MASTER

An injection seeding neuron: towards a fully optical neural network

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An Injection Seeding Neuron
Towards a fully optical
neural network

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Abstract

The work described in this thesis is part of a Laser Neural Network project that aims at realising a neural network in optics. A possible application for such an optical neural network is a fully optical cross connect in a telecommunication network. The injection seeding neuron is a new concept for a fully optical neuron, the basic element of a neural network. In this thesis the concept of the injection seeding neuron is described and verified both theoretically and experimentally.

The injection seeding neuron realises the non-linear function, necessary for the implementation of a neuron, by injecting light into a semiconductor laser. The output power of the laser shows a sigmoid like behaviour as function of the injected power. When this injected power is made proportional to a weighed sum of inputs, an optical neuron is realised. By applying controllable feedback to the laser, the threshold of the neuron can be influenced.

Simulations are done to examine the behaviour of the injection seeding neuron theoretically. The model used for these simulations is the multi-mode rate equations model. Procedures to find the steady state solution and the time dependent solution are described. The steady state solution is used to examine the non-linear function of the neuron. The results from these simulations are consistent with the principle of operation.

An experimental setup is built to verify the principle of operation of the injection seeding neuron. A source laser is wavelength tunable and provides the signal to be injected in the neuron. A neuron laser has controllable feedback for each mode of the laser diode individually and a way of injecting external light. The experiments done with this setup show that the injection seeding neuron indeed shows a sigmoid like behaviour, but further experiments are necessary for a complete verification of the operation principle.

A more stable experimental setup is necessary for future experiments and recommendations are made on how this can be achieved.
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Chapter 1

Introduction

This report is written as 'afstudeerscriptie' (master thesis) for the degree of 'Elektrotechnisch Ingenieur' (Master of Electrical Engineering) at the Eindhoven University of Technology (TUE). The work described in this thesis is part of the Laser Neural Network project. This project aims at the realisation of a neural network in optics, i.e. a neural network where the inputs and outputs are defined by optical signals and the necessary 'calculations' are performed in the optical domain. The project is a cooperation of the Telecommunications group of the Department of Electrical Engineering at the TUE and Philips Research. The work for this thesis was carried out at Philips Research in Eindhoven from February 1997 until October 1997.

One of the features of optical neural networks is their potential speed. Speed is also an important factor for one of the possible applications of optical neural networks: a fully optical cross connect for optical telecommunication. A cross connect is a junction in a fiber network. One way of doing optical telecommunication is sending packages of information with a header and a destination address over the network. A cross connect could be made with an optical neural network, which could detect the headers and route the package to the output corresponding to the destination address of the package. It is expected that because of its high speed [1], the optical neural network can perform this task up to very high bit rates.

The Laser Neural Network is a way of realising an optical neural network. Currently there is a setup that uses the sensitivity of a semiconductor laser for reflections of its own output [1]. This setup realises one specific type of neural network ('Winner-Take-All'-network) with a single laser diode. Successful experiments have been done with this setup [2].

A limitation of this setup is that the inputs are not defined by optical powers but by optical transmissions. There are a number of possibilities to realise optical inputs defined by optical power. One possibility is to use the sensitivity of semiconductor lasers to injected light from another laser ('injection seeding'). In this concept one neuron (the basic element of a neural network) with optical inputs and output is realised with one laser diode. The neuron has been named 'Injection
Seeding Neuron’. The fact that only a single neuron is realised with one laser has the advantage that the type of neural network can be chosen freely. However, a disadvantage is the increasing complexity of the neural network, since multiple lasers are necessary.

The work underlying this thesis could be described as 'The development and verification of the concept of the injection seeding neuron'. In the remaining part of this chapter a general introduction to neural networks and lasers is given. Chapter 2 describes the operation principle of the injection seeding neuron and the semiconductor laser phenomena used for its realisation. Chapter 3 describes (numerical) simulations that can be used for a detailed theoretical examination of the neuron. In chapter 4, the experimental setup is described that is used to verify the operation principle of the injection seeding neuron. This chapter also describes measurements and results done with this setup. In chapter 5 conclusions are drawn and recommendations are given for further research on the injection seeding neuron.

1.1 Neural networks

Neural networks, or rather artificial neural networks, are built in analogy with the biological nerve system. The basic element of the artificial neural network is the neuron, the artificial analogon of an animal nerve cell. Such a nerve cell has multiple dendrites ('inputs') connecting to it and it has one axon ('output'). Through the dendrites the nerve cell receives stimuli. When all stimuli together exceed a certain threshold level, the nerve cell sends out a stimulus itself over its axon. The stimuli coming from the different dendrites do not have the same influence on the 'decision' of the nerve cell whether a stimulus is send or not.

![Figure 1.1: A biological neuron and its artificial analogon.](image)
Figure 1.1 shows a biological and an artificial neuron. The artificial neuron is a model of the biological neuron. The inputs of the artificial neuron $x_i$ are weighed by the weight vector $\vec{w}$. The weighed sum of inputs $\sum_i w_i \cdot x_i$ is mapped onto the output with a non-linear function $f$. This leads to the expression for the output $y$ of the neuron:

$$y = f \left( \sum_i w_i \cdot x_i \right)$$

(1.1)

The neuron can be used with both analog and digital inputs and output. Already with a single neuron simple functions can be realised. In figure 1.2 two digital examples are shown: a logic OR-gate and a logic NAND-gate. In these examples the nonlinear function $f$ is a step function with threshold level $T$. If the weighed sum of inputs is below threshold, the output $y$ will be 0, if the weighed sum of inputs is above threshold the output will be 1. Figure 1.2 also shows the weighed sum of inputs and the resulting output.

(a) Logic OR-gate

(b) Logic NAND-gate

Figure 1.2: Examples of simple functions that can be realised with a single neuron.

By connecting multiple neurons a neural network can be made. There are several types of neural networks, each with their own typical architecture. Well-known examples are feedforward networks (with only forward connections) and networks with recurrent connections. These two examples are shown in figure 1.3a and figure 1.3b respectively.

The functionality of a neural network depends on its weights. These weights are set for a particular functionality by means of a learning algorithm and a set of input-output vectors that describe the desired functionality (the training set).
Often the initial weights are chosen randomly. Then the learning algorithm shows the input vectors to the network one by one. For each input an error value, being the difference between the desired output vector and the actual output vector, is calculated. Depending on this error value the learning algorithm adapts the weights. This is repeated until the desired functionality is trained to the network. A neural network should generalize, meaning that after learning it should also give the correct output value for an input value that was not part of the training set. In order to be able to test afterwards whether this is true, the available set of input-output relations is divided into two sets: a training set and a test set. The first set is used to train the network. The second set is used to see if the network has learned its desired functionality properly.

Neural networks are good at solving problems that are inherently parallel (such as pattern recognition) or where a lot of known input-output combinations are related by some unknown function (which is often the case with statistical problems). An interesting example of the use of neural networks for statistical problems is given in reference [3]. In this example a neural network is used to predict the sales of newspapers and magazines. Ideally, the number of copies delivered at each vending location should just exceed the number of copies sold, so each customer is able to buy a copy. The number of papers sold depends on a lot of parameters, e.g. place of sale, weather, day of the week etc. These are the inputs to the neural network. After training the network with statistical data of the past few years, the network appeared to be able to predict the number of copies sold more accurately than could be done before. The number of shops sold out could be reduced by 30%, whereas the total number of unsold copies remained the same.

The most common realisation of neural networks is in software. Because computers are sequential in general, the outputs of the neurons are calculated one by
one and therefore the parallelism is lost. This disadvantage can be overcome by using dedicated integrated circuits (IC) to realise a neural network. As can be seen from figure 1.3 neural networks have a lot of crossing connections. This becomes a problem when implementing complex neural networks in IC's. Optical neural networks finally lack both these disadvantages. They are parallel and since light rays can cross without interacting the connection problem disappears as well.

1.2 Laser theory

1.2.1 Lasers in general

Lasers are well known for their capability of emitting monochromatic, coherent light. Laser is an abbreviation of Light Amplification by Stimulated Emission of Radiation. There are many different types of lasers but their basic operating principles are the same. In general a laser consists of a medium with optical gain inside an optical cavity as shown in figure 1.4. The energy needed for laser operation is supplied by an external source: the energy pump. Depending on the type of laser, this source can be an electrical current, a chemical reaction or light. The cavity is terminated on each side by mirror planes that partially reflect and transmit the radiation. The end mirrors form a so called Fabry-Pérot resonator.

![Figure 1.4: Schematic representation of a laser cavity formed by two mirror planes with a medium with optical gain in between.](image)

The frequency selectivity required for monochromatic operation is supplied by an optical filter. This can be the Fabry-Pérot cavity. To explain the principle of operation of such a cavity, let us consider a light wave that is propagating through the cavity. While doing so it is amplified and at the end of the cavity it is (partially) reflected by the end mirrors. When the wave has been reflected and is propagating in the opposite direction, it will interfere with the initial wave. When the wave has been reflected a number of times, all the reflected parts of the wave will interfere. If the round trip phase of the wave in the cavity is an integer
number of $2\pi$ the interference will be constructive, if not it will be destructive. This leads to the phase condition for a Fabry-Perot cavity:

$$m\lambda = 2L \quad m \in \mathbb{N}$$

Where $L$ is the length of the cavity and $m$ an integer number. For each value of $m$ this equation describes a so called Fabry-Perot mode. The laser will only emit light at wavelengths that correspond to one of these (longitudinal) cavity modes.

The threshold for lasing is defined as the point where the optical losses during one round trip through the cavity are exactly compensated by the optical gain. Or in other words when the cavity has a round trip gain of 1:

$$R_1R_2e^{2gL} = 1$$

Where $R_1$ and $R_2$ are the reflection coefficients of the cavity end mirrors, $L$ is the length of the cavity and $g$ is the net power gain per unit of length of the laser material. From this equation we get an expression for the threshold gain:

$$g_{\text{threshold}} = \frac{1}{2L} \ln \frac{1}{R_1R_2}$$

To explain the operation principle of optical gain we first consider a simple two-level system with energy levels $E_1$ and $E_2$ respectively and $E_2 > E_1$. In the unpumped situation, the laser material will be in thermal equilibrium and the majority of the electrons will be in the lower energy state. The more the material is pumped, the more electrons will be excited and will occupy the higher energy state.

There are a number of processes in which an electron makes a transition between these energy bands. They can be divided into radiative processes, that involve the emission or absorption of a photon, and non-radiative processes. Here we will only consider the radiative processes. When an electron makes a transition from energy level $E_2$ to $E_1$, a photon with energy $\Delta E = E_2 - E_1$ will be emitted. An electron transition from $E_1$ to $E_2$ absorbs a photon with energy $\Delta E$. The energy $\Delta E$ is proportional to the frequency $\nu$ of the photons:

$$\Delta E = h\nu$$

where $h$ is Planck’s constant. The processes that involve the emission or absorption of a photon are shown in figure 1.5a to 1.5c. The spontaneous emission process shown in figure 1.5a, in which an electron makes a spontaneous transition from $E_2$ to $E_1$, emits a photon with a random phase. The process in figure 1.5b is called stimulated emission. Here an electron transition from $E_2$ to $E_1$ is initiated by an incident photon with energy $\Delta E$. An extra photon is created which has the same energy $\Delta E$ and the same phase as the original photon. In the absorption process shown in figure 1.5c a photon of energy $\Delta E$ is absorbed while an electron makes a transition from $E_1$ to $E_2$. 
To start laser operation, some initial photons are necessary because the gain mechanism can only produce photons when some photons are already present. The initial photons are supplied by the spontaneous emission mechanism. The gain mechanism is determined by the stimulated emission and absorption mechanisms. According to [4] the absorption/emission rates $Z$ (per second per unit volume) can be defined as follows:

\[
Z_{\text{stimulated}} = B_{21} n_2 \rho(\nu_{21})
\]

\[
Z_{\text{absorption}} = B_{12} n_1 \rho(\nu_{21})
\]

Where $B_{12}$ and $B_{21}$ are transition probabilities with $B_{12} = B_{21} = B$, $n_1$ and $n_2$ are the electron densities in the bands $E_1$ and $E_2$ respectively and $\rho(\nu_{21})$ represents the optical field at frequency $\nu_{21} = \frac{\Delta E}{h}$. The net optical gain (per second per unit volume) can be defined as:

\[
g_{\text{optical}} = Z_{\text{stimulated}} - Z_{\text{absorption}} = B(n_2 - n_1) \rho(\nu_{21})
\]

So the net optical gain $g_{\text{optical}}$ is positive when $n_2 > n_1$. This situation ($n_2 > n_1$) is called population inversion. Population inversion can be achieved by pumping the laser material with a pump energy that is above a certain threshold level.

In most lasers the two level system representation is too simple. In solid state and semiconductor lasers the discrete levels are broadened to bands, so in stead of two energy levels $E_1$ and $E_2$, there are two bands of energy levels close together, one around $E_1$ and one around $E_2$. So there is not one specific $\Delta E$, but a whole range of possible $\Delta E$'s. This means the laser will have gain (and spontaneous emission) over a range of optical frequencies. The width of this gain spectrum depends on the type of laser. Gas lasers for instance only have two discrete energy levels between which radiative electron transitions take place and therefore their gain spectrum consists of discrete frequencies only. Dye lasers on the other hand can have a typical broad gain spectrum of about hundred THz.
1.2.2 Semiconductor lasers

Semiconductor lasers, also called diode lasers, are the smallest and most inexpensive type of lasers available. This makes them very popular for low power applications (< 500 mW).

