that the influence of a congestion decreases as the uncertainty of the state of the congestion at the time of arrival increases. Note that the dynamic cost of a congested road segment a at the time of arrival t is a stochastic variable with a certain expectation $\mathbb{E}(c(a,t))$ and a variance $\text{Var}(c(a,t))$ which depend on the course of the congestion. Note that the expected time of arrival is known once the congested edge is expanded.
   The influence function I can have the following form:

$$I(a) = \begin{cases} 1 & \text{Var}(c(a)) \leq \beta; \\ \frac{\beta}{\text{Var}(c(a))} & \text{Var}(c(a)) > \beta. \end{cases} \tag{7.4}$$

   Like in the previous strategy the parameter α, in this strategy the parameter β can be used to determine the speed with which the influence decreases as the variance increases.

   + The less certain the cost of a congestion gets, the less influence the congestion will have during planning.
   – Sometimes it may be hard to determine the variance of the cost of a congested road segment.

– A consequence of an influence function based on variance is that a congestion nearby is not taken into account if the variance is high. Take for example a bridge. If the bridge is open for traffic there will be no delay. If the arrival is at a time the bridge is closed for traffic, most often a great delay is experienced. Because of the great variance the bridge will be ignored during planning. It is not that obvious that this is desirable.

4. With an influence function based on the variance a boundary for that variance can be chosen, the same way as with an influence function based on distance. The idea is that if the state of the congestion at the time of arrival is too uncertain, the congestion will be ignored all together rather than taking it into account only a little as is the case in the previous strategy.

   The fourth strategy combines this boundary with a linear decrease of the influence as the variance increases.
   The above reasoning leads to the following influence function (in which the boundary is set at V):

$$I(a) = \begin{cases} 1 - \frac{\text{Var}(c(a))}{V} & \text{Var}(c(a)) \leq V; \\ 0 & \text{Var}(c(a)) > V. \end{cases} \tag{7.5}$$

   ± This strategy has the same advantages and disadvantages as the previous strategy.

Note that both an influence function based on distance as an influence function based on variance has its advantages. It might be possible to combine the advantages through an influence function which is based on both distance and variance.
Without going further into the matter it can be said that there are several combinations of distance and variance possible for an influence function. For example, the influence can run down a in linear way or it can approach 0 for infinity. The distance and variance can be summed, but can also be multiplied.

+ Both the distance and the variance are taken into account. The result is a possible realistic determination of the influence of a congestion.

– The advantage mentioned above can also be considered a disadvantage. Because both distance and variance are taken into account, the algorithm may become complex.

7.2 Choosing An Alternative Strategy

An objective of this research is to find an good strategy for taking congestions far away into account. Therefore, one of the strategies mentioned in the previous section will be chosen. The chosen strategy will be compared with the current strategy.

As was said, the choice will be made from the strategies 1 through 4 from the previous section.

One of the most important objectives of the comparison will be getting an idea of what is and what is not important for a *good* strategy with respect to answering the question whether or not a certain congestion should be taken into account during planning. Therefore, the alternative strategy that will be chosen should be a simple one. The strategy can not be to complicated. However, all the alternative strategies are reasonably simple. Note that this is also a reason why a strategy with a influence function based on a combination of distance and variance is not considered.

An assumption is made that the variance of the cost of a road segment is dependent with the distance from the car to the congestion (the further the distance, the bigger the variance). This assumption is based on the fact that if the distance between the car and the congestion increases, so does the time needed to get to the congestion. The longer the time needed to get to the congestion, the more uncertain the state of the congestion at the time of arrival. And therefore, the bigger the variance will be. Of course, their are exceptions, for example a blockade. But those are considered exceptions that prove the rule.
A consequence of this assumption is that the variance of the cost of a congestion is also taken into account by strategies based on sorely distance. Therefore, such a strategy based on distance will be chosen. The strategies based on variance, strategies 3 and 4 will be removed as possible alternative strategy. Note that another reason for choosing either strategy 1 or 2 is that the variance of the cost of a congestion is not always known, opposed to the distance, which can easily be calculated.

What remains is a choice between strategy 1 and 2. It is decided to a non-linear decrease of the influence function, and thus strategy 2.
This decision is based on the fact that with a linear decrease of the influence, the difference of the influence of two congestion is based only on the distance between each other and not on their respective distances to the car. For example, let four congestions be given at distances 1, 31, 71 and 101 km from the car. With a linear decrease, the difference of the influences of the first two congestions is equal to the difference of the influences of the last two congestions. This will probably not be realistic.

The current strategy will be compared with the strategy that uses an influence function which decreases (non linear) with distance. The influence function I of a congested road segment a has the following form:

$$I(a) = \begin{cases} 1 & d(CCP,a) \leq \alpha; \\ \frac{\alpha}{d(CCP,a)} & d(CPP,a) > \alpha. \end{cases}$$

The value of α will be varied in the comparison of the strategies.

Chapter 8

Planning: Testing the Strategies

This chapter describes the testing of the strategy chosen in Chapter 7. First there will be a discussion of the properties of a 'good' strategy in Section 8.1. That section also contains information about errors that can be made by the route planner. In Section 8.2 the purpose of the simulations is explained. Section 8.3 describes the framework in which the simulations are done. After that, the test design is given in Section 8.4. Which settings of the parameters will be used in the simulations is explained in Section 8.5. The results of the simulations are discussed in Section 8.6. Finally, the conclusions are given in Section 8.7.

8.1 Properties Good Strategy

In this section first a description is given of some of the properties which can indicate a 'good' route. After that follows a discussion about the several situations that can occur (expected versus average length of a congestion) and the possible errors in planning.

Ask ten different people about their conception of what the properties of a 'good' route should be and they will probably provide you with ten different answers. Next is described some of the properties a 'good' route can have. We are considering the situation with congestions. Therefore, the actual optimal route is not known in advance.

- Optimal route.
  The best property a dynamic route can have, is being the best possible route. The best possible (optimal) route can only be calculated afterwards, since the course of congestions is in general not known in advance. Trying to use the optimal route thus requires that all traffic information for the future is known.

- Near-optimal route.
  Sometimes the driver does not want to wait until the optimal route is found. It is even

possible that the dynamic information makes it impossible to find the optimal route. In such a case, the driver might be more interested in a near-optimal route which can be found quickly. Such a route should have costs lower than or equal to $(100+a)\%$ times the costs of the optimal route, with a a certain (small) number.

- Little uncertainty.
  Because the dynamic behaviour of the traffic is not known in advance, there will always be some uncertainty about the costs of the determined route. A driver may prefer to minimize this uncertainty over minimizing the costs of the route. He may be more interested in a route A with 42 ± 2 minutes needed than in a route B with 40 ± 10 minutes needed, even though the expected time needed is the smallest for the second route. However rarely, a driver might even prefer a route with little uncertainty even when the 'worst scenario' for the optimal route still leads to a route with less costs that the 'best scenario' for the preferred route. Such situations can occur if for the driver it is more important to arrive *on* time rather than *in* time.

- Worst case.
  Last but not least, a driver could be interested in the route which is optimal from a negative point of view. With this is meant a route assumes everything *goes wrong*. If at a certain road segment the possible travel time lies between 2 and 5 minutes, during the search the travel time is set at 5 minutes. Note that with this objective also route A with 42 ± 2 minutes needed is preferred over route B with 40 ± 10 minutes needed (as was the case in the example in the previous item).

A route must be found by the route planner based on the traffic information provided. Since it was already shown that this information is not flawless (see Section 5.1.2), the severity of some possible errors will be discussed.
Assume that for certain CCP and DES there are two routes possible. Route 1 has cost f_1 and route 2 has cost f_2, with $f_1 < f_2$. Assume that on route 1, there might be a congestion. The cost of the congestion f_c can range from 0 (no congestion) till f_{max}, with $f_1 + f_{max} > f_2$. Thus, if the cost of the congestion is larger than a certain threshold, route 2 should be preferred over route 1. There are several situations that can happen during the planning of the routes, which is based on the expected course of the congestion. Next, we list these situations and evaluate whether the correct route is given by the route planner. In the listing, f_{exp} is the expected cost of the dynamic route. Also, the *actual congestion* is the congestion at the time the car arrives at the congestion. The *expected congestion* is the estimation of this *actual congestion* done during planning. Afterwards the optimal route can be determined based on the actual course of the congestion. The cost of this route will be denoted by f_{opt}.

- The actual congestion equals the expected congestion. In this situation $f_{exp} = f_{opt}$, so the correct route is found.

- The actual congestion is smaller than expected, but the congestion is nevertheless large enough for the optimal route to avoid it. So, both the dynamic route planner and the optimal route planner give route 2 as the currently best route. So, again $f_{exp} = f_{opt}$.
- The actual congestion is smaller than the expected congestion. The congestion is small enough to be ignored by the optimal route planner, but based on the expected congestion, the dynamic route planner advises to avoid it. Hence, the dynamic route planner advises route 2, while the optimal route is route 1. So, $f_{exp} > f_{opt}$.
- The actual congestion is larger than expected, but the expected congestion was nevertheless large enough for the dynamic route planner to avoid it. So, both the dynamic route planner as well as the optimal route planner gives route 2. Hence, $f_{exp} = f_{opt}$.
- The actual congestion is larger than expected, but it is still small enough for the optimal route to be through it. Hence, both the dynamic route planner as well as the optimal route gives route 1. So, $f_{exp} = f_{opt}$.
- The actual congestion is larger than expected, and the optimal route is around it, thus route 2. This in contrast with the dynamic route, which is planned through the congestion. So, $f_{exp} > f_{opt}$.

Summarizing, one can see that there are two situations in which the dynamic route planner fails to find the optimal route. One is if the congestion is smaller than expected and the dynamic route planner wrongly advises the route avoiding the congestion. The other is if the congestion is larger than expected and the dynamic route planner wrongly advises the route through the congestion. We will define these two situations as type I and type II errors respectively:

Definition 8.1. *The situations in which the dynamic route planner wrongly advises a route avoiding the congestion is called a type I error.*

Definition 8.2. *The situation in which the dynamic route planner wrongly advises a route through the congestion is called a type II error.*

If in the above described situation error type *I* occurs, the maximum *superfluous cost*, i.e. the cost that would be omitted if the other route would have been chosen, equals the difference between the costs of route 2 and the costs of route 1 without congestion. Thus, the maximum superfluous cost is $f_2 - f_1$. Note that in this case we have made the assumption that there is no congestion on route 2.

If in the above described situation error type *II* occurs, the maximum superfluous cost will be equal to $f_1 + f_c - f_2$, with f_c the cost of the congestion. Since theoretically the cost of the congestion can reach infinity (for example in case of a road block), the superfluous cost has no upper bound.

Since the superfluous cost has no upper bound in case of a type *II* error, the safe choice to make is to always take route 2. In that case, the maximum superfluous cost is known. The driver can not be struck by unexpected high cost.
However, as the difference between f_1 and f_2 grows, choosing route 1 no matter the congestion grows more attractive. This choice is based on two observations. First, the chance of large superfluous cost when route 2 is chosen increases. And second, the chance of (large) superfluous cost when route 1 is chosen decreases.

A good strategy is a strategy which makes good use of the difference of f_1 and f_2.

8.2 Purpose of the Simulations

This section explains the purpose of the simulations.

The purpose of the simulations is to gain some insight in the effect of a certain way to take congestions in the road network into account. We are especially interested in the effect of congestions far away. The question is how to balance between the risk that the route will be planned through a congestion on the one side and the risk that an unnecessary detour will be planned on the other side. An unnecessary detour can be planned in case during planning a congestion is taken into account which turns out to be smaller than expected. The congestion can even be gone by the time the car arrives at the possibly congested road segment.
The insight is attempted to be gained by multiple plannings of a number of routes and comparing the results. During planning, each time a different strategy to take congestions into account is used.
One of the ways to compare the strategies is to test the cost of the routes planned by each of the strategies against the cost of the optimal route. The optimal route is determined afterwards, when the real course of the congestion is known.

8.3 Framework of the Simulations

This section describes the framework in which the simulations are done. When we talk about planning a route, it is done in this framework.

We next list the important differences between the current route planner and the route planner which is used for the simulations.

- Prototype.
  The prototype route planner will be used in the simulations instead of the current route

planner. The difference is that the prototype does not have all the functionality of the current route planner. The advantage of the prototype over the current route planner is that it is more easily modified because of the simple structure.

- HL1 top level.
  As explained in Section 2.3, the route planner uses different road maps with each its own level of detail. In general the congestions all lie in the part of the map in which a route is searched on the *HL*0 level, the map with the least detail. This could cause a problem since we are interested in small local detours (a detour is local if the maximum distance between the route with the congestion and the detour is small). But these detours are most often via minor roads, and those roads are not 'visible' on the *HL*0 level. Therefore, the choice is made to do the simulations omitting the *HL*0 levels. Thus, the *HL*1 level is the least detailed level.
  The fact that with *HL*1 as least detailed level most of the 'good' local alternatives will be considered during planning strengthens the choice not to use *HL*2 as the least detailed map. Furthermore, local alternatives not found on the *HL*1 level will be via even more minor roads. A route via those roads is not preferable since the cost of a road increases as the road class increases (for more information about road classes, see Section 2.3). Last but not least, the more detailed the least detailed road map, the longer it takes the route planner to plan a route.

- Forward search.
  Another difference between the current route planner and the prototype is that with the prototype forward search can be applied. This means that a route will be planned from the car towards the destination. In the current route planner backwards planning (from destination to car) is applied to avoid the car catching problem. For more information, again see Section 2.3.
  In the prototype forward search is preferred because during the search it is important to know the time at which the car arrives at a certain road segment.
  During simulation fixed departure times are taken, as will be explained in Section 8.5. In practice the current time can not be taken as departure time in the search. After all, planning a route takes time, and thus the route planned is based on a departure time in the past. Note that the error will not be large since the state of the road network does not change fast and in general the planning of the route does not take long.

Note that more information about the differences between the route planner of VDO and the one used in this research can be found in Section 2.3.

8.4 Test Design

This section describes what kind of routes will be planned in this simulation.

8.4.1 Static Routes

First of all we are interested in information about the static routes. These are routes planned in the situation in which there are no congestions. This information is important to be able to compare the results in case of a congestion with the results in case of no congestions.
Since the costs are denoted with the somewhat vague g value, the results contain information about the travel time and the distance of the routes.
These costs will be determined by planning the routes without congestions.

8.4.2 Routes Based on Expected Course of Congestion

Once the costs of the static routes are known, the next focus is on the information about the routes planned based on the expected course of the congestions. This means that all parameters of the congestion model have their expected values. Recall that all parameters are stochastic variables (see Section 5.2). The specific means and variances of the distribution of these stochastic variables will be discussed in Section 8.5.

The *expected* cost of the planned route is not always equal to the *planned* cost (g value which is used during planning). This is caused by the fact that during planning an influence function (see Section 7.1) is used to multiply the dynamic cost, this in contrast with the expected cost.
The *simulated* cost, cost based on realizations of the distributions of the parameters of the congestion model, is again different from both the expected cost and the planned cost. By comparing the expected costs with several simulated costs it will be tested whether the expected costs are realistic. A criterium is the average difference and the dispersion of the difference.

The following tests have been carried out:

- For all possible settings of the parameters as will be discussed in Section 8.5, routes will be planned based on the expected course of the congestions. This has been done for each of the strategies.
- For all possible settings, simulated costs of the planned route will be determined for 100 realizations of the distribution of the course of the congestions. We assume that for infinity, the simulated costs will converge to the actual costs of the planned routes.

8.4.3 Distribution Optimal Route

Ultimately, we are interested in the distribution of the optimal route in case the actual course of the congestions is exactly known. This is important to be able to check whether the routes planned by the several strategies (based on expected course of the congestions) matches the optimal route.

8.5 Parameters

In this section is explained which settings of the parameters will be used. The important parameters are the departure times of the car, the parameters of the distribution of the congestion and the parameter of the strategies.

- Departure times of the car.
  The moment at which the car departs will be varied. At first, the departure times where selected based on two assumptions. First there is an assumption about the course of the congestions. All the congestions were assumed to start at 07u00, and last for approximately two hours. The other assumption is that it takes the car two hours to reach the congestion.
  Based on these assumptions, departure times were chosen to be able to simulate four situations. For this cause, the departure times TO_1, TO_2, TO_3 and TO_4, are taken as 0, 60, 120 and 180 respectively. Note that these departure times are all according to the origin of time, which is assumed to be at $04u00$ am. and these values have minutes as unit of time. From the first departure time it follows that the car drives through the possible congested road segment long before the congestion starts. With TO_2, the car is supposed to arrive at the congestion the moment it starts. The third departure time causes the car to arrive during the period in which the congestion has its maximum length. And finally, TO_4 causes the car to arrive at the moment the congestion vanishes.

  However, during the research it became apparent that the assumptions made are not accurate. For most of the routes, the car needs far less than two hours to reach the location of the congestion. The other assumption is not accurate either. This has another reason. The way the congestions are stored by the route planner causes the lengths to have a certain maximum.

  Because the assumptions were not accurate the departure times had to be changed.
  The departure times have been chosen to simulate certain situations. The first departure time (t_{d_1}) ensures the car to arrive at the congestion the moment it is expected to start. The second departure time (t_{d_2}) causes the car to arrive at the moment the congestion is expected to reach its maximum length. Departing at the third departure time (t_{d_3}), the car

Route	t_{d_1}	t_{d_2}	t_{d_3}	t_{d_4}
1	47	72	132	157
2	85	110	170	195
3	72	83	143	154
4	95	111	171	187
5	74	99	159	184
6	97	123	183	208
7	72	97	157	183
8	83	94	154	165
9	89	114	174	199
10	70	88	148	166

Table 8.1: New Departure Times Used in Simulation

arrives at the congestion when its length is expected to start decreasing. The fourth and last departure time (t_{d_4}) is chosen so that the car arrives at the congestion the moment it is expected to end.
These departure times are different for each pair of start and end coordinates. In Table 8.1 the departure times are listed. Note that the values in Table 8.1 have minutes as unit. This is important because the start times of the congestion is set at $07u00$ am., as will be shown when we talk about the parameters of the congestions.

- Parameters of the congestions.
  In the future different parameter settings might be used during simulations. However, in this simulation each congestion will use one setting. The parameter settings are first of all based on databases used in [13]. Most of the congestions have a setting based on [13]. Some congestions have an adapted setting, because of a restriction on the maximum length of the congestions.
  We next list the settings of the important parameters of the congestions.

  - Start time of the congestion.
    The point in time at which a congestion starts is independent of the rest of the course of the congestion. The start time of a congestion (T_s) is a stochastic variable with follows a normal distribution. To be more precise, $T_s \sim N(\mu_{T_s}, \sigma^2_{T_s})$. For each congestion, $\mu_{T_s} = 180$ and $\sigma^2_{T_s} = 56\frac{1}{4}$. Both values are in minutes. This means that on the average a congestion starts at $07u00$ am. (180 minutes is 3 hours). A requirement is that 95% of the congestions start in the interval $[06u45, 07u15]$. From the normal distribution is known that 95% of the distribution lies in the interval $[\mu - 2\sigma, \mu + 2\sigma]$. Hence, the choice for $\sigma = 7\frac{1}{2}$ and thus $\sigma^2 = 56\frac{1}{4}$.
    Note that there is a theoretical chance that the start time of the congestion will be negative. But since 99% of the distribution lies above $\mu - 3\sigma = 157\frac{1}{2}$, in practice a negative value will not occur. This chance of a negative value do apply also to the

Route	μ_L (km)	σ_L (km)
1	5	1
2	5	1
3	2.2	0.05
4	3.2	0.1
5	5	0.4
6	5	1
7	5	1
8	2.2	0.05
9	5	0.4
10	3.6	0.1

Table 8.2: Distribution Length Congestions

other parameters. But the settings for those parameters are taken also such that that chance is also only theoretical.

- Maximum length of the congestion.
  According to the databases from [13] the maximum length of the congestion lies between 3 and 7 km.. Therefore, for most of the congestion applies $L \sim N(5,1)$. For some of the routes, the congestion has a maximum length smaller than 7 km., see Appendix B.3. Therefore, for these routes other parameters are chosen.
  Table 8.2 shows for each of the routes the μ_L and σ_L of the corresponding congestion. These values are chosen in such a way that the maximum length according to Section B.3 equals $\mu + 3\sigma$ (for example, for route 3 the maximum length of the congestion is $2.2 + 3 \times 0.05 = 2.35$ km.). Note that 99% of the normal distribution lies beneath $\mu + 3\sigma$ (and 98% lies within $\mu \pm 3\sigma$). To be sure that the length of the congestion does not exceed its maximum, the distribution is cut off at $\mu \pm 3\sigma$.
- Increase period of the congestion.
  The time needed by the congestion to reach its maximum length depends on this maximum length and is set on an average of 25 minutes. This value is based on the assumption that an average congestion with a maximum length of 5 km lasts for approximately 2 hours (in this case 110 minutes).
  With an average length of the congestion of 5 km., it follows that the increase rate is 0.2 km per minute. From this 0.2 km/min it follows that the time needed for the congestion to reach its maximum length lies between 15 minutes (in case the length is 3 km.) and 35 minutes (in case the length is 7 km.). To be sure that 95% of the realizations lies between 10 and 40 minutes, $\sigma_{\delta_{s,ts}}$ is set to be equal to $2\frac{1}{2}$ (and thus $\sigma^2 = 6\frac{1}{4}$).
  Concluding we can say that $\Delta_{s,ts} \sim N(\frac{l}{0.2}, 6\frac{1}{4})$, with l the realization of the maximum length of the congestion.
- Period in which the congestion has its maximum length.

The length of the interval in which the congestion has its maximum length is assumed to be independent of the rest of the course of the congestion. This time lies in general between 40 and 80 minutes. Hence, $\Delta_{ts,te} \sim N(60,100)$.

- Decrease period of the congestion.
  For the length of the time in which the length of the congestion decreases the same reasoning as with the length of the time in which it increases is valid. Hence, these distributions are equally distributed. Thus, $\Delta_{te,e} \sim N(\frac{l}{0.2}, 2\frac{1}{2})$.

- Parameters of the strategies.
  For the current strategy as well as the alternative strategy the values of the parameters has to be set.
  It is decided that the parameters of the current strategy will be set to equal the parameters the current route planner uses and not to vary with the boundary distance. This means that 150 km. will be used as boundary distance.
  The influence function used if the current strategy is applied will be as follows (with a the road segment under consideration):

$$I(a) = \begin{cases} 1 & d(CCP, a) \leq 150; \\ 0 & d(CPP, a) > 150. \end{cases} \tag{8.1}$$

The parameter α of the alternative strategy will have four different values. These four values will be indicated with α_1, α_2, α_3 and α_4 and will have the values 25, 50, 100 and 250 respectively. Thus, $\alpha_1 = 25$, $\alpha_2 = 50$, $\alpha_3 = 100$ and $\alpha_4 = 250$.
Recall that the influence function has the following form (with a again the road segment under consideration):

$$I(a) = \begin{cases} 1 & d(CCP, a) \leq \alpha; \\ \frac{\alpha}{d(CCP,a)} & d(CPP, a) > \alpha. \end{cases} \tag{8.2}$$

To be able to plan routes taking all congestions fully into account, and thus using no strategy, an influence function which is always equal to 1 is defined. Note that this can be used to plan optimal routes.

For convenience, the strategies will be numbered. The numbering is given in Table 8.3. Throughout the remainder of this report, this numbering will be used, unless stated otherwise.

Strategy	Number
all congestions into full account	1
current strat.	2
altern. strat., $\alpha = 25$	3
altern. strat., $\alpha = 50$	4
altern. strat., $\alpha = 100$	5
altern. strat., $\alpha = 250$	6

Table 8.3: Numbering the Strategies

Route	Time
1	161.2
2	160.0
3	157.2
4	154.8
5	188.9
6	186.0
7	179.4
8	177.8
9	173.5
10	171.9

Table 8.4: Static Routes

8.6 Test Results

This section contains the results of the simulations.
Recall from Section 8.5 that at first the departure times were taken wrongly. But since the tests were already done, these results are also present in this section. Some of the results are rather trivial, since according to the departure time, the car has no or little chance to have any trouble caused by the congestion. The car has passed the road segment either long before the congestion starts or only after the congestion ends.
These trivial results will be shaded.

First, we recall the information about the static routes. This information was already given in Section 6.3, but is repeated here because it is important for the comparison. Not the entire table is repeated. We are interested in the time needed for a certain route, and therefore the information about the cost and the distance is omitted.

Now that we know the information about the static routes, it is important to know which route will be planned by the strategy. Important is the question whether the strategy plans a route via the location of the congestion or not.

Strategy\ Departure	0	60	120	180	t_{d_1}	t_{d_2}	t_{d_3}	t_{d_4}
1	1.0	0.9	0.0	0.7	1.0	0.0	0.0	1.0
2	1.0	1.0	0.3	0.8	1.0	0.3	0.3	1.0
3	1.0	1.0	0.7	0.9	1.0	0.7	0.7	1.0
4	1.0	1.0	0.6	0.8	1.0	0.6	0.6	1.0
5	1.0	1.0	0.0	0.7	1.0	0.0	0.0	1.0
6	1.0	0.9	0.0	0.7	1.0	0.0	0.0	1.0

Table 8.5: Fractions of Routes Planned Through Congestion by Strategies

Since we have six strategies, eight different departure times and ten different routes, the full results require a table with 480 entries. Therefore, it is chosen to split this table in three parts and place these parts in the appendix D. Here, a summary will be given in Table 8.5. For each of the strategies and each of the departure times, the percentage is given of the routes planned by the strategy which are planned through the congestion. The value is between 0 and 1. A value of 1 means that for all ten pairs of start and end coordinates, the combination of strategy and departure time leads to a route through the location of the congestion (a value of 0 means the opposite). Recall the numbering of the strategies from Table 8.3 in Section 8.5.
Note that Table 8.5 does not give a conclusion whether the decision made by the strategy is correct.

In Table 8.6 for each of the pairs of start and end coordinates the average cost and the variance is given of the route through the congestion and the route avoiding the congestion. The route through the congestion is denoted with an A and the detour is denoted with a B.
The values in Table 8.6 are based on the routes planned in the simulation. For some combinations of departure time and start and end coordinates all the strategies planned the same route (either A or B). Therefore, some of the entries of Table 8.6 are blank.
Note that the entries in the table are all *extra* costs. The costs of the static route are subtracted from the total costs. Furthermore, note that the values are given in seconds rather than in minutes.

Table 8.7 contains information for each combination of departure time, strategy and pair of start and end coordinates about whether the route through the congestion is planned or a detour is planned. An A stands for a route through the congestion and a B for a detour (like in Table 8.6). Because the total table would be too large, trivial parts are omitted. For example, the values for departure time 0 are all zero. Also, for all combinations of departure times and pairs of start and end coordinates, strategy 1 and strategy 6 have the same results. Therefore, strategy 6 is omitted. Furthermore, in case a certain row in the table is precisely equal to the previous row, this row will be omitted. For example, the row for the first pair of start and end coordinates and strategy 3 is omitted since the results are exactly the same as for the same pair of start and end coordinates and strategy 2.

As last result we have Table 8.8, which contains the percentages of the optimal routes which goes through the congestion. However, the table only contains information for departure times 0, 60,

	Old departure times			
Route	0	60	120	180
1,*A*	0 (0)	777 (428)	1474 (359)	48 (186)
1,*B*		472 (0)	473 (6)	
2,*A*	0 (0)	0 (0)		939 (645)
2,*B*			233 (0)	233 (0)
3,*A*	0 (0)			3 (34)
3,*B*		13 (48)	87 (0)	
4,*A*	0 (0)	0 (0)	947 (96)	470 (391)
4,*B*			503 (0)	
5,*A*	0 (0)	4 (24)	1527 (118)	478 (532)
5,*B*			681 (0)	
6,*A*	0 (0)	0 (0)	1209 (346)	1323 (514)
6,*B*			711 (0)	711 (0)
7,*A*	0 (0)	20 (85)	1535 (283)	502 (614)
7,*B*			595 (0)	
8,*A*	0 (0)	0 (0)	664 (14)	56 (148)
8,*B*			263 (0)	
9,*A*	0 (0)	0 (0)		
9,*B*			45 (0)	45 (0)
10,*A*	0 (0)	27 (71)	1070 (29)	69 (183)
10,*B*			148 (0)	
	New departure times			
Route	t_{d_1}	t_{d_2}	t_{d_3}	t_{d_4}
1,*A*	233 (293)	1277 (344)	1264 (545)	407 (568)
1,*B*		472 (0)	473	
2,*A*	234 (300)			413 (572)
2,*B*		233 (0)	233 (0)	
3,*A*	211 (256)			226 (270)
3,*B*		87 (0)	87 (0)	
4,*A*	242 (306)	782 (248)	716 (361)	287 (370)
4,*B*		503 (0)	503 (0)	
5,*A*	236 (302)	1289 (293)	1247 (440)	347 (479)
5,*B*		681 (0)	681 (0)	
6,*A*	230 (297)	1308 (323)	1247 (556)	387 (556)
6,*B*		711 (0)	711 (0)	
7,*A*	150 (181)	1092 (400)	1285 (527)	410 (570)
7,*B*		595 (0)	595 (0)	
8,*A*	200 (250)	461 (231)	455 (275)	236 (272)
8,*B*		263 (0)	263 (0)	
9,*A*	225 (293)			358 (486)
9,*B*		45 (0)	45 (0)	
10,*A*	192 (243)	809 (289)	810 (373)	302 (392)
10,*B*		148 (0)	148 (0)	

Table 8.6: Cost (Variance) of Route Through Congestion and of Detour Route

	Old departure times			New departure times			
Route, strat.	60	120	180	t_{d_1}	t_{d_2}	t_{d_3}	t_{d_4}
1,1	B	B	A	A	B	B	A
1,2	A	A	A	A	A	A	A
1,5	A	A	A	A	B	B	A
2,1	A	B	B	A	B	B	A
2,3	A	B	A	A	B	B	A
2,4	A	B	B	A	B	B	A
3,1	A	B	A	A	B	B	A
4,1	A	B	A	A	B	B	A
4,3	A	A	A	A	A	A	A
4,5	A	B	A	A	B	B	A
5,1	A	B	A	A	B	B	A
5,2	A	A	A	A	A	A	A
5,5	A	B	A	A	B	B	A
6,1	A	B	B	A	B	B	A
6,3	A	A	A	A	A	A	A
6,5	A	B	B	A	B	B	A
7,1	A	B	A	A	B	B	A
7,3	A	A	A	A	A	A	A
7,5	A	B	A	A	B	B	A
8,1	A	B	A	A	B	B	A
8,3	A	A	A	A	A	A	A
8,5	A	B	A	A	B	B	A
9,1	A	B	B	A	B	B	A
10,1	A	B	A	A	B	B	A
10,2	A	A	A	A	A	A	A
10,4	A	B	A	A	B	B	A

Table 8.7: Route Planned Through Congestion or Detour

Route\ Dep.times	0	60	120	180
1	1,00	0,31	0,01	0,97
2	1,00	1,00	0,00	0,20
3	1,00	0,94	0,00	0,99
4	1,00	1,00	0,01	0,51
5	1,00	1,00	0,00	0,67
6	1,00	1,00	0,06	0,14
7	1,00	1,00	0,00	0,66
8	1,00	1,00	0,00	0,90
9	1,00	1,00	0,00	0,08
10	1,00	0,93	0,00	0,85

Table 8.8: Fractions of Optimal Routes Through Congestion

120 and 180. For the new departure times, these simulations are not done.

8.7 Conclusions

This section contains the conclusions that can be drawn from the results shown in the previous section.

The first conclusion is that departure time 0 has obvious results. The car crosses the possible congested road segments long before the congestion starts. All strategies plan the static route. Another conclusion is that in all cases the strategy in which all congestions are considered gives the same results as the alternative strategy with $\alpha = 250$. This is also trivial since all congestions lie within 250 kilometers from the car. Therefore, strategy 6 also considers all congestions.

If we focus on the pairs of start and end coordinates, we see that for pair 3 and pair 9 an alternative route is available with only a slightly higher cost than the optimal route. This follows from the small extra cost in case an alternative route is planned.
For such routes, a good strategy should prefer the alternative route over taking the risk of a large congestion (recall the upper bound for a type I error as described in Section 8.1). And indeed, all strategies plan the alternative route in case the expected length of the congestion is too large.

The chance of a type II error should be minimized. This is done by only taking the route through the congestion if the extra cost of the alternative route is high. However, this is where several strategies fail. As can be seen in Table 8.7 strategies 3 and 4 plan through the congestion even for departure times t_{d_2} and t_{d_3}. This results in high average costs, much higher than for the alternative route. In the cases in which the distance between the car and the congestion is more than 150 kilometer, strategy 2 also fails. In those cases, the congestion is neglected by strategy 2.

Overall, we conclude that strategy 5 (alternative strategy with $\alpha = 100$) finds the best balance between the risk of a error and planning time. Strategy 1 and 6 give the same results as strategy 5. But we prefer strategy 5 since the planning time is shorter (since fewer congestions will be considered).
Furthermore, strategy 3 and 4 seem useless, but could prove their strengths in case monitoring is applied.

Chapter 9

Monitoring: Why, When and How

In this chapter the concept of monitoring will be discussed. In Section 9.1, the reasons for replanning will be discussed. Explained is why monitoring is needed. After that, in Section 9.2 the possible intervals between consecutive replannings will be discussed. The last section contains information about the ways in which the replanning can be done.

9.1 Reasons for Replanning

While guiding the driver along the currently best route, new (or updated) traffic information will become available. This new information may change the nature of the best route. The navigation system will have to be able to notice the suboptimality of the currently presented route, and will have to be able to give the driver the option to replan the route. The driver is asked whether the route has to be replanned, since the driver might prefer not to switch to a new route even if the current route is suboptimal. Sometimes it is necessary to replan, whether the driver wants it or not. For example, if the road ahead is blocked completely.
This section is concerned with the monitoring part of the objective. Probable causes for route suboptimality will be discussed.

9.1.1 Causes of suboptimality

Once old traffic information is replaced with new traffic information, in general, there are four possible kinds of changes in the traffic situation. These four kinds of changes correspond with the following list:

1. Traffic congestion on current best route dissolved/decreased;

2. Traffic congestion on current best route arose/increased;
3. Traffic congestion off current best route dissolved/decreased;
4. Traffic congestion off current best route arose/increased.

Note that these changes are based on the expected course of the congestion. For example, a congestion might be expected to have a length of 3 kilometer at the time the car would reach it. Now information could be provided that the congestion is increasing faster than expected and that the new expected course of the congestion resolves in a length of 4 kilometer at the time the car arrives. This can be a type 2 or type 4 change (see list above).

For monitoring, only (2) and (3) are interesting. These changes could lead to the current route to become suboptimal. Take the above example in which the expected length of the congestion increases from 3 to 4 kilometer. If the current best route is through the congestion, the increased expected length could make an alternative route preferable.
The other two kind of changes cause the current route not to get worse compared to other routes. Type (1) causes the current route to become better. An other route may also become better, but will never improve more than the current route. In situation (4) the current route stays the same, but (an) other route(s) become worse. So, the current route will stay the best possible route.
Like was said, in case of a type (2) or (3) change, the current route becomes worse compared to other routes, so the current route might become suboptimal.
An interesting role is played by traffic congestions far away. Since we have to cut off the horizon, traffic congestions far away become more and more important while the car drives towards it. A case in which this is important is the current strategy. As long as the distance between the car and the congestion is more than 150 kilometer, the congestion will be ignored. But once the congestion gets into the circle with a radius of 150 kilometer around the car, the congestion will be taken into full account.

9.1.2 Possible Benefits

This section contains a description of the possible benefits of monitoring. Two possible benefits will be discussed.

More Certainty

Suppose a route is replanned when the car has travelled 20 kilometer along the route found during the previous search and suppose that this replanning is triggered by new information on the course of a congestion. Since the distance from the car to the congestion has decreased, the course of the congestion has less time to change unexpectedly. Furthermore, when only the current length of the congestion is provided, it is difficult to model an expected course of the congestion. It is not known whether the congestion is increasing, decreasing or staying constant. When new information is provided, this information can be used to increase the accuracy of the model of the expected course of the congestion. For example, it can be seen whether the congestion is increasing, decreasing or staying constant.

More Detail

When a replanning is done after a certain time or distance since the previous planning, the distance between the car and the congestion has been decreased. Since the distance has decreased, it is possible that the congestion moved into an other search level. For example, the congestion could move from $HL0$ level to $HL1$ level. Such a transaction causes the route planner to search more (minor) roads near the congestion. More roads near the congestion will be visible. Therefore, there is more chance that a local detour will be found.

9.2 Possible Intervals between Consecutive Replannings

This section contains a description of the possible intervals between consecutive replannings. In the previous section is explained why monitoring is done. Now we know that monitoring is needed, it has to be investigated when this monitoring should take place. Several intervals are possible. These intervals are described in the following list.

- Time.
  One of the possible intervals between consecutive replannings is *time*. For example, each ten minutes the route will be replanned. An advantage of *time* as interval is that at any time it is known when the next replanning will be done. The question is what the time interval should be. There should be found a balance between the number of replannings that will be done and the risk of driving along a suboptimal route too long. A big interval corresponds with few replannings, but with a great chance of driving along an suboptimal route. Take for instance an interval of 10 minutes. It might happen that new information about a congestion will be provided 1 minute after the last replanning. This means that the car drives along a possible suboptimal route for 9 minutes. A short interval corresponds with many replannings, but with a small chance of driving along an suboptimal route.

- Distance.
  An other possible interval is based on distance. This corresponds for example with a replanning each x kilometer. Since distance and time are strongly related (through speed), taking distance as interval has the same advantages and disadvantages as taking time as interval.

- Important Junctions.
  A third possible interval is based on the important junctions along the route. The most important junction is the junction at which the optimal route and the *first alternative* (the route which would be optimal if the optimal route would be blocked) starts to differ. In general, an important junction is a junction which separates the optimal route with a global or local alternative.

- New Information.
  The last possible interval is based on the traffic information provided. New information can be reason for suboptimality, and therefore reason for a replanning. A problem with new information as trigger for a replanning is that it is never clear when the next replanning will be done.

A note on an additional disadvantage of using a short interval, which goes for all the possible intervals. The disadvantage is the possibility that too many switches are made. There is even a (very) small chance to put the car into a loop which can go on for a long time. This happens when the alternative route leads back to the departure point of the car. Once the car is back on the original departure point, a replanning is done in which possibly the original route is planned. This loop will end once the course of the congestion is changed so much that either an other route will be planned all together or the original route will not get suboptimal anymore.
Of course, too many switches is confusing for the driver as well. Especially when the direction is inverted totally and the car has to go back to where it came from. The final argument for preferring driving along a suboptimal route over too many replannings is the fact that a replanning is too time and processor consuming.

9.3 Replan Methods

Once is decided that a replanning is needed, it has to be decided how this replanning will be done. One of the questions is whether we replan the whole route or only a section around the newly appeared congestion.
The answer to this question should be based on the cause of the suboptimality of the route. If the congestion on route has increased, it could be enough to search for a local alternative.
If a congestion off route has decreased, it can also be sufficient to research only a small part. To be more precise, the road network near the decreased congestion has to be searched for a possible shorter route via the formerly congested roads.

Whether it is possible to research only a part of the road network, depends on the information kept from the former search. If only the optimal route is kept, the whole route should be replanned. Replanning only a part of the route incorporates a risk that an optimal route is neglected.
If additional information is stored from the previous search, it might not be needed to replan the

whole route. If for example besides the optimal route also the first alternative route with its cost is stored, in case a congestion on the optimal route appears, it suffices to compare the new cost of that route with the stored cost of the first alternative.

Chapter 10

Monitoring: Testing the Strategies

This chapter will contain a discussion of the simulations done to test the strategies when monitoring is applied. In the first section, Section 10.1, the properties of a good strategy in case of monitoring are described. After that, in Section 10.2, the purpose of the simulations is explained. The framework used in the simulations is the same as with the simulations without monitoring, see Section 8.3. Therefore, a section about this framework is omitted. In Section 10.3, the test design is given. Like in Chapter 8, in Section 10.4 is explained which settings of the parameters will be used in the simulations. In Section 10.5, first the results of the simulations done are discussed and second, the conclusions that can be drawn from the simulation will be discussed.

10.1 Properties Good Strategy

In this section a description is given of what properties a 'good' strategy should have in case monitoring is applied.

First of all it is important to note that when monitoring is applied, the criteria for a good strategy are different from the criteria when a planning is done only once. For example, when monitoring is applied, the decision to avoid the congestion can be postponed.
Postponing the decision to take a detour (if possible) can be a good property of a strategy. It decreases the possibility of a type II error. Recall that a type II error occurs when the route planner wrongly advises to avoid the route with the congestion.
Another advantage of postponing the decision to avoid the congestion is that when the distance between the car and the congestion decreases, the possibility that a local detour will be found increases. This possibility increases since the congestion might move into a more detailed transition box, and therefore more roads near the congestion are considered. Recall that during the first planning most of the congestions are outside the outer most transition box and therefore near the congestion only roads of road class 0 (the major roads, for example highways) are considered

during the search.
However, postponing the decision to avoid the congestion does not only have good properties. A disadvantage is that if the decision to avoid the congestion is postponed too long, a good detour may not be available anymore.

We conclude with the statement that a good strategy should find the perfect balance between postponing the decision to avoid the congestion and the risk of missing a possible good detour.

10.2 Purpose of the Simulations

In this section is explained what the purpose of the simulations is.

Like stated in Section 8.2, the purpose of the simulations is to gain some insight in the effect of a certain way to take congestions in the road network into account.
With monitoring, we are especially interested in the balance between the risk of missing a good detour which is currently visible and the chance of finding a better detour which becomes visible as the car approaches it.
The insight is attempted to be gained by multiple plannings of a number of routes and comparing the results. For planning, each time a different strategy is used to take congestions into account.

10.3 Test Design

In this section is explained which tests are done and what the settings of the parameters were. Also, some important issues are addressed.

The static routes are not planned with monitoring since at the time the simulations were done, we assumed that the same routes will be found as in the case of planning only once. However, this might not be correct since with monitoring short cuts that were not visible the first planning will probably be found. Note that most of the static routes can be deduced from the test results by looking at the routes in which the expected cost of the congestion is small enough for the strategy to plan the same route as the static route planner.

One of the important differences between our simulations and reality is that we assume that an expected course of the congestion is provided which does not change in time. In our simulations new information about the course of that congestion will not be provided. Hence, all the possible modifications of the route planned by the route planner will be based on newly visible roads.
Note that the increase in the certainty about the state of the congestion at the arrival of the car is implicitly used by the influence function.

Another important issue that needs to be addressed is the kind of interval that will be used to decide when the replannings will be done.
The choice is made to use *distance* as interval. *New information* is not provided and thus can not be used to trigger a replanning. If *important junctions* would be used as interval, those important junctions would have to be identified after the first planning (and after each replanning). However, if the distance until the next important junction is large, it is possible that the junction which separates the current optimal route from the optimal route determined afterwards will be unnoticed because the junction might be on a too detailed level of the road network. Therefore, *important junctions* have not been chosen as interval. This leaves us with the choice between *time* and *distance*. Since time and distance are heavily related (through *speed*), the choice for any of the two is rather arbitrary.

For the choice which distance would be used as interval between two consecutive replannings, a balance has to be found between the consumption of processor time and the risk of missing a possible detour. Ideally, the route planner would be replanning continuously, checking the optimality of the current driven route again and again. However, this would consume too much processor time. From the point of view of the processor time planning would be done ideally only once and monitoring would not be applied.
The choice is made to use an interval of 25 kilometer. Hence, each time the car has travelled (another) 25 kilometer along the route currently considered optimal a replanning is done. A smaller interval is too processor time consuming. Furthermore, the changes in the road network between two consecutive replannings would be too small. The chance that the roads near the congestion move from one road level towards another level would be too small. An interval of 25 kilometer is chosen as smallest interval for which the situation changes significantly. A larger interval is not chosen since with an interval of 25 kilometer processor time is not an issue and above it is argued that the interval should be as small as possible.

As will be explained in appendix A and was mentioned in Section 6.3, during collecting information from a certain route errors are made. Important in this section is the problem of double chains. Double chains stands for counting some road segments twice when a switch is made from a certain level of detail to a less detailed level.
The problem of double chains is important when monitoring is applied, since after every replanning the route is started at the most detailed level. Hence, after every replanning double chains will occur. During the simulations a correction is made to try to solve the problem of double chains.
Note that *gaps* (fail to count certain road segments, see also appendix A) only occur once, near the destination.

Because we assume that no new information is provided, we assume that once a route avoiding the congestion is planned by a strategy, no more monitoring is needed. Theoretically, it is possible that the congestion moves into a more detailed search level. However, we will prove that this is not likely. There are two kinds of detours, namely global detours and local detours. In

case of a global detour, the distance between the congestion and the route is mostly great enough to cause the congestion to be outside the outermost transition box. In case of a local detour, the distance between the congestion and the detour is mostly small enough to cause area near the congestion to be searched on the most detailed level. However, also the local detour will be on the most detailed level, and therefore the detour is the best detailed detour as well.
Hence, in both cases it is not likely that the congestion moves into a more detailed search level after the decision to avoid the congestion is made.

In the first place monitoring is applied because the dynamic nature of the road network. Therefore, monitoring is no longer interesting once the remaining part of the route is static. This is not only the case when a route is planned avoiding the congestion, but is also valid once the car has past the congestion on the current route. Therefore, once the congestion is passed by the car, monitoring is no longer applied.

For all pairs of start and end coordinates combined with all departure times and strategies, routes are planned and monitored. Due to lack of time these routes are planned 10 times, and not 100 times as was done in the previous simulations. At first, it was believed that only the increase in the influence of the congestions were cause of planning a route different from the one planned at the previous monitoring point. Since using strategy 1 amounts to full influence of the congestion from the start, monitoring would never lead to an other route planned during monitoring if the above reasoning was true. Therefore, at first strategy 1 was also omitted from the simulations. When it became clear there was a flaw in the above reasoning, since more detail can also reveil a better route, only one day of testing remained for my research. Therefore, there was time for the simulation of monitoring for all pairs of start and end coordinates, departure times and strategy 1 only once.

10.4 Parameters

This section explains which settings of the parameters will be used.

For the largest part the parameters are the same as the settings of the parameters in the simulations in which the plannings were done only once. For example, the parameters of the congestions are set equal to the values they had during the previous simulations. However, there are also some differences.
First, an error has been made with the departure times of the car. For the fourth pair of start and end coordinates the first departure times should be equal to 95. However, this departure time is mistakenly set to 0, which would have been correct if the old departure times would have been used. Second, the last setting of the parameter α of the alternative strategy ($\alpha = 250$) is omitted. This setting can be omitted since from the results of the previous simulations it could be concluded that using $\alpha = 250$ leads to the same results as using no strategy (taking all congestions into full account).

10.5 Test Results and Conclusions

In this section first the results of the simulations are given and discussed. After that, the conclusions that can be drawn from the simulation will be discussed.

The results of the simulations are given in Table 10.1. Note that from the original results the cost of the optimal static route is subtracted.
When looking at the results in Table 10.1, the first things that stand out are the negative entries. These negative entries mean that the cost of routes driven by the car during monitoring is less than the cost of the route found if the planning is done once in a static road network. This can have two causes. The first possible cause is the problem of double chains. The program tried to correct all occurrences of double chains by subtracting an estimation of the extra distance, time and cost. However, since it is only an estimate, this could lead to errors in the total distance. The other possible cause is the fact that with each replanning a bigger part of the roads are visible for the route planner. This means that a possible shorter route which is not visible the first planning can be planned a later planning.

Before taking a closer look note that for strategy 1 only 1 simulation is done in contrast with the 10 simulations done for the other strategies. Therefore, even if strategy 1 plans the same route as some other strategy, the results could still be different.
It is also important to recall that simulations for strategy 6 are not done since the results would be the same as for strategy 1.

Taking a closer look leads to the conclusion that the pairs of start and end coordinates are of great variety. There are pairs for which it turns out to be profitable to wait with the decision to avoid the congestion, for example pair 8. For this pair, the best results are given for the strategy which gives the congestions the least influence. There are also pairs for which this lack of influence is punished (for example pairs 4 and 7). For these pairs postponing the decision to avoid the congestion leads to higher costs.

For the most part, strategy 1 (and thus also strategy 6) gives the best results. However, as was already stated in the previous simulations, these strategies take almost every congestion into full account. Therefore, even though the results are slightly worse for some instances, strategy 5 is preferred above strategies 1 and 6. Note that this strategy (the alternative strategy with $\alpha = 100$) was already considered the best in case no monitoring was applied.
Strategy 2 is not chosen since the hard boundary is uses. For the most routes (in which the decision to avoid the congestion should be postponed) strategy 2 gives the same results as strategy 1 and 6 (since the congestion would move into the field of vision of strategy 2).
Strategies 3 and 4 do not prove their strengths as was hoped after the first simulation in which planning was done only once and monitoring was not applied. Apparently, $\alpha = 100$ is small enough to make sure that all local detours will be searched before the decision whether to avoid the congestion can not be postponed any longer.

	Departure times					Departure times			
	47	72	132	157		97	123	183	208
1,1	21	450	472	0	6,1	0	711	711	0
1,2	50,5	447	447	-32	6,2	207	854	854	176
1,3	38,7	447	447	-32	6,3	207	895	895	176
1,4	32,8	447	447	-32	6,4	207	903	903	176
1,5	62,3	447	447	-32	6,5	207	854	854	176
	85	110	170	195		72	97	157	183
2,1	0	223	223	1	7,1	0	595	595	0
2,2	-67	159	159	-51	7,2	-18	602	602	-18
2,3	-67	159	159	-51	7,3	-18	1105	711	-18
2,4	-67	159	159	-51	7,4	-18	714	600	-18
2,5	-67	159	159	-51	7,5	-18	602	602	-18
	72	83	143	154		83	94	154	165
3,1	22	87	87	0	8,1	0	263	0	0
3,2	-37	54	54	-37	8,2	-14	263	-14	-14
3,3	-37	54	54	-37	8,3	-14	261	-14	-14
3,4	-37	54	54	-37	8,4	-14	264	-14	-14
3,5	-37	54	54	-37	8,5	-14	264	-14	-14
	95	111	171	187		89	114	174	199
4,1	24	469	469	24	9,1	-12	55	55	74
4,2	24	993	985	24	9,2	-12	55	55	28
4,3	24	469	469	24	9,3	-12	55	55	28
4,4	24	469	469	32	9,4	-12	55	55	74
4,5	0	503	503	0	9,5	0	45	45	19
	74	99	159	184		70	88	148	166
5,1	3	681	681	0	10,1	2	148	148	0
5,2	20	659	659	56	10,2	63	202	202	30
5,3	20	659	659	56	10,3	63	202	204	30
5,4	20	659	659	56	10,4	63	149	149	30
5,5	7	691	691	43	10,5	63	149	149	30

Table 10.1: Average Extra Time Route 1 Through 10 (seconds)

Chapter 11

Conclusions and Recommendations

This chapter contains a summary of the important conclusions that can be drawn from the results of the simulations that are done. This summary is given in Section 11.1. The achieved results are discussed also in that section. Recommendations for future work are given in Section 11.2

11.1 Summary and Conclusions of This Report

The objective of my graduation assignment was to *find planning and monitoring algorithms which can deal effectively with long range, time dependent traffic information within the framework available in the VDO navigation systems*. The issues that we had to take into account were *imperfect traffic information* and *resource constraints*. The traffic information provided can be incorrect, incomplete and old (not up to date). Furthermore, the algorithm used should be compatible with the current available system resources. This means the algorithm can not use too time consuming calculations.
First such an algorithm is searched in the literature. However, no effective algorithm is known for the problem at hand, i.e. finding a minimum cost path in a stochastic dynamic graph. Therefore, an algorithm will be searched by means of a simulation.

The current route planner takes congestions into account during planning by means of an influence function. The output of this influence function is a value between 0 and 1. This value is used to multiply the expected dynamic cost of a congestion. The current route planner neglects all congestions outside a circle around the car with a diameter of 150 kilometer. For every congestion inside the circle, the value of the influence function is 1.
A certain form of the influence function is called a *strategy*.

Alternative strategies are investigated. One alternative strategy is chosen to be compared with the current strategy. This alternative strategy has an influence function which output decreases (in a non linear way) as the distance from the car to the congestion increases. The influence function $I(a)$, with a the congestion, used by the alternative strategy depends on the parameter α, and has the following form ($d(CCP, a)$ is the distance between the car (Current Car Position) and the congestion):

$$I(a) = \begin{cases} 1 & d(CCP, a) \leq \alpha; \\ \frac{\alpha}{d(CCP, a)} & d(CPP, a) > \alpha. \end{cases}$$

For a better comparison, also a strategy in which all congestions are taking into full account is added to the comparison. Note that this strategy is too processor time consuming and therefore can not be implemented in the framework available in the VDO navigation systems.

The comparison of the strategies is done for the case in which a planning is done only once as well as for the case in which monitoring is applied. Monitoring means that the quality of the route currently presented to the driver is monitored and the route is replanned at certain intervals.

For the comparisons, a general model for the course of a congestion has been developed. This model states that a congestion has a certain standard course.

From the results of the simulations that are done it can be concluded that in case of planning once the alternative strategy with $\alpha = 100$ finds the best balance between the risk of a wrong route planned and planning time. The alternative strategy with $\alpha = 250$ gives the same results, but $\alpha = 100$ is preferred since the planning times are shorter.
In case monitoring is applied, the alternative strategy with $\alpha = 250$ give the best results. The 'second' strategy is the alternative strategy with $\alpha = 100$. The results for the strategy with $\alpha = 100$ is only slightly worse than the results for the strategy with $\alpha = 250$. Because the planning time and processor time consumption is less for $\alpha = 100$, this parameter setting is preferred, even though the results for $\alpha = 250$ are better.
The strategy currently used by the route planner performs worse than the alternative strategy for the situations in which the congestion lies outside the 'circle of influence'. Also the alternative strategies with $\alpha < 100$ ($\alpha = 25$ and $\alpha = 50$) perform worse because the influence of congestions far away is too small with these strategies.

Concluding it can be said that the algorithm which deals most effectively with long range, time dependent traffic information within the framework available in the VDO navigation systems is the alternative strategy with $\alpha = 100$.

11.2 Recommendations

The research in the area of dynamic route planning and how to take congestions far away into account is far from finished. Some of the recommendations for the further research are listed below.

- As is also mentioned in [2], a congestion on a certain road segment also effects the roads in the near distance of that road segment. Therefore, in further research, a traffic problem should be modelled as a region rather than a single road segment. Otherwise, it is likely that a strategy which neglects a congestion until the car is almost at the congested road segment will perform better during simulations than in reality.

- In the simulations done, an expected course of the congestion is given. In reality this is only true for structural congestions, i.e. congestions which occur frequently. Further research can be done in the area of incidental congestions, in which the expected development over time is not known.

- Replanning routes proved to be beneficial because of the more detailed map that can be used. Therefore monitoring can also be applied in static situations (in case there is no congestion). Shortcuts can perhaps be found.

- As was explained in Appendix C, an adapted graph of the course of congestion is needed if the departure times has to be chosen in such a way that the congestion is met at a certain phase.

- In the simulations done, only one possible congestion is considered. Further research can be done to see how the strategies perform in case there are more possible congestions.

- It should also be investigated what happens if the information about a congestion becomes more certain with time. For example, what happens when at the time of the first planning a certain stochastic variable is normally distributed as $N(10,5)$ and at the time of the second planning (for example 10 minutes later) it is normally distributed as $N(8,3)$?

- The use of the congestion while determining the h values (see algorithm A*, Section 2.2.2) should be investigated. This will probably lead to several problems. For example, what is the chance the remaining part of the route goes via the congestion and what is the expected arrival time at the congestion if the route is via the congestion?

- During the simulations done, one general model of the course of a congestion is used. It is clear that not all congestions will have the same expected course. To get better models for all kind of congestions an investigation could be done to identify certain groups of congestions. Each group would contain types of congestions with a similar known expected course. The criteria of the groups can be determined through an investigation.

- Most of the routes locals detours are found once the congestion moves into a more detailed road map. Therefore, it might be interesting to check the quality of the routes found when an area around the congestion is manually moved towards a more detailed road map.

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Appendix A

Modifications Source Code Route Planner

In this appendix it is first described how congestions are taken into account in the dynamic route planner by VDO. After that, the modifications that are made to the source code of the current static route planner on behalf of this research are described. The complications caused by these modifications are discussed in the last section.

A.1 Dynamic Route Planner VDO

VDO gets information about the length of a current congestion together with its begin and end coordinates. The dynamic route planner of VDO uses *static congestions*. This means that VDO assumes that the state of the congestion will not change over time. The route planner draws a straight line segment from the given begin coordinate to the end coordinate. Then, a rectangle is formed around the line segment. The extra costs that are caused by the congestion on a certain road segment in the rectangle is based on the position of the road segment in the rectangle and the angle the road segment makes with the congestion. This is done because otherwise extra costs would be added on the congested road in both directions. So, even if the congestion is in the opposite direction, extra costs would be added.

A.2 Modifications for Research

In this section is described how the source code of the route planner is modified to be able to take congestions into account during this research.

One of the first differences with the dynamic route planner from VDO is that in this research we want to take the changes in the state of the congestion into account. Therefore, we should be

able to work with different lengths of the congestion.
Because we want to work with changes in the length of the congestion, a fixed start and end coordinate of the congestion can not be used. The choice is made to use a fixed end coordinate of the congestion. The start coordinate (i.e. the place the car drives into the congestion) follows at any time from the length of the congestion at that time and the end coordinate.
To take a congestion into account all the possibly congested road segments are stored in a file. Note that this is done for each different level network. To determine which road segments are possibly congested, from the end coordinate of the congestion the road is followed backwards until the absolute maximum length of the congestion is reached.
During planning, for each road it is checked whether it is on the list of possibly congested road segments. If so, the current time is used to determine the current length of the congestion after which the extra cost for the road segment is determined .

Another difference is that we will only consider frequent congestions. From frequent congestions it is known what can be expected. This information is used to set the parameters of the congestion.

The last difference is a assumption already implicitly used. This difference is that only the roads with the congestion on it get extra costs. No spread-out effect like the rectangle of the dynamic route planner is used.

A.3 Complications due to Modifications

All the modifications made to the source code of the route planner are also cause for some complications.
One of the complications is the occurrence of double chains. What double chains are and what the causes are for those double chains will be described in the first subsection. The second subsection describes a similar difficulty, namely gaps in the route.

A.3.1 Double Chains

After planning a route, information about the route found is wanted. However, since the g values used during planning are not entirely accurate (the used strategy factor the dynamic costs), the information about the route is obtained by checking all road segments of the route and add their information to the overall information. With the checking of the road segments is where the errors occur. Especially the points where the route planner makes a switch from one road level network to another are cause for failures. When the switch is made between two road level networks, the first road segment of the second level is added to the route rather than the last road segment of the first level. Most of the times though, the first road segment of the higher level includes more than just the last road segment of the lower level. The road segments of the lower level that are also include by the first road segment of the higher level are counted twice while gathering information about the route found by the route planner. Those roads are considered to be double chains.

In Figure A.1 the principle of double chains is shown. The costs of the segment of the route shown in this figure should be $c_a + c_b + c_c$. However, since road segment (u_j, u_{j+1}) is added rather than (v_{i+2}, v_{i+3}) ,c_a and c_b are counted twice, the costs of the segment of the route shown in this figure will be set at $2(c_a + c_b) + c_c$.

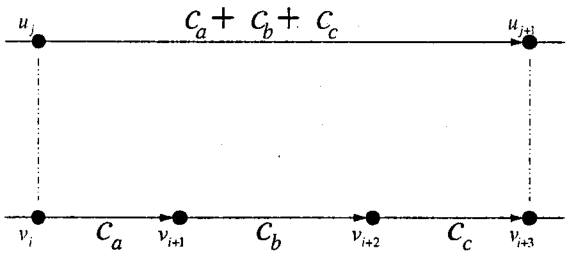

Figure A.1: Double Chains

A.3.2 Gaps in the Route

In the previous section the principle of double chains is explained. Double chains are caused by changing from a certain level of the road network to a network with a lower level of detail. Further down the route (as the car approaches the destination) the network changes again to a higher level of detail. This has an inverted effect as double chains, namely gaps. These gaps are caused because the last road segment on the first level is omitted for the first segment of the second, more detailed, level.

Figure A.1 can also be used to illustrate the principle of gaps. In this case, road segment (u_j, u_{j+1}) is substituted by only (v_{i+2}, v_{i+3}) rather than by (v_i, v_{i+1}), (v_{i+1}, v_{i+2}) and (v_{i+2}, v_{i+3}). Therefore, c_a and c_b will not be counted. Due to this gap, the costs of the segment of the route shown in this figure will be set at c_c.

Appendix B

Simulation: Area, Coordinates and Congestions

In every simulation that is done during the research the same set of start and end coordinates are used. In this appendix first an indication is given of the area in which the start and end coordinates lie. This area will have to meet several requirements. After that, all the pairs of start and end coordinates will be described. In the last section, a list is given of the congestions on the routes between each of the start and end coordinates.

B.1 Simulation Area

In this section an indication is given of the area in which the start and end coordinates lie. This area will have to meet several requirements.

For all pairs of coordinates there will have to be several possible ways to get from the start coordinate to the end coordinate. This requirement will be met if the area has a high road density.
Along all routes that will be planned it has to be possible to insert a realistic traffic congestion. This requirement will be met since the area which we will use is known as an area with a lot of traffic jams each day.
The last requirement is that the area will have to be fairly large. This is needed since we are particularly interested in long routes. For long routes, the start and end coordinates should be at least 200 kilometers apart.

We found that all these requirements are met using an area in the German Ruhrgebiet.

Route	Start city	End city
1	Eindhoven	Wetzlar
2	Wetzlar	Eindhoven
3	Trier	Warner Bros Movie World
4	Warner Bros Movie World	Trier
5	Luik	Kassel
6	Kassel	Luik
7	Kleve	Frankfurt
8	Frankfurt	Kleve
9	Eindhoven	Kassel
10	Kassel	Eindhoven

Table B.1: Start and End Cities in Simulations

B.2 Start and End Coordinates

Now that the area is chosen a description is given of the different start and end coordinates. All start and end coordinates are chosen at the border of the area and represent several cities. There is one exception, namely a coordinate which represents an amusement park, Warner Bros Movie World, rather than a city. For convenience this will be considered a city as well. Eight cities are chosen. Four pairs are made and routes will have to be planned from the first of each pair to the other of that pair and vice versa. For the remaining two start and end coordinates (we want ten different start and end coordinates) we combine two cities from the former pairs. This results in the following list of start and end cities/ coordinates displayed in Table B.1.

B.3 Traffic Congestions

Since we are interested in planning dynamic routes, a possible congestion should be on the otherwise static route between each pair of coordinates. In Table B.2 we list the positions of the congestions of each pair of coordinates.

Route	Highway	Position	Direction
1	A45	Near Siegen	South-East
2	A3	Just above junction with A46	North
3	A3	Just beneath junction with A4	North
4	A61	Near Waldorf	South
5	A1	Just right of junction with A45, near Schwerte	North-East
6	A1	Just right of junction with A45, near Schwerte	South-West
7	A3	Near Krunkel	South-East
8	A3	Just beneath junction with A4	North
9	A1	Just right of junction with A45, near Schwerte	North-East
10	A40	Near Essen	West

Table B.2: Positions of the Traffic Congestions

Appendix C

Improvement Departure Times Aiming at Certain Arrival Time

In this appendix it is described why the departure times taken during this research are not exactly accurate. Furthermore, an improvement for these departure times will be discussed.

Departure Time During Simulations

The departure times are chosen in such a way that the car is to arrive at the starting time of a certain phase of the course of the congestion. Four departure times are taken, t_{d_1}, t_{d_2}, t_{d_3} and t_{d_4}, corresponding with the four phases of the congestion (the fourth phase meaning that the congestion is vanished).
Given the phases of the congestion to start at time t_s, t_{t_s} and t_{t_e} respectively and given that the congestion is vanished at time t_e. Given that the time needed to travel from the car to the congestion is equal to T_{cc}. Now, the departure times have been chosen as follows:

$$t_{d_1} = t_s - T_{cc} \tag{C.1}$$

$$t_{d_2} = t_{t_s} - T_{cc} \tag{C.2}$$

$$t_{d_3} = t_{t_e} - T_{cc} \tag{C.3}$$

$$t_{d_4} = t_e - T_{cc} \tag{C.4}$$

However, there is a flaw in the reasoning on which these departure times are based. Once the congestion increases, the car can not drive all the way up to the end coordinate of the congestion. Therefore, the travel time from the car to the point where the car 'hits' the congestion is less than T_{cc}. To be more precise, given that at a certain time the length of the congestion is equal to l and given that the free flow speed at the congested road is equal to v, the travel time from the car to the congestion equals $T_c c - \frac{l}{v}$. A consequence is that, if for example the car departs at time t_{d_2}, it will reach the congestion at time T, which is the solution of the following equation:

$$T = t_{d_2} + T_{cc} - \frac{L(T)}{v} \tag{C.5}$$

with $L(T)$ the length of the congestion at time T.
Notice that in case the car departs at time t_{d_2}, the end coordinate of the congestion will not be reached at time t_{t_s}, but at time $t_{t_s} - \frac{l}{v} + \frac{l}{v_c}$ (with v_c the speed driven at the congestion). This time is later than t_{t_s}, since the free flow speed is higher than the speed driven at the congestion.

Now, given the course of a congestion fixed by the start times t_s, t_{t_s} and t_{t_e}, end time t_e and maximum length l, it is shown which four departure times should be used to assure the car to arrive at the congestion at each of those times.
Given T_{cc}, the time needed to travel to the end coordinate of the congestion, and given v and v_c, the free flow and the congestion speed at the congested road segments respectively, the departure times t_{d_1}, t_{d_2}, t_{d_3} and t_{d_4} should be chosen to satisfy the following four equations:

$$t_{d_1} = t_s - T_{cc}, \tag{C.6}$$

$$t_{d_2} = t_{t_s} - T_{cc} + \frac{l}{v}, \tag{C.7}$$

$$t_{d_3} = t_{t_e} - T_{cc} + \frac{l}{v}, \tag{C.8}$$

$$t_{d_4} = t_e - T_{cc}, \tag{C.9}$$

Hence, for departure times t_{d_2} and t_{d_3} the arrival time at the congestion is a little bit earlier than intended.

Departure Time vs. Congestion Length

A graph can be drawn reflecting the departure time and the corresponding length of the congestion at the moment the car arrives at the congestion. Between the departure times in equations C.6 through C.9 the graph will be a straight line.
In Figure C.1 the graph is shown. For convenience, this graph will be called graph A. For completeness, the schematic overview of the length of the congestion versus time is shown in Figure C.2. That graph will be called graph B.

Note that both graphs have the same form, which is only logical. However, though the length of the period in which the graphs has its maximum values is equal, the rate of growth is bigger in graph B than in graph A. On the other side, the rate at which the graph decreases is less in graph B than in graph A.

Graphs like graph A can be used to determine the appropriate departure times for future simulations.

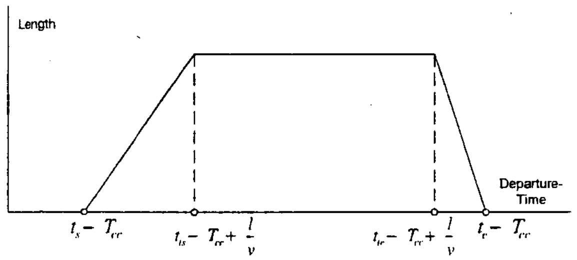

Figure C.1: Departure Time vs. Congestion Length

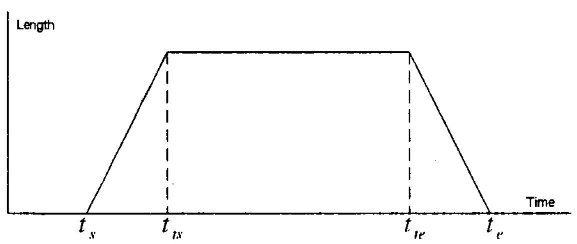

Figure C.2: Time vs. Congestion Length

Appendix D

Full Test Results

In this appendix the full results of the test done for the case in which the planning is done only once. In the tables, every row starts with an indication of a combination of route and strategy. For example, x,y indicates route x and strategy y. Recall the numbering of the strategy from Table 8.3.

	Old departure times								New departure times							
	0		60		120		180		47		72		132		157	
1,1	0	0	472	0	473	6	48	186	233	293	472	0	473	6	407	568
1,2	0	0	777	428	1474	359	48	186	233	293	1277	344	1264	545	407	568
1,3	0	0	777	428	1474	359	48	186	233	293	1277	344	1264	545	407	568
1,4	0	0	777	428	1474	359	48	186	233	293	1277	344	1264	545	407	568
1,5	0	0	777	428	1474	359	48	186	233	293	472	0	473	6	407	568
1,6	0	0	472	0	473	6	48	186	233	293	472	0	473	6	407	568
	0		60		120		180		85		110		170		195	
2,1	0	0	0	0	233	0	233	0	234	300	233	0	233	0	413	572
2,2	0	0	0	0	233	0	233	0	234	300	233	0	233	0	413	572
2,3	0	0	0	0	233	0	939	645	234	300	233	0	233	0	413	572
2,4	0	0	0	0	233	0	233	0	234	300	233	0	233	0	413	572
2,5	0	0	0	0	233	0	233	0	234	300	233	0	233	0	413	572
2,6	0	0	0	0	233	0	233	0	234	300	233	0	233	0	413	572
	0		60		120		180		72		83		143		154	
3,1	0	0	13	48	87	0	3	34	211	256	87	0	87	0	226	270
3,2	0	0	13	48	87	0	3	34	211	256	87	0	87	0	226	270
3,3	0	0	13	48	87	0	3	34	211	256	87	0	87	0	226	270
3,4	0	0	13	48	87	0	3	34	211	256	87	0	87	0	226	270
3,5	0	0	13	48	87	0	3	34	211	256	87	0	87	0	226	270
3,6	0	0	13	48	87	0	3	34	211	256	87	0	87	0	226	270

Table D.1: Average Cost and Variance Route 1 Through 3

	Old departure times								New departure times							
	0		60		120		180		95		111		171		187	
4,1	0	0	0	0	503	0	470	391	242	306	503	0	503	0	287	370
4,2	0	0	0	0	503	0	470	391	242	306	503	0	503	0	287	370
4,3	0	0	0	0	947	96	470	391	242	306	782	248	716	361	287	370
4,4	0	0	0	0	947	96	470	391	242	306	782	248	716	361	287	370
4.5	0	0	0	0	503	0	470	391	242	306	503	0	503	0	287	370
4,6	0	0	0	0	503	0	470	391	242	306	503	0	503	0	287	370
	0		60		120		180		74		99		159		184	
5,1	0	0	4	24	681	0	478	532	236	302	681	0	681	0	347	479
5,2	0	0	4	24	1527	118	478	532	236	302	1289	293	1247	440	347	479
5,3	0	0	4	24	1527	118	478	532	236	302	1289	293	1247	440	347	479
5,4	0	0	4	24	1527	118	478	532	236	302	1289	293	1247	440	347	479
5,5	0	0	4	24	681	0	478	532	236	302	681	0	681	0	347	479
5,6	0	0	4	24	681	0	478	532	236	302	681	0	681	0	347	479
	0		60		120		180		97		123		183		208	
6.1	0	0	0	0	711	0	711	0	230	297	711	0	711	0	387	556
6.2	0	0	0	0	711	0	711	0	230	297	711	0	711	0	387	556
6,3	0	0	0	0	1209	346	1323	514	230	297	1308	323	1247	556	387	556
6,4	0	0	0	0	1209	346	1323	514	230	297	1308	323	1247	556	387	556
6,5	0	0	0	0	711	0	711	0	230	297	711	0	711	0	387	556
6,6	0	0	0	0	711	0	711	0	230	297	711	0	711	0	387	556
	0		60		120		180		72		97		157		183	
7,1	0	0	20	85	595	0	502	614	150	181	595	0	595	0	410	570
7.2	0	0	20	85	595	0	502	614	150	181	595	0	595	0	410	570
7.3	0	0	20	85	1535	283	502	614	150	181	1092	400	1285	527	410	570
7,4	0	0	20	85	1535	283	502	614	150	181	1092	400	1285	527	410	570
7.5	0	0	20	85	595	0	502	614	150	181	595	0	595	0	410	570
7.6	0	0	20	85	595	0	502	614	150	181	595	0	595	0	410	570

Table D.2: Average Cost and Variance Route 3 Through 7

	Old departure times								New departure times							
	0		60		120		180		83		94		154		165	
8,1	0	0	0	0	263	0	56	148	200	250	263	0	263	0	236	272
8,2	0	0	0	0	263	0	56	148	200	250	263	0	263	0	236	272
8,3	0	0	0	0	664	14	56	148	200	250	461	231	455	275	236	272
8,4	0	0	0	0	664	14	56	148	200	250	461	231	455	275	236	272
8,5	0	0	0	0	263	0	56	148	200	250	263	0	263	0	236	272
8,6	0	0	0	0	263	0	56	148	200	250	263	0	263	0	236	272
	0		60		120		180		89		114		174		199	
9,1	0	0	0	0	45	0	45	0	225	293	45	0	45	0	358	486
9,2	0	0	0	0	45	0	45	0	225	293	45	0	45	0	358	486
9,3	0	0	0	0	45	0	45	0	225	293	45	0	45	0	358	486
9,4	0	0	0	0	45	0	45	0	225	293	45	0	45	0	358	486
9,5	0	0	0	0	45	0	45	0	225	293	45	0	45	0	358	486
9,6	0	0	0	0	45	0	45	0	225	293	45	0	45	0	358	486
	0		60		120		180		70		88		148		166	
10,1	0	0	27	71	148	0	69	183	192	243	148	0	148	0	302	392
10,2	0	0	27	71	1070	29	69	183	192	243	809	289	810	373	302	392
10,3	0	0	27	71	1070	29	69	183	192	243	809	289	810	373	302	392
10,4	0	0	27	71	148	0	69	183	192	243	148	0	148	0	302	392
10,5	0	0	27	71	148	0	69	183	192	243	148	0	148	0	302	392
10,6	0	0	27	71	148	0	69	183	192	243	148	0	148	0	302	392

Table D.3: Average Cost and Variance Route 8 Through 10