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Measurements and models on an Erbium-doped fibre amplifier

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FACULTY OF ELECTRICAL ENGINEERING  
TELECOMMUNICATIONS DIVISION (EC)

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**MEASUREMENTS AND MODELS ON  
AN ERBIUM-DOPED FIBRE AMPLIFIER**

by

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Eindhoven, June 1<sup>st</sup>, 1994

Report of graduation study

performed from Sept. 1993 until June 1994

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Coach: prof.dr.ir. W.C. van Etten

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## Summary

This graduation report deals with the properties of Erbium-doped fibre amplifiers. The Erbium-doped fibre amplifier is an optical amplifier that is used in the 1.5  $\mu\text{m}$  window of optical fibres. A study is made on the amplification mechanism in the amplifier. This results in a numerical model which can be used to calculate the optical power in the fibre at all wavelengths. This includes the pump power, the signal power and the noise power that is generated in the amplifier. Using this model a computer program is developed to calculate the amplifiers gain and noise figure under arbitrary conditions. The loss and gain spectra of the Erbium-doped fibre are used as input parameters along with the so-called saturation parameter.

Measurements are performed on an amplifier setup to study its performance and to verify the computer model. The noise figure is accurately modelled. At high gains there is a deviation between the measured and simulated gain curves. This effect is due to the saturation of the amplifier by its own noise. This effect was not included in the computer program. Using the Lycom fibre, net gains up to 24 dB were measured with a quantum-limited noise figure.

The Erbium-doped fibre amplifier as a pre-amplifier shows a constant gain versus the input power. If it is used as a booster however, the gain will saturate, yielding a virtually constant output power. In a two-channel configuration with the first channel at 1538 nm, the second channel should be close to the first one to minimize gain compression.

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## 1. Introduction

In modern telecommunications, optical communication systems have become very important. This is mainly due to the high bandwidth and large transmission distance that can be achieved. For very long transmission links, the attenuation of the fibre can be a problem. One can apply a number of repeaters in the link to regenerate the signal. This solution is quite expensive and inflexible. It is often better to use an optical amplifier to compensate for the loss. Optical amplifiers can also be used as a pre-amplifier to improve receiver sensitivity or as a booster amplifier at the transmitter end. Optical amplification is also required in a long haul soliton transmission link, to maintain the required soliton power. Except for these applications, the amplifier can be applied in numerous other situations where optical amplification is needed.

Optical amplifiers can be classified into two groups: Semiconductor laser amplifiers and optical fibre amplifiers. Semiconductor laser amplifiers have some disadvantages compared to optical fibre amplifiers. A larger loss is introduced when they are coupled to optical fibres. Their polarisation dependency requires measures to maintain or control the state of polarisation. Optical fibre amplifiers are based on a certain host material for the optically active fibre. This can be  $\text{SiO}_2$  or some other type of glass. This material is doped with optically active material. For the  $1.3 \mu\text{m}$  window, Praseodymium seems to be in favour. In the  $1.5 \mu\text{m}$  window, Erbium proved to be most suitable.

This report deals with the characteristics of the Erbium-doped fibre amplifier (EDFA). In chapter 2 a study is made on the amplification mechanism within the EDFA. A numerical model is described to calculate the power spectrum within the Erbium-doped fibre. In chapter 3 a computer program is presented that can be used to simulate the gain and noise figure. It is based on the model of chapter 2. Using this program, the amplifiers behaviour under several conditions can be studied. These include signal wavelength, signal power, fibre length, pump wavelength and pump power.

In chapter 4 the practical amplifier system is discussed. Here, the influence of input and

output losses on the noise figure is investigated. This is necessary to be able to compare measurements to the results from the model. A method for accurate gain and noise figure measurements is described as well.

During the graduation period measurements were performed on two different Erbium-doped fibres. In chapter 5 the results of the York EDF are presented. In chapter 6, a fibre of Lycom was used. The measurements on this fibre are compared to results of the simulation program. Some extra simulations are carried out to investigate the behaviour of the EDFA as a booster or pre-amplifier. The EDFA in a WDM system is simulated as well. This is done by applying a second signal at the input.

## 2. Numerical model for Erbium-doped fibre amplifiers

It is quite difficult to predict the characteristics of Erbium-doped fibre amplifiers. This is because they depend on a large number of parameters and on the conditions of use. It is therefore virtually impossible to find an empirical relation that holds in each situation. A theoretical model is needed to find the characteristics. The accuracy of a model is primarily dependent on the assumptions that were made to simplify it. A complete description that accounts for all effects that take place in the EDFA is virtually impossible. A numerical model will be described that is most suitable for our purposes. It calculates the power levels at the output of the amplifier for every wavelength. Under the assumption that the amplifier is not saturated by ASE, it can be further simplified. Using the results from the model, the noise figure can easily be calculated.

### 2.1 Characteristics of an Erbium-doped fibre amplifier

An optical fibre amplifier has several characteristics that can be of importance in a practical application. All characteristics are dependent on a large number of factors. First there are the fibre parameters. They depend on the type of glass that is used and the Erbium concentration. The distribution of the Erbium over the fibre core is also of influence. Apart from the parameters, the operating conditions are also very important. The signal power at the input will be of significant influence on the gain. The same goes for the pump power. Other conditions are: The wavelength of the signal, the wavelength of the pump laser and the fibre length. The influence of multiple signals in a WDM application has to be considered as well. The following characteristics can be considered:

- Gain
- Noise Figure
- Gain compression
- Crosstalk

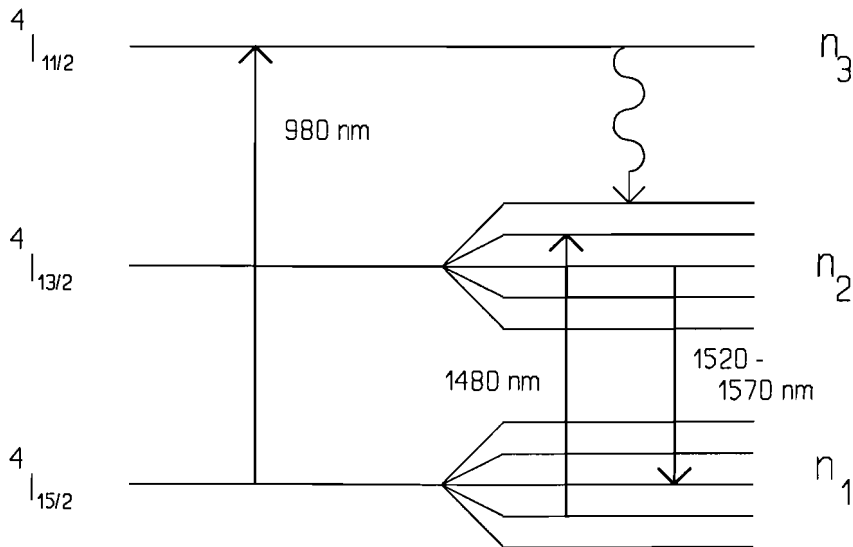


The gain of an EDFA can be as high as 50 dB in some cases. The noise figure is limited to a theoretical value of 3 dB as will be shown in section 2.5. In a WDM application a signal can be affected by a signal in another channel. This can lead to a decreased gain (gain compression). It is also possible that two channels show crosstalk. In practice this does not appear to be a problem. The bit time in a transmission system is generally much smaller than the lifetime of the metastable level of the Erbium atom, so the amplifiers population inversion will not change during one bit time. Crosstalk can therefore be neglected in most cases.

## 2.2 Modelling strategies

In the past, several models have been developed to describe the characteristics of optical amplifiers and Erbium-doped fibre amplifiers in particular. They are all based on the laser rate equations. When the steady state condition is concerned, the population of the laser levels can be determined. The three energy levels of the Erbium atom that are relevant for 980 nm or 1480 nm pumped EDFA's are shown in fig. 1. These levels are broadened when it is doped in a glass host. Homogeneous broadening is due to the electric field surrounding each Erbium atom. A further broadening is caused by the random glass structure. This effect is called inhomogeneous broadening. The latter is often neglected to simplify the model.

The  ${}^4I_{15/2} - {}^4I_{11/2}$  transition can be used to pump the amplifier. This corresponds to a pump wavelength of 980 nm. When the electrons are excited they rapidly decay to the  ${}^4I_{13/2}$  level, without radiation being emitted. This level is called the metastable level, because the electrons remain here for a relatively long time (about 10 ms). In this way a population inversion is created. When a signal is applied whose photon energy corresponds to the  ${}^4I_{15/2} - {}^4I_{13/2}$  transition (1520 - 1570 nm), stimulated emission takes place and the signal is amplified. The amplifier can also be pumped using the  ${}^4I_{15/2} - {}^4I_{13/2}$  transition. A pump wavelength of approximately 1480 nm is needed for this. This is possible due to the Stokes shift. This means that absorption is dominant at the lower end of the wavelength spectrum, whereas emission is dominant at the higher end.



*Fig. 1: Energy levels of the  $Er^{3+}$  ion, doped in a glass host*

The type of model that is used depends on the specific needs. If an Erbium-doped fibre has to be designed, one needs a model that includes the erbium distribution across the fibre core. This is the so-called spatial-mode model. It uses an equivalent noise bandwidth that can not be determined accurately. It is much easier to integrate over the fibre cross-section, introducing the effective overlap integral. The parameters that are required can be measured directly on the fibre by a loss and a fluorescence measurement. The Erbium has to be well confined to the centre of the core, which is the case for all well-designed amplifiers. A very simple model was presented by Saleh et al. [1]. This model however, can not be used for noise figure modelling, because it does not include ASE. It consists of a set of equations that can be solved analytically.

In many papers the assumption is made that the pump-power is well above the threshold level, all along the fibre. Then the inversion of the fibre is uniform. This is generally not the case. In an amplifier that is optimized for maximum gain, the pump-level at the output end will be equal to this level. If it were still above threshold, increasing the fibre length would improve the gain. If the fibre is made too long, part of the gain will be lost by

attenuation in the last part of the fibre. Noise-figure calculations are also effected by the assumption made in this paragraph.

For our purpose we need a model that includes non uniform inversion due to a low pump-level as well as saturation effects. To find the noise figure it has to include ASE as well. The spectrally resolved model is the simplest model that meets these requirements. It will be discussed in the next section.

### 2.3 Spectrally resolved model

All amplifier characteristics are strongly dependent on the signal wavelength and the pump wavelength. This dependence is manifest by the loss spectrum  $\alpha(\lambda)$  and the gain spectrum  $g^*(\lambda)$ , both in [Np/m] ( $1 \text{ dB} \triangleq 0.23 \text{ Np}$ ). They can be written as:

$$\begin{aligned} \alpha(\lambda) &= \sigma_a(\lambda)\Gamma(\lambda)n_t \\ g^*(\lambda) &= \sigma_e(\lambda)\Gamma(\lambda)n_t \end{aligned} \quad (1)$$

In this formula  $\Gamma(\lambda)$  is the overlap integral as defined in (8). It is a measure for the overlap of the optical mode and the Erbium distribution. The total erbium density is  $n_t$ .  $\sigma_a(\lambda)$  and  $\sigma_e(\lambda)$  are the absorption and emission cross sections. They are functions of the glass composition, because the broadening of the energy-levels is partly caused by the electric fields in the glass (Stark splitting). They can be calculated once  $\alpha(\lambda)$ ,  $g^*(\lambda)$ ,  $\Gamma(\lambda)$  and  $n_t$  are known. The loss and gain spectra can easily be measured. The overlap integral however, is difficult to determine. It is therefore advisable to incorporate the measured loss and gain spectrum in the model, instead of the absorption and emission cross sections.