From an electrical point of view, semiconductor lasers are diodes, consisting of n-doped and p-doped semiconductor materials that join at a so-called pn-junction. One of the junction properties is that it only conducts current in one direction. In general, current is conducted by electrons in the conduction band in n-doped material and by empty electron states ('holes') that move through the valence band in p-doped material. When a current is applied in the forward direction, electrons are injected in the conduction band at the n-side and holes are injected...
in the valence band at the p-side. The injected electrons and holes drift to the pn-junction where they recombine. In the case of a laser diode the area where the carriers recombine is called the active region.

Depending on the type of semiconductor material (GaAs for instance), the recombinations in the active regions can be radiative: photons are emitted. This is used in Light Emitting Diodes (LED's) and in laser diodes. More specific information about the properties of diodes and semiconductors in general can be found in a lot of text books (for example reference [6]).

In the active region electrons in a high energy state (conduction band) make a transition to a lower energy state (valence band) and emit photons as described in the previous section. This implies that the emission wavelength is determined by the energy difference between the conduction and valence band, the bandgap of the semiconductor. Semiconductor lasers typically emit at wavelengths between the green-blue part of the electromagnetic spectrum (300–400 nm) up to the far infrared (a few microns). The width of the gain spectrum is determined by the distribution of electron states in the bands. The typical width of the gain spectrum is some tens of nanometers, depending on the semiconductor materials.

For laser operation, the net optical gain should exceed its threshold value. So the density of electrons in the conduction band should be higher than in the valence band (population inversion). With an increasing current the densities of electrons in the conduction band and of holes in the valence band increase. At a certain current, population inversion is obtained and when the current is increased even more the threshold gain is reached. This current is called the threshold current \( I_{th} \) of the laser diode. Typical values for \( I_{th} \) are in the 20 to 80 mA range.

The mirror planes of the cavity are formed by the semiconductor/air interface. When a wave is incident on an interface between two materials with a different refractive index, Fresnel's law applies. The reflection coefficient is given by:

\[
R = \left( \frac{n_2 - n_1}{n_2 + n_1} \right)^2
\]  

(1.9)

Where \( n_1 \) and \( n_2 \) are the refractive indexes of the materials 1 and 2 respectively.

A commonly used semiconductor material for optical purposes is GaAs. GaAs has a refractive index of approximately 3.5. Air has a refractive index of 1, so the reflectivity of the mirror planes is about 30%. The typical length of semiconductor laser cavities is 300 \( \mu \)m. They have a typical threshold gain (equation 1.4) of 40 cm\(^{-1}\). Their typical gain at normal operating conditions is 100 cm\(^{-1}\).
Chapter 2

Injection seeding neuron

The injection seeding neuron is a concept for an all-optical neuron. Optical, because it has optical inputs and an optical output. The first section of this chapter shows a simplified analysis of such an injection seeding based neuron in order to describe its basic principles of operation. In the second section the laser phenomena involved are described in more detail.

2.1 Principle of operation

As seen in the introduction, the output of a neuron can be described by:

\[ y = f \left( \sum w_i \cdot x_i \right) \] (2.1)

with output \( y \), non-linear function \( f \), weight vector \( \vec{w} \) and input vector \( \vec{x} \).

The non-linear function is realised by injecting light into a non-lasing mode of a laser. Assume that mode \( m_0 \) is the lasing mode. If light from an external source is injected in a non-lasing mode, say \( m_1 \), then this mode will start lasing when the injected power is above a certain threshold level. At the same time mode \( m_0 \) is turned off (stops lasing) due to mode competition. This is shown in figure 2.1. When the injected power is made proportional to the weighed sum of inputs \( \sum_i w_i \cdot x_i \), this system realises a neuron with the power in mode \( m_1 \) as output.

To be able to control the threshold and shape of the sigmoid, controllable optical feedback is applied to the laser diode for each mode separately. The laser is operated such that it can only lase when external optical feedback is applied. Both optical feedback and external injection of light...
in a mode can, in this simple analysis, be represented by a reduction of the losses for that particular mode. A schematic representation of the gain/loss profile with controllable losses (controllable feedback/external injection) is shown in figure 2.2. A mode will start lasing when the losses for that mode are compensated by the gain as shown for the second mode in figure 2.2.

Figure 2.2: Schematic representation of the gain/loss profile with adjustable losses (by feedback and/or injection) for each mode separately. The resulting spectrum is shown in the lower part of the figure.

Figure 2.3: Neuron operation with controllable feedback and injection of light. The feedback in mode n is held constant in all three situations (note that the gain curve has shifted down in figure c). The white bars represent a decrease of losses by optical feedback, the grey bars a decrease of losses by injection of light.

Neuron operation is shown in figures 2.3a to 2.3c. In these figures two (arbitrary) modes are considered: the negative mode (n-mode) and the positive mode (p-mode). The reason for these names will be explained in the next section. In the initial situation feedback is given in mode n such that this mode will start lasing. A signal with a power proportional to the weighed sum of inputs is in-
2.1 Principle of operation

JECTED in mode p. When the weighed sum of inputs is larger than zero but still below threshold, we get the situation of figure 2.3a. When the neuron threshold is reached as shown in figure 2.3b, both the n-mode and the p-mode will lase. Finally, when the weighed sum of inputs is above threshold, the p-mode will be the only lasing mode as shown in figure 2.3c. These three situations are also indicated in the sigmoid function in figure 2.1 with a, b and c respectively.

![Diagram](image)

**Figure 2.4:** Changing the threshold level by controlling the feedback levels.

By controlling the feedback levels in both modes separately the threshold of the neuron can be influenced. When more feedback is applied to mode n in the initial situation, more injected power in mode p will be necessary to reach the threshold for mode p (figure 2.4b). On the other hand, when initial feedback is also applied to mode p such that mode n still is the active mode when no injection is applied, less injected power in mode p will be necessary to reach threshold (figure 2.4c).

### 2.1.1 Negative weights

In optics only positive signals exist and consequently these optical signals can only be weighed with positive factors. In neural networks however a lot of function classes require negative weights (see for instance figure 1.2b). These weights can reduce the weighed sum of inputs (assumed that the inputs are positive). So when a neuron is above threshold an extra input with a negative weight can bring the neuron below threshold again without any of the other inputs being changed. This is similar to changing the threshold level of the neuron.

In the previous section we have seen that the threshold of the sigmoid function can be changed by changing the feedback level of the n-mode. This threshold can also be changed by injecting light from an external source into the n-mode. To obtain a functionality that is comparable with negative weights two signals instead of one are injected: a signal $z_p$ in the p-mode (positive mode) and a signal $z_n$ in the n-mode (negative mode). $z_p$ decreases the threshold and therefore has a positive influence on the output (power in mode p). $z_n$ on the other hand increases the threshold resulting in a negative influence on the output power. Inputs with a positive weight should contribute to $z_p$ and inputs with a negative weight to $z_n$. Therefore the weight vector of the neuron is divided into a $w^+$ and a $w^-$ vector as follows:
With these weight vectors two positive signals can be constructed:

\[ w_i^+ = \begin{cases} w_i & \text{if } w_i > 0 \\ 0 & \text{if } w_i \leq 0 \end{cases} \quad (2.2) \]

\[ w_i^- = \begin{cases} -w_i & \text{if } w_i < 0 \\ 0 & \text{if } w_i \geq 0 \end{cases} \quad (2.3) \]

\[ z_p = \sum_i w_i^+ \cdot x_i \quad (2.4) \]

and

\[ z_n = \sum_i w_i^- \cdot x_i \quad (2.5) \]

Figure 2.5: Neuron operation with a signal \( z_n \) bringing the neuron below threshold, without a change in the signal \( z_p \).

To explain the operation principle we start with the last situation in figure 2.3, also shown in figure 2.5a, where the neuron is above threshold. The signal \( z_p \) is held constant. When \( z_n \) is increased the neuron will first reach threshold again (figure 2.5b) and will then go below threshold (figure 2.5c). So by applying an extra input with a 'negative' weight, the neuron can go below threshold again. This means we indeed have obtained a negative weight functionality.

### 2.1.2 Neural network using the injection seeding neuron

To make a neural network with injection seeding neurons, the signals \( z_p \) and \( z_n \) have to be constructed from the inputs. These inputs go into two so called optical vector matrix multipliers, one to construct \( z_p \) for each neuron and the other to construct \( z_n \) for each neuron. These multipliers are shown schematically in figure 2.6. The inputs are defined by optical powers at wavelength \( \lambda_p \) for the \( W^+ \)-multiplier and \( \lambda_n \) for the \( W^- \)-multiplier. Note that the two sets of inputs \( x_1 \ldots x_3 \) in the figure
both represent the same optical powers, only at a different wavelength. In the multipliers transmission elements provide the weighing. The transmission of an element is proportional to its corresponding weight. The weight vector of a neuron is on the row corresponding to that neuron. The inputs are projected on the columns of the transmission matrix. By adding the signals in each row, the signals $z_p$ ($w^+$-matrix) and $z_n$ ($w^-$-matrix) are constructed for each neuron separately. These signals are injected into the neuron lasers. For simplicity it is assumed that the (converted) signals from the different inputs add coherently. In reality this might be hard to achieve. This is discussed in section 5.2.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2_6}
\caption{Optical vector matrix multiplier for an injection seeding neural network with inputs $x_1 \ldots x_3$, positive weight matrix $W^+$, negative weight matrix $W^-$ and the constructed signals $z_{p,1} \ldots z_{p,4}$ and $z_{n,1} \ldots z_{n,4}$.
}
\end{figure}
2.2 Laser phenomena

A schematic representation of a possible setup of the injection seeding neuron is shown in figure 2.7. It consists of a laser diode ('neuron laser') with one anti-reflection coated facet. The setup of the external cavity is such that the feedback (reflectivity) can be controlled separately for each longitudinal cavity mode of the laser diode. Through the beam splitter light from another laser can be injected into the neuron laser. The injection seeding neuron uses three laser phenomena that are well-known in literature [5, 7]: optical feedback, injection seeding (injection locking/mode locking) and mode competition. These will be discussed below.

![Figure 2.7: Schematic representation of the injection seeding neuron.](image)

2.2.1 Optical feedback

Laser diodes are extremely sensitive to light reflected back into the laser cavity. These reflections can alter the operation characteristics of the laser enormously. Therefore this phenomenon has been studied thoroughly in literature, especially because of its negative influence on the performance of optical communication systems (for instance [7, 8]).

According to Tkach and Chraplyvy [9] five feedback regimes can be distinguished, depending on the amount of feedback. In figure 2.8 the different regimes are shown as a function of external cavity round trip time ($\tau_{ext}$) (which is proportional to the distance of the external reflector) and effective external reflectivity ($R_{ext}$) for a distributed feedback (DFB) laser. Although we use a different type of laser, the effects of optical feedback are comparable. In regime I the feedback only causes the spectral width of the laser emission to be somewhat broadened or narrowed (depending on the feedback phase). In regime II the laser can lase at multiple external cavity modes. Depending on the feedback phase the laser hops between these modes or has stable single mode emission at the external mode with the smallest spectral line width (because this is the most stable mode [7]). In this last situation there is also a considerable spectral line width narrowing. Regime III shows a single mode emission at the external mode with the smallest spectral line width, independent of the feedback phase. Regime IV is called the coherence collapse regime and shows mode hopping over several external cavity modes, a
very large spectral line width broadening and an enormous increase in intensity noise. Finally in regime V the feedback reaches a level such that the instabilities of regime IV disappear again and the lasing modes are imposed by the external cavity.

Since the characteristics of the laser emission depend strongly on the feedback regime, it is important to know in what feedback regime the system is operating. The neuron laser has an anti-reflection coating on one side so the feedback will be strong and the transition from regime IV to V should be considered. The system will operate in regime V when $R_{ext} \gg R_{laser}$ ($R_{ext}$ accounts for all the losses in the external cavity and not only the reflectivity of the end mirror). The neuron laser diode should only lase when external feedback is applied, so it has to be operated well below its threshold current (which is high because of the applied coating). This means that laser operation is determined by the external cavity. This implies that the reflectivity of the external cavity $R_{ext}$ has to be larger than the reflectivity of the laser facet with the anti-reflection coating. Hence as long as the current through the laser diode is smaller than its threshold current, the system is operated in regime V and the cavity is formed by a laser facet on one side and the external mirror on the other side. Frequencies close to an internal mode frequency of the laser diode will have a higher gain than other frequencies because of the residual reflectivity of the coated laser facet. This means that the minimum feedback level necessary for laser operation will be smaller for external cavity modes close to an internal cavity mode.

### 2.2.2 Injection seeding

Injection locking (or injection seeding) is a technique that was first described by Van der Pol in 1927 for electronic circuits [10]. The technique applies in general
to oscillating circuits with a saturable gain that are driven by an external source. Under the right circumstances, the circuit will lock to the external signal meaning that it will oscillate at the same frequency as the external source. This technique can also be applied to semiconductor lasers.

