MASTER

Towards a plasma simulation model for the Philips QL-lamp: collisional radiative model and Thomson scattering measurements

Herben, P.G.J.M.

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TOWARDS A PLASMA SIMULATION MODEL FOR THE PHILIPS QL-LAMP

Collisional Radiative Model & Thomson scattering measurements

Ing. P.G.J.M. Herben

Technische Universiteit Eindhoven
Faculteit der Technische Natuurkunde
Vakgroep Deeltjesfysica
Werkgroep Evenwicht en Transport in Plasma's

Begeleiders:
Ir. J. Jonkers
Ir. J. van Dijk
Dr. J.A.M. van der Mullen
Prof. Dr. Ir. D.C. Schram
SUMMARY

Our goal is creating a Plasma Simulation Model for the Philips QL-lamp. In order to do this, a Collisional Radiative Model (CRM) for mercury has been developed. This CRM calculates the production of light (also referred to as the 'radiative losses') and the free electron production, as a function of the electron density and temperature. This data is implemented in the general Plasma Simulation Model, PLASIMO, and then PLASIMO can calculate plasma-specific parameters.

The CRM consists of 2 independent levels (the atom and ion ground state), and all dependent levels up until a cut-off level, determined by the boundary between Corona Balance and Electron Saturation Balance. An independent level is a level which density does not depend on the densities of other levels. The density of a dependent level is determined by the independent levels. In this model stepwise and jump excitation are included, as well as transitions due to radiative decay. To account for the reabsorption of resonant radiation an escape factor has been included. Also the validity of the assumption that the EEDF is Maxwellian is examined. It turns out that for an electron density higher than $5 \times 10^{18}$ m$^{-3}$ the Maxwellian EEDF is valid.

The influence of the metastable-metastable and metastable-resonant (heavy-heavy) collisions has also been investigated. The rate of production of free electrons due to these heavy-heavy collisions is in the same order as the free electron production due to electron collisions. To include this effect into PLASIMO a CRM with six independent levels has been developed, in which the metastable and resonant levels are taken to be independent. However, implementing this CRM into PLASIMO causes a numerical instability in PLASIMO. This CRM with six independent levels can also be used to calculate the non-local treatment of resonant radiation, in stead of using an escape factor.

High-current TL measurements have been done to measure the I-V characteristics. These characteristics can be compared with PLASIMO simulations. Up until now, it has proven to be difficult to correctly simulate the experimental results. This is caused by the numerical instability of PLASIMO, and not taking the non-local treatment of resonant radiation and heavy-heavy collisions into account.

To examine the validity of PLASIMO, an attempt has been made to determine the electron density and temperature in the QL-lamp, using the method of Thomson scattering. However, it turns out that the stray light, caused by firing a laser at the lamp, is much more intense than the Thomson signal. Because of this it is not possible to determine the electron density and temperature.
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1 INTRODUCTION

1.1 PREFACE

In the group "Equilibrium and Transport in Plasmas" a computer program is being developed to describe a new type of lamp, the Philips QL-lamp. To develop this simulation model an existing computer model, PLASIMO, describing the ICP is being extended to describe the QL-lamp. This model was designed by D. Benoy, and was used to describe two-dimensionally an Argon plasma, at atmospheric pressure. Later on, this model was extended by M. Tas to include more species, such as mercury. The model describing the QL-lamp consists of a gas discharge and an inductive energy transfer part. The QL-lamp produces light in the same way the TL-lamp does, so that we can use the experience gained in TL-models and literature ([Ele72], [Way71] and [Rai91]). However, there are two major differences between the TL-lamp and the QL-lamp.

The main difference between the TL-lamp and the QL-lamp is the energy transfer. The QL-lamp is an inductively coupled lamp, which means that there is no direct contact between the electrode and the plasma, but that the energy is being transferred inductively. In the TL-lamp on the other hand the electrodes are in direct contact with the plasma. The main advantage is the extended lifetime (more than 60,000 hours). The inductive energy transfer part of PLASIMO is described extensively in the report of M. van der Velden [Vel97].

The other difference is the current through the lamp. The TL-lamp has a typical current of about 400 mA, whereas the QL-lamp has a typical current of about 10 A. Because of this the electron density is higher and therefore the production of electrons is different. This means that the TL-model has to be modified for this much higher current. To examine if this part has been modified correctly PLASIMO will calculate I-V characteristics, which can be verified by doing I-V measurements on high current TL-lamps.

If both parts have been modified, and the model has been adjusted for the different geometry of both lamps, we should have a model describing the QL-lamp. In order to test the model simulations have to be compared with experimental data.

This graduation project consists of two major parts. The first part is to obtain experimental values of the electron density $n_e$ and the electron temperature $T_e$ of the Philips QL-lamp. This will be done to compare these results with the data obtained by PLASIMO. To measure these quantities the method of Thomson scattering will be used. It has already been attempted to measure $n_e$ and $T_e$ using other methods (Absolute Line Intensities ALI, Continuum Emission Measurements CEM and indirectly via Diode Laser Absorption DLA [Bak96]) but the results of these methods all differ, sometimes with a factor of 4. Even the trends they predict are different. This could be explained by the fact that all three methods are based on certain hypothesis, and if these hypothesis are wrong, the results will be wrong. The average electron density, calculated using the methods mentioned above, is about $10^{19}$ m$^{-3}$, and the electron temperature is about 2 eV.

To solve this problem the method of Thomson scattering will be used. This method has the advantage that the electron density and temperature can be measured directly, no assumption has to be made. Photons, originating from a Nd:YAG laser, are scattered on free electrons in the plasma. Because all free electrons have different velocities, the detected signal will be Doppler-broadened. The width of the Thomson spectrum is a measure for the electron temperature, the area under the curve is a measure for the electron density.
The other part of this graduation project consists of examining the plasma part of PLASIMO. This has already been described by M. Tas, but until now the description of the electron production coefficient $S_{CR}$ is too simple. After that the model will be verified with measurements on and simulations of TL-lamps operating at high current (1A - 5A), so the possible errors in the EM coupling will not effect the simulations.

When the part describing the gas discharge is implemented well, the EM part can be implemented, PLASIMO can be adapted for the different geometry and then the QL-lamp can be simulated. For the model to be complete the simulations have to be perfected and other parts, such as the radiation transport and the effect of the tillings, have to be examined and simulated. For a final check of the model we can compare the electron density and temperature from the Thomson scattering experiments with the data obtained by PLASIMO.

### 1.2 TECHNOLOGY ASSESSMENT

In the world about 25% of all produced energy is electrical energy. About 25% of all electrical energy is used for lighting purposes. This means that it is very important to create lamps with a high efficacy (the intensity of the light coming out of the lamp as compared to the input power [lm/W]). If highly efficient lamps can be created less energy is needed to produce the same amount of light. This can be done by making and testing different lamps. Because this is a time and money consuming task, other methods to construct more efficient lamps are under investigation. This is the main reason a Plasma Simulation Model PLASIMO for the Philips QL-lamp is being developed. With this model it will be possible to calculate the effect of several parameters, such as the mercury and buffer gas pressure and design, and to create a more efficient lamp without having to build and test dozens of different lamps. An additional effect is the better understanding of the plasma in the lamp.