A numerical model is derived in [2] by dividing the relevant part of the optical spectrum in a number of bands of width  $\Delta\nu_k$ , centred at  $\nu_k$  (Fig. 2). The number of bands is determined by the spectral resolution at which the ASE spectrum is to be resolved.  $P_k$  is the power in band  $k$ ,  $I_k$  is the light intensity distribution. They are related by:

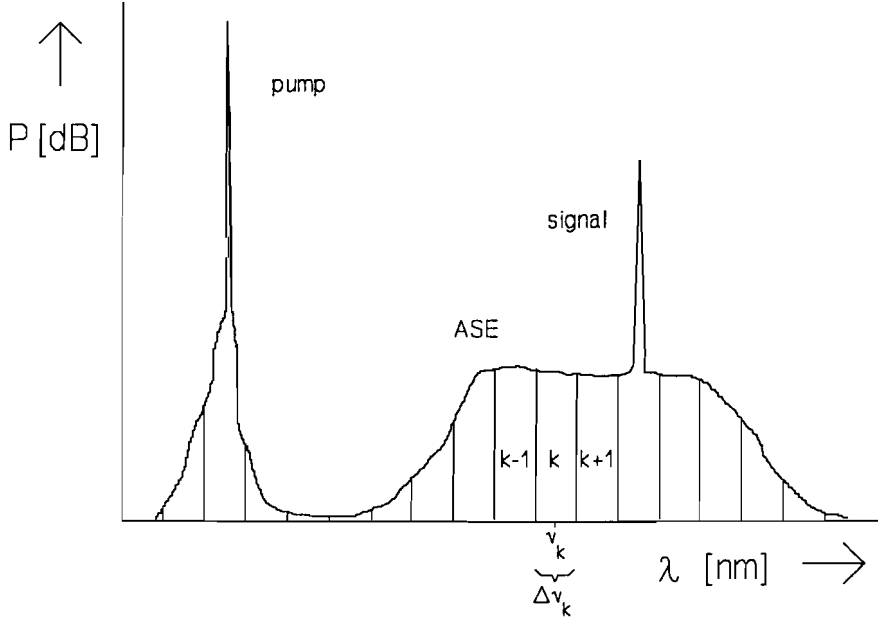


Fig. 2: Optical spectrum divided in wavelength bands

$$P_k(z) = \int_0^{2\pi} \int_0^{\infty} I_k(r, \phi, z) r dr d\phi \quad (2)$$

The intensity distribution can be normalized by:

$$i_k(r, \phi) = I_k(r, \phi, z) / P_k(z) \quad (3)$$

Using the normalized optical intensity the basic rate equation is expressed as [2]:

$$\frac{dn_2}{dt} = \sum_k \frac{P_k i_k \sigma_{ak}}{h\nu_k} n_1(r, \phi, z) - \sum_k \frac{P_k i_k \sigma_{ek}}{h\nu_k} n_2(r, \phi, z) - \frac{n_2(r, \phi, z)}{\tau} \quad (4)$$

where  $n_1$  is the density of ground level erbium ions [ $\text{m}^{-3}$ ] and  $n_2$  is the density of ions in the metastable state. The first term on the right-hand side of (4) is due to the absorption of light power. This includes pump absorption as well as signal (re)absorption. The second term is the stimulated emission term which is responsible for amplification. The last term is the spontaneous emission contribution. This is a decay of the metastable level with a

time constant  $\tau$ . Because of the long lifetime of this level compared to the lifetime of the pump-level, the population of the pump-level can be neglected, so:

$$n_t(r, \phi, z) = n_1(r, \phi, z) + n_2(r, \phi, z) \quad (5)$$

Since we are only interested in the z-dependence of (4) it is integrated over the  $r$ - and  $\phi$ -coordinates. For this purpose we define the equivalent radius of the erbium dopant as:

$$b_{eff} \doteq \sqrt{2 \int_0^{\infty} \frac{n_t(r)}{n_t(0)} r dr} \quad (6)$$

The average of the linear erbium density [ $m^{-1}$ ] can now be expressed as:

$$\bar{n}_i(z) = \frac{\int_0^{2\pi} \int_0^{\infty} n_i(r, \phi, z) r dr d\phi}{\pi b_{eff}^2} \quad (7)$$

where  $i = 1, 2$  or  $t$ , to denote the ground level, metastable level or the sum of both levels.

The definition for the overlap integral is:

$$\Gamma_{k,i}(z) \doteq \frac{\int_0^{2\pi} \int_0^{\infty} i_k(r, \phi) n_i(r, \phi, z) r dr d\phi}{\bar{n}_i} \quad (8)$$

It is a measure for the overlap of the optical beam with the erbium dopant. Generally the values  $\Gamma_{k,1}$  and  $\Gamma_{k,2}$  are not the same. If the erbium is well confined to the centre of the core however, they are almost equal and are replaced by a single constant  $\Gamma_k$ . Using these results the rate equation becomes:

$$\frac{d\bar{n}_2}{dt} = \sum_k \frac{P_k(z) \sigma_{ak} \Gamma_k \bar{n}_1}{h\nu_k \pi b_{eff}^2} - \sum_k \frac{P_k(z) \sigma_{ek} \Gamma_k \bar{n}_2}{h\nu_k \pi b_{eff}^2} - \frac{\bar{n}_2}{\tau} \quad (9)$$

Next, we consider the steady-state situation by setting the time derivative to zero. Substituting (5) in (9) gives:

$$\sum_k \frac{P_k(z) \sigma_{ak} \Gamma_k \bar{n}_t}{h\nu_k \pi b_{eff}^2} = \sum_k \frac{P_k(z) \sigma_{ak} \Gamma_k \bar{n}_2}{h\nu_k \pi b_{eff}^2} + \sum_k \frac{P_k(z) \sigma_{ek} \Gamma_k \bar{n}_2}{h\nu_k \pi b_{eff}^2} + \frac{\bar{n}_2}{\tau} \quad (10)$$

By introducing the saturation parameter as:

$$\zeta = \pi b_{eff}^2 \bar{n}_t / \tau \quad (11)$$

and substituting (1), the final equation for the population inversion is found to be:

$$\frac{\bar{n}_2(z)}{\bar{n}_t} = \frac{\sum_k \frac{P_k(z) \alpha_k}{h\nu_k \zeta}}{1 + \sum_k \frac{P_k(z) (\alpha_k + g_k^*)}{h\nu_k \zeta}} \quad (12)$$

Next, we need an expression for the propagation of the beams in both directions. The general formula is:

$$\frac{dP_k}{dz} = u_k \sigma_{ek} \int_0^{2\pi} \int_0^{\infty} i_k(r, \phi) n_2(r, \phi, z) (P_k(z) + mh\nu_k \Delta\nu_k) r dr d\phi - u_k \sigma_{ak} \int_0^{2\pi} \int_0^{\infty} i_k(r, \phi) n_1(r, \phi, z) P_k(z) r dr d\phi \quad (13)$$

where  $u_k = 1$  for a forward propagating beam and  $u_k = -1$  for a backward beam. The first term gives the increase in power due to emission. The term  $mh\nu_k \Delta\nu_k$  is the spontaneous emission contribution. For a single-mode fibre  $m = 2$  has to be taken because there are two orthogonal polarisation states. Using (8), (13) can be written as:

$$\frac{dP_k}{dz} = u_k \sigma_{ek} \Gamma_k \bar{n}_2 (P_k(z) + 2h\nu_k \Delta\nu_k) - u_k \sigma_{ak} \Gamma_k \bar{n}_1 P_k(z) \quad (14)$$

Substituting (1) and (5) in (14) gives the final result:

$$\frac{dP_k(z)}{dz} = u_k(\alpha_k + g_k^*) \frac{\bar{n}_2(z)}{\bar{n}_t} P_k(z) + u_k g_k^* \frac{\bar{n}_2(z)}{\bar{n}_t} 2h\nu_k \Delta\nu_k - u_k \alpha_k P_k(z) \quad (15)$$

Formula (15) is in fact a set of simultaneous differential equations. There is an equation for both the co-propagating and the counter-propagating direction and for each wavelength band used. The equations are coupled by the population inversion of (12). The set can be numerically solved on a computer, once the loss spectrum, the gain spectrum and the saturation parameter are known. The boundary conditions which have to be satisfied depend on the signal and pump power that are applied. If a co-propagating pump is used they are:  $P_{k=s}(0) = P_{signal}$  and  $P_{k=p}(0) = P_{pump}$ . For the remaining forward beams we have  $P_k(0) = 0$ , because there is no ASE generated at length 0. The backward beams all have  $P_k(L) = 0$ .

## 2.4 Spectrally resolved model without ASE saturation

Spontaneous emission noise in an EDFA tends to accumulate towards the output. In some cases the ASE reaches such high levels that it saturates the amplifier. In this case the whole ASE spectrum has to be calculated, because all spectral components contribute to the saturation effect, as can be seen from equation (12). If the ASE does not lead to gain saturation the model can be simplified substantially. Now the ASE has to be calculated only at the wavelength of interest, the signal wavelength. This is because the noise figure of an optically filtered EDFA depends on the ASE level at the signal wavelength only, as shown in section 2.5. Equation (12) has to be evaluated for the pump and signal powers only, because the ASE has no influence on the population inversion.

$$\frac{\bar{n}_2(z)}{\bar{n}_t} = \frac{\sum_{k=p,s} \frac{P_k(z)\alpha_k}{h\nu_k\zeta}}{1 + \sum_{k=p,s} \frac{P_k(z)(\alpha_k + g_k^*)}{h\nu_k\zeta}} \quad (16)$$

The propagation equation has to be solved for 3 bands only: The signal band  $s$ , the pump band  $p$  and the noise (ASE) band  $n$ . For the signal and the pump band equation (15) can be written as:

$$\frac{dP_k(z)}{dz} = (\alpha_k + g_k^*) \frac{\bar{n}_2(z)}{n_1} P_k(z) - \alpha_k P_k(z) \quad k=p,s \quad (17)$$

It has been assumed that the pump and signal beams are co-propagating. For the noise band the original equation (15) has to be used. The boundary conditions are:  $P_s(0) = P_{signal}$ ,  $P_p(0) = P_{pump}$  and  $P_n(0) = 0$ .

## 2.5 Noise figure calculations

In this section a method will be discussed to calculate the noise figure. It uses the ASE level that can be found from the model in section 2.3. The noise figure can be measured in two ways. An optical spectrum analyzer measures the ASE level directly, whereas an electrical spectrum analyzer measures the signal-spontaneous beat noise. A noise figure formula will be derived for both methods.

In [3] the signal-to-noise ratio at the output of an Erbium-doped fibre amplifier was evaluated. An ideal fotodetector ( $\eta = 1$ ) is assumed, so it is not of influence on the noise characteristics:

$$(S/N)_{out} = \frac{\left( \frac{GP_s e}{h\nu} \right)^2}{2e \frac{(GP_s + P_{sp}) e}{h\nu} B_e + \frac{4e^2}{h\nu} P_s n_{sp} G (G-1) B_e + 2n_{sp}^2 (G-1)^2 e^2 (B_o B_e - \frac{1}{2} B_e^2)} \quad (18)$$

$P_s$  is the input signal power,  $G$  is the amplifier gain and  $n_{sp}$  is the population inversion parameter defined as:  $n_{sp} = \bar{n}_2 / (\bar{n}_2 - \bar{n}_1)$ .  $B_o$  is the bandwidth of the optical filter at the



output of the EDF and  $B_e$  is the electrical bandwidth. The first term in the denominator consists of the shot noise contribution of both the signal and the ASE. The second term represents signal-spontaneous beat noise and the third term is the spontaneous-spontaneous beat noise. If the output is sufficiently filtered (small  $B_e$ ), the spontaneous-spontaneous beat noise can be neglected, as well as its shot noise. The signal shot noise is maintained, because it dominates at very low gains. We find:

$$(S/N)_{out} = \frac{\left(\frac{GP_s e}{h\nu}\right)^2}{2eGP_s \frac{e}{h\nu} B_e + \frac{4e^2}{h\nu} P_s n_{sp} G(G-1) B_e} \quad (19)$$

The signal-to-noise ratio at the input of the amplifier is limited only by the shot noise of the signal. It is expressed as:

$$(S/N)_{in} = \frac{\left(\frac{P_s e}{h\nu}\right)^2}{2e \frac{P_s e}{h\nu} B_e} = \frac{P_s}{2h\nu B_e} \quad (20)$$

Using (19) and (20) the noise figure is expressed as:

$$F \triangleq \frac{(S/N)_{in}}{(S/N)_{out}} = \frac{1 + 2n_{sp}(G-1)}{G} \quad (21)$$

For a good amplifier with  $G \gg 1$  the signal shot noise is negligible, and the noise figure is simply  $F \approx 2n_{sp}$ . From [3] the ASE output power in an optical bandwidth  $\Delta\nu_k$ , including both polarisation states, is found to be:

$$P_n = 2n_{sp}(G-1)h\nu\Delta\nu_k \quad (22)$$

Substituting (22) in (21) gives:

$$F = \frac{1 + \frac{P_n}{h\nu\Delta\nu_k}}{G} \quad (23)$$

If an optical spectrum analyzer is used, the power in a certain bandwidth  $\Delta\lambda$  at the signal wavelength will be measured. This can be scaled to the bandwidth  $\Delta\nu_k$ . If the gain is also measured, the noise figure can be found from (23).

The noise figure can also be measured using an electrical spectrum analyzer. An expression for the signal-spontaneous beat noise spectral density [ $A^2/Hz$ ] is [3]:

$$N_{s-sp} = \frac{4e^2\eta^2}{h\nu} n_{sp} G(G-1)P_s \quad (24)$$

Substituting (24) in (23) yields:

$$F = \frac{1 + \frac{N_{s-sp}h\nu}{2(e\eta)^2GP_s}}{G} \quad (25)$$

If the spectral density is measured in [ $W/Hz$ ], it has to be divided by the impedance of the analyzer to find  $N_{s-sp}$  in [ $A^2/Hz$ ]. The gain and the input signal power have to be measured as well.

The minimum attainable noise figure is determined by the minimum of  $n_{sp}$ . If the signal does not saturate the amplifier, (16) can be written as:

$$\frac{\overline{n_2(z)}}{\overline{n_1}} = \frac{P_p(z)/P_{th}}{1 + \frac{\alpha_p + g_p^*}{\alpha_p} P_p(z)/P_{th}} \quad (26)$$

where  $P_{th} = h\nu_p\zeta/\alpha_p$  is the pump threshold. If the pump level is well above threshold at

every  $z$ -coordinate (pump absorption bleaching), the spontaneous emission factor becomes:

$$n_{sp} \doteq \frac{\overline{n_2}}{\overline{n_2 - n_1}} = \frac{\alpha_p}{\alpha_p - g_p^*} \quad (27)$$

For a 980 nm pump  $g_p^* = 0$ , so the spontaneous emission factor equals 1. This results in a noise figure of 3 dB. This is called the quantum limit. If  $g_p^* \neq 0$ , which is the case for 1480 nm pumping, the quantum limit is higher than 3 dB. For the Lycom Erbium-doped fibre, we have  $\alpha_p = 3.11$  dB/m and  $g_p^* = 1.05$  dB/m (Fig. 10). With these parameters the spontaneous emission factor is found to be 1.51. This corresponds to a quantum limit of 4.8 dB.

### 3. Simulation program for Erbium-doped fibre amplifiers

In chapter 2, a numerical model was described, that can be used to evaluate the power spectrum at the output of an Erbium-doped fibre amplifier. Using this spectrum, the gain and noise figure can easily be calculated. A program is needed that can solve the model on a computer. It should be able to calculate the gain as well as the noise figure as a function of a number of simulation settings. Such a program will be presented in this chapter.

#### 3.1 Solving simultaneous differential equations

Calculating the whole spectrum takes a lot of computing time. This is because several tens of differential equations have to be solved simultaneously. This is even more complicated when counter-propagating beams are considered as well, because boundary conditions will then apply to both ends of the fibre. The equations have to be solved by iteration, until a required accuracy is obtained. This will not be needed in our situation. A co-propagating pump is used because this setup shows better noise performance. The simplified model from section 2.4 is used to reduce computing time. The model is solved by using 4 bands only:

- Signal band
- Pump band
- ASE band
- Second signal band

An extra band was used for a second signal channel. This allows us to model gain compression in a WDM system. Using these four bands the gain and noise figure can be calculated as a function of the following settings:

- Fibre length

- Signal power
- Signal wavelength
- Signal power of second channel
- Signal wavelength of second channel
- Pump power
- Pump wavelength

The differential equations are solved by the fourth-order Runge-Kutta method. The stepsize is adapted at each step, to meet a predefined accuracy. The routines are taken from [4] and modified to meet our specific needs. During the integration, intermediate results are stored. These are used for the simulation where the fibre length is the independent variable. It is not needed then to repeat the integration for several lengths of fibre. This type of simulation is therefore much faster than the other types.

### 3.2 Structure of the program

The program was written in Borland Pascal<sup>®</sup> for Windows<sup>®</sup>. It runs under the Windows environment. A fairly new programming technique, called Object Oriented Programming (OOP), is used. This means that program code (methods) and the data (fields) it acts upon, are no longer separated, but combined in an object. This enables programmers to develop programs in a very structured way. It also makes the code more readable. In the OOP approach, an object can use another object as a data field. We therefore get a hierarchic structure of objects owning other objects. The structure of the program is depicted in appendix A. It only shows the most important fields and methods.

The source code is divided in a number of units. Each unit corresponds to a major object. The main program is called EDFASIM.PAS. The only thing it does is open the main window of the application. The main window is defined in OMYWIN.PAS. It serves as a control panel for the simulator. It enables the user to make selections from the menu and press the start and stop buttons. The simulator which is defined in OSIM.PAS, contains the settings already mentioned in section 3.1. It also has a field to store options for the

simulator. The user can select if he wants to calculate the gain and/or the noise figure. There is also a field called SimType to store the type of simulation that is required. All the settings from the list in section 3.1 can be used as the independent variable, whose range is stored in StartValue and StopValue.

The fibre object is in the OFIBRE.PAS unit. It stores the fibre parameters in the LossList, GainList and SatParm fields. The lists can hold any number of points. There are routines that get the loss or gain parameter at a certain wavelength by interpolating from the lists. The OGRAPH.PAS unit holds the Graph object. This is in fact a window that is put on top of the main window. The window is used for drawing the graph with the gain and noise figure results. Except for the units mentioned earlier, the program uses an include file with global definitions. This is the GLOBAL.PAS file.

Two more files are needed for the program. The EDFA\_RES.RES file is a resource file. Resources are used by Windows programs to store several graphical items, such as bitmaps, icons, dialog boxes and strings. They are created by a special program called a resource workshop and linked to the executable file by the linker. The EDFA\_RES.INC file contains identifiers that are used in the resource.

### 3.3 Using the program for simulations

The simulation program that was written is very easy to use. The user can interactively change the options and simulation settings. He can then run the program and interpret the results. If not satisfactory, the options or settings can be changed and the simulation can be run again. The main window is depicted in fig. 3. On top there are the Window Control-menu box, the title bar and the Window Minimize button. These are common for all windows applications. The menu bar is beneath the title bar. The rest of the main window is called the client area. There are a RUN button to start the simulator and a STOP button to interrupt the simulator when running. When the RUN button is pressed the program checks if all the required data is available, to make sure the program will yield meaningful results. When the simulator is running, the progress indicator will blink

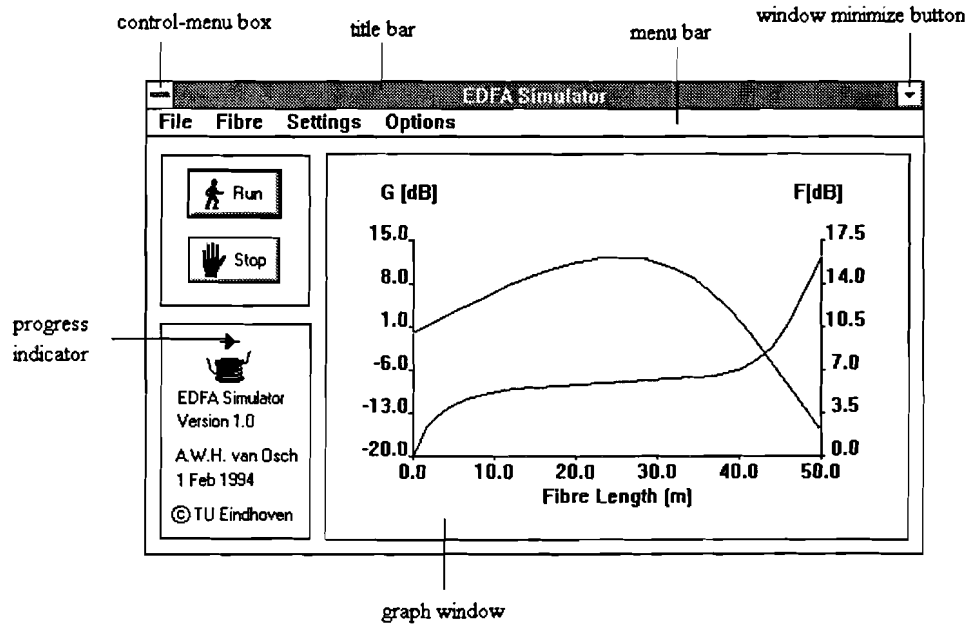


Fig. 3: Main window of the simulation program

to show activity. Next to the buttons is the graph window that is used to represent the results. The available menu items are shown in fig. 4.

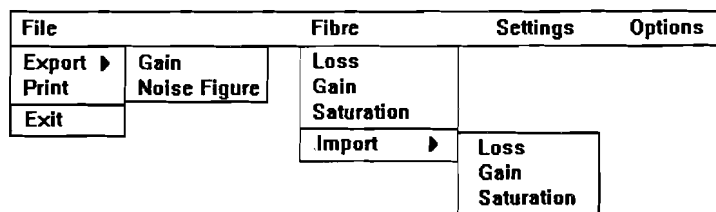


Fig. 4: Menu structure of the program

The first item from the FILE menu is EXPORT. It can be used to export simulation results to a text file. This can be done for the gain as well as the noise figure. The results are written in two columns. The first is the value of the independent variable, the second is the gain or noise figure value. A file name can be entered in a dialog box. The default file extension for gain and noise figure results are .G and .F, respectively. The units are the

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same as the units used in the graph on the screen. This means that wavelengths are expressed in nm, signal powers in dBm and pump powers in mW. The gain and noise figure are both in dB.

The results can also be printed on any available printer by the PRINT option. It copies the contents of the graph window to the default windows printer. The default printer can be set up using the Windows Print Manager. This enables us to print the image to a file as well. The last option from the FILE menu is EXIT. It closes the application and removes it from memory.

The FIBRE menu is used for all operations concerning the fibre parameters. The LOSS item brings up a dialog box with a table of loss parameters. This is a collection of points that are sampled from the loss spectrum that was measured on the fibre. The smoothness of the simulated curves will be determined by the number of points that is used. If no simulation versus the wavelength is required, there need only be two points in the list. One for the signal wavelength and one for the pump wavelength. The list can be edited if necessary. The GAIN option is used in the same way to edit the gain list. The saturation parameter is just a single number that can be edited using the SATURATION menu item.

In many cases, the parameter lists will be quite long. It is easier then to read the parameters from a file. This is done by the IMPORT option. The files consist of two columns separated by one or more spaces. The first column is the wavelength in meters. The second is the loss or gain value in dB/m. The saturation parameter can be imported as well. This feature is included for the sake of completeness. A certain fibre type can now be fully described by three files with the same name but with a different extension. The default file extensions are .LOS for the loss list, .GAI for the gain list and .SAT for the saturation parameter. The names of the files can be selected using a dialog box.