One of the most important parameters in injection seeding is the detuning, the frequency difference between the injected signal and the natural frequency of the oscillator. When this detuning is within the so-called locking bandwidth, the oscillator will lock to the injected signal. The optimal detuning is the detuning that will result in the highest output power. In most injection seeding applications the locking bandwidth is symmetric with respect to this optimal detuning. In semiconductor lasers however the locking bandwidth is asymmetrical due to the dependence of the refractive index on the carrier density [11]. Another difference is that part of the (statically stable) locking bandwidth is not dynamically stable. In general dynamic instability occurs when a system has an equilibrium in which small perturbations (of the carrier number and/or the photon number in our case) are amplified and the system will not return to this equilibrium (compare a pendulum in the upright position).

A semiconductor laser has multiple resonance frequencies (cavity modes) at which it can lase. Under the right conditions injection locking is possible for each of these modes. Two types of injection seeding can be distinguished: main-mode and side-mode (or intermodal) injection seeding. Main-mode injection seeding means that light is injected in the lasing mode. When light is injected in a non-lasing mode this is called side-mode or intermodal injection seeding. When the laser locks in the case of side-mode injection seeding, the main mode will go off and the laser will start lasing at the injected side-mode.

For the injection seeding neuron the most important case is side-mode injection seeding, but also the main-mode case is interesting. For side-mode injection seeding, it is shown both theoretically [12, 13] and experimentally [14] that three injection seeding regimes can be distinguished:

1. **Stable single mode operation (locked operation).** In this regime the laser emits single mode at the injected side-mode.

2. **Unstable operation.** In this regime chaotic behavior can occur.

3. **Stable two signal operation.** Here the laser emits at the injected frequency and at the frequency of the main mode simultaneously. The relative strength of the modes depends on the injected power and the power of the free-running mode [12].

In which of these regimes the laser operates, depends on a number of parameters, the most important being the injected power, the laser current (power of the lasing mode) and the injected mode (the gain difference between the main mode and the injected mode). To give an impression of the shape and size of the
different regimes and their dependence on the parameters mentioned above, the results of the calculations on the locking bandwidth as done by Debernardi [12] are shown in figures 2.9a to 2.9c. In these figures the black areas represent stable single mode operation, the white areas represent unstable operation and the grey areas represent stable two signal operation. Figure 2.9a shows that the laser will not lock to the injected signal anymore above a certain current value. The magnitude of this current depends on the injected power. One can say that the laser will not lock anymore when the main mode is too strong to be overcome. It is expected that adjusting the feedback level for the lasing mode of the neuron laser has a similar effect as changing the current since it also changes the power in this mode. In figure 2.9b the same is plotted as in figure 2.9a but now as a function of the injected power and detuning at different current levels. In figure 2.9c the different regimes are shown as a function of injected power and detuning at different injected modes. The different locking bandwidth is caused by the gain difference between the main (lasing) mode and the injected side-mode. It is expected that changing the feedback level in the injected mode of the neuron laser has a similar effect.

When applying injection seeding to the neuron laser, the neuron should have a well defined spectrum with one emission line in both the n-mode and the p-mode. Therefore the neuron should not operate in the unstable regime. To achieve this the detuning should be chosen such that the unstable regime does not exist.

2.2.3 Mode competition

There are several processes that contribute to mode competition. Here the most important process is discussed.

The gain of all modes is caused by stimulated emission. The amount of the gain depends on the difference between the density of electrons in the conduction band and the density of holes in the valence band (population inversion). When an optical field is amplified it will in general decrease the population inversion. Therefore the amplification of this optical field will decrease the gain for all modes. When two optical fields at different frequencies (two modes) exist they will use carriers from the same 'stock', so they have to compete for carriers. The mode with the smallest threshold gain will win. This mode will 'eat away' the most carriers and so it decreases the number of carriers that can be used by the weaker mode, which will get even weaker. This will go on until the weaker mode is under threshold. This weaker mode now has been turned off by mode competition. Mode competition can also be understood if one realises that there is only a limited amount of power available (at a constant current), that has to be shared by all modes.
Injection seeding neuron

Figure 2.9: Dependence of the injection regimes on the injected power \( P_{loc} \), the detuning and the current through the laser (after Debernardi [12]). \( m \) denotes the number of the injected mode (\( m = 0 \) is the main mode).
Chapter 3

Numerical simulations

Calculations on and simulations of the injection seeding neuron can provide a good (theoretical) understanding of its behaviour. Performing these calculations and simulations requires a model that describes the behaviour of a semiconductor laser under injection seeding. With this model the principle of operation of the injection seeding neuron can be verified theoretically. In the first section the multi-mode rate equations model is presented. In the second section two methods of using this model in simulations are described. In the third section the model is applied to the injection seeding neuron and the sigmoid function is examined by using one of the methods described in the second section.

3.1 Multi-mode rate equations model

The multi-mode rate equations model is a well-known and widely used model that has proven to be accurate [5, 7, 15, 16, 17, 18, 19]. It consists of a set of coupled differential equations. Each mode \( m \) has two equations associated with it: a photon equation and a phase equation. There is one equation that describes the number of carriers in the active region of the laser diode. The form of these equations depends on what phenomena are included. In their most simple form the equations describe a solitary laser diode. Terms that describe noise, injection seeding and weak optical feedback can be added to these simple equations.

There are a lot of different conventions and notations used to describe the properties of the laser in the model. The model explained in this section uses most of the conventions as defined by Agrawal [5]. An overview of all the symbols used in this chapter can be found in table 3.2 on page 43.

First the model for a solitary laser diode is given with the definitions and assumptions used. Furthermore, additional terms that account for noise, injection seeding and weak optical feedback are introduced, one at a time. The solitary rate equations are given by [5]:
Numerical simulations

\[ \dot{P}_m = (G_m - \gamma_m) P_m + R_{sp} \]  \hspace{1cm} (3.1)

net optical gain \hspace{1cm} spontaneous emission rate

\[ \phi_m = -\frac{\bar{\mu}}{\mu_g} (\omega_m - \Omega_m) + \frac{1}{2} \beta_c (G_m - \gamma_m) \]  \hspace{1cm} (3.2)

frequency shift relative to the frequency at threshold \hspace{1cm} phase change due to a carrier induced change of the refractive index

\[ \dot{N} = \frac{I}{q} - \frac{\gamma_e}{\gamma_c} N - \sum_m G_m P_m \]  \hspace{1cm} (3.3)

injected carriers \hspace{1cm} carrier losses due to spontaneous recombination \hspace{1cm} carrier losses due to stimulated emission

In the phase equation \( \frac{\bar{\mu}}{\mu_g} \) accounts for the frequency dependence of the refractive index (see below). \( N \) denotes the carrier number in the active region and is related to the carrier density \( n \) by:

\[ N = V n \]  \hspace{1cm} (3.4)

where \( V \) is the volume of the active region.

The most important phenomena described by these equations are:

- relaxation oscillations and modulation characteristics
- mode competition
- threshold current
- carrier dependent wavelength shifts (frequency chirping)

The following definitions and assumptions are used in this model:

**gain** The expression for the (main-mode) gain is:

\[ G_0(n) = \Gamma v_g g(n) \]  \hspace{1cm} (3.5)

where \( \Gamma \) is the confinement factor, \( v_g \) the group velocity and \( g(n) \) the carrier dependent gain. Only the part of the optical field that propagates through the active region of the laser diode experiences gain. The confinement factor \( \Gamma \) indicates what part of the optical field propagates through the active region. \( g(n) \) relates the gain to the carrier density \( n \). The gain is assumed
to be linear with respect to \( n \) around the working point \( n_0 \). \( n_0 \) is the carrier density at the onset of population inversion (also called transparency). This leads to the following expression for \( g(n) \):

\[
g(n) = a(n - n_0)
\]  

(3.6)

where \( a \) is the gain coefficient. This coefficient can be determined both theoretically and experimentally.

The spectral dependence of the gain is assumed to be parabolic. The gain at frequency \( \omega \) within the gain spectrum is:

\[
G(\omega - \omega_0) = G(\omega_0) \left( 1 - \left( \frac{\omega - \omega_0}{\Delta \omega_g} \right)^2 \right)
\]  

(3.7)

where \( \Delta \omega_g \) is half the width of the gain spectrum. Now the gain for a mode \( m \) can be defined as

\[
G_m = \eta_m G_0
\]  

(3.8)

with gain roll-off factor \( \eta_m \):

\[
\eta_m = 1 - \left( \frac{\omega_m - \omega_0}{\Delta \omega_g} \right)^2 = 1 - \left( \frac{m}{M} \right)^2
\]  

(3.9)

where \( m \) is the mode number relative to the main mode and \( M \) the number of modes in the gain spectrum on each side of the main mode (so there are \( 2M + 1 \) modes within the gain spectrum).

**photon decay rate** Two loss mechanisms contribute to the photon decay in the active region of the laser diode: the internal losses (such as scattering) and the facet losses. The photon decay rate \( \gamma \) is given by:

\[
\gamma = v_g (\alpha_m + \alpha_{int}) = \tau_p^{-1}
\]  

(3.10)

where \( v_g \) is the group velocity, \( \alpha_m \) are the facet losses, \( \alpha_{int} \) the internal losses and \( \tau_p \) is the average photon life time.

The facet losses are distributed over the cavity length:

\[
\alpha_m = \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)
\]  

(3.11)

where \( L \) is the length of the active region (cavity) and \( R_1 \) and \( R_2 \) are the facet reflectivities.
**carrier decay rate** There are several processes that contribute to the spontaneous carrier decay rate $\gamma_c$: non-radiative recombination, radiative recombination and Auger recombination.

The carrier decay rate is given by:

$$\gamma_c = A_{nr} + Bn + Cn^2 = \tau_e^{-1}$$

(3.12)

where $A_{nr}$ is the non-radiative recombination rate, $B$ is the radiative recombination coefficient, $C$ is the Auger recombination coefficient and $\tau_e$ is the average carrier life time.

Often a much simpler model is used for the carrier decay rate (for instance in reference [7]). In this model $\gamma_c$ is taken independent of $n$ and all recombinations are assumed to be radiative.

**spontaneous emission** The spontaneous emission rate is equal to the rate at which carriers recombine radiative (spontaneously). If we consider an arbitrary mode, only a fraction of the total spontaneous emission will be injected in that particular mode. This is expressed by the spontaneous emission factor $\beta_{sp}$. The expression for the spontaneous emission yields:

$$R_{sp} = \beta_{sp}\eta_{sp}\gamma_c n = \beta_{sp}Bn^2$$

where $\eta_{sp} = \frac{Bn}{\gamma_c}$

(3.13)

where $\gamma_c n$ is the total number of recombinations per unit time and $\eta_{sp}$ expresses the part of the recombinations that are radiative.

In the simple case where $\gamma_c$ is independent of $n$ and $\eta_{sp} = 1$, the spontaneous emission is given by:

$$R_{sp} = \beta_{sp}\gamma_c n = \beta_{sp}\frac{n}{\tau_e}$$

(3.14)

**laser threshold** The laser threshold is defined as the point where the gain equals the losses for the main mode: $G_0 = \gamma_0$. From this expression the carrier density at threshold $n_{th}$ follows:

$$n_{th} = n_0 + \frac{\alpha_m + \alpha_{int}}{a\Gamma}$$

(3.15)

**refractive index** The refractive index $\mu$ depends on the carrier density and the frequency. These dependencies are assumed to be linear:

$$\mu = \bar{\mu} + \frac{\partial \mu}{\partial n}(n - n_{th}) + \frac{\partial \mu}{\partial \nu}(\nu - \Omega)$$

(3.16)

where $\Omega$ is the threshold frequency (see below).
**line width enhancement factor** The line width enhancement factor $\beta_c$ expresses the phase-amplitude coupling. This coupling is caused by the fact that both the refractive index $\mu$ and the gain $g$ depend on the carrier density $n$. The line width enhancement factor $\beta_c$ is defined by:

$$\beta_c = -2k_0 \left( \frac{\partial \mu / \partial n}{\partial g / \partial n} \right)$$

(3.17)

where $\mu$ is the refractive index, $n$ the carrier density, $g$ the gain and $k_0$ the wavenumber in vacuum ($k_0 = \omega / c$). Note that $\partial g / \partial n = a$.

**mode spacing** The general expression for the modes of the laser is:

$$\nu_m = m \frac{c}{2\mu L}$$

(3.18)

where $\nu_m$ is the mode frequency, $c$ the speed of light, $\mu$ the refractive index of the gain medium and $L$ the length of the laser cavity ($\mu L$ is the optical length of the cavity). Since $\mu$ varies with the carrier density and frequency, the (angular) mode frequencies at threshold $\Omega_m$ are defined by:

$$\Omega_m = 2\pi m \frac{c}{2\mu L}$$

(3.19)

where $\bar{\mu}$ is the mode index at threshold. Note that $\Omega_m$ is defined at the laser threshold for that particular mode: $G_m = \gamma_m$.

Taking into account the frequency dependence of $\mu$, the mode spacing of the laser is given by:

$$\Delta \nu = \frac{c}{2\mu_g L}$$

(3.20)

where $\mu_g$ is the group refractive index.

**output power** The output power from facet $x$ in mode $m$ is given by:

$$P_{m}^{\text{out}} = \frac{R_x}{R_1 + R_2} h \frac{\omega_m}{2\pi} v_g \alpha_m P_m$$

(3.21)

where $P_{m}^{\text{out}}$ is the output power, $h$ Planck's constant, $\omega_m$ the mode frequency, $v_g$ the group velocity and $P_m$ the photon number in the cavity. Note the difference between the photon number and the output power.

### 3.1.1 Noise-driven rate equations

A laser is a physical system so it inevitably exhibits noise. To account for noise, a noise term is added to each equation. The two most important noise sources in the physical system are added to the model: spontaneous emission noise and
the noise due to the discrete nature of carrier generation and recombination (shot noise). The spontaneous emission noise affects $P_m$ and $\phi_m$, while $N$ is affected by both the carrier shot noise and the noise in the photon numbers $P_m$. This leads to the following stochastic (noise-driven) rate equations [5]:

\[
\begin{align*}
\dot{P}_m &= (G_m - \gamma_m)P_m + R_{sp} + F_{P_m}(t) \\
\dot{\phi}_m &= -\frac{\bar{\mu}}{\mu_B}(\omega_m - \Omega_m) + \frac{1}{2}\beta_e(G_m - \gamma_m) + F_{\phi}(t) \\
\dot{N} &= \frac{I}{q} - \gamma_e N - \sum_m G_m P_m + F_N(t)
\end{align*}
\]  

(3.22) (3.23) (3.24)

The noise sources $F_{P_m}$, $F_{\phi}$, and $F_N$ are Langevin noise sources. The noise correlation time is assumed to be much shorter than the photon and carrier relaxation times. This is known as the Markovian assumption. The mean values and the auto- and cross correlation terms using this assumption are given by [5, 20]:

\[
\begin{align*}
E(F_i(t)) &= 0 \\
E(F_i(t)F_j(t - t')) &= 2D_{ij}\delta(t - t')
\end{align*}
\]  

(3.25) (3.26)

where $E(\ldots)$ depicts the expectation and $F_i(t)$ is one of the noise sources $F_{P_m}$, $F_{\phi}$, or $F_N$.

The correlation coefficients $D_{ij}$ are given by [5, 20]:

\[
\begin{align*}
D_{P_mP_m} &= R_{sp}\langle P_m \rangle \\
D_{P_mN} &= -R_{sp}\langle P_m \rangle \\
D_{NN} &= R_{sp}\sum_m \langle P_m \rangle + \gamma_e\langle N \rangle \\
D_{\phi\phi} &= \frac{R_{sp}}{4\langle P_m \rangle}
\end{align*}
\]

with $\langle P_m \rangle$ and $\langle N \rangle$ being the average (steady state) values of the photon numbers and the carrier number. All other coefficients $D_{ij}$ are 0.

Because of the noise sources the model now also describes the following laser phenomena:

- intensity noise
- phase noise
- line width

Some important properties concerning injection seeding and weak optical feedback are also due to noise. They will be presented in the next two sections.
3.1 Multi-mode rate equations model

3.1.2 Rate equations with injection seeding

When an external signal is injected in the laser, the injected light will change the operating characteristics of the laser. This is expressed in the following set of rate equations [18]:

\[
\dot{P}_m = (G_m - \gamma_m)P_m + R_{sp} + F_{P_m}(t) + 2k_c\sqrt{P_m(t)}\sqrt{P_i(t)}\frac{\cos(\phi_i(t) - \phi_m(t))}{\text{coherent adding of laser signal and injected signal}}
\]

\[
\dot{\phi}_m = -\frac{\bar{\mu}}{\mu_g}(\omega_m - \Omega_m) + \frac{1}{2}\beta_c(G_m - \gamma_m) + F_{\phi_m}(t) - \frac{\omega_i - \Omega_m}{k_c\sqrt{P_i(t)}} + \frac{\sin(\phi_i(t) - \phi_m(t))}{\sqrt{P_m(t)}} + \text{phase change due to the difference between the mode phase and the phase of the injected signal}
\]

\[
\dot{N} = \frac{I}{q} - \gamma_eN - \sum_m G_mP_m + F_N(t)
\]

In these equations, \(P_i\) is the intensity of the injected signal, \(\phi_i\) the phase of the injected signal and \(k_c\) the coupling coefficient. This coefficient expresses the resulting injected power in the cavity when light with a certain intensity incidents on the laser facet. This coefficient is given by [7]:

\[
k_c = \frac{1 - R}{\tau_L\sqrt{R}}
\]

where \(R\) is the reflectivity of the facet involved in the coupling and \(\tau_L\) the laser diode cavity round trip time.

The following injection seeding phenomena are described by this model [18]:

- frequency shift due to the injected signal
3.1.3 Rate equations with weak optical feedback

For modelling purposes the feedback regimes as described in section 2.2.1 are divided into two categories: weak feedback (regimes I to IV) and strong feedback (regime V). In the case of strong feedback each external cavity mode is treated as a separate mode with its own photon and phase equations. In the case of weak feedback however there is only a photon equation and a phase equation for each internal cavity mode. The effects of weak feedback are accounted for by adding a term to these equations that represents one reflection from the external cavity (multiple reflections from the external cavity are neglected). The equations in the case of weak optical feedback yield [5]:

\[ \dot{P}_m = (G_m - \gamma_m)P_m + R_{sp} + F_{P_m}(t) + \]

\[ 2 \frac{k_c}{f_{ext}} \sqrt{P_m(t - \tau_{ext})} \sqrt{P_m(t)} \cos(\Omega_m \tau_{ext} + \phi_m(t) - \phi_m(t - \tau_{ext})) \]

\[ \dot{\phi}_m = -\frac{\hbar}{\mu_g} (\omega_m - \Omega_m) + \frac{1}{2} \beta_c (G_m - \gamma_m) + F_{\phi_m}(t) - \]

\[ k_c \sqrt{f_{ext}} \frac{P_m(t - \tau_{ext})}{\sqrt{P_m(t)}} \sin(\Omega_m \tau_{ext} + \phi_m(t) - \phi_m(t - \tau_{ext})) \]

\[ \dot{N} = \frac{I}{q} - \gamma_e N - \sum_m G_m P_m + F_N(t) \]

In these equations \( \tau_{ext} \) is the external cavity round trip time, \( f_{ext} \) denotes what fraction of the output power is reflected back into the laser diode and \( k_c \) is the coupling coefficient as described in the previous section.
3.2 Methods of simulation

3.2.1 Steady state solution

Finding the steady state solution is a fast way to perform a simple analysis of the rate equations. Since noise essentially does not have a steady state, its time dependent character is neglected and only its mean value is taken into account. Some important instabilities (feedback regimes, dynamically instable region of the locking bandwidth) incorporated by the noise-driven equations in the case of injection seeding and weak optical feedback disappear when neglecting the time dependence of the noise. The steady state solution can however still be interesting in these cases because the (statically stable) locking bandwidth and the possible external cavity modes can be determined from it [5, 7].

To find the steady state solution the left hand side of the rate equations system is set to zero ($P_m = 0$, $\dot{\phi}_m = 0$ and $N = 0$). The steady state solution can be obtained in a simple way when there is no mutual coupling between the photon and phase equations. This is the case for the solitary laser diode equations and the equations with weak optical feedback. In the case of injection seeding however, the equations do not become independent unless $\cos(c\phi_i(t) - c\phi_m(t))$ is taken constant. The situation where $\cos(c\phi_i(t) - c\phi_m(t)) = 1$ is called optimum detuning, i.e. the detuning where the output power is at its maximum. When neglecting the mutual coupling is not desirable, the equations have to be solved time dependent as shown in the next section.

The equations can be solved by writing the photon equations in the form $P_m = f_m(N)$ and substituting these expressions into the carrier equation. This leaves us with the problem of finding $N$ for which $\dot{N} = 0$ is valid. With this $N$ the photon numbers $P_m$ can be calculated. This procedure is described in more detail in appendix A.

3.2.2 Time dependent solution

The rate equations form a complex non-linear system that is hard to solve analytically. Therefore they are solved time dependent by numerical integration. There are a number of different methods to perform the integration, but they all have in common that they use the differential equations to obtain the derivatives $\dot{P}_m$, $\dot{\phi}_m$ and $\dot{N}$ and use these to calculate the signals $P_m(t)$, $\phi_m(t)$ and $N(t)$.

There are two issues that require some special attention: the calculation of the phase derivative $\phi_m$ and the noise.
The phase equations contain the term \( -\frac{\bar{\mu}}{\mu_g} (\omega_m - \Omega_m) \) in which \( \omega_m \) is unknown because it still has to be determined. This problem can be solved by bringing this term to the left side of the equation and redefining \( \phi \) as:

\[
\dot{\phi}_{\text{new}} = \dot{\phi} + \frac{\bar{\mu}}{\mu_g} (\omega_m - \Omega_m)
\]

So, in steady state \( \dot{\phi} \) now represents the cavity frequency shift relative to the cavity frequency at threshold \( (\Omega_m)' \).

Since the numerical integration is done on a digital computer, a discrete version of the noise sources has to be used. Discrete versions of the noise terms are obtained by using a zero-order-hold with sample time \( \Delta t \). If the sample time is taken small enough \( (\frac{1}{\Delta t} \text{ larger than the bandwidth of the system}) \), the fact that discrete noise sources are used instead of real continuous noise, has no effect on the simulation results. The discrete noise sources are given by [16]:

\[
F_i[n] = \sqrt{D_{ij}} \sqrt{\frac{1}{\Delta t}} x_i
\]

where \( x_i \) is an independent Gaussian noise source with average value 0 and variance 1. This noise sequence is made with the random generator of the computer. The discrete noise sources for the photon, phase and carrier equations can be constructed by using equation 3.35:

\[
F_{Pm}[n] = \sqrt{\frac{2R_{sp}(P)}{\Delta t}} x_{Pm}
\]

\[
F_N[n] = \sqrt{2\gamma_e(N)} \Delta t x_N - \sqrt{\frac{2R_{sp}}{\Delta t}} \sum_m \sqrt{(P_m)} x_{Pm}
\]

\[
F_{\phi_m}[n] = \sqrt{\frac{R_{sp}}{2(P_m \Delta t)}} x_{\phi_m}
\]

To calculate the noise variances and auto- and cross-correlation coefficients \( (D_{ij}) \) the steady state values of \( P_m \) and \( N \) should be known. They can be obtained by finding the steady state solution of the equations as shown in the previous section. Because of the noise every time dependent solution of the rate equations will be different. To obtain accurate results multiple (equivalent) solutions have to be averaged. The number of equivalent solutions needed for accurate results can be determined by trial and error.

From the simulation results the relative intensity noise spectrum (RIN) and the spectrum of the signal can be calculated. The RIN is defined by [5]:

\[
\text{RIN} = \frac{S_p(\omega)}{(P)^2}
\]
where $S_p(\omega)$ is given by:

$$S_p(\omega) = \langle |\delta P(\omega)|^2 \rangle$$

(3.40)

$\delta P(\omega)$ is the Fourier transform of $\delta P = P - \langle P \rangle$, the perturbation of $P$ from its average value.

The spectrum of the signal is the power spectral density of the slowly varying amplitude of the optical field $E(t)$ [20]:

$$E(t) = \sqrt{P(t)} e^{-i\phi(t)}$$

(3.41)

The implementation of these calculations and the numerical integration in Matlab and Simulink is described in appendix A.

3.3 Simulation of the injection seeding neuron

3.3.1 Model

The injection seeding neuron from the previous chapter can be described as a two-mode system. Both modes have optical feedback and injection seeding as shown schematically in figure 3.1. Since the system is operating in feedback regime V, both modes have a photon and a phase equation like those in equations 3.27 and 3.28 in section 3.1.2. According to Agrawal [5] feedback from a passive external cavity can be seen as an effective reflectivity of the laser facet. This reflectivity includes the laser facet reflectivity, the external end mirror reflectivity and all losses inside the external cavity. $R_n$ and $R_p$ are the effective reflectivities for the n-mode and the p-mode respectively. These reflectivities are used to calculate the photon decay rate $\gamma_m$ (equation 3.10) for both modes.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.1.png}
\caption{Schematic representation of the injection seeding neuron.}
\end{figure}
3.3.2 Determining the neuron sigmoid function

The function of the output power in the p-mode $P_p$ versus the injected power in the p-mode $z_p$ should be a sigmoid function. To determine the non-linear (sigmoid) function a steady state analysis is used. This analysis is simplified by assuming that there is no noise (inherent to a steady state analysis), that both modes have a gain equal to the main-mode gain (because the modes are close together they only have a very small gain difference) and that the detuning is at its optimum for both injected signals. This leads to the following set of equations:

\[ \dot{P}_n = (G_0 - \gamma_n)P_n + R_{zp} + 2k_c \sqrt{P_n(t)P_{i,n}(t)} \]  

\[ \dot{P}_p = (G_0 - \gamma_p)P_p + R_{zp} + 2k_c \sqrt{P_p(t)P_{i,p}(t)} \]  

\[ \dot{N} = \frac{I}{q} - \gamma_c N - G_0 (P_n + P_p) \]

(3.42)  

(3.43)  

(3.44)

The phase equations are left out because they do not influence this set of equations and they do not add extra information about the sigmoid function.

For each set of $R_n$, $R_p$, $I$, $z_n$ and $z_p$ the steady state solution of the system is found as described in appendix A. The output power of both modes can be calculated by using equation 3.21. This equation can also be used reciprocal to calculate the injected power from the injected photon number.

The parameters used in the simulations are listed in table 3.1 on page 42.

\[ \text{Figure 3.2: Powers in mode } n \text{ and } p \text{ as function of the injected power in mode } p \text{ } z_p \text{ (} R_n = 0.32, R_p = 0.25, z_n = 0 \text{ and } I = 30 \text{ mA).} \]

In figure 3.2 the powers in mode $n$ and $p$ resulting from a simulation are plotted as function of the injected power in mode $p$, $z_p$. It can be seen that when the
injected power in mode p increases the power in mode p increases and the power in mode n decreases. The threshold power is defined as the minimum injected power in the p-mode necessary to make the p-mode the dominant mode. This threshold power is indicated with the dashed line in figure 3.2. At the point where mode p has the same power as mode n in the situation without injection, the power in mode n is almost 0 and the power in mode p start increasing less steep. This point is indicated in the figure with the dotted lines.

The threshold power and shape of the sigmoid function are influenced by the amount of feedback (\(R_n\) and \(R_p\)), the power injected in mode n (\(z_n\)) and the laser diode current \(I\). These influences are examined and discussed below.

![Figure 3.3: Output power as function of the injected power in mode p, \(z_p\), for different initial powers in mode n (\(R_p = 0.25, I = 30 \, mA\) and \(z_n = 0\)).](image)

![Figure 3.4: Threshold power as function of the initial power in mode n, \(P_n\) at \(z_p = 0\), for different laser diode currents (\(R_p = 0.25\) and \(z_n = 0\)).](image)

In figure 3.3 the sigmoid function is shown at different initial levels of \(P_n\) (dashed lines) with \(R_p = 0.25, I = 30 \, mA\) and \(z_n = 0\). With increasing initial power in mode n, the threshold power increases and the slope of the sigmoid
Numerical simulations

Figure 3.5: Output power as function of the injected power in mode p, \( z_p \), for different effective reflectivities in mode p, \( R_p \) \((R_n = 0.32, I = 30 \, mA \, \text{and} \, z_n = 0)\).

Figure 3.6: Threshold power as function of the effective reflectivity in mode p, \( R_p \), for different laser diode currents \((R_n = 0.32 \, \text{and} \, z_n = 0)\).

function will get less steep. In figure 3.4 the threshold power is shown as a function of the initial n-mode power at different currents with \( R_p = 0.25 \) and \( z_n = 0 \). It can be seen that there is a certain value of the initial n-mode power at which the threshold power starts increasing rapidly. At this point mode p is just under (laser) threshold in the initial situation \((z_p = 0)\). It can also be seen that this point moves to higher values when the current is increased. If the initial power in mode n is held constant while the current is increased (this requires a decrease of \( R_n \)), the threshold power will decrease. This is not surprising since the difference between \( R_n \) and \( R_p \) gets smaller in this way, so the laser thresholds of these modes are closer together and less injected power is needed to switch from mode n to mode p.

In figure 3.5 the sigmoid function is shown at different effective reflectivities \( R_p \) of mode p with \( R_n = 0.32, I = 30 \, mA \, \text{and} \, z_n = 0 \). With an increasing reflectivity the threshold power decreases and the slope of the sigmoid function gets steeper. In figure 3.6 the threshold power is shown as function of \( R_p \) at different currents...
3.3 Simulation of the injection seeding neuron

![Graphs showing output power as function of injected power in mode p, zn, for different injected powers in mode n, zn (R_n = 0.32, R_p = 0.25 and I = 30 mA).](image)

**Figure 3.7:** Output power as function of the injected power in mode p, zn, for different injected powers in mode n, zn (R_n = 0.32, R_p = 0.25 and I = 30 mA).

![Graph showing threshold power as function of injected power in mode n, zn, for different laser diode currents (R_n = 0.32 and R_p = 0.25).](image)

**Figure 3.8:** Threshold power as function of the injected power in mode n, zn, for different laser diode currents (R_n = 0.32 and R_p = 0.25).

and R_n = 0.32, zn = 0. In this case there is no sharp bend in the curve like in figure 3.4.

In figure 3.7 the sigmoid function is shown at different injected powers in mode n (zn) with R_n = 0.32, R_p = 0.25 and I = 30 mA. The figure shows that an increasing zn leads to an increasing threshold power and a decreasing steepness. In figure 3.8 the threshold power as function of zn is shown. For ideal neuron operation the threshold power should vary linearly with the injected power in the n-mode. It can be seen from the figure that the curve is almost linear at higher injection levels and it is not very sensitive to current changes.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity length</td>
<td>( L )</td>
<td>250 ( \mu )m</td>
</tr>
<tr>
<td>Active-region width</td>
<td>( w )</td>
<td>2 ( \mu )m</td>
</tr>
<tr>
<td>Active-layer thickness</td>
<td>( d )</td>
<td>0.2 ( \mu )m</td>
</tr>
<tr>
<td>Active-region volume</td>
<td>( V )</td>
<td>100 ( \mu )m(^3)</td>
</tr>
<tr>
<td>Confinement factor</td>
<td>( \Gamma )</td>
<td>0.3</td>
</tr>
<tr>
<td>Effective mode index</td>
<td>( \bar{\mu} )</td>
<td>3.4</td>
</tr>
<tr>
<td>Group refractive index</td>
<td>( \mu_g )</td>
<td>4</td>
</tr>
<tr>
<td>Line-width enhancement factor</td>
<td>( \beta_c )</td>
<td>5</td>
</tr>
<tr>
<td>Facet reflectivity (not a.r.-coated)</td>
<td>( R_1 )</td>
<td>0.32</td>
</tr>
<tr>
<td>Facet reflectivity (a.r.-coated)</td>
<td>( R_2 )</td>
<td>0.01</td>
</tr>
<tr>
<td>Facet losses ((R_1 = R_2 = 0.32))</td>
<td>( \alpha_m )</td>
<td>45 ( \text{cm}^{-1} )</td>
</tr>
<tr>
<td>Internal losses</td>
<td>( \alpha_{\text{int}} )</td>
<td>40 ( \text{cm}^{-1} )</td>
</tr>
<tr>
<td>Gain constant</td>
<td>( a )</td>
<td>2.5 ( \times ) 10(^{-16}) ( \text{cm}^2 )</td>
</tr>
<tr>
<td>Carrier density at transparency</td>
<td>( n_0 )</td>
<td>1 ( \times ) 10(^{18}) ( \text{cm}^{-3} )</td>
</tr>
<tr>
<td>Nonradiative recombination rate</td>
<td>( A_{\text{nr}} )</td>
<td>1 ( \times ) 10(^8) ( \text{s}^{-1} )</td>
</tr>
<tr>
<td>Radiative recombination coefficient</td>
<td>( B )</td>
<td>1 ( \times ) 10(^{-10}) ( \text{cm}^3/\text{s} )</td>
</tr>
<tr>
<td>Auger recombination coefficient</td>
<td>( C )</td>
<td>3 ( \times ) 10(^{-29}) ( \text{cm}^6/\text{s} )</td>
</tr>
<tr>
<td>Carrier lifetime at threshold</td>
<td>( \tau_e )</td>
<td>2.2 ( \text{ns} )</td>
</tr>
<tr>
<td>Photon lifetime</td>
<td>( \tau_p (\gamma^{-1}) )</td>
<td>1.6 ( \text{ps} )</td>
</tr>
</tbody>
</table>

Table 3.1: Typical parameter values for a 1.3 \( \mu \)m buried-heterostructure laser (after Agrawal [5]).
### 3.3 Simulation of the injection seeding neuron

<table>
<thead>
<tr>
<th>Device parameters</th>
<th>Operating parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>refractive index</td>
</tr>
<tr>
<td>$\bar{\mu}$</td>
<td>mode refractive index at threshold</td>
</tr>
<tr>
<td>$\mu_g$</td>
<td>group index</td>
</tr>
<tr>
<td>$v_g$</td>
<td>group velocity</td>
</tr>
<tr>
<td>$\tau_L$</td>
<td>internal cavity roundtrip time</td>
</tr>
<tr>
<td>$\alpha_{int}$</td>
<td>internal photon losses</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>photon losses at laser facets</td>
</tr>
<tr>
<td>$L$</td>
<td>length of the active region</td>
</tr>
<tr>
<td>$V$</td>
<td>volume of the active region</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>confinement factor</td>
</tr>
<tr>
<td>$R_1$, $R_2$</td>
<td>facet reflectivities</td>
</tr>
<tr>
<td>$A_{nr}$</td>
<td>non-radiative recombination rate</td>
</tr>
<tr>
<td>$B$</td>
<td>radiative recombination coefficient</td>
</tr>
<tr>
<td>$C$</td>
<td>Auger recombination coefficient</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>linewidth enhancement factor</td>
</tr>
<tr>
<td>$a$</td>
<td>gain coefficient</td>
</tr>
<tr>
<td>$\Delta \omega_g$</td>
<td>width of the gain spectrum</td>
</tr>
<tr>
<td>$M$</td>
<td>number of modes in the gain spectrum on each side of the main mode</td>
</tr>
<tr>
<td>$q$</td>
<td>elementary charge</td>
</tr>
<tr>
<td>$h$</td>
<td>Planck’s constant</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of light</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.2:** Overview of the symbols used in the rate equation model.
Chapter 4

Experiments

To demonstrate the operation principle of the injection seeding neuron, an experimental setup has been built, as described in the first section of this chapter. With this setup the first injection seeding neuron experiments have been performed. The measurements and results of these experiments are presented in the second section.

4.1 Experimental setup

The experimental setup for the realisation of the injection seeding neuron as described in chapter 2 can be divided into three parts:

1. A source laser setup to provide the light to be injected in the neuron. It should be possible to choose any mode of the neuron laser as the p-mode. Since the light of the source laser has to be injected in the p-mode, the source laser should be wavelength tunable.

2. The actual neuron, so a laser with adjustable feedback for each mode and a possibility to inject light. This part of the setup is called the neuron laser.

3. Measuring equipment to measure the spectrum and total output power of both lasers.

These three parts will be described in the next sections. An overview of the complete experimental setup is shown in appendix B.3.

4.1.1 Source laser

The required wavelength tunability of the source laser is obtained by using an anti-reflection coated laser diode with an external cavity. Because of the anti-reflection coating the laser diode acts as a bright, superluminescent light source. Only when the optical feedback from the external cavity mirror is strong enough, the laser will
lase with the external cavity acting as the laser cavity. The anti-reflection coating is applied to make the laser diode more sensitive to optical feedback, so less power has to be fed back into the laser diode before lasing operation starts. When the optical feedback is filtered such that only one wavelength is fed back into the laser diode, the laser will lase at the selected wavelength. Since the external cavity acts as a laser cavity, the mode spacing is determined by the length of the external cavity. It can be calculated from equation 3.20 with $\mu_g = 1$ (air) and $L$ the length of the external cavity.

![Figure 4.1: Reflection grating with groove spacing $d$, angle of incidence $i$ and angle of reflection $\phi$.](image)

The wavelength filtering in the external cavity is supplied by a reflection grating. Such a grating is a dispersive element, which in this case means that the angle of reflection depends on the wavelength of the incident beam. It consists of parallel grooves in a substrate of a reflecting material. Due to diffraction, the reflections from the different grooves will interfere constructively or destructively, depending on the differences in path length. This leads to the well-known grating equation:

$$d(\sin i + \sin \phi) = m\lambda$$

(4.1)

where $d$ is the groove spacing, $i$ the angle of incidence, $\phi$ the angle of reflection (both angles are relative to the normal of the grating), $m$ the order of the diffracted beam and $\lambda$ the wavelength. These parameters are shown in figure 4.1.

At a certain orientation of the grating, part of the first order of the diffracted laser beam will be reflected back into the laser diode. The wavelength this corresponds to is determined by the angle of the grating, so by changing this angle the laser can be tuned in wavelength. The relation between the angle and the reflected wavelength can be calculated from equation 4.1. The diffracted beam that is reflected back into the laser diode has the opposite direction of the incident beam, so $i = \phi$. With $m = 1$ this leads to an expression for $\phi$:

$$\sin \phi = \frac{\lambda}{2d}$$

(4.2)
In the experimental setup a certain wavelength range $\Delta \lambda$ is reflected back into the laser instead of one discrete wavelength. The width of this range is determined by the lens system used and the grating resolution. $\Delta \lambda$ can be minimized in the setup by choosing appropriate values for the groove spacing $d$ of the grating and the focal length $f$ of the lens. This is shown below.

The resolution of the grating $\delta \lambda$ is given by:

$$\frac{\delta \lambda}{\lambda} = \frac{m}{K}$$

where $K$ is the number of illuminated grooves of the grating.

**Figure 4.2:** Maximum allowed angle $\Delta \varphi$ for which an incident light ray is still projected in the active region of the laser with width $w$.

For a one lens system the maximum angle that is still reflected back into the laser (see figure 4.2) is given by (assuming $\sin \Delta \varphi = \Delta \varphi$):

$$\Delta \varphi = \frac{w}{f}$$

where $w$ is the width of the active region (typically 5-10 $\mu$m) and $f$ is the focal length of the lens (typically a few mm). $\Delta \varphi$ can be related to a wavelength range $\Delta \lambda$ by determining the dispersion of the grating $\frac{\partial \lambda}{\partial \varphi}$ from equation 4.2:

$$\Delta \lambda = \frac{\partial \lambda}{\partial \varphi} \Delta \varphi = 2d \cos \varphi \Delta \varphi$$

Substituting equations 4.2 and 4.4 in this equation and assuming that $\sin \Delta \varphi = \Delta \varphi$ gives an expression for $\Delta \lambda$:

$$\Delta \lambda = 2d \cos \left( \frac{\lambda}{2d} \right) \frac{w}{f}$$

The actual setup of the source laser is given in figure 4.3. The laser used in this setup is a commercially available Philips Optoelectronics, CQL-806 series, multiple quantum well laser with a nominal wavelength of 675 nm. One facet is anti-reflection coated with a residual reflectivity $\leq 1\%$ (The laser with the anti-reflection coating is only available for research purposes and not commercially available.)
The temperature of the laser diode is held constant with a Peltier element connected to a temperature controller. The external cavity is terminated by a grating with 2400 lines/mm. The reflection efficiency of the grating in the first order is optimal when the incident light has a horizontal polarisation. The laser however emits light with a vertical polarisation (indicated by the encircled arrow in figure 4.3). The polarisation of the laser could be rotated by rotating the laser diode, but because the cross section of the laser beam has an oval shape this would mean that less grooves of the grating would be illuminated, resulting in a worse grating resolution. Instead the polarisation state of the laser beam is rotated with a λ/2 plate. Depending on its angular position, such a λ/2 plate can rotate the polarisation of a beam over an arbitrary angle. Here, the angular position of the plate is adjusted such that the polarisation is rotated 90 degrees.

A beam splitter with a reflection coefficient of 30% is used to couple power out of the cavity. Part of this power is injected into the neuron laser. The optical isolator prevents light from the neuron laser being injected into the source laser cavity. The

**Figure 4.3:** Setup of the source laser. The encircled arrows indicate the polarisation of the light.
light coming from the isolator (in the forward direction) has a polarisation of 45°. The output power is controlled with a transmission element consisting of a one pixel LCD-display and a polariser. The LCD-display can rotate the polarisation of the incident light between 90° and 0°, depending on the applied voltage. The polariser only transmits the component in one (fixed) direction, so the power after the polariser depends on the amount of polarisation rotation by the LCD-display. The voltage for the LCD-display is supplied by a function generator that can be controlled by a PC.

The neuron laser is only sensitive for the vertical component of the polarisation of the injected light. In order to be able to measure the magnitude of the injected power correctly, the injected light should also be polarised vertically. The light transmitted from the polariser is already linearly polarised, so the polarisation state only has be be rotated to become vertically polarised. Again this rotation is done with a $\frac{1}{2}\lambda$ plate.

The distance between the laser and the grating $L$ is 20 cm. This leads to a mode spacing of 750 MHz (equation 3.20). The lens has a focal length of 4 mm and a diameter of 4 mm. This diameter determines the number of illuminated lines of the grating, which in turn determines the grating resolution. With this beam diameter the resolution of the grating $\delta \lambda$ is 0.041 nm. The range of wavelengths reflected back into the laser diode through the one lens system is 0.61 nm ($u = 5 \mu m, \lambda = 675 \text{ nm}$). Since this is larger than the grating resolution, the resulting $\Delta \lambda$ for the setup is 0.61 nm.

Ideally $\Delta \lambda$ should be less than or equal to the mode spacing. With a mode spacing of 750 MHz this requires a $\Delta \lambda$ of 0.0011 nm. This is hard to achieve with this setup however, so in our case $\Delta \lambda$ is larger than the mode spacing. Therefore the laser is allowed to start lasing on a set of external cavity modes within the feedback wavelength range $\Delta \lambda$. The mode with the highest total gain, determined by the feedback level and the laser gain, will start lasing.

### 4.1.2 Neuron laser

The neuron laser should have controllable feedback for each mode and it should have a way of injecting light into it. A setup to realise this is shown in figure 4.4. The neuron laser resembles the source laser and consists of a temperature controlled, anti-reflection coated laser diode with an external cavity. However, the optical filter in the external cavity is realised in a different way. This filter consists of a grating, a line LCD-display, an end mirror and two lenses as shown in figure 4.4. The modes are split and for each mode the power reflected back into the laser diode (the feedback efficiency) can be controlled with the LCD-display. So multiple modes can be reflected back into the laser diode simultaneously. With a beam splitter, light from the source laser is injected into the neuron laser. The $\frac{1}{2}\lambda$ plate is inserted for a better efficiency of the grating (see previous section).

To explain the operation principle of the mode splitting in the external cavity,
Figure 4.4: Setup of the neuron laser. The optical filter in the external cavity allows multiple modes to be reflected back into the laser diode simultaneously.

the part of the cavity between the grating and the end mirror is drawn schematically in figure 4.5.

Two issues are important to look at: all wavelengths (modes) should be separated spatially at some point, so the feedback efficiency can be controlled for all modes individually. Also each mode should be reflected back into the laser diode to provide optical feedback. Each parallel beam coming from the grating will have a focus in the focal plane of lens 1. The position of this focus depends on the angle of the dispersed laser beam and is therefore a function of the wavelength. Lens 1 projects point $S_1$ in point $S_2$ in the back focal plane of lens 2. The foci in the focal plane of lens 1 are projected on the cavity end mirror by lens 2. In this way

Figure 4.5: Paths of the laser beams in the filtering part of the neuron laser cavity.
4.1 Experimental setup

each wavelength (mode) will be reflected back following the same path as before, providing optical feedback for all modes. These conditions result in the following equation describing the optical system:

\[ b_1 + f_2 = f_1 + v_2 \]  (4.7)

where \( v_1 \) and \( v_2 \) are the object distances of lens 1 and 2 respectively, \( b_1 \) and \( b_2 \) are the respective image distances. In general the image distance and the object distance are related to the focal length of the corresponding lens by:

\[ \frac{1}{f} = \frac{1}{b} + \frac{1}{v} \]  (4.8)

When the focal lengths of the lenses and the position of the grating are known, these conditions specify the positions of the lenses and the end mirror unambiguously.

The LCD (line) display and the polariser are used to control the feedback for each mode individually. The LCD is placed in the focal plane of lens 1, because at this point all modes have a focus at a different position. The distance \( \Delta X \) between the foci of two beams, depends on the angle between these beams \( \Delta \varphi \) and is given by:

\[ \Delta X = f_1 \sin \Delta \varphi \]  (4.9)

\( \Delta \varphi \) can be related to a wavelength difference by using the dispersion of the grating \( \frac{\partial \varphi}{\partial \lambda} \) determined from equation 4.1:

\[ \Delta \varphi = \frac{\partial \varphi}{\partial \lambda} \Delta \lambda = \frac{m}{d \cos \varphi} \Delta \lambda \]  (4.10)

If the feedback is to be controlled for each mode then \( \Delta X \) for two adjacent modes (the mode pitch) should be equal to the line pitch of the display. The line pitch of the available LCD-display is 0.3 mm. The distance between the laser and the end mirror is approximately 1.5 m, resulting in a mode spacing of 100 MHz. Under these conditions it is not possible to make the mode pitch equal to the line pitch. Instead the pitch for the internal mode frequencies of the laser diode is made equal to line pitch. At these frequencies the gain will have a local maximum, so the laser will start lasing at the external cavity mode closest to an internal mode frequency. By applying appropriate voltages to the lines of the display, the feedback can be controlled for each laser diode cavity mode separately. These voltages are controlled with an LCD-driver with one channel (voltage) for each line. This driver can be controlled through the serial port of a PC.

4.1.3 Measuring equipment

In both the source laser and neuron laser light is coupled out for measuring purposes. The optical power and the spectrum for both lasers are measured. Because
Experiments

the detectors that are used for measuring the optical power are only linear in a limited part of their range, a calibration curve is measured that can be used to compensate for this non-linearity. The amount of light coupled into the detectors depends strongly on the alignment of the setup. Each time the alignment is changed the detectors are calibrated with another, calibrated detector. The calibration curves and the calibration procedure are described in detail in appendix B.

The spectrum can be measured on a wide wavelength scale with a low resolution and on a narrow wavelength scale with a high resolution. The measurement on the wide wavelength scale is done with an Optical Multichannel Analyser (OMA). In such an analyser the light is coupled into a monochromator containing a set of high quality diffraction gratings. The first order diffracted light is projected on a CCD-array. Hence each pixel of this array contains information about the optical power for a certain wavelength range. The analyser used, an EG&G Princeton Applied Research model 1460, has a CCD array with 1024 pixels and a total wavelength range of approximately 28 nm.

On a narrow wavelength scale the spectrum can be measured with a Fabry-Pérot type spectrum analyser. This device consists of a Fabry-Pérot interferometer containing mirrors with a very high reflectivity (> 99 %). One of the mirrors is mounted on a piezo element, so the length of the interferometer cavity can be changed, i.e. scanned. Like a laser cavity, this cavity only supports (transmits) a discrete number of modes with a certain width. The spacing of these modes is called the free spectral range and is determined by the interferometer cavity length, whereas the width of the transmission modes is determined by the finesse of the resonator. With the piezo element the transmission peaks (modes) can be scanned over the spectrum of the incident signal. The power transmitted through the cavity is measured with a detector. Displaying the voltage of the detector on an oscilloscope as a function of the piezo voltage results in the spectrum of the incident signal. The scanning of the piezo and the triggering of the oscilloscope are synchronised by a controller. The Fabry-Pérot spectrum analyser used in this setup is a Tropel model 240 with a free spectral range of 1.5 GHz, which equals 0.0034 nm at \( \lambda = 675 \) nm, and a resolution of about 7.5 MHz.

4.2 Measurements and results

4.2.1 Laser spectra

To get an impression of the emission region of both lasers, the spontaneous emission spectra of the source laser and the neuron laser as measured with the OMA are shown in figure 4.6a and figure 4.6b respectively. These figures show that the peak of the emission is centered around 684 nm.

The spectrum of the source laser as measured with the Fabry-Pérot spectrum analyser is shown in figure 4.7. From this figure it can be seen that although the
4.2 Measurements and results

The bandwidth $\Delta \lambda$ of the light coupled back into the laser is much wider than the mode spacing, the laser is still emitting a single mode signal.

Figure 4.8 shows the spectrum of the neuron laser when it is lasing in multiple longitudinal cavity modes. There are approximately 15 mode spacings within the free spectral range of 1.5 GHz, so the mode spacing is 100 MHz as expected from the calculations in section 4.1.2. In order to obtain this multi-mode spectrum, the alignment of the neuron laser had to be disturbed. When the setup is aligned properly, the neuron laser emits a single mode signal.

Figure 4.6: Spontaneous emission spectra at $I = 40 \ mA$.

Figure 4.7: Spectrum of the source laser.

Figure 4.8: Spectrum of the neuron laser with multiple (external) cavity modes lasing.
4.2.2 Optical amplification

When light from the source laser is injected in the neuron laser when the external cavity is not active (i.e. blocked), the laser will operate as an optical amplifier. The amount of amplification depends on the laser current and the injected wavelength.

The measurement of the optical amplification as a function of the wavelength can show the profile of the gain curve and the residual influence of the laser diode cavity of the neuron laser. Therefore two different measurements of the amplification as function of the wavelength are done:

1. Over a small wavelength region (1 to 2 mode spacings). This measurement will show the residual effect of the laser diode cavity. In our setup the wavelength tuning of the source laser was not accurate enough to use it as a tunable source for this measurement. Therefore the injected wavelength is held constant and the modes of the neuron laser are shifted by varying the temperature of the laser. As a result the internal modes of the neuron laser diode are scanned over the source wavelength. This method uses the dependence of the mode frequencies on the refractive index, which in turn depends on temperature.

2. Over a broad wavelength region (gain width of the laser). This measurement will show the profile of the gain curve. The source laser is tuned such that it is lasing at a frequency near an internal mode frequency of the neuron laser diode. The output power of the neuron laser is measured as a function of the injected wavelength. The injected power is held constant during these measurements.

The dependence of the gain of the laser on temperature is assumed to be small in these measurements.

The results of the two different wavelength dependence measurements are shown in figure 4.9. For the temperature control of the laser a temperature dependent resistor is used to measure the temperature. The resistor used in these measurements is not calibrated however and no absolute temperature measure can be defined. Figure 4.9a shows the dependence of the output power of the neuron laser as a function of the neuron laser temperature (arbitrary units) at an injected power of 91 $\mu$W and a current of 52.4 mA. The total temperature range in the figure is estimated to be around 2 °C. This estimation is based on the calibration of a similar resistor which has a temperature dependence of approximately 1°C/kΩ. As can be seen from this figure, the amplification is periodic with temperature. This is caused by the fact that subsequent (internal) modes are scanned over the injected wavelength when the temperature is changed. Figure 4.9b shows the wavelength dependence of the amplification on a larger wavelength scale. The injected power is 46 $\mu$W and the current is 52.3 mA. The ripples in the curve are
4.2 Measurements and results

(a) Output power of the neuron laser as function of its temperature ($P_{\text{inj}} = 91 \mu\text{W}$ and $I=52.4 \text{ mA}$).

(b) Output power of the neuron laser as function of the injected wavelength ($P_{\text{inj}} = 46 \mu\text{W}$ and $I=52.3 \text{ mA}$).

Figure 4.9: Cavity power as function of wavelength.

due to the measurement procedure: the source laser is wavelength tuned by rotating the grating until the output power of the neuron laser is at a local maximum (so the injected frequency is near an internal mode frequency of the neuron laser diode, see figure 4.9a). Since the tuning of the source laser is not continuously, the maximum output power found does not have to be the real local maximum.

Figure 4.10: Neuron laser output power as function of the injected power at different neuron laser diode currents $I$.

The optical amplification also depends on the neuron laser current. The output of the neuron laser is measured as a function of the injected power from the source laser. This measurement is done for several different currents. For each measurement the temperature of the neuron laser is tuned such that the injected
wavelength is at an internal mode wavelength of the neuron laser diode.

Figure 4.10 shows the results of the measurement of the output power as function of the injected power. The larger the current, the more the injected signal is amplified. For a certain neuron laser current, the amplification gets less at higher injected powers. This is due to the power saturation of the neuron laser output.

4.2.3 Sigmoid function

![Figure 4.11: Sigmoid functions of the neuron. The reflectivity of the p-mode, $R_p$, is higher in figure a than in figure b.](image)

The sigmoid function of the neuron is measured by measuring the power in the n- and the p-mode (see figure 3.1) with the OMA at increasing power values of the injected signal in mode p. The setup appeared to be extremely sensitive to vibrations. The neuron laser only locked to the injected signal for a short time (a few seconds in most cases) before it drifted away. Due to this stability problem only a few good measurements could be done. The instabilities are analysed in detail in the next section. The measurements are shown in figure 4.11. The detectors were not calibrated during these measurements, so the results are plotted as function of the relative injected power (transmission of the source laser LCD-display). The powers of the modes are plotted relative to the power of mode n without injection in mode p. In figure 4.11a the reflectivity of the p-mode was higher than in figure 4.11b, or in other words mode p was closer to threshold in the left figure and hence less injected power is needed in the p-mode for the p-mode to take over lasing operation from the n-mode. The neuron threshold power is indicated in these figures with a dashed line. When the slope of the power in the p-mode is considered well above threshold, i.e. when the power in mode n is almost 0, it can be seen that this slope is higher with a higher p-mode reflectivity. This can be
explained by the fact that the optical amplification in the p-mode increases when its reflectivity increases.

To give an impression how much injected power is needed to switch the output power of the neuron laser completely from the n-mode to the p-mode, an attempt was made to measure this injected power as function of the initial n-mode power. The closer the injected power came to its minimum value for switching, the harder is was to measure whether it was still possible to switch all the power from the n-mode to the p-mode. This is so because the allowed vibrations of the cavity length $L$ get smaller due to a decrease of $\frac{P}{P}$ as will be shown in the next section. In this setup 12.5 mW could be switched to the p-mode completely by injecting a power of 40 $\mu$W into the p-mode. This 40 $\mu$W is probably not the lowest possible injected power for switching 12.5 mW, but it gives an indication of the magnitude of the injected power needed to switch a certain amount of power, or in other words of the injected power needed to put the neuron above threshold (threshold as defined in section 3.3.2).

During the measurements, a number of different locking spectra were observed. These spectra correspond to the three categories as described in section 2.2.2 and in reference [14]. For each regime a spectrum is shown, as measured with the Fabry-Pérot spectrum analyser, in figure 4.12. The arrows indicate the injected frequency. An interesting remark is that in the unstable regime and the two signal regime external cavity modes were lasing at different frequencies than the injected signal. Without the injected signal however, the modes are below threshold. No explanation has been found for this phenomenon.

![Spectra of the neuron laser under injection seeding in different detuning regimes. The arrows indicate the frequency of the injected signal.](image)

**Figure 4.12:** Spectra of the neuron laser under injection seeding in different detuning regimes. The arrows indicate the frequency of the injected signal.

### 4.2.4 Stability of the setup

As mentioned before the setup suffers from instabilities. To minimize the mechanical vibrations of the optical table on which the setup is fixed, the table is mounted
Experiments on pneumatic isolators. The vibrations caused by air flow are reduced by putting covers over both the neuron laser and the source laser. These measures improved the stability, but they were not sufficient to eliminate the instabilities totally.

The remaining instabilities can be understood if the locking bandwidth is considered. If the length of the neuron cavity \( L \) changes, the frequencies of the modes also change. The shift of a mode frequency also means a shift of the locking band. So if the change in \( L \) is large enough, the injected signal can go out of the locking range and locking is no longer obtained. According to reference [21] the statically stable locking range for main mode injection locking is given by:

\[
-\rho \sqrt{1 + \beta_c^2} < \Delta \nu < \rho
\]  

and

\[
\rho = \frac{c}{2 \mu g L} \sqrt{\frac{P_i}{P}}
\]

where \( P_i \) is the injected power and \( P \) the power in the injected mode. Although only a part of this locking bandwidth is also dynamically stable, the bandwidth from equation 4.11 is used to estimate the required stability. If \( P \) is assumed to be a 100 times larger than \( P_i \) than \( \sqrt{\frac{P}{P_i}} \) is 0.1. This leads to a \( \rho \) of 0.1 times the mode spacing. If \( \beta_c \) is assumed to be around 4, the width of the locking bandwidth is around \( 5\rho = 0.5 \Delta \nu \approx 50 \text{ MHz} \). The mode frequencies are given by:

\[
\nu_m = \frac{m c}{2 L}
\]  

For stable locking, \( \nu_m \) may only shift within the locking band, so \( \Delta \nu_m \) may not be larger than \( \Delta \nu \). The allowed \( \Delta L \) with this restriction is given by:

\[
\Delta L = \left| \frac{\partial L}{\partial \nu_m} \Delta \nu_m \right| = \frac{2 L^2}{mc} \Delta \nu
\]  

\( m \) can be calculated from equation 4.12 with \( \nu = c/\lambda \). With equation 4.11 and \( \mu_g = 1 \) (air), this leads to an expression for \( \Delta L \) which is independent of \( L \):

\[
\Delta L = \frac{1}{2} \lambda (1 + \sqrt{1 + \beta_c^2}) \sqrt{\frac{P_i}{P}}
\]  

Substituting the appropriate values in this equation results in a \( \Delta L \) of 0.17 \( \mu \text{m} \). This is a very severe restriction. If the dynamically stable locking band is considered the allowed \( \Delta L \) will be even smaller. On the other hand, according to reference [27] the locking bandwidth is broadened for side mode injection locking due to the gain difference between the lasing mode and the injected mode. This will result in a larger \( \Delta L \). What the resulting change in \( \Delta L \) will be due to these effects is hard to predict.

The change in \( L \) that could cause the setup to drift from its desired setting, can be caused by an expansion of the optical table. This table is made of steal,
which has a typical temperature coefficient of about $10^{-5}$ m°C$^{-1}$. The length of the cavity is 1.5 m, so a temperature change of 1 °C will change the cavity length by 15 μm. The condition for $\Delta L$ is that it should be less than 0.17 μm. This corresponds to a temperature change of the optical table of approximately 0.01 °C. Another reason for a change in $L$ that is too large could be the drift of the cavity end mirror holder or the grating holder. This is probably not the case however, because different holders were tried both for the cavity end mirror and for the grating. None of these changes resulted in a more stable setup.
Chapter 5

Conclusions and recommendations

5.1 Conclusions

The simulations of the injection seeding neuron in section 3.3.2 show that the injection seeding neuron as described in chapter 2 indeed exhibits a sigmoid like non-linear behaviour. The threshold of this sigmoid function can be influenced by changing the feedback in the n-mode or the p-mode. The simulations also show that injecting a signal in the n-mode changes the threshold power almost linearly with the power of the injected signal in mode n. This means that a negative weight functionality is also possible with the injection seeding neuron. From these simulations can be concluded that when the wavelength detuning of both injected signals \( z_n \) and \( z_p \) is at its optimum value (maximum output power), the operating characteristics of the injection seeding neuron are consistent with the operation principle as described in chapter 2.

From the experiments on the sigmoid function (section 4.2.3) can be seen that the neuron does exhibit a sigmoid like behaviour. It also seems that the threshold can be influenced by changing the reflectivity of mode p. So from these measurements can be concluded that the injection seeding neuron indeed realises a sigmoid function by means of injection seeding. Hence the basic principle of operation is verified successfully. However in order to verify if the reflectivity of the p-mode really does influence the threshold of the neuron, still further experiments are required. The influence of injecting a signal in mode n \( z_n \) has not been investigated experimentally due to the instabilities as described in the previous chapter and a lack of time to tackle these instabilities.

The sigmoid functions obtained from the simulations seem to be consistent with the measured sigmoids. Because the number of measurements is very small, one has to be careful when drawing conclusions about the predictive value of the simulations from these experiments.
5.2 Recommendations

The locking characteristics of the neuron laser were neglected in the simulations of the steady state. These characteristics are however expected to influence the behaviour of the injection seeding neuron because of the dependence of the output power on the wavelength detuning and the dependence of the locking range on the injected power (see section 2.2.2). Further simulations that include a non-optimal detuning and noise sources can be done to investigate the influence of the detuning on the operating characteristics of the injection seeding neuron and to examine the width of the dynamically stable locking range.

Further experiments have to be done for a complete verification of the operation principle of the injection seeding neuron. For this purpose the setup has to be improved to overcome the instabilities as discussed in section 4.2.4. One measure that can be taken is to reduce the length of the cavity. This will improve the stability of the cavity in general, but it will also make the cavity less sensitive to expansion of the optical table due to temperature changes as discussed in section 4.2.4.

One way to reduce the cavity length is to use a different LCD-display. The new display should have a smaller line pitch and it should preferably operate in reflection instead of in transmission. A smaller line pitch reduces the focal length of lens 1 (figure 4.5) needed to achieve the necessary mode pitch. An LCD-display in reflection works as a mirror with variable reflection coefficients. This eliminates the need for the part of the setup between the LCD-display and the end mirror. A possible setup with a reflection LCD-display is shown in figure 5.1. In this setup the different wavelengths are focused on the LCD-display using only one lens. The total length of the wavelength selective part of the cavity (between the grating and the display in this case) is twice the focal length of the lens.

If equations 4.9 and 4.10 are combined the necessary focal length of the lens is

![Figure 5.1: A possible setup for the wavelength filtering part of the cavity of the neuron laser with an LCD-display in reflection.](image-url)
given by:
\[ f = \frac{\Delta X d \cos \varphi}{m \Delta \lambda} \]  (5.1)

There are line displays commercially available with a line pitch of 12 μm. If we assume a grating with 2400 lines/mm and \( \varphi = 75^\circ \) (which is true when \( i = 41^\circ \) and \( \lambda = 675 \) nm) then a lens with a focal length of 8.6 mm is necessary to project two adjacent internal modes (with a mode spacing of approximately 0.15 nm) on two adjacent lines of the display. This is the minimum necessary mode pitch. For a larger mode pitch, a lens with a larger focal length is necessary. Due to the physical sizes of the available equipment it will be hard to use a lens with a focal length of 8.6 cm. If a lens with a focal length of 5 cm is used instead, the total cavity length (including the part between the laser and the grating) is estimated to be around 25 cm. As a result the effect of the expansion of the optical table due to temperature differences becomes 6 times smaller.

In order to achieve injection locking, the setup has to be controlled such that the source laser wavelength is within the locking bandwidth of an external cavity mode of the neuron laser. In the current setup this is done by adjusting the length of the neuron cavity by a displacement of the end mirror. The present mirror holder however can not be moved accurately enough for a reliable tuning of the cavity length and hence of the wavelength. A solution would be to replace this holder by one that can be positioned more accurately.

![Figure 5.2: Possible new setup for the source laser with a Fabry-Perot cavity as the wavelength selective element in the laser cavity.](image)

Another way of achieving injection locking would be to shift the frequency of the source laser. However for this purpose the wavelength tuning of the current source laser setup is not accurate enough. This tuning can be improved by using a Fabry-Perot interferometer instead of a grating as the wavelength selective element in the external cavity of the laser. This is shown in figure 5.2. A Fabry-Perot interferometer consist of two mirrors, carefully aligned such that interference of the optical field travelling back and forth between those mirrors allows only certain wavelength components to be transmitted. The width of these transmission peaks is typically some tens of MHz, depending on the reflection coefficients of the mirrors (the higher these reflection coefficients, the smaller the transmission bandwidth).
Conclusions and recommendations

One of the mirrors of the Fabry-Pérot cavity is mounted on a piezo element. By controlling the voltage applied to the piezo, the distance between the two Fabry-Pérot mirrors can be tuned, and hence the wavelength of the transmission peak. Since the external cavity laser will lase at the wavelength of this transmission peak, the tuning of the interferometer mirror position results in a wavelength tuning of the laser.

To make the injection seeding neuron suitable for application in a neural network for a telecommunication application some issues have to be examined:

- The locking bandwidth of the neuron laser.
  This bandwidth limits the allowed wavelengths of the injected signal. The locking range has a typical width of a few GHz when the cavity is a few centimeters long. The wavelength of an optical signal in telecommunications is known with an accuracy of about 1 nm ($\approx 150$ GHz). In WDM (Wave-length Division Multiplexing) applications this accuracy is 10-20 GHz. So the restrictions on the wavelength of the injected signal due to the locking bandwidth of the neuron laser are much stricter than the wavelength accuracy of telecommunication signals permits. A possible solution would be to use a wavelength converter that has a well defined output wavelength, adapted to be within the locking bandwidth of the neuron laser and an input sensitivity for a wide range of wavelengths.

- The construction of the signals $z_n$ and $z_p$ from the inputs.
  In figure 2.6 the inputs $x_1 \ldots x_3$ for one vector matrix multiplier ideally have the same wavelengths ($\lambda_p$ or $\lambda_n$). In this ideal case the signals from the different inputs will add in the neuron laser. The behaviour of the neuron laser when such signals are injected is the same as when only one signal would be injected. In reality however there will be a small difference between the wavelengths of the inputs. The behaviour of the neuron laser when signals with a small wavelength difference are injected should be investigated. When the input signals are optical communication signals, the inputs will probably vary greatly in wavelength, so wavelength converters are necessary. It should be investigated with what accuracy the different input signals can be converted to the same wavelength.

- The physical size of the neuron.
  When making a neural network with injection seeding neurons, these neurons should be small in order to keep the size of the neural network limited. Therefore the injection seeding neuron should be miniaturized. By injecting the light at the non-coated facet of the laser diode, the beam splitter in the cavity becomes redundant. This and the use of a LCD-display with a small pixel pitch, should make it possible to reduce the cavity of the neuron laser to a few centimeters.
• Use of fiber optics
The use of fiber optics instead of free space interconnects should be examined. At telecommunication wavelengths optical amplifiers are available with a broad optical gain bandwidth. These amplifiers can be accessed by fibers from two sides and can be used to construct a neuron laser in a rather straightforward manner.
Bibliography


Appendix A

Numerical simulations

A.1 Steady state solution

For calculating the steady state the photon equations with $\dot{P}_m = 0$ have to be written as a function of $N$: $P_m(N) = f_m(N)$. This can be done readily for the case of the solitary equations and the equations with weak optical feedback. For the case of injection seeding the phase equation has to be neglected and a steady state phase has to be assumed in order to solve the equations simply. Here we assume optimum detuning, so the phase of the mode is equal to the phase of the injected signal. This leads to the following expressions for the photon number for the solitary case, the case with injection seeding and the case with weak optical feedback respectively:

$$P_m(N) = \frac{R_{sp}}{\gamma_m - G_m} \quad (A.1)$$

$$P_m(N) = \left( \frac{k_c \sqrt{P}_i}{\gamma_m - G_m} + \sqrt{\left( \frac{k_c \sqrt{P}_i}{\gamma_m - G_m} \right)^2 + \frac{R_{sp}}{\gamma_m - G_m}} \right)^2 \quad (A.2)$$

$$P_m(N) = \frac{R_{sp}}{\gamma_m - G_m - 2k_c \sqrt{f_{ext}} \cos(\Omega_m t_{ext})} \quad (A.3)$$

For each mode one of these expressions can be substituted into the carrier equation with $\dot{N} = 0$:

$$f(N) = \frac{I}{q} - \gamma_c N - \sum_m G_m(N) P_m(N) = 0 \quad (A.4)$$

This leaves us with the problem of solving this equation in $N$. Since the denominator of all three expressions will get zero for a certain $N$, $f(N)$ will have an asymptote for each mode. In the case of a solitary laser and injection seeding
Numerical simulations

Figure A.1: $\hat{N}$ as function of $N$ with three modes.

An example of $f(N)$ for a three mode system is given in figure A.1. As can be seen from this figure, the function $f(N)$ has $M+1$ zero crossings, where $M$ is the number of modes. $M$ zero crossings are related to an asymptote. The $M+1^{th}$ zero crossing is related to the steady state in the case that the laser is operated below threshold. A mode that has an asymptote on the left of this point is above threshold, when its asymptote is on the right the mode is below threshold. The system will stabilize at the zero crossing that has the smallest value of $N$.

To find this zero crossing, the positions of the asymptotes are calculated with equation A.5 or A.6, depending on the 'type' of mode (solitary, injection seeding, weak optical feedback). The $M+1^{th}$ possible zero crossing is approximated by solving the carrier equation without the optical field:

$$N = \frac{I}{q\gamma_e} \quad (A.7)$$
A.2 Solving the rate equations with Matlab and Simulink

The zero crossing with the smallest $N$ is found by starting at the smallest $N$ as calculated above. If this is one of the asymptotes, $N$ can be found by stepping to the left (smaller $N$), calculating $f(N)$ at each step, until a positive $f(N)$ is found. This is the first approximation of $N$. A more accurate $N$ can be found by repeating the procedure with a smaller step size from the smallest $N$ found where $f(N)$ is still negative.

When the smallest initial $N$ is the one found with equation A.7 the same procedure can be followed, only the direction of stepping has to be determined first: positive when $f(N)$ is positive and negative when $f(N)$ is negative.

A.2 Solving the rate equations with Matlab and Simulink

A.2.1 Simulink implementation of the injection seeding neuron model

The body of the Simulink model consists of an implementation of the carrier equation. The modes are represented in this implementation with blocks containing the photon and phase equations. The implementation is shown in figure A.2. Each mode is implemented with a separate block. There are three different type of blocks: a solitary mode block (figure A.3), a block for a mode with injection seeding (figure A.4) and a block for a mode with weak optical feedback (A.5). Each block takes as input a vector with two elements: the main-mode gain $G_0$ and the spontaneous emission rate $R_{sp}$. The block for a mode with injection seeding has a second input (vector) for the photon number and phase of the injected signal. The values for this vector can come from another simulated laser. The laser properties are defined in global variables.

The blocks for the modes are so called 'masked blocks', which means that the structure of the block (as shown in figures A.3 to A.5) is hidden and instead only input fields for a number of parameters are visible. Some of these parameters are common to all types of blocks and some are specific to a specific type of block.
Numerical simulations

The parameters that all blocks have in common are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode number</td>
<td>mode_nr</td>
<td>The mode number is equal to the distance in mode spacings of the considered mode and the main-mode. It is used to determine $\eta_m$</td>
</tr>
<tr>
<td>Initial state</td>
<td>initial_state</td>
<td>Vector with two elements, the first element being the initial photon number and the second the initial phase</td>
</tr>
<tr>
<td>Noise power</td>
<td>noise_power</td>
<td>Vector with two elements, the first element being the photon noise power and the second the phase noise power (see equations 3.36 and 3.38)</td>
</tr>
<tr>
<td>Effective facet reflectivity</td>
<td>R2_eff</td>
<td>Effective facet reflectivity</td>
</tr>
</tbody>
</table>

The extra parameters for the injection seeding block are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detuning</td>
<td>detuning</td>
<td>Detuning of the injected signal and the cavity resonance at threshold ($\omega_i - \Omega_m$)</td>
</tr>
</tbody>
</table>

The extra parameters for the weak optical feedback block are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>External cavity round trip time</td>
<td>tau_ext</td>
<td>External cavity round trip time ($\tau_{ext}$)</td>
</tr>
<tr>
<td>Feedback phase</td>
<td>feedback_phase</td>
<td>Phase of the reflected signal ($\Omega_m \cdot \tau_{ext}$)</td>
</tr>
<tr>
<td>Feedback fraction</td>
<td>f_ext</td>
<td>Fraction of the emitted power reflected back on the laser facet ($f_{ext}$)</td>
</tr>
</tbody>
</table>
The laser properties should be defined in the following global variables:

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Gain coefficient</td>
</tr>
<tr>
<td>beta_c</td>
<td>Line width enhancement factor ( \beta_c )</td>
</tr>
<tr>
<td>beta_sp</td>
<td>Spontaneous emission coefficient ( \beta_{sp} )</td>
</tr>
<tr>
<td>GAMMA</td>
<td>Confinement factor ( \Gamma )</td>
</tr>
<tr>
<td>k_c</td>
<td>Coupling coefficient ( k_c ) ([\text{s}^{-1}])</td>
</tr>
<tr>
<td>L</td>
<td>Cavity length ( L ) ([\text{m}])</td>
</tr>
<tr>
<td>modes_in_spectrum</td>
<td>Number of modes in the gain spectrum of the laser</td>
</tr>
<tr>
<td>n_0</td>
<td>Carrier density at transparency ( n_0 ) ([\text{m}^{-3}])</td>
</tr>
<tr>
<td>noise_sample_time</td>
<td>Noise sample time ([\text{s}])</td>
</tr>
<tr>
<td>q</td>
<td>Elementary charge ([\text{C}])</td>
</tr>
<tr>
<td>R1</td>
<td>Facet reflectivity</td>
</tr>
<tr>
<td>tau_e</td>
<td>Average carrier lifetime ( \tau_e ) ([\text{s}])</td>
</tr>
<tr>
<td>V</td>
<td>Volume of the active region ( V ) ([\text{m}^3])</td>
</tr>
<tr>
<td>v_g</td>
<td>Group velocity ( v_g ) ([\text{m/s}])</td>
</tr>
</tbody>
</table>

**A.2.2 Calculating the spectrum and RIN**

The calculations of the spectrum and the RIN both require a calculation of a power spectral density. Consider the general case where \( X(f) \) is the power spectral density of a signal \( x(t) \). \( X(f) \) can be calculated from its corresponding time dependent signal, \( x(t) \), by using a discrete Fourier transform. Assume that the variable \( x \) contains \( N \) (Matlab: \( N \)) samples of the signal \( x(t) \) with a sample time \( \Delta t \) (Matlab: \( \text{delta}_t \)). The following Matlab-code will calculate the \( N \)-point power spectral density \( X \) of \( x \):

\[
X = \text{abs(fft}(x)) \cdot 2 \cdot \text{delta}_t/N; \\
X = \text{fftshift}(X);
\]

This can be applied to the RIN and the spectrum. Assume that the variables \( p \) and \( \phi \) contain \( N \) samples of the photon number and phase of a mode respectively with a sample time \( \Delta t \). Then the spectrum of the signal defined by \( p \) and \( \phi \) \( S(f) \) (Matlab: \( S \)) can be calculated with the following Matlab code:

\[
\%
\text{calculate slow varying amplitude of electrical field} \\
E = \text{sqrt}(p) \cdot \text{exp}(-\text{sqrt}(-1) \cdot \phi); \\
\%
\text{sqrt}(-1) \text{ is a robust implementation of the imaginary unit } i \\
S = \text{abs(fft}(E)) \cdot 2 \cdot \text{delta}_t/N; \\
S = \text{fftshift}(S);
\]
The same can be done for the RIN:

\[
p_{av}=\text{mean}(p); \quad \% \text{ average of } p
\]
\[
delta_p=p-p_{av};
\]
\[
delta_P=\text{abs}(\text{fft}(\delta_p))^2*\delta_t/N;
\]
\[
delta_P=\text{fftshift}(\delta_P);
\]
\[
\text{RIN}=\delta_P^2/p_{av}^2;
\]
A.2 Solving the rate equations with Matlab and Simulink

Figure A.2: Main block of the Simulink implementation of the injection seeding neuron.
A.2 Solving the rate equations with Matlab and Simulink

Figure A.4: Simulink block for a mode with injection seeding.
Figure A.5: Simulink block for a mode with weak optical feedback.
Appendix B

Experiments

B.1 Detector calibration

The power of both the injected signal and the output power of the neuron laser are measured with a detector. The detectors suffer from saturation and therefore they have a non-linear optical power-voltage curve. To compensate for this non-linearity, a calibration curve is used. The curves have been measured for both detectors and are shown in figure B.1. The optical powers are measured with a third, calibrated detector. The alignment has a strong effect on the optical power that is coupled into the detector. Each time the alignment is changed the detectors have to be calibrated. The alignment is assumed to have a linear effect on the optical power coupled into the detectors, so it is not necessary to measure a whole calibration curve each time the detectors have to be calibrated. Instead at each calibration one voltage - optical power combination is measured to determine the optical power coupled into the detector. When a measurement has to be done that involves measuring optical powers, the following procedure is followed to calibrate the detectors and to convert measured voltages to optical powers:

1. measure one voltage - optical power \((V_{\text{ref}}, P_{\text{ref}})\) combination

2. determine what optical power \(P_{\text{cal}}\) corresponds to \(V_{\text{ref}}\) in the calibration curve

3. calculate the power factor \(a_{\text{ref}} = \frac{P_{\text{ref}}}{P_{\text{cal}}}\)

4. perform the measurement

5. for each element of the measured voltages \(V_{\text{meas}}\): determine the optical power in the calibration curve corresponding to \(V_{\text{meas}}\)

6. multiply this optical power with the power factor \(a_{\text{ref}}\) to find the real optical power \(P\)
Experiments

500
T ~400
~
3000
Cl.
200
Cl.
100

Figure B.1: Calibration curves for the detectors for the source laser and the neuron laser.

B.2 Transmission curves of the LCD-displays

Both transmission elements consisting of an LCD-display and a polariser have a non-linear voltage-transmission characteristic. AC voltages have to be applied to the LCD-displays. The transmissions are plotted relative to the maximum transmission. The transmission of the source laser LCD-polariser combination is plotted as function of the effective voltage on the LCD-display. The incident light has a 45° polarisation (0° corresponds to vertically polarised light). The curve is shown in figure B.2a.

The same curve is measured for one line of the neuron LCD-display - polariser combination. Here the relative transmissions are plotted as function of the driver amplitude. Amplitude 1000 corresponds to 15 V peak-peak, so an effective voltage of 10.6 V. All lines are similar, so their curves are assumed to be similar as well. The curve is shown in figure B.2b.

Figure B.2: Relative transmissions of LCD-display, polariser combinations as function of the applied voltage.
B.3 Overview of the experimental setup