The advantage of the QL-lamp is the expanded lifetime of the lamp. Because there are no electrodes in the plasma, the lifetime of the lamp is limited by the electronics only. This means a lifetime of 60,000 hours continuous use, and this also means that the relative costs of maintenance and replacement can be greatly reduced. Furthermore the environment benefits from the fact that now less lamps have to be produced.
2 THEORY

2.1 THE TL-LAMP

2.1.1 WORKING PRINCIPLE

A TL-lamp is depicted in figure 2.1. An electric field is applied over the electrodes, and this e-field accelerates the free electrons in the plasma. Part of the mercury atoms (the active medium) will be excited. The decay of these excited atoms mostly occurs while emitting ultraviolet radiation. This radiation is transformed into visible light by a fluorescent powder. Another part of the mercury atoms will be ionised, which causes the creation of free electrons.

![Figure 2.1: Cross-section of a TL-lamp [Vos97].](image)

The magnitude of the mercury pressure is important for the efficiency of the lamp. The optimal mercury pressure for a burning lamp is about 5 mTorr (0.7 Pa). The density of the resonant states, i.e. the states which cause over 99% of the light, are proportional to the density of the ground state, so if the mercury pressure (and thus the ground state density) is decreased, the density of the resonant states and the emitted light is decreased as well. If the mercury pressure is too high, the reabsorption will increase, and so the efficiency decreases.

If mercury was the only gas present in the lamp, the losses of free electrons due to diffusion to the wall would become too large. Because of this a buffer gas (argon, 33 Pa for the QL-lamp, 400 Pa for the TL-lamp) is added to the lamp.

2.1.2 GENERAL ELECTRIC PROPERTIES

The main light emitting component of the TL-lamp is the ‘positive column’. PLASIMO can only simulate this positive column. Other components are the cathode fall at the negative electrode and the anode fall at the positive electrode. The cathode fall originates from the fact that the electrons, created by the cathode, are drawn away by the electric field much more quickly than the positive ions. Because of this a positive space charge is created.
This means that not all of the voltage applied to the lamp is across the positive column. This is an energy loss in the light production of the system. Increasing the current, and thus the temperature of the electrodes decreases the cathode fall, whereas the anode fall remains almost constant ([Ele72] and [Rai91]). With the used frequency of 2.65 MHz the anode-fall will become negligible [Koe65]. If the frequency is high enough the positive ions which remain from the cathode-period can satisfy the need for positive ions during the anode-period during the whole anode-phase.

The cathode fall decreases with increasing electrode temperature because the creation of electrons rises exponentially with increasing temperature. This means that if the current is doubled, the number of created electrons is more than doubled, and so the velocity of the electrons decreases. Because of this the positive space charge, and thus the cathode fall is decreased.

A fluorescent lamp operated in a circuit does not behave like a resistance. A fluorescent lamp has a negative voltage current characteristic which means that when operating the lamp at a higher RMS current, a lower RMS voltage across the lamp appears. Electronic ballasts serve mainly to stabilise the lamp and to prevent a current runaway.

2.1.3 I-V CHARACTERISTIC

The general current density - electric field characteristic of a gas discharge lamp is depicted in fig. 2.2. In this characteristic we can distinguish four different regions:

![Fig. 2.2: General j-E characteristic of a gas discharge lamp.](image)

1) In this region the only ionisation process is direct ionisation. The ambipolar diffusion of electrons and ions to the wall is compensated by electron production via direct ionisation. The ionisation process has virtually no influence on the neutral mercury particles density. The ambipolar diffusion is determined by the tube radius (Schottky theory), the ionisation is a function of the electron temperature. For a given tube radius the ambipolar diffusion is determined. The electron temperature adjusts itself in such a way that the electron production and ambipolar diffusion are equal.
So for a certain tube radius, the electron temperature remains constant (equation 2.1).

\[
\nabla D_e \nabla n_e = D_a \frac{n_e}{\Lambda^2} = n_e n_i K_{1e}(T_e) \Rightarrow K_{1e}(T_e) = \frac{D_a}{n_i \Lambda^2}
\]

(2.1)

Where \( \Lambda \) is the diffusion length \( (\Lambda = r / 2.4 \) [Rai91]), \( D_a \) the ambipolar diffusion coefficient \([m^2/s]\), \( n_e \) the electron density \([m^{-3}]\), \( n_i \) the neutral particles density \([m^{-3}]\) and \( K_{1e} \) the direct ionisation rate \([m^3/s]\).

An electric field applied to an ionised gas will generate a current density \( j \) given by:

\[
 j = -n_e e \mu_e E + n_e q_e \mu_e E \equiv -n_e e \mu_e E = \sigma E
\]

(2.2)

with \( \mu_e \) the electron mobility:

\[
\mu_e = \frac{e}{m_e \nu_{eh}} = \frac{e}{m_e \sum_h (n_h \sigma_m \nu)}
\]

(2.3)

with \( \nu_{eh} \) the collision frequency for momentum transfer for electrons with heavy particles, \( m_e \) the electron mass \([9.1 \times 10^{-31} \text{ kg}]\), \( n_h \) the heavy particle density \([m^{-3}]\), \( \sigma_m \) the momentum transfer cross section \([m^2]\) and \( \nu \) the electron velocity \([m/s]\). The notation between brackets indicates an averaging over the electron energy distribution function. The summation runs over the different types of heavy particles, both neutrals and ions. The momentum cross section is a rising function of the electron temperature \( T_e \), for \( T_e \leq 10 \text{ eV} \) [Vos97], so the electron mobility decreases with increasing electron temperature (2.3). The electric field is also determined by the requirement that the electrical energy input per unit volume equals the energy loss per unit volume [Way71]:

\[
 jE = n_e W_e(T_e)
\]

(2.4)

with \( W_e(T_e) \) the average total energy loss per electron at electron temperature \( T_e \). Combining equations 2.2 and 2.4 we get:

\[
 E = \left( \frac{W_e(T_e)}{e \mu_e(T_e)} \right)^{1/2}
\]

(2.5)

If \( T_e \) remains constant, all terms on the right hand side remain constant, and so does \( E \). If \( T_e \) decreases, \( \nu_{eh} \) decreases and so \( \mu_e \) increases (2.3). \( W_e \) decreases, so according to equation 2.5 \( E \) decreases.

2) In the previous region the condition is that only direct ionisation occurs. This is valid only for a very low electron density, and so a very low current density (2.2). If the electron density is increased, the possibility of stepwise ionisation increases. For stepwise ionisation electrons with a temperature less than the temperature needed for direct ionisation can also participate. So, with increasing current, and so increasing \( n_e \), the electron temperature decreases, and so the electric field.