The third item from the menu bar (SETTINGS) is provided to enter the settings to be used by the simulation. At each setting there is an edit control to enter a value. The changes will be accepted by pressing the OK button. The OPTIONS item is used to set the independent variable, by selecting one of the so-called radio buttons. The range for the



variable has to be entered in the required unit as well. The units are the same as those used in the settings dialog box. Finally one or two check boxes can be checked. They are used to enable or disable gain and noise figure calculation.

## 4. The practical amplifier system

The theoretical model of chapter 2 describes the behaviour of the Erbium-doped fibre as amplifying medium. In a practical system a few additional components are needed to realise an amplifier. These components introduce losses, which have to be considered when an amplifier is designed or characterised. In this chapter, the influence of this non-ideal behaviour will be discussed. When measurements are performed on the EDFA, the influence of laser RIN has to be taken into account. The noise figure can then be determined correctly.

### 4.1 Coupling loss due to mode field mismatch

If two standard single mode fibres are coupled by a connector, the signal is attenuated by approximately 0.4 dB. If a connection is made between a standard fibre and an Erbium-doped fibre, extra loss is introduced. This is due to the mode field mismatch between the fibres. Erbium-doped fibres are designed with a smaller core to optimize gain. This results in a smaller mode field diameter. The normalized frequency of a standard single-mode fibre at 1.5  $\mu\text{m}$  is approximately  $\nu = 1.95$ . The core radius is  $a = 4.5 \mu\text{m}$ . The mode field parameter  $w_0$  can be approximated by the following equation [5]:

$$w_0 = (0.65 + 1.619\nu^{-3/2} + 2.879\nu^{-6}) a \quad (28)$$

We find:  $w_0 = 5.84 \mu\text{m}$ . The DF1500 Erbium-doped fibre of York has a numerical aperture  $NA = 0.20$ . The core radius is  $a = 1.4 \mu\text{m}$ . The normalized frequency can be calculated as:

$$\nu = \frac{2\pi a}{\lambda} NA = \frac{2\pi \cdot 1.4 \cdot 10^{-6}}{1.5 \cdot 10^{-6}} 0.2 = 1.17 \quad (29)$$

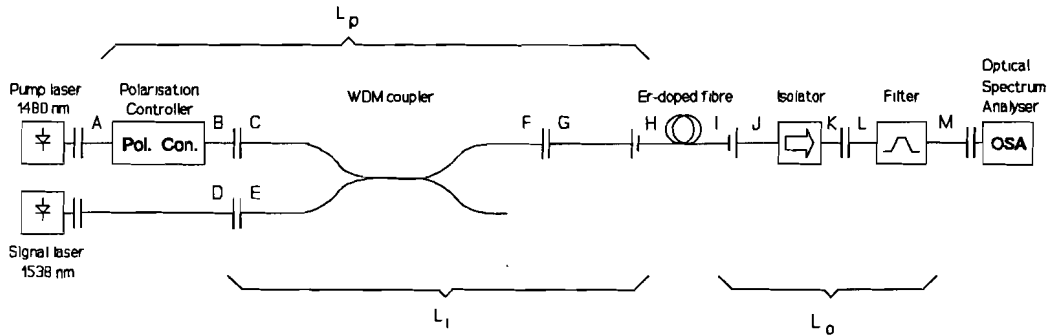
Equation (28) is not accurate for such low  $\nu$ , so we use the exact results from fig. 5.10 of [5]. We find  $w_0 = 2.5 \cdot 1.4 = 3.5 \mu\text{m}$ . The coupling efficiency can now be calculated from [5]:

$$\eta_c = 4 \left( \frac{w_1 w_2}{w_1^2 + w_2^2} \right)^2 \quad (30)$$

where  $w_1$  and  $w_2$  are the mode field parameters of both fibres involved. The coupling efficiency is 0.78. This corresponds to a loss of 1.1 dB. The Lycom Erbium-doped fibre has a mode field parameter  $w_0 = 2.4$ . From (30) the coupling loss to a standard fibre is found to be 3.1 dB.

## 4.2 Influence of input and output losses on the noise figure

In section 2.5 an expression was derived for the noise figure of an Erbium-doped fibre amplifier. Except for the fibre, all amplifier components were considered to be ideal. In practice however, there are various losses in the system as can be seen from fig. 5. At the



*Fig. 5: Configuration of the Erbium-doped fibre amplifier*

input the signal is attenuated by connection losses and the insertion loss of the WDM-coupler. Further attenuation is caused by the mode field mismatch at the interface with the EDF, as discussed in the previous section. At the output the signal is attenuated due to the mode field mismatch as well. The insertion loss of the isolator and filter cause further signal attenuation. The signal path of the Erbium-doped fibre amplifier with input and

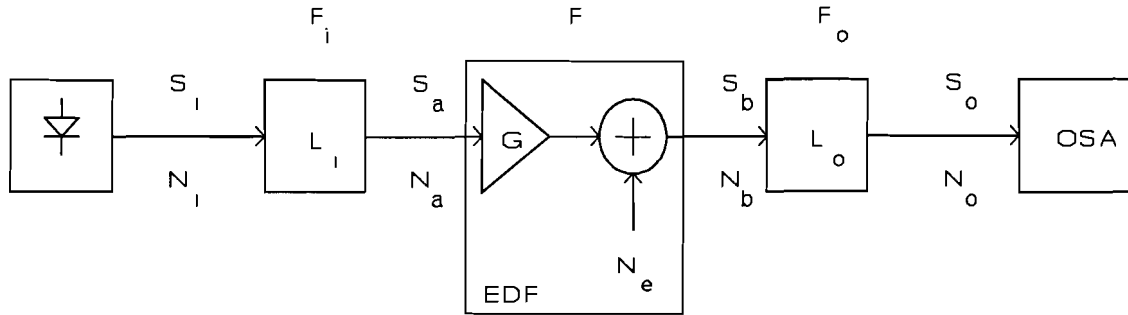


Fig. 6: Signal path with input and output losses

output losses is depicted in fig. 6. A shot-noise limited source is considered, so equation (20) applies for the signal-to-noise ratio. It is clear from this equation that any loss at the laser output will result in a corresponding decrease in signal-to-noise ratio. We can therefore write:

$$F_i = \frac{(S/N)_i}{(S/N)_a} = L_i \quad (31)$$

At the output, the situation is more complicated. Here the noise originates from different sources. The two most important components are the shot-noise and the signal-spontaneous beat noise. In section 2.5 the signal-to-noise ratio at the output of the EDF was calculated to be:

$$(S/N)_b \approx \frac{\left( \frac{GP_s e}{h\nu} \right)^2}{2eGP_s \frac{e}{h\nu} B_e + \frac{4e^2}{h\nu} P_s n_{sp} G(G-1) B_e} \quad (32)$$

The electrical power at the output of a photo-detector is proportional to the square of the photo-current. The attenuation at the output of the amplifier will therefore decrease the signal power and ASE power by a factor  $L_o^2$ . The shot-noise, however, is not actually a photo-current, but due to the statistical fluctuations of the photo-current. The shot-noise appears to be proportional to the signal level. It is therefore attenuated by a factor  $L_o$  only.

We can therefore write:

$$(S/N)_o = \frac{\frac{1}{L_o^2} \left( \frac{GP_s e}{h\nu} \right)^2}{\frac{1}{L_o} 2eGP_s \frac{e}{h\nu} B_e + \frac{1}{L_o^2} \frac{4e^2}{h\nu} P_s n_{sp} G(G-1) B_e} \quad (33)$$

The noise figure of the output attenuator can be found by dividing (32) by (33):

$$F_o = \frac{(S/N)_b}{(S/N)_o} = \frac{L_o + 2n_{sp}(G-1)}{1 + 2n_{sp}(G-1)} \quad (34)$$

Using the result of (21), it can be expressed as:

$$F_o = \frac{L_o + FG - 1}{FG} \quad (35)$$

The noise figure degradation due to output attenuation depends on the amplifiers gain and noise figure. If the gain is very large  $F_o$  will tend towards 1. This means that the overall noise figure will not be affected. Now the contributions to the noise figure of the individual components are known, the overall noise figure can be expressed as:

$$F_g = F_i \cdot F \cdot F_o \quad (36)$$

Combining equations (31), (35) and (36) the net noise figure of the EDF is found to be:

$$F = \frac{F_g}{L_i} + \frac{1-L_o}{G} \quad (37)$$

From the derivation above, it can be concluded that the input loss of the amplifier will degrade the noise performance considerably. The WDM-coupler should be carefully designed and low loss connections should be used, if possible. In most situations the amplifier gain will be quite high, so the output loss is less critical.

### 4.3 Cancelling laser RIN in measurements

When measurements are performed on the setup of fig. 5, the noise that is generated by the laser, has to be taken into account. Up to now an ideal laser source has been assumed. In practice the laser always generates noise. This laser RIN must be corrected for when measuring gain or noise performance. The power of the signal at the input of the amplifier must be corrected by:

$$P_{in} = P_{peak,in} - N_{in} \quad (38)$$

$P_{peak,in}$  is the peak power spectral density that was measured by the OSA.  $N_{in}$  is the noise PSD at the signal wavelength. This is determined by measuring the PSD at the immediate left and right of the peak. These two measured values are interpolated to find the noise PSD at the required wavelength. The output signal power is determined in the same way:

$$P_{out} = P_{peak,out} - N_{out} \quad (39)$$

Again, the signal power is found by subtracting the noise PSD from the measured peak PSD. The gain is now calculated as:

$$G = \frac{P_{out}}{P_{in}} \quad (40)$$

The spectral density of the noise that is generated by the amplifier is found by subtracting the amplified input noise PSD from the measured output noise PSD:

$$P_n = N_{out} - N_{in}G \quad (41)$$

The noise figure can now be calculated by substituting the results from (40) and (41) in equation (23). The procedures described above, are automatically performed when the 'EDFA Personality' (option 51) is used with the HP71450A optical spectrum analyzer. First the input signal is measured. After the amplifier is inserted, another measurement is carried out. The EDFA test personality now calculates the gain and noise figure.

## 5. Measurements on the York fibre

Using the setup of Fig. 5 several measurements were performed on the York DF1500 fibre. Because a high signal level at the input of the amplifier can cause the amplifier to saturate, the gain and noise figure as functions of the input signal level are first investigated. From these measurements a sufficiently low signal level is chosen for the other measurements. Here, the gain is measured as a function of pump power. This is done for three different fibre lengths. The noise figure of the longest fibre was also measured as a function of pump power. For all measurements the 1480 nm pump laser was used. The signal is provided by a 1538 nm DFB laser.

### 5.1 Input and output losses

The losses in the setup were measured using an optical power meter. The pump is attenuated by the WDM-coupler and polarisation controller by 2.1 dB ( $L_{A \rightarrow G} = 2.1$  dB). The signal is attenuated by 1.8 dB ( $L_{D \rightarrow G}$ ). The loss at the connection to the Erbium-doped fibre cannot easily be measured because the fibre itself exhibits a considerable loss. It is however possible to measure the connector loss at a wavelength where the fibre is transparent (e.g. 1300 nm). This method is not accurate because the mode fields in both fibres are a function of the wavelength. This results in different connector losses. The loss is therefore estimated by the calculations in section 4.1. The loss due to the mode field mismatch is 1.1 dB. An additional loss of 0.4 dB is assumed for each connector regardless of the fibre types involved. The loss is thus  $L_{G \rightarrow H} = L_{I \rightarrow J} = 1.5$  dB. The input loss is therefore  $L_i = L_{D \rightarrow H} = 1.8 + 1.5 = 3.3$  dB. The loss of the isolator, filter and two connectors was measured as 4.3 dB. We can therefore write  $L_{J \rightarrow M} = 4.3 - 0.4 = 3.9$  dB. Using these results, the total output loss is  $L_o = L_{I \rightarrow M} = 1.5 + 3.9 = 5.4$  dB.

## 5.2 Gain and noise figure versus input signal power

In fig. 7 the gain and noise figure are shown versus the input signal power. A fibre length of 17 meters was used.

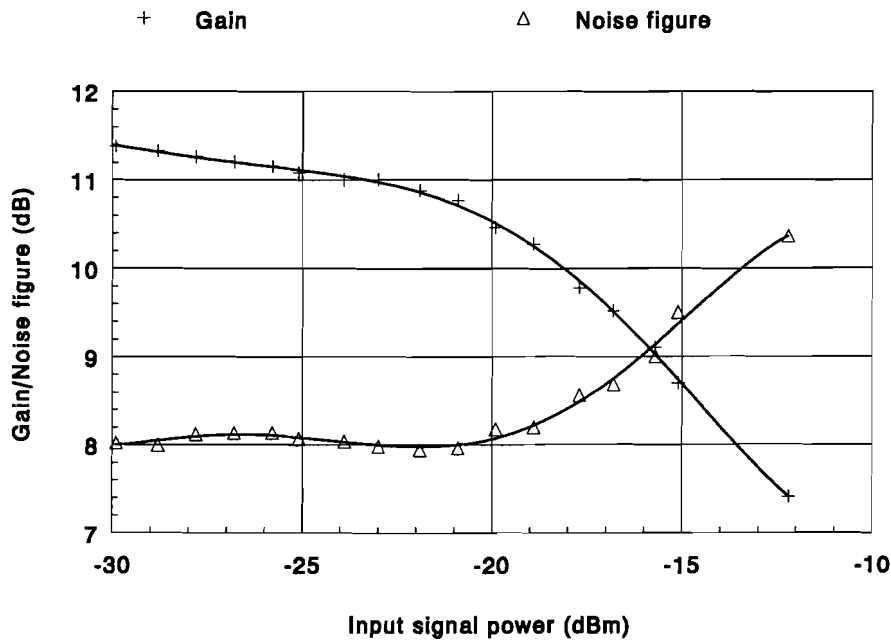


Fig. 7: Gain and noise figure versus input signal power

The pump power was set at 30 mW at point A. The signal level is measured just before the WDM-coupler (point D). Gain and noise figure were measured including both input and output losses. It can be seen that the gain decreases with increasing signal power. At low signal levels the slope is low, but at a certain point the amplifier shows considerable saturation and the gain falls rapidly. Here the population inversion is severely degraded, so the noise figure increases. For low input levels, the noise figure is about 8 dB. This was measured including both input and output losses. The net noise figure of the bare Erbium-doped fibre can be calculated using (37):

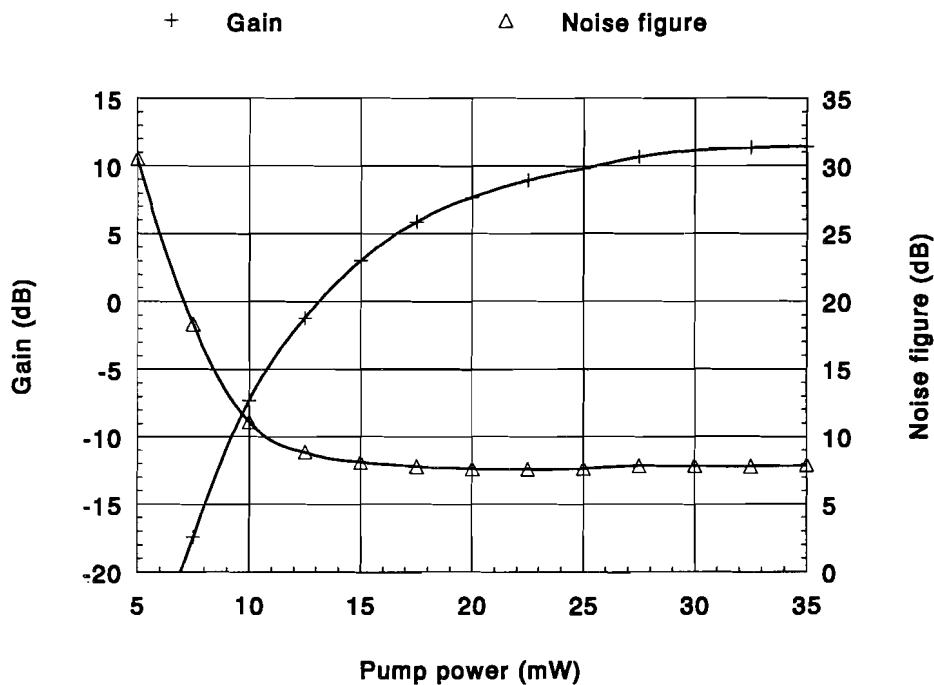
$$F = \frac{F_g}{L_i} + \frac{1-L_o}{G} = \frac{10^{0.8}}{10^{0.33}} + \frac{1-10^{0.54}}{10^{1.1}} = 2.76 \text{ (4.4 dB)} \quad (42)$$



A noise figure of 4.4 dB is a reasonable value for a 1480 nm pumped amplifier as discussed in section 2.5.

### 5.3 Gain and noise figure versus pump power

In this section the gain and noise figure for small input signals is measured. A signal is considered to be small when it has no influence on the population inversion. This means that the amplifier operates in the linear region. From the previous section it is clear that



*Fig. 8: Gain and noise figure versus pump power*

this holds for powers up to approximately -21 dBm. For this measurement a signal level of -26.5 dBm was chosen. The fibre length was 17 m. For low pump powers the amplifier is in fact an attenuator (Fig. 8). A positive gain is found only when the pump power exceeds 13 mW. For higher powers the gain curve flattens. This is because complete population inversion is reached for high pump powers. Almost all Erbium atoms are in the

metastable level, so a further increase in pump power can not excite any extra atoms. This pump light leaves the fibre at the output end. This is known as pump absorption bleaching. For low pump powers the noise figure is very high. This is because the signal is attenuated. In fact, in this region the noise figure approximates the loss value. At approximately 15 mW the noise figure reaches the final value, although the gain still increases. This might seem unlogical, but this is just what the quantum limit is about. As we have seen in section 2.5, the noise figure can not go below this limit.

#### 5.4 Gain versus pump power for different fibre lengths

When the gain is measured for different lengths of Erbium-doped fibre, conclusions can be drawn regarding the optimum fibre length. The gain is measured as a function of the pump power. The signal power is -26.5 dBm. In contrast to the measurements in the previous two paragraphs, the signal and pump powers were measured at the output of the WDM-coupler (point G). The output loss was still included. The results are depicted in fig. 9.

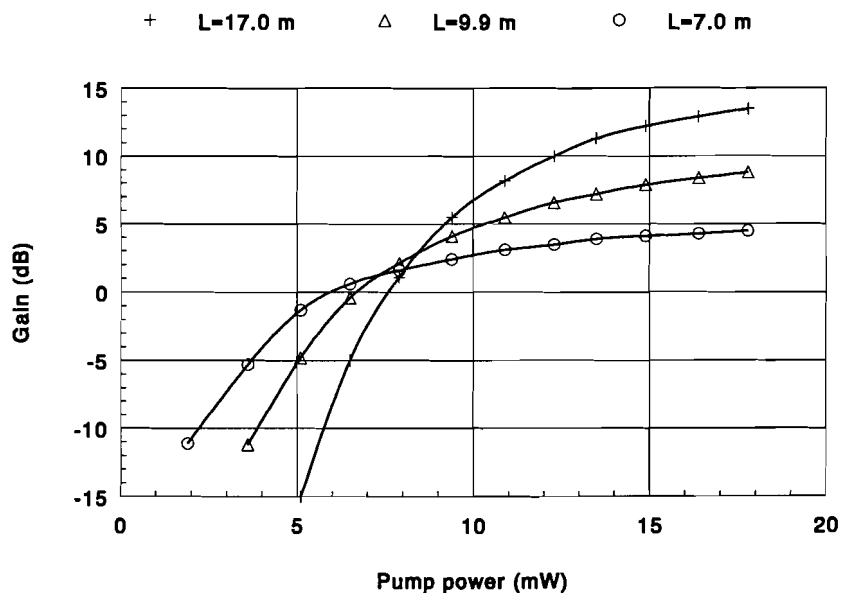
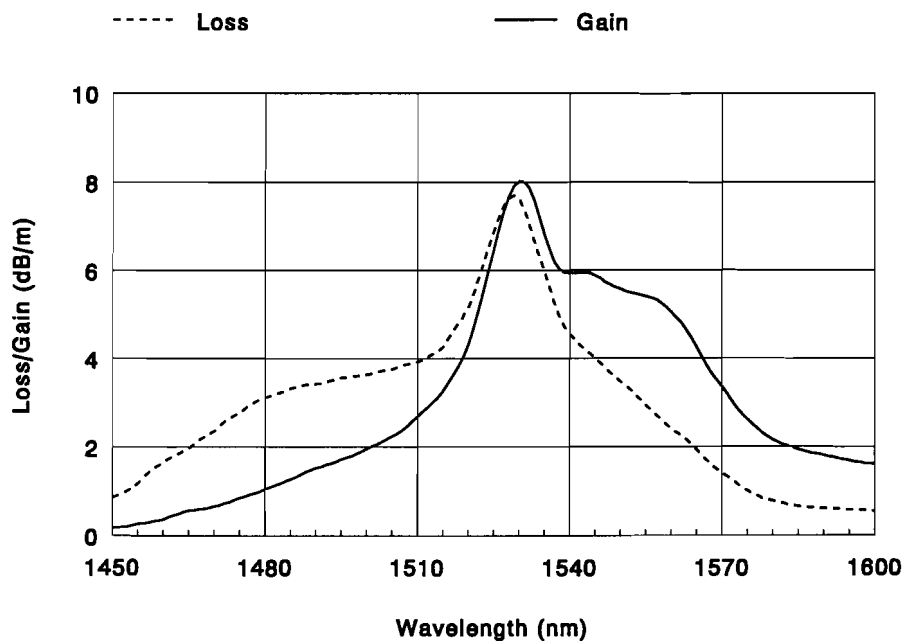


Fig. 9: Gain versus pump power for different fibre lengths

The shape of the curves was already discussed in the previous section. For longer fibre length, the gain at maximum pump power is higher. This is not the case for low pump powers. Here it is better to choose a shorter fibre length. The above indicates the existence of an optimum fibre length for each pump power. In this case it is obvious that a longer fibre would yield a higher gain, at 18 mW pump power. One must take care that the fibre is not made too long. The signal will then be reabsorbed at the output end of the fibre, where the pump power is below the threshold level. This will also degrade the noise figure considerably.

## 6. Measurements and simulations on the Lycom fibre

In this chapter the characteristics of the Lycom R37003 Erbium-doped fibre will be discussed. The fibre is co-doped with Al/La to improve Erbium solubility. The Erbium concentration is  $0.5 \cdot 10^{25} \text{ m}^{-3}$  (120 ppm). Fibre parameters were supplied by the manufacturer as absorption and emission cross-sections. From equation (1) it is clear that they can be converted to the loss and gain parameters by multiplying them with a constant. This constant is determined from the peak absorption of 7.7 dB/m at 1528 nm. This parameter was supplied by the manufacturer as well (Appendix C). The loss and gain spectra are depicted in fig. 10.



*Fig. 10: Loss and gain spectra of the Lycom fibre*

Some characteristics are measured on a practical setup and compared to the results of the simulation program of chapter 3. These are the gain and noise figure as functions of the pump power. The amplifiers wavelength dependency can only be simulated because no tunable laser is available. Furthermore the performance of a pre-amplifier and booster amplifier will be discussed.

## 6.1 Input and output losses

For the measurements in this chapter, the setup of fig. 5 was used again. This means that the pump is attenuated by 2.1 dB ( $L_{A \rightarrow G} = 2.1$  dB). The signal is attenuated by 1.8 dB ( $L_{D \rightarrow G}$ ) in the WDM-coupler. The loss due to the mode field mismatch was found to be 3.1 dB (section 4.1). An additional loss of 0.4 dB is assumed for each connector regardless of the fibre types involved. The loss is thus  $L_{G \rightarrow H} = L_{I \rightarrow J} = 3.5$  dB. The input loss is therefore  $L_i = L_{D \rightarrow H} = 1.8 + 3.5 = 5.3$  dB. The loss of the isolator, filter and two connectors is  $L_{J \rightarrow M} = 3.9$  dB. Using these results, the total output loss is  $L_o = L_{I \rightarrow M} = 3.5 + 3.9 = 7.4$  dB.

From the above it is clear that the input and output losses are quite high. The total loss amounts to 12.7 dB. The contribution of the mode field mismatch to the losses is considerable. Both input and output losses will be accounted for when measurements are made on the fibre.

## 6.2 Gain saturation measurement

The signal level at the input of an EDFA can be of strong influence on the amplifiers gain. The gain performance is optimal for a weak input signal. If the signal level is increased beyond a certain value, the gain will start to fall. For the measurement in this section, a fibre length of 15 m. was used. The signal is provided by a 1538 nm DFB laser. The saturation curves ( $G_{D \rightarrow M}$ ) are measured for different values of the pump power (9.6 mW, 6.9 mW, 5.5 mW and 4.1 mW). The specified pump power is the power at the input of the fibre (point H). Tables with the results are given in appendix D. The net fibre gain ( $G_{H \rightarrow I}$ ) is found by adding 12.7 dB to the measured values. The signal level at the input of the fibre is calculated as well. The characteristics of the EDF are depicted in fig. 11. Up to -30 dBm no considerable saturation takes place. Of course the gain is still strongly dependent on the pump power. At high signal levels, saturation is most serious for high pump powers. This is due the fact that the pump power will be consumed at a higher rate when the gain is high.

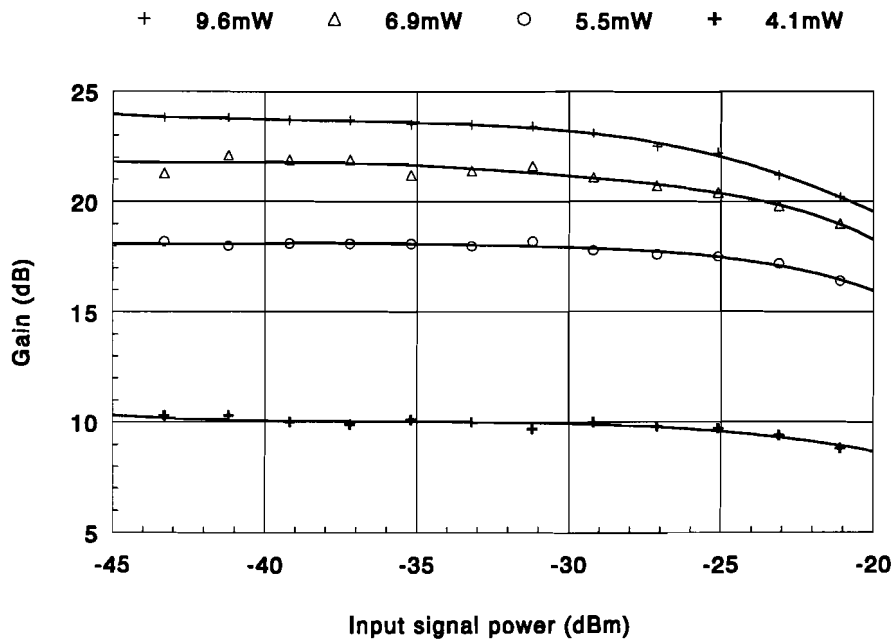


Fig. 11: Gain versus input power for different pump levels

### 6.3 Gain and noise figure versus pump power

In this section, the gain and the noise figure are measured versus the pump power. A signal level of -35 dBm (in the EDF) was chosen, since the amplifier will then not saturate. Again the signal is provided by a 1538 nm DFB laser. The results can be found in appendix E, table 1. The Gain is corrected by adding 12.7 dB to each value. The noise figure is corrected by means of equation (37). The corrected values are depicted in table 2 of appendix E. From this table the graph of fig. 12 was constructed. The gain and noise figure are represented by the crosses and triangles respectively. The gain increases with a fixed slope for low pump powers. At higher powers, the fibre will be bleached by the pump. Some of the pump power will then leave the fibre at the output. This power is lost in the output filter. It is therefore advisable not to drive the amplifier too far in the bleached region.

The noise figure for low pump powers is very high. This is because the EDFA is in fact

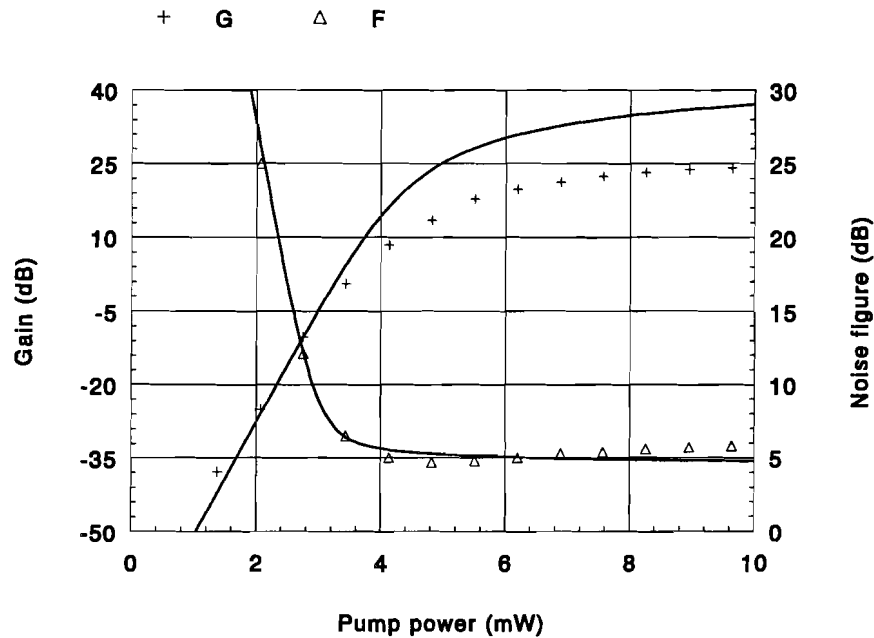


Fig 12: Measured (scatter graph) and simulated (solid lines) gain and noise figure

an attenuator in this region. An attenuator degrades the noise figure with the same amount as the attenuation. With increasing pump power, the noise figure falls to 4.8 dB. For some unknown reason it increases slightly for even higher pump powers.

Using the simulation package described in chapter 3, the performance of the amplifier was investigated using the same conditions as in the measurement. The loss and gain parameters were taken from fig. 10. The only parameter that was not known, is the saturation parameter. It was chosen as  $3.5 \cdot 10^{15}$  for closest fit to the measurements. The results of the simulation are shown in fig. 12 as solid lines. The noise figure is quite accurately modelled for low as well as for high pump powers.

For low pump powers, the gain is accurately modelled too. For higher pump powers however, the simulation predicts a considerable higher gain. This might be due to the fact that the model does not include saturation effects caused by the spontaneous emission. While the noise power in the fibre increases, it consumes part of the pump light. The pump level and gain along the fibre will then be less than expected. The background loss

of the fibre has been neglected as well. This amounts to some 12 dB/km. This is not very much, but one has to bear in mind that both the signal and the pump are attenuated. The effect will therefore be doubled. The point at which the pump starts bleaching the fibre is correctly predicted. This happens at approximately 5 mW.

## 6.4 The amplifier as booster and pre-amplifier

In this section the amplifiers behaviour as booster amplifier and pre-amplifier is investigated. This is done by means of simulation. In fig. 13, the gain versus the input signal power is shown for different pump powers. The signal and pump wavelengths are 1538 nm and 1480 nm respectively. The fibre length is 15 m. If the EDFA is used as a pre-amplifier the signal level will be very low. In this region the gain curve is flat. If the amplifier is used as a booster however, it will be used in the falling region of the curve. The gain is now a function of the signal level. This will cause non-linear distortion. The slope at which the curves fall approaches -1. This means that the output power will be nearly independent of the signal power. If the slope were exactly -1, the output power would have been constant.

The noise figure of the amplifier is depicted in fig. 14. It can be seen that it hardly increases for input powers up to -20 dBm. At this point the gain has already entered the saturated region. This means that a booster amplifier can be used in the quantum-limited region too. The amplifier can be used to maintain a certain power level along a long fibre span. If repairs are made that increase the transmission loss, the booster will automatically adjust its gain and the output level will remain virtually unchanged.

The influence of the signal wavelength on the amplifiers performance as pre-amplifier or booster can be important in a WDM system, where several channels have to be amplified at the same time. A simulation was made of the gain and noise figure versus the signal wavelength. The pump power was set at 9.6 mW. The rest of the settings is the same as before. The simulation was performed for 6 different input levels: -50 dBm, -40 dBm, -30 dBm, -20 dBm and -10 dBm. Fig. 15 shows that the gain of the preamplifier is at its



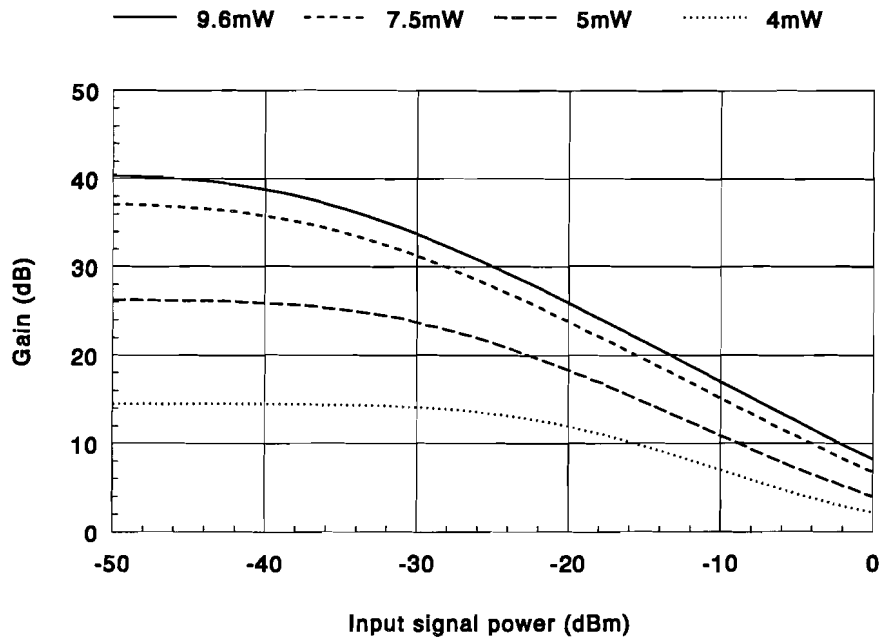


Fig. 13: Gain versus input power for different pump powers

maximum at 1532 nm. It is strongly wavelength dependent, even in the region 1530 nm - 1560 nm. When the input power is increased (booster amplifier), the gain curve is flattened. With  $P_s = -20$  dBm, the gain fluctuation remains within 1 dB over a range of 40 nm.

The noise figure is depicted in fig 16. Longer wavelengths are in favour for low-noise operation. In fact, the noise figure can go below the quantum limit for long wavelengths. This is because the loss spectrum as well as the gain spectrum tend towards zero (fig. 10). The fibre will finally become transparent, resulting in a noise figure of 0 dB. It can be seen that the noise figure does not significantly deteriorate for input powers below -20 dBm. The curves for  $P_s = -50$  dBm, -40 dBm and -30 dBm are indistinguishable. Only the noise figure curve for  $P_s = -10$  dBm is significantly higher.

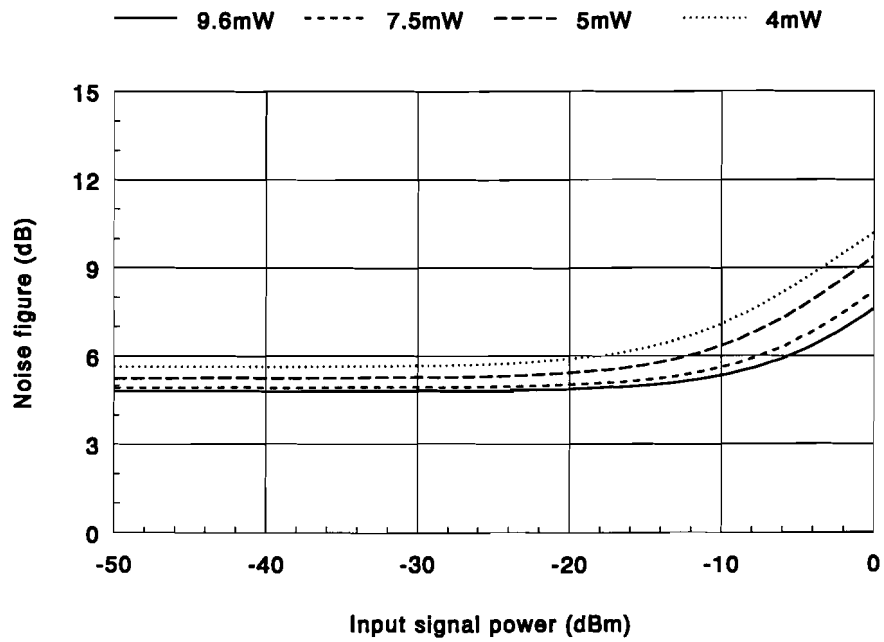


Fig. 14: Noise figure versus input power for different pump powers

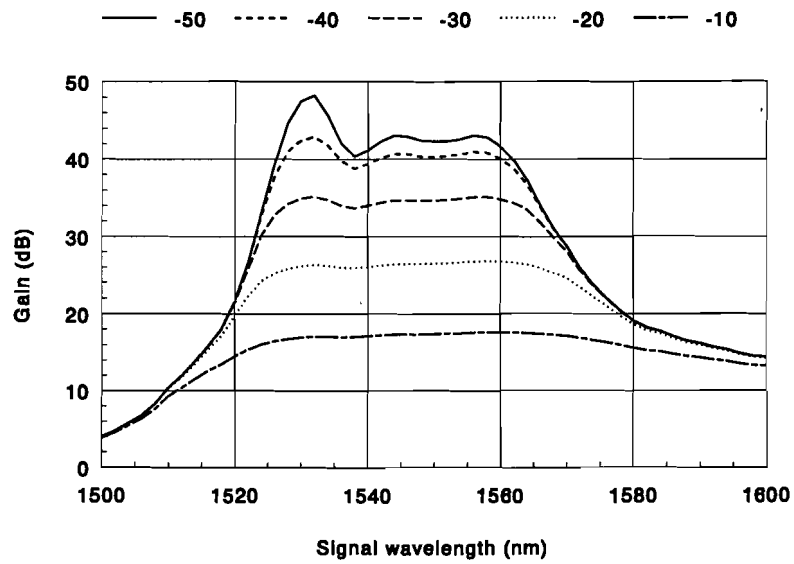
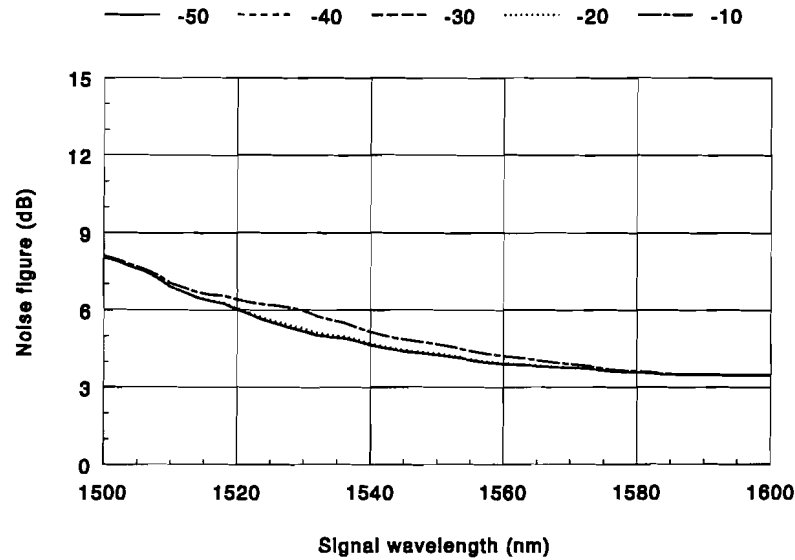


Fig. 15: Gain versus signal wavelength for different input levels

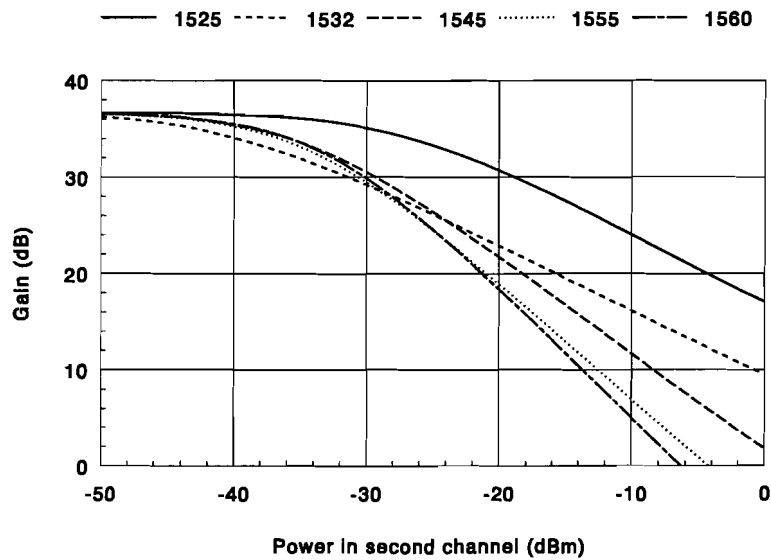


*Fig. 16: Noise figure versus signal wavelength for different input levels*

## 6.5 Gain compression in WDM systems

In some applications the EDFA is used for the amplification of more than one input signal. Each signal is modulated on another carrier wavelength (WDM). Up to now only one input signal has been considered. Of course the presence of a second signal will be of influence on the amplifiers behaviour. As we have already noted in section 2.1, the crosstalk between two channels is negligible. There is however an interaction between two channels in the form of gain compression. This means that the gain in one channel will decrease when the signal power in another channel is increased. This effect can be quite troublesome when an extra channel is introduced to upgrade the transmission capacity or channels have to be switched on and off.

In fig. 17, the result is shown of a simulation of the gain versus the power in a second channel. The signal wavelength is 1538 nm. and has a power of -35 dBm. The pump power is 9.6 mW at 1480 nm and the fibre length is 15 m. The gain curves are shown for five different wavelengths for the second signal. For 1525 nm the gain is not so severely saturated as for the other wavelengths. This is logical, because the gain of a signal at that



*Fig. 17: Gain versus signal power in channel 2 for different channel 2 signal wavelengths*

wavelength is not in the optimum range of 1530 - 1560 nm. It will therefore use less pump power. At the wavelength with optimal gain (1532 nm) the saturation starts at very low levels. The slope at which the gain falls is very low though. At 1545 nm, 1555 nm and 1560 nm the slope is considerably higher. From these three wavelengths, 1545 nm shows the least saturation. From the above it is clear that the second channel should be close to the first one. If a wavelength smaller than 1538 nm is chosen, saturation will take effect at low signal powers but the gain will fall slowly. If it is chosen higher than 1538 nm, saturation will occur at higher signal levels, but will be more severe. It depends on the signal powers which one of the two wavelengths is in favour.

## 7. Conclusions and recommendations

A Study on the Erbium-doped fibre amplifier has shown that its behaviour is quite difficult to predict. This is because it depends on a number of fibre parameters, which are wavelength dependent. Furthermore, the conditions under which the amplifier is used, is of influence on it's performance. These conditions include the power levels of the signal(s) as well as the pump. The wavelength of each light beam has to be known as well. A numerical model was described to calculate the amplifiers gain and noise figure under arbitrary conditions. It was implemented in a computer program that runs under Windows® 3.1.

The influence of losses in a practical amplifier system was investigated. It appears that any loss at the input will degrade the noise figure with the same amount. Losses at the output are less severe if the gain of the amplifier is high. A large part of the losses is due to the mismatch between the mode field of the Erbium-doped fibre and the mode field of the standard single mode fibre. It is recommended that these losses are reduced to improve the amplifiers performance.

Measurements were performed on two types of Erbium-doped fibre. It was shown that saturation occurs when the signal power is too high. The amplifiers gain and noise figure versus the pump power were measured as well. The noise figure of the bare Erbium-doped fibre was measured to be as low as the quantum limit for 1480 nm pumping. Net gains up to 24 dB were measured. The results were compared to the results of the simulation program. The noise figure was accurately modelled. The gain at low pump powers was accurate too. For large gains, however, the amplifier is saturated by the amplifiers noise. This effect was not included in the computer program to reduce computing time. The program will have to be adapted to obtain accurate results under all conditions.

By means of simulation the amplifiers behaviour as pre-amplifier and as booster amplifier was investigated. A pre-amplifier operates at low input powers and has a constant gain. The boosters gain, however, falls rapidly as the signal power is increased. In fact, the

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output power will remain virtually constant. It was shown that the booster can be operated with a quantum-limited noise figure as well. The booster is less wavelength sensitive than the pre-amplifier. This makes it more suitable for applications involving multiple signal wavelengths.

Research on Erbium-doped fibre amplifiers has led to more sophisticated amplifier setups, employing multiple optical filters and isolators. They are used for spectral gain equalisation and noise figure improvement. The performance of these setups can be the subject of future studies on the Erbium-doped fibre amplifier. The amplifiers performance can also be increased by applying a 980 nm pump laser, which has recently become available at the telecommunication division.

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## List of abbreviations

ASE	Amplified Spontaneous Emission
DFB	Distributed FeedBack laser
EDF	Erbium-Doped Fibre
EDFA	Erbium-Doped Fibre Amplifier
NA	Numerical Aperture
OOP	Object Oriented Programming
OSA	Optical Spectrum Analyser
PSD	Power Spectral Density
RIN	Relative Intensity Noise
WDM	Wavelength Division Multiplex



## List of constants and variables

### Constants

$c$	Speed of light in vacuum = $2.99792 \cdot 10^8$	[m/s]
$e$	Electron charge = $1.60210 \cdot 10^{-19}$	[C]
$h$	Planck's constant = $6.62559 \cdot 10^{-34}$	[Js]

### Variables

$a$	Core radius	[m]
$b_{eff}$	Equivalent radius of Erbium dopant	[m]
$B_e$	Electrical bandwidth	[Hz]
$B_o$	Bandwidth of optical filter	[Hz]
$F$	Noise figure of EDF	
$F_b$	Noise figure of EDFA	
$F_i$	Noise figure of input loss	
$F_o$	Noise figure of output loss	
$G$	Gain of EDF	
$g^*(\lambda)$	Gain parameter	[dB/m]
$I_k(r, \phi, z)$	Optical intensity in band $k$	[W/m <sup>2</sup> ]
$i$	Energy state of Erbium atom.	$i=1$ : ground state $i=2$ : metastable state $i=t$ : any state
$i_k(r, \phi)$	Normalized optical intensity in band $k$	[m <sup>-2</sup> ]
$L$	Fibre length	[m]
$L_i$	Input loss	
$L_o$	Output loss	
$m$	Number of modes in the fibre	
$NA$	Numerical aperture	

$N_{s-sp}$	Signal-spontaneous beat noise spectral density	[A <sup>2</sup> /Hz]
$n_1(r, \phi, z)$	Density of Er-atoms in ground level	[m <sup>-3</sup> ]
$n_2(r, \phi, z)$	Density of Er-atoms in metastable level	[m <sup>-3</sup> ]
$n_t(r, \phi)$	Total Erbium density	[m <sup>-3</sup> ]
$n_{sp}$	Population inversion parameter	
$\bar{n}_i(z)$	Average linear Erbium density, $i=1,2$ or $t$	[m <sup>-1</sup> ]
$P_k(z)$	Optical power in band $k$	[W]
$P_n$	ASE power at output of EDF	[W]
$P_n(z)$	ASE power	[W]
$P_p(z)$	Pump power	[W]
$P_s$	Signal power at input of EDF	[W]
$P_s(z)$	Signal power	[W]
$P_{sp}$	ASE power at output of the EDF	[W]
$P_{th}$	Pump threshold power	[W]
$(S/N)_a$	Signal-to-noise ratio at input of EDF	
$(S/N)_b$	Signal-to-noise ratio at output of EDF	
$(S/N)_i$	Signal-to-noise ratio at input of EDFA	
$(S/N)_{in}$	Signal-to-noise ratio at input of EDF	
$(S/N)_o$	Signal-to-noise ratio at output of EDFA	
$(S/N)_{out}$	Signal-to-noise ratio at output of EDF	
$u_k$	Direction of light in band $k$ , $u_k = 1$ : pos. z-dir. $u_k = -1$ : neg. z-dir.	
$w_0$	Mode field parameter	[m]
$\alpha(\lambda)$	Loss parameter	[dB/m]
$\Gamma(\lambda)$	Overlap integral	
$\Gamma_{k,i}(\lambda)$	Overlap integral for band $k$ with $i=1,2$ or $t$	
$\Gamma_k(\lambda)$	Overlap integral for band $k$	
$\Delta\nu_k$	Width of frequency band	[Hz]
$\zeta$	Saturation parameter	[m <sup>-1</sup> s <sup>-1</sup> ]
$\eta$	Quantum efficiency of fotodetector	

$\eta_c$	Coupling efficiency	
$\lambda$	Wavelength	[m]
$\nu$	frequency	[Hz]
$\nu_k$	Center frequency of band $k$	[Hz]
$\sigma_a(\lambda)$	Absorption cross-section	[m <sup>2</sup> ]
$\sigma_{ak}(\lambda)$	Absorption cross-section for band $k$	[m <sup>2</sup> ]
$\sigma_e(\lambda)$	Emission cross-section	[m <sup>2</sup> ]
$\sigma_{ek}(\lambda)$	Emission cross-section for band $k$	[m <sup>2</sup> ]
$\tau$	Time constant of metastable level	[s]
$\nu$	Normalized frequency	

# Appendix A: Structure of EDFASIM simulation program

## Main Window

### Sim

<i>GainResult</i>	<i>Gain points</i>
<i>NoiseFigureResult</i>	<i>Noise figure points</i>
<i>IntResult</i>	<i>Intermediate results</i>
EditSettings	Dialog box to edit the simulator's settings
EditOptions	Dialog box to edit the simulator's options
ExportGain	Write <i>GainResult</i> to a text file
ExportNoiseFigure	Write <i>NoiseFigureResult</i> to a text file
Solve	Integration of differential equations over the length
Clear	Clear the results
Check	Check if simulator can be started
Gain	Calculate gain from signal level
NoiseFigure	Calculate noise figure from ASE level
RunLength	Run simulator with length on x-axis
RunSigPower	" " " signal power " "
RunSigWavel	" " " signal wavelength " "
RunSatPower	" " " saturating signal power " "
RunSatWavel	" " " saturating signal wavelength " "
RunPumpPower	" " " pump power " "
RunPumpWavel	" " " pump wavelength " "

### Settings

<i>Length</i>	<i>Fibre length</i>
<i>SigPower</i>	<i>Signal input power</i>
<i>SigWavel</i>	<i>Signal wavelength</i>
<i>SatPower</i>	<i>Saturating signal input power</i>
<i>SatWavel</i>	<i>Saturating signal wavelength</i>
<i>PumpPower</i>	<i>Pump input power</i>
<i>PumpWavel</i>	<i>Pump wavelength</i>

### Options

<i>SimType</i>	<i>Type of simulation</i>
<i>DoGain</i>	<i>True if gain has to be calculated</i>
<i>DoNoiseFigure</i>	<i>True if noise figure has to be calculated</i>
<i>StartValue</i>	<i>Start value for x-axis</i>
<i>StopValue</i>	<i>Stop value for x-axis</i>

### Fibre

<i>LossList</i>	<i>Loss spectrum of the fibre</i>
<i>GainList</i>	<i>Gain spectrum of the fibre</i>
<i>SatParm</i>	<i>Saturation parameter of the fibre</i>
GetLoss	Interpolate loss from <i>LossList</i>
GetGain	Interpolate gain from <i>GainList</i>
EditLoss	Dialog box to edit loss spectrum
EditGain	Dialog box to edit gain spectrum
EditSat	Dialog box to edit saturation parameter
ImportLoss	Import loss spectrum from a file
ImportGain	Import gain spectrum from a file
ImportSat	Import saturation parameter

### Graph

Clear	Clear the graph
Paint	Repaint the graph
DrawAxis	Draw axis with an appropriate scale
DrawGainPoint	Draw gain curve to a new point
DrawNoiseFigurePoint	Draw noise figure curve to a new point
PrintGraph	Print the graph on the default printer

## Appendix B: Data of the York DF1500 Erbium-doped fibre

Batch: ND712-01  
Core composition: Germano-Silicate glass

### Fibre geometry:

Cladding diameter: 125.8  $\mu\text{m}$   
Cladding ovality: 1.3 %  
Core diameter: 2.8  $\mu\text{m}$   
Concentricity: 0.93  $\mu\text{m}$

### Fibre parameters:

Dopant concentration: 120 ppm  
Numerical Aperture: 0.20  
Cut-off wavelength: 850-970 nm

## Appendix C: Data of the Lycom R37003 Erbium-doped fibre

ID: 940731750001  
Dopants in the core: Al, La, Er

### Fibre geometry:

Cladding diameter: 125  $\mu\text{m}$   
Core diameter: 3.1  $\mu\text{m}$

### Fibre parameters:

Erbium concentration:  $0.5 \cdot 10^{25} \text{ m}^{-3}$   
Numerical Aperture: 0.27  
Mode field diameter at 1550 nm: 4.8  $\mu\text{m}$   
Cut-off wavelength: 1050 nm

### Attenuation

at 950-1050 nm  
Peak absorption: 2.7 dB/m  
Peak wavelength: 975 nm

at 1500-1550 nm  
Peak absorption: 7.7 dB/m  
Peak wavelength: 1528 nm

Background loss at 1100 nm: <12 dB/km

## Appendix D: Results of saturation measurements on the Lycom Erbium-doped fibre

$P_{s,D}$ (dBm)	$P_{s,H}$ (dBm)	$G_{D \rightarrow M}$ (dB)	$G_{H \rightarrow I}$ (dB)
-40.0	-45.3	11.3	24.0
-38.0	-43.3	11.1	23.8
-35.9	-41.2	11.1	23.8
-33.9	-39.2	11.0	23.7
-31.9	-37.2	11.0	23.7
-29.9	-35.2	10.8	23.5
-27.9	-33.2	10.8	23.5
-25.9	-31.2	10.7	23.4
-23.9	-29.2	10.4	23.1
-21.8	-27.1	9.8	22.5
-19.8	-25.1	9.5	22.5
-17.8	-23.1	8.5	21.2
-15.8	-21.1	7.5	20.2

*Table 1: Measured gain and net gain versus signal power ( $L = 15$  m,  $P_p = 9.6$  mW)*

$P_{s,D}$ (dBm)	$P_{s,H}$ (dBm)	$G_{D \rightarrow M}$ (dB)	$G_{H \rightarrow I}$ (dB)
-40.0	-45.3	9.3	22.0
-38.0	-43.3	8.6	21.3
-36.0	-41.3	9.4	22.1
-34.3	-39.6	9.2	21.9
-32.0	-37.3	9.2	21.9
-29.9	-35.2	8.5	21.2
-27.9	-33.2	8.7	21.4
-25.9	-31.2	8.9	21.6
-24.1	-29.4	8.4	21.1
-22.0	-27.3	8.0	20.7
-19.9	-25.2	7.7	20.4
-18.0	-23.3	7.1	19.8
-15.8	-21.1	6.3	19.0

Table 2: Measured gain and net gain versus signal power ( $L = 15$  m,  $P_p = 6.9$  mW)



$P_{s,D}$ (dBm)	$P_{s,H}$ (dBm)	$G_{D \rightarrow M}$ (dB)	$G_{H \rightarrow I}$ (dB)
-40.1	-45.4	5.4	18.1
-38.0	-43.3	5.5	18.2
-36.0	-41.3	5.3	18.0
-34.3	-39.6	5.4	18.1
-31.8	-37.1	5.4	18.1
-29.9	-35.2	5.4	18.1
-27.8	-33.2	5.3	18.0
-25.8	-31.1	5.5	18.2
-23.8	-29.1	5.1	17.8
-21.8	-27.1	4.9	17.6
-19.9	-25.2	4.8	17.5
-17.8	-23.1	4.5	17.2
-15.7	-21.0	3.7	16.4

Table 3: Measured gain and net gain versus signal power ( $L = 15$  m,  $P_p = 5.5$  mW)

$P_{s,D}$ (dBm)	$P_{s,H}$ (dBm)	$G_{D \rightarrow M}$ (dB)	$G_{H \rightarrow I}$ (dB)
-40.0	-45.3	-2.5	10.2
-38.0	-43.3	-2.4	10.3
-36.0	-41.3	-2.4	10.3
-34.3	-39.6	-2.7	10.0
-31.9	-37.2	-2.8	9.9
-29.9	-35.2	-2.6	10.1
-27.9	-33.2	-2.7	10.0
-25.9	-31.2	-3.0	9.7
-24.0	-29.3	-2.7	10.0
-22.0	-27.3	-2.9	9.8
-20.0	-25.3	-3.0	9.7
-18.0	-23.3	-3.3	9.4
-16.0	-21.3	-3.9	8.8

Table 4: Measured gain and net gain versus signal power ( $L = 15$  m,  $P_p = 4.1$  mW)

## Appendix E: Results of pump power measurements on the Lycom Erbium-doped fibre

$P_{p,A}$ (mW)	$G_{D \rightarrow M}$ (dB)	$F_{D \rightarrow M}$ (dB)
5.0	-50.5	50.1
7.5	-37.7	37.7
10.0	-23.0	23.4
12.5	-12.2	14.6
15.0	-4.3	11.1
17.5	0.7	10.3
20.0	5.1	10.2
22.5	7.1	10.4
25.0	8.6	10.6
27.5	9.7	10.7
30.0	10.5	10.9
32.5	11.1	11.0
35.0	11.4	11.1

*Table 1: Measured gain and noise figure versus pump power ( $L = 15$  m,  $P_s = -35$  dBm)*

$P_{p,H}$ (mW)	$G_{H \rightarrow I}$ (dB)	$F_{H \rightarrow I}$ (dB)
1.38	-37.8	34.9
2.07	-25.0	25.0
2.75	-10.3	12.1
3.44	0.5	6.5
4.13	8.4	5.0
4.82	13.4	4.7
5.51	17.8	4.8
6.20	19.8	5.0
6.89	21.3	5.3
7.57	22.4	5.4
8.26	23.2	5.6
8.95	23.8	5.7
9.64	24.1	5.8

Table 2: Net gain and noise figure versus pump power ( $L = 15$  m,  $P_s = -35$  dBm)