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3) If the current density, and so \( n_e \), is increased even more, the ionisation increases rapidly. Because of this the ion transport to the wall increases as well. In stationary state the ion transport to the wall equals the diffusion of neutral particles to the centre. However, the ion ambipolar diffusion coefficient \( D_a \) to the wall is about a factor of 10 larger than the diffusion coefficient \( D \) of neutral particles to the centre.

\[
\frac{\nabla (D_a \nabla n_+)}{\nabla (D 
abla n_1)} = \frac{D_a \Delta n_+}{D \Delta n_1} = 1 \implies D_a = 10 D = \frac{\Delta n_1}{\Delta n_+} = 10
\]  \hspace{1cm} (2.6)

The density of the mercury particles in the centre will decrease, this is called depletion (2.6). If the depletion is total, i.e. if no mercury atoms are left in the centre of the discharge, electron production can only occur through ionisation of noble gas particles. The energy needed to ionise a noble gas particle is much higher than the energy needed to ionise a mercury particle, so a higher electron temperature is needed.

4) Region four is the same as region two, i.e. stepwise ionisation becomes important, but now only noble gas particles can cause electron production.

2.2 THE QL-LAMP

2.2.1 INTRODUCTION

The QL-lamp (figure 2.3 [Jon97]) is an electrodeless low pressure gas discharge lamp. The main advantage of the absence of electrodes is that the lifetime of the lamp increases considerably, to a value of 60,000 hours. This is because the lifetime is no longer determined by the electrodes, but by the electronics. Other advantages are the more continuous efficacy during the lifetime.

![Diagram of the Philips QL-lamp](image)

Fig. 2.3: The Philips QL-lamp.
2.2.2 THE WORKING OF THE QL-LAMP

The working principle of the QL-lamp is mainly the same as the TL-lamp. The major difference between the lamps is that in the QL-lamp the energy is coupled inductively into the plasma by an alternating electromagnetic field (2.65 MHz, [Net91] and [Sch91]). The QL-lamp exists of three components: the induction coil, the discharge vessel and the electronics.

The induction coil is located in the centre of the bulb to avoid blocking of the light. The coil is not located inside the discharge vessel; it is positioned within a glass cavity in the discharge vessel (fig. 2.3). In the discharge vessel no electrodes are present, but it contains a noble gas filling and an amalgam-controlled mercury vapour. The inside of this vessel is coated with a protective coating on which a fluorescent powdertex has been applied. For our setup, however, no fluorescent powder has been applied, because this would interfere with our measurements. The inner part of the vessel has a cylinder-shaped cavity that is also coated with fluorescent powder.

The power coupler has an induction coil which protrudes from a mounting flange. The coil is connected by a coaxial cable to the high frequency generator which is located inside a separate housing.

The electronics supply an alternating current of 1A (frequency 2.65 MHz) to the induction coil, and because of this an alternating magnetic field is induced. The system of induction coil and plasma can be interpreted as a transformer, with the induction coil as the primary coil (15 turns) and the electron current in the plasma (10A) as the secondary coil (1 turn). The plasma current in the QL-lamp is about a factor of 25 higher than in the TL-lamp.
3 COLLISIONAL RADIATIVE MODEL

3.1 INTRODUCTION

The goal of the QL-project is to develop an engineering model, i.e. a model which predicts macroscopic parameters such as electrical characteristics and the light and heat production, and the local parameters such as the electron density $n_e$ and the electron temperature $T_e$. To do this, the local production of new free electrons and of radiation has to be known. This can be calculated with the coefficients from table 3.1. A Collisional Radiative Model (CRM) is necessary to calculate these coefficients.

The plasma consists of two atomic species, argon (Ar) and mercury (Hg). For each species we need a CRM. For argon there is an extensive model available [Ben93]. For mercury, up to now a three-step model was used, consisting of the ground state, a radiating “metastable” level and the ion ground state [Vos97]. This model is not valid for higher $n_e$, because stepwise excitation, that is becoming increasingly important for increasing $n_e$, is neglected in this model.

Table 3.1: Coefficients calculated by CRM.

| Coefficients                                      | $S_{CR}$ | $\alpha_{CR}$ | $\varepsilon_{||}$ | $\varepsilon_{\perp}$ |
|--------------------------------------------------|----------|---------------|---------------------|------------------------|
| Ionisation coefficient                           | $[m^3s^{-1}]$ |               |                     |                        |
| Recombination coefficient                        | $[m^3s^{-1}]$ |               |                     |                        |
| Radiation losses coefficient related to the ground state | $[eVm^3s^{-1}]$ |               |                     |                        |
| Radiation losses coefficient related to the ion ground state | $[eVm^3s^{-1}]$ |               |                     |                        |

With the ionisation and recombination coefficients the net production of electrons $H_e$, the net production of ions $H_i$ and the net destruction of neutral particles $-H_i$ can be calculated (3.1). With the radiation losses coefficients the radiation losses can be calculated, which are an important term in the electron energy balance since we are modelling a lamp (3.2).

$$H_e = H_i = -H_i = n_e n_i S_{CR} - n_e n_+ \alpha_{CR} \quad [m^3s^{-1}]$$  \hspace{1cm} (3.1)

$$P_{\text{rad}} = n_e n_i \varepsilon_{||} + n_e n_+ \varepsilon_{\perp} \quad [eV/m^3s]$$  \hspace{1cm} (3.2)

PLASIMO solves particle- and energy balances in a control volume for steady-state, such as the electron balance:

$$n_i n_e S_{CR} - n_e n_+ \alpha_{\text{CR}} = -\nabla(D \nabla n_e) \quad [m^3s^{-1}]$$  \hspace{1cm} (3.3)

The net production of electrons equals the diffusion losses of electrons. The right hand side of equation 3.3 is already inserted in PLASIMO. The left hand side has to be determined by the CRM. Also the electron energy balance is solved in PLASIMO:

$$\sigma E^2 = P_{\text{elast}} + P_{\text{inel}} + \nabla \cdot k \nabla T$$  \hspace{1cm} (3.4)

with $\sigma E^2$ the locally dissipated power, $P_{\text{elast}}$ the power lost in elastic collisions, $P_{\text{inel}}$ the power lost in inelastic collisions and $\nabla \cdot k \nabla T$ the conductive losses. The inelastic collisions term consists of the ionisation and excitation processes and the radiation part.
The problem is that we have to know the ASDF (Atomic State Distribution Function) if we want to be able to calculate these coefficients. The ASDF describes the distribution of the bound electrons over the excited states. This would not be difficult if the plasma is in Local Thermodynamic Equilibrium (LTE). In this case the information about the plasma can be obtained from the laws of statistical mechanics, see § 3.2. The distribution of the electrons over the excited states is given by the laws of Boltzmann and Saha, which all depend on one single temperature. Unfortunately, the plasma is not in LTE. Because of this we have to use another method to calculate the ASDF.

A Collisional Radiative Model (CRM) is used to compute the ASDF. This can be done directly from the rate coefficients of the relevant elementary processes (table 3.2). We start with the assumption that the transitions between the ground state, the excited states and the ion ground state are due to emission or electron induced collisions.

Table 3.2: Overview of the relevant elementary processes.

<table>
<thead>
<tr>
<th>Elementary processes</th>
<th>Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>collisional (de)excitation</td>
<td>$A_p + e \leftrightarrow A_q + e$</td>
</tr>
<tr>
<td>line radiation emission and absorption</td>
<td>$A_p \leftrightarrow A_q + \nu$</td>
</tr>
<tr>
<td>collisional ionisation and three-particle recombination</td>
<td>$A_p + e \leftrightarrow A^* + e + e$</td>
</tr>
<tr>
<td>radiative recombination</td>
<td>$A^* + e \rightarrow A_p + \nu$</td>
</tr>
</tbody>
</table>

The program which calculates the ASDF was made by Bart Hartgers [Har96]. In this program the atom-specific parameters, which are the different energy levels, the statistical weights, the excitation cross-sections, the radiative transition probabilities, the electron density and temperature, the neutral density and temperature and the ion density have to be implemented. Once the coefficients are known, they can be implemented in PLASIMO, and used to compute plasma-specific parameters such as the local electron density $n_e$, the local electron temperature $T_e$, the I-V characteristic etc.
3.2 THE ASDF FOR LTE

If we assume that the plasma is in Local Thermodynamic Equilibrium (LTE), the ASDF is given by the laws of Boltzmann and Saha ([Sch96] and [Mul86]). Boltzmann's law (3.5) gives a relation between the density of an excited state p and the ground state density, and is valid if the Boltzmann balance (3.6) is in equilibrium and the free electrons obey a Maxwellian Electron Energy Distribution Function (EEDF) (3.7).

\[
\frac{n_p}{g_p} = \frac{n_e}{g_1} e^{(-E_p/T_e)} \tag{3.5}
\]

\[
A_1 + e \leftrightarrow A_p + e \tag{3.6}
\]

\[
f_M(E)dE = \frac{2\pi\sqrt{E}}{(\pi T_e)^{3/2}} e^{(-E/T_e)} \tag{3.7}
\]

\[
K_{pq} = \int_{E_{pq}}^{\infty} \sigma_{pq} f_\text{e}(E_\text{e})(E_\text{e})dE_\text{e} \tag{3.8}
\]

with \(k_B\) Boltzmann’s constant \([1.38 \times 10^{-23}\, \text{J/K}]\), \(T_e\) the electron temperature \([\text{eV}]\), \(n_p\) the density of an excited level \(p\) \([\text{m}^{-3}]\), \(g_p\) the statistical weight of the excited level \(p\), \(n_e\) the density of the ground state \([\text{m}^{-3}]\), \(g_1\) the statistical weight of the ground state, \(v_\text{e}\) the electron velocity and \(E_{pq}\) the energy difference between the excited level \(p\) and the ground level \([\text{eV}]\). Equation 3.8 describes the transition rate coefficient \(K_{pq}\) \([\text{m}^3\,\text{s}^{-1}]\).

Saha’s law (3.9) gives a relation between the density of an atomic excited state \(p\) and the density of the ion ground state, and is valid if the Saha balance (3.10) is in equilibrium and again the Maxwellian EEDF is valid:

\[
\frac{n_p}{g_p} = \frac{n_\text{e}_\text{i}}{g_\text{i}} \left( \frac{\hbar^2}{2\pi m_e k_B T_e} \right)^{3/2} e^{(I_p/T_e)} \tag{3.9}
\]

\[
A_p + e \leftrightarrow A^+ + e + e \tag{3.10}
\]

with \(n_\text{i}\) the density of the ion ground state \([\text{m}^{-3}]\), \(n_\text{e}\) the electron density \([\text{m}^{-3}]\), \(g_\text{i}\) the statistical weight of the ion ground state, \(k_B T_e\) the electron temperature \([\text{J}]\), \(g_\text{e} = 2\) the statistical weight of the free electron state and \(I_p\) the ionisation energy of a level \(p\). The laws of Boltzmann and Saha predict the same ASDF, when the plasma is in LTE.

However, in case of losses of electrons due to diffusion, the Saha balance is not in equilibrium, \(A_p + e \rightarrow A^+ + e + e\). Because of this the Boltzmann balance is also not in equilibrium, \(A_{\text{low}} + e \rightarrow A_{\text{high}} + e + e\). Now the plasma is not in LTE and a CRM is needed to calculate the ASDF.
3.3 COLLISIONAL RADIATIVE MODEL

3.3.1 THEORY

Collisional radiative models relate the densities of the excited levels to the densities of the atom and ion ground state. These models are formed by creating and solving mass balances for all the excited states. To solve these balances, the Quasi Steady State Solution (QSSS) ([Mul86] and [Har96]) method is used. The basic idea for QSSS is that the atom and ion ground states can be considered as two large particle reservoirs, which are filled or drained on a relative long time scale \(10^4\) s. From these large reservoirs a small amount is distributed over the system of excited levels. This distribution happens on the small time scale of atomic processes (typically \(10^7\) s). Thus the two basic assumptions to be made for the QSSS are:

- The diffusion- or plasma decay times must be much longer than the lifetime of the atomic excited states.
- The total number of atoms in the excited states must be much smaller than the number of ground state atoms or ions.

Collisional Radiative Models are formed by mass balances (3.11) for 'all' excited states:

\[
\frac{\partial n_p}{\partial t} + \vec{V} \cdot (n_p \vec{w}_p) = \left( \frac{\partial n_p}{\partial t} \right)_{CR}
\]  

(3.11)

According to this equation, the increase of the density of an excited level in time is related to the divergence of the flow and the net result of populating and depopulating processes. When the QSSS approximation is applied, these populating and depopulating processes are assumed to be fast compared to the changes in time and by transport for all excited levels. This means:

\[
\left( \frac{\partial n_p}{\partial t} \right)_{CR} = 0 \quad \Rightarrow \quad n(p)D(p) = P(p)
\]  

(3.12)

The destruction of a level \(p\) must be equal to the production of that level. For the atom and ion ground state, the full mass balances still apply:

\[
\frac{\partial n_i}{\partial t} + \vec{V} \cdot (n_i \vec{w}_i) = \left( \frac{\partial n_i}{\partial t} \right)_{CR}
\]  

(3.13)

\[
\frac{\partial n_+}{\partial t} + \vec{V} \cdot (n_+ \vec{w}_+) = \left( \frac{\partial n_+}{\partial t} \right)_{CR}
\]  

(3.14)

The left hand side of these equations are solved by PLASIMO. Because densities of both ground states are typically much larger than those of the excited states, the atom and ion densities can be considered to be equal to the atom and ion ground state respectively.
In the QSSS population from and depopulation to the atom and ion ground state are considered to be instantaneously. These populating processes scale with the densities of the atom and ion ground state. This means that the density of a level \( p \) can be expressed as a superposition of contributions from the ion and atom ground state, or of the Saha density \( n_p^S \) and the Boltzmann density \( n_p^B \) respectively:

\[
n_p = r_p^+ \cdot n_p^S + r_p^- \cdot n_p^B
\]

(3.15)

The Saha density, described in equation 3.9, is the density of a level \( p \) if it would be in equilibrium with the ion ground state. The Boltzmann density, equation 3.5, is the density of a level \( p \) if it would be in equilibrium with the atom ground state. The coefficients \( r_p^+ \) and \( r_p^- \) express the difference between \( n_p \) and \( n_p^S \) and \( n_p^B \). Another way of describing this is by saying that there are two independent levels, the atom ground state and the ion ground state. By independent we mean that the density of that level is independent of the densities of all other levels. All other levels are dependent, which means that their density depends on the densities of the independent levels. The density of a dependent level \( p \) can now be written as the superposition of a contribution from the ground state and a contribution from the ion ground state.

### 3.3.2 SOLVING THE ASDF

The CR coefficients can be found by solving a set of linear equations representing the mass balances for the excited states.

\[
\begin{align*}
    n_e \sum_{q \neq p} n_q K_{qp} \\
    + \sum_{q > p} n_q A_{qp} A_{qp} \\
    + n_e n_p (n_e K_{ep} + A_{ep}) \\
    - n_p \sum_{q < p} A_{pq} A_{pq} \\
    - n_e n_p K_{pq} \\
    - n_e n_p S^p_p = 0
\end{align*}
\]

(3.16)

(a) represents population of a level \( p \) caused by (de)excitation of other levels, (b) the cascade radiation, i.e. the population of a level \( p \) by the effective radiative decay of the upper lying levels \( q \), (c) three and two particle recombination to level \( p \), (d) depopulation of a level \( p \) due to the effective radiative decay to all lower lying levels \( q \), (e) depopulation of a level \( p \) by collisional transitions and (f) by ionisation.

Equation 3.16 can be solved in three ways, analytical, numerical and a combination of the latter two, the hybrid method. The first method has the disadvantage that the analytical expressions for the excitation coefficients and transition probabilities are based on a hydrogen approximation. A highly excited atom can see the nucleus and the orbiting electrons as a nucleus with charge \( e^+ \). Obviously, this can not be correct for the lower levels of a non-hydrogen atom, for which deviations from hydrogen can be substantial.
The second method, the numerical one, suffers from a different problem. In this case it is not clear what the highest considered level should be. It is impossible to take all levels into account, because there are infinitely many levels. If too many levels are neglected, stepwise ionisation will be underestimated, and so the ionisation coefficient will be underestimated.

The third method attempts to use the best of both worlds. Because the numerical method suffers from uncertainties with respect to the top of the system, and the analytical method is imprecise for low internal energies, it seems a good idea to extend the numerical method with an analytical top.

First a distinction has to be made between two regions:

region 1 (numerical): the levels for which radiative decay and deexcitation dominate. This means that an electron which is excited to a higher level in that region, usually falls back to a lower lying level. So most of the levels in this region are situated in Corona Balance (CB).

region 2: the levels for which those processes mentioned above can be neglected, and excitation and ionisation dominate. This means that each electron which is situated in that region will be excited to even higher levels, and eventually becomes a free electron. All levels in this region have to be situated in Electron Saturation Balance (ESB).

An important condition for dividing all levels in two regions is that all levels in region 2 have to be situated in ESB. This is because we assume that every bound electron that crosses the border between the two regions will eventually become a free electron. The boundary will be determined so that all levels in region 2 are situated in ESB. This means that the highest levels in region 1 could also be situated in ESB. Now we only need to calculate the densities of the levels in region 1. The net flow $J$ from region 1 to region 2 can be calculated analytically with the next equation:

$$J(p, p+1) = n_e n_p K_{p,p+1} - n_e n_{p+1} K_{p+1,p}$$

$$= b_p n_p K_{p,p+1} - b_{p+1} n_{p+1} n_p K_{p+1,p}$$

(3.17)

in which $p$ is the highest level in region 1 ($p$ is situated in ESB). The parameter $b_p$ is defined as:

$$b_p = \frac{n_p}{n_p^s}$$

(3.18)

with $n_p^s$ the density of a level $p$ according to Saha (3.9). With Detailed Balancing [Mul86] equation 3.17 can be rewritten as:

$$J(p, p+1) = (b_p - b_{p+1}) n_e n_p^s K_{p,p+1}$$

(3.19)

In § 3.4 the boundary between region 1 and region 2 is determined. Even if we only look at the energy levels in region 1 there are still 31 energy levels. To reduce this number of levels we will group several energy levels together to a total of 13 levels. This will be done in § 3.5.
The calculation of the densities of the excited states and the accompanying r-values can be done using the method described in [Mul86] by solving equation 3.16 for every excited state. This can be represented by:

\[
M_N = S^+ + S^i
\]  

(3.20)

in which \( N \) is an N-1 dimensional vector whose components are the \( n_p \) values. \( N \) is the number of the cut-off level (in our case, \( N=13 \)). The vectors \( S^+ \) and \( S^i \) give the direct population contribution for the levels \( p \) from the continuum and from the ground state, respectively.

\[
S_p^+ = n_e n_s D_{sp} = n_e n_s \left[ n_e K_{sp} + A_{sp} \Lambda_{sp} \right]
\]  

(3.21)

\[
S_p^i = n_e n_s K_{ip}
\]  

(3.22)

The matrix \( M \) has components:

\[
M_{np} = -D_{qp} = -n_e K_{qp} - A_{qp} \Lambda_{qp}
\]  

(3.23)

\[
M_{pp} = D(p) = n_e K(p) + A(p)
\]  

(3.24)

with \( D_{qp} \) the destruction of a level \( q \) to a level \( p \), and \( D(p) \) the total destruction of a level \( p \). The solution of equation 3.20 can be written as equation 3.25. The density of an excited state can be written as the sum of contributions from the atom and ion ground state and the other excited states. Since this is possible for every excited state, the population of a state \( p \) can also be expressed in population from the ground states (fig. 3.1). For doing so it is necessary to solve the system of equations (3.20).

\[
n_p = n_p^+ + n_p^i
\]  

(3.25)

Figure 3.1: The production/destruction balance of a level \( p \) [Mul86].
The contributions from the ground state and ion ground state can be written as:

\[ n_p^+ = (M^{-1} S^+)_p \]  
\[ n_p^I = (M^{-1} S^I)_p \]  

(3.26)  

(3.27)

\( M^{-1} \) is the inverse matrix of \( M \), and \( n_p^+ \) and \( n_p^I \) the densities \( [m^{-3}] \) of the excited state \( p \) as a result of the contribution from the ion and atom ground state respectively. With the \( \text{ASDF} \) known, the \( r \)-values can now be calculated:

\[ r_p^i = \frac{n_p^i}{n_i} \]  

(3.28)

with \( i = 1 \) (the atom ground state) or \(+\) (the ion ground state). We can now use these \( r \)-values to calculate the effective ionisation (\( \alpha_{CR} \)) and recombination (\( \alpha_{CR} \)) coefficients:

\[ \alpha_{CR} = n_e K_{q1} + A_{i1} + \sum_{q \neq l} \frac{n_p^q r_p^q}{n_e n_+} (n_e K_{q1} + A_{q1} \Lambda_{q1}) \]  

[\( m^3 s^{-1} \)]  

(3.29)

\[ S_{CR} = \sum_{q \neq l} K_{iq} + S_i - \sum_{q \neq l} \frac{n_p^q r_p^q}{n_e n_+} (n_e K_{q1} + A_{q1} \Lambda_{q1}) \]  

[\( m^3 s^{-1} \)]  

(3.30)

Also the radiation losses coefficients can now be calculated:

\[ \varepsilon_i = \frac{1}{n_i n_e} \sum_p n_p^i \sum_{q \neq l} A_{pq} \Lambda_{pq} \times E_{pq} \]  

[\( eV m^3 s^{-1} \)]  

(3.31)

with \( i \) again \( 1 \) or \(+\) and \( E_{pq} \) the energy difference in \( eV \).
3.4 THE BOUNDARY BETWEEN CB AND ESB

As mentioned before, it is necessary to calculate the boundary between Corona Balance and Electron Saturation Balance. In region 1 radiative decay and deexcitation has to dominate over excitation (CB). In region 2 excitation has to dominate (ESB). To do this we have meet with two demands [Mul86].

In Corona Balance radiative decay dominates, in ESB stepwise excitation. On the boundary the processes are by definition almost equal. So the first demand is that the rate of excitation has to be almost equal to the rate of radiative decay. In order to calculate \( I_p^{\text{CR}} \), i.e. the ionisation energy at the boundary between the radiative and collisionally dominated parts of the ASDF, we will have to use the hydrogen-approximation (an atom is described as a hydrogen atom, so one electron \( e^- \) circling around the core, with a net charge of \( e^+ \)). If this approximation is used, the value of \( I_p^{\text{CR}} \) is constant for every atom.

\[
n_e n_p K(p) = n_p A(p) \Rightarrow n_e K(p) = A(p) \quad (3.32)
\]

The radiative decay \( A(p) \) and the excitation rate \( K(p) \) can be written as:

\[
A(p) = 2 \gamma Z^4 p^{-4.5} \quad (3.33)
\]

\[
K(p) = 2.6 \times 10^{-14} Z^4 p^4 \sqrt{T_e} \quad (3.34)
\]

with \( \gamma = 7.87 \times 10^9 \text{s}^{-1} \), \( p \) the principle quantum number, \( T_e \) the electron temperature (= 1 eV) and \( Z \) the charge number of the core (here: \( Z=1 \)). Using the ionisation energy \( I_p = R/p^2 \) we get [Mul97]:

\[
I_p^{\text{CR}} = \left( \frac{n_e}{9 \times 10^{18}} \right)^{2/9} \times R = 0.6 \text{ eV} \quad (3.35)
\]

with \( n_e \) the lowest expected electron density \( (10^{18} \text{ m}^{-3}) \) and \( R \) the Rydberg constant \( (13.6 \text{ eV}) \). For increasing \( p \) the excitation coefficient \( K(p) \) increases, because the energy difference \( \Delta E \) decreases. The radiative decay \( A(p) \) decreases with increasing \( p \).

The second demand is the hot-cold criteria. If an electron is situated in ESB excitation has to dominate over deexcitation. The ionisation energy for this boundary can be calculated with equation 3.36:

\[
n_e n_p K_{p,p+1} = n_e n_p K_{p,p-1} \Rightarrow \frac{I_p^{\text{HC}}}{T_e} = 1 \Rightarrow I_p^{\text{HC}} = 1 \text{ eV} \quad (3.36)
\]

With \( T_e \) the estimated electron temperature in eV. From these two demands we can determine at which ionisation energy the boundary between CB and ESB is situated. For an estimated lowest electron density of about \( 10^{18} \text{ m}^{-3} \) and an estimated electron temperature of about 1 eV the boundary ionisation energy \( I_p \) is about 0.6 eV. All the levels which have an ionisation energy of less than 0.6 eV are situated in ESB (region 2), and the levels with an ionisation energy of more than 0.6 eV are situated in CB (region 1). However, we assume that the electron density is larger than \( 10^{18} \text{ m}^{-3} \), so the highest excited levels in region 1 are probably situated in ESB. When we are going to make an energy level scheme for mercury we will only have to take into account those levels with an ionisation energy of more than 0.6 eV.
3.5 GROUPING THE ENERGY LEVELS

In table 3.3 the ionisation energy for a number of levels is written down. The data in the first, second and fourth column originates from [Mos78]. The results in the third column result from the data from [Mos78], or are calculated using the quantum defect (discussed further on in this section).

Table 3.3: The different energy levels for mercury as given by [Mos78].

<table>
<thead>
<tr>
<th>Energy level</th>
<th>Energy [cm(^{-1})]</th>
<th>Ionisation energy (I_0) [eV]</th>
<th>Statistical weight (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(^1)S(_0)</td>
<td>10.4375</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>6(^1)P(_0)</td>
<td>5.77</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>6(^1)P(_1)</td>
<td>5.55</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>6(^3)P(_2)</td>
<td>4.98</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>6(^3)P(_3)</td>
<td>3.74</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>7(^3)S(_1)</td>
<td>62350</td>
<td>2.71</td>
<td>3</td>
</tr>
<tr>
<td>7(^3)S(_0)</td>
<td>63928</td>
<td>2.51</td>
<td>1</td>
</tr>
<tr>
<td>7(^1)P(_0)</td>
<td>69517</td>
<td>1.82</td>
<td>1</td>
</tr>
<tr>
<td>7(^1)P(_1)</td>
<td>69662</td>
<td>1.80</td>
<td>3</td>
</tr>
<tr>
<td>7(^1)P(_2)</td>
<td>71208</td>
<td>1.67</td>
<td>5</td>
</tr>
<tr>
<td>7(^1)P(_3)</td>
<td>71295</td>
<td>1.60</td>
<td>3</td>
</tr>
<tr>
<td>6(^1)D(_2)</td>
<td>71333</td>
<td>1.60</td>
<td>5</td>
</tr>
<tr>
<td>6(^1)D(_1)</td>
<td>71336</td>
<td>1.60</td>
<td>3</td>
</tr>
<tr>
<td>6(^3)D(_2)</td>
<td>71396</td>
<td>1.59</td>
<td>5</td>
</tr>
<tr>
<td>6(^3)D(_3)</td>
<td>71396</td>
<td>1.58</td>
<td>7</td>
</tr>
<tr>
<td>8(^3)S(_1)</td>
<td>73961</td>
<td>1.27</td>
<td>3</td>
</tr>
<tr>
<td>8(^3)S(_0)</td>
<td>74405</td>
<td>1.21</td>
<td>1</td>
</tr>
<tr>
<td>8(^1)P(_0)</td>
<td>76824</td>
<td>0.91</td>
<td>1</td>
</tr>
<tr>
<td>8(^1)P(_1)</td>
<td>76863</td>
<td>0.91</td>
<td>3</td>
</tr>
<tr>
<td>7(^1)D(_2)</td>
<td>77064</td>
<td>0.89</td>
<td>5</td>
</tr>
<tr>
<td>7(^1)D(_1)</td>
<td>77085</td>
<td>0.88</td>
<td>3</td>
</tr>
<tr>
<td>7(^1)D(_2)</td>
<td>77108</td>
<td>0.88</td>
<td>5</td>
</tr>
<tr>
<td>7(^1)D(_3)</td>
<td>77130</td>
<td>0.88</td>
<td>7</td>
</tr>
<tr>
<td>5(^1)F(_2,1,4) 5(^1)F(_3)</td>
<td>0.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9(^1)S(_1)</td>
<td>78216</td>
<td>0.74</td>
<td>3</td>
</tr>
<tr>
<td>9(^1)S(_0)</td>
<td>78404</td>
<td>0.72</td>
<td>1</td>
</tr>
<tr>
<td>9(^3)P(_0)</td>
<td>0.76</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>9(^3)P(_1)</td>
<td>0.61</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>9(^3)P(_2)</td>
<td>0.58</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>9(^1)P(_3)</td>
<td>0.58</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>8(^1)D(_2)</td>
<td>79661</td>
<td>0.56</td>
<td>5</td>
</tr>
<tr>
<td>8(^1)D(_1)</td>
<td>79679</td>
<td>0.56</td>
<td>3</td>
</tr>
<tr>
<td>8(^1)D(_2)</td>
<td>79690</td>
<td>0.56</td>
<td>5</td>
</tr>
<tr>
<td>8(^1)D(_3)</td>
<td>79690</td>
<td>0.56</td>
<td>7</td>
</tr>
<tr>
<td>6(^1)F(_2,3,4) 6(^1)F(_1)</td>
<td>79745</td>
<td>0.55</td>
<td>28</td>
</tr>
<tr>
<td>5(^1)G(_2,4,5) 5(^1)G(_4)</td>
<td>0.54</td>
<td></td>
<td>36</td>
</tr>
<tr>
<td>10(^1)S(_0) 10(^3)S(_1)</td>
<td>0.47</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>10(^1)P(_1)</td>
<td>81154</td>
<td>0.34</td>
<td>3</td>
</tr>
<tr>
<td>10(^3)P(_0,1,2)</td>
<td></td>
<td>0.39</td>
<td>9</td>
</tr>
</tbody>
</table>
With this data the levels with an ionisation energy of more than 0.6 eV can be determined. As we can see in table 3.3 the highest energy level with an ionisation energy more than 0.6 eV is the $9^3\text{P}_1$ level. As mentioned before, there are still many energy levels. If all the transition rates between all the different energy levels have to be calculated, and put in our model, that would be a time-consuming task. What we are going to do is reducing the number of energy levels, by grouping them. An energy level can be described as follows:

$$E_{\text{level}} = p^{2s+1}L_l$$  \hspace{1cm} (3.37)

All energy levels with the same $p$ and $L$ quantum numbers are grouped (3.37). This means for instance that all the $7^3\text{P}$ levels ($7^3\text{P}_1$, $7^3\text{P}_0$, $7^3\text{P}_2$) are grouped to form the $7^3\text{P}$ group. This is done for all energy levels, with the exception of the $6^1\text{S}_0$ level (the ground state) and the $6^3\text{P}_0$, $6^3\text{P}_1$ and $6^3\text{P}_2$ levels. These levels are not grouped because the $6^3\text{P}_0$ and $6^3\text{P}_2$ levels are metastable levels, they play an important role in the ionisation process. The $6^3\text{P}_1$ and $6^1\text{P}_1$ levels are resonant levels. The transitions from these levels to the ground state $6^1\text{S}_0$ are responsible for the largest part of the radiation.

The ionisation energy and the statistical weight for a certain group are determined by:

$$I_{\text{group}} = \frac{\sum I_p \cdot g}{\sum g}, \quad g_{\text{group}} = \sum g \hspace{1cm} (3.38)$$

With $I_p$ the ionisation energy for a certain level, and $g$ the statistical weight for that level.

There is a problem with determining the ionisation energy for certain groups, like the $6\text{D}$, $10\text{S}$, $10\text{P}$, $5\text{F}$, $6\text{F}$, $7\text{F}$ and $5\text{G}$ groups. For these groups not all, or even none, of the energy levels are known. The ionisation energy for these groups is calculated by determining the quantum defect $\Delta_l$. The quantum defect is the difference of the effective principal quantum number ($p_{\text{eff}}$) of a level ($p$) and the principal quantum number $p$ according to the hydrogen approximation.

$$\Delta_l = p - p_{\text{eff}} = p - \sqrt{\frac{R}{I_p}} \hspace{1cm} (3.39)$$

In the energy level notation $p^{2s+1}L_l$, $p$ is the value of the pqn according to the hydrogen approximation. The value of the effective pqn for a non-hydrogen atom is determined by the equation $p_{\text{eff}} = \sqrt{\frac{R}{I_p}}$. In table 3.4 the quantum defects of the $S$, $P$, $D$ and $F$ shell are determined. We now know the quantum defect for each shell, so that the value of the effective pqn for the unknown groups can be determined, $p_{\text{eff}} = p - \Delta_l$. 


Table 3.4: Quantum-defect of mercury.

<table>
<thead>
<tr>
<th>Shell</th>
<th>Effective pqn √R/I₀</th>
<th>pqn (H-model)</th>
<th>Quantum-defect Δ₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-Shell (L=0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6⁰S₀</td>
<td>1.14</td>
<td>6</td>
<td>4.86</td>
</tr>
<tr>
<td>7⁰S₀</td>
<td>2.33</td>
<td>7</td>
<td>4.67</td>
</tr>
<tr>
<td>8⁰S₀</td>
<td>3.35</td>
<td>8</td>
<td>4.65</td>
</tr>
<tr>
<td>9⁰S₀</td>
<td>4.36</td>
<td>9</td>
<td>4.64</td>
</tr>
<tr>
<td>P-Shell (L=1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 6¹P₁</td>
<td>1.91</td>
<td>6</td>
<td>4.09</td>
</tr>
<tr>
<td>7¹P₁</td>
<td>2.92</td>
<td>7</td>
<td>4.08</td>
</tr>
<tr>
<td>8¹P₁</td>
<td>3.87</td>
<td>8</td>
<td>4.13</td>
</tr>
<tr>
<td>9¹P₁</td>
<td>4.85</td>
<td>9</td>
<td>4.15</td>
</tr>
<tr>
<td>D-Shell (L=2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6²D₁</td>
<td>2.92</td>
<td>6</td>
<td>3.08</td>
</tr>
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<td>3.06</td>
</tr>
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<td>6.94</td>
<td>10</td>
<td>3.06</td>
</tr>
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</tr>
<tr>
<td>6³F₁</td>
<td>4.97</td>
<td>6</td>
<td>1.03</td>
</tr>
<tr>
<td>7³F₁</td>
<td>5.96</td>
<td>7</td>
<td>1.04</td>
</tr>
<tr>
<td>8³F₁</td>
<td>6.96</td>
<td>8</td>
<td>1.04</td>
</tr>
</tbody>
</table>

The ground state 6⁰S₀ usually is not hydrogen like, so the quantum defect will be larger than for an average S-shell (L=0). We can see that for L=0 the quantum defect Δ₁ is about 4.7, for L=1 about 4.1, for L=2 about 3.1 and for L=3 about 1.0. In general, for increasing L Δ₁ approaches zero. This is because for increasing L the excited electron describes a more circular orbit, and because of this it will “see” the nucleus and orbiting electrons more as a hydrogen nucleus. For decreasing L the bound electron can dive into the electron cloud and so the shielding is less effective [Mul86].

If the quantum defect belonging to a certain L-value is known, the ionisation energy I₀ can be calculated with equation 3.39. For instance, the ionisation energy for the 5F group can now be calculated. We assume that the quantum defect for the F-shell (L=3) is about 1. For the 5F group the principal quantum number equals 5, and so the ionisation energy according to equation 3.39 is 13.6/(5-1)² = 0.85 eV.

Now we can determine the ionisation energies for the different groups. The results can be seen in tables 3.5 and 3.6 and figure 3.3. Figure 3.2 gives the electron excitation scheme.
Table 3.5: Grouped energy levels for mercury.

<table>
<thead>
<tr>
<th>Energy-level</th>
<th>( I_p ) [eV]</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 6^3S_0 )</td>
<td>10.43750</td>
<td>1</td>
</tr>
<tr>
<td>( 6^3P_0 )</td>
<td>5.77</td>
<td>1</td>
</tr>
<tr>
<td>( 6^1P_1 )</td>
<td>5.55</td>
<td>3</td>
</tr>
<tr>
<td>( 6^3P_2 )</td>
<td>4.98</td>
<td>5</td>
</tr>
<tr>
<td>( 6^3P_1 )</td>
<td>3.74</td>
<td>3</td>
</tr>
<tr>
<td>( 7S )</td>
<td>2.66</td>
<td>4</td>
</tr>
<tr>
<td>( 7P )</td>
<td>1.67</td>
<td>12</td>
</tr>
<tr>
<td>( 6D )</td>
<td>1.59</td>
<td>20</td>
</tr>
<tr>
<td>( 8S )</td>
<td>1.25</td>
<td>4</td>
</tr>
<tr>
<td>( 8P )</td>
<td>0.96</td>
<td>12</td>
</tr>
<tr>
<td>( 7D )</td>
<td>0.88</td>
<td>20</td>
</tr>
<tr>
<td>( 5F )</td>
<td>0.85</td>
<td>28</td>
</tr>
<tr>
<td>( 9S )</td>
<td>0.73</td>
<td>4</td>
</tr>
<tr>
<td>( 9P )</td>
<td>0.60</td>
<td>12</td>
</tr>
<tr>
<td>( 8D )</td>
<td>0.56</td>
<td>20</td>
</tr>
<tr>
<td>( 6F )</td>
<td>0.55</td>
<td>28</td>
</tr>
<tr>
<td>( 5G )</td>
<td>0.54</td>
<td>36</td>
</tr>
<tr>
<td>( 10S )</td>
<td>0.47</td>
<td>4</td>
</tr>
<tr>
<td>( 10P )</td>
<td>0.38</td>
<td>12</td>
</tr>
<tr>
<td>( 9D )</td>
<td>0.34</td>
<td>20</td>
</tr>
<tr>
<td>( 7F )</td>
<td>0.34</td>
<td>28</td>
</tr>
</tbody>
</table>

Fig. 3.2: Electron excitation scheme [Ben90].

As will be discussed in § 3.7 there are certain selection rules for transitions. One of those selection rules is \( \Delta L = 0, \pm 1 \). This means that a transition from an S-shell (\( L=0 \)) to a D-shell (\( L=2 \)) is not allowed. In the CRM, only stepwise excitation is included, so only transitions to an above lying level are allowed.
Fig. 3.3: Energy scheme for mercury (not on scale).

Table 3.6: Possible excitation transitions for mercury after [Ben90].

<table>
<thead>
<tr>
<th>Initial level</th>
<th>Final levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6^1S_0$</td>
<td>$6^3P_0$</td>
</tr>
<tr>
<td>$6^3P_0$</td>
<td>$6^3P_1$</td>
</tr>
<tr>
<td>$6^3P_1$</td>
<td>$6^3P_2$</td>
</tr>
<tr>
<td>$6^3P_2$</td>
<td>$6^1P_1$</td>
</tr>
<tr>
<td>$6^1P_1$</td>
<td>$7S$</td>
</tr>
<tr>
<td>$7S$</td>
<td>$7P$</td>
</tr>
<tr>
<td>$7P$</td>
<td>$6D$</td>
</tr>
<tr>
<td>$6D$</td>
<td>$8P$</td>
</tr>
<tr>
<td>$8S$</td>
<td>$8P$</td>
</tr>
<tr>
<td>$8P$</td>
<td>$7D$</td>
</tr>
<tr>
<td>$7D$</td>
<td>$9P$</td>
</tr>
<tr>
<td>$5F$</td>
<td>$8D$</td>
</tr>
<tr>
<td>$9S$</td>
<td>$9P$</td>
</tr>
</tbody>
</table>
3.6 EXCITATION RATES AND CROSS-SECTIONS

An important part of the CRM are the cross-sections for excitation. Using these cross-sections the excitation rates can be determined, using equation 3.8 (§ 3.2).

\[ K_{pq} = \int_{E_m} \sigma_{pq} f(E) v(E) dE \]  \hspace{1cm} \text{[m}^3\text{s}^{-1}] \tag{3.8} \]

An extensive study of this subject is done in [Mas96]. These cross-sections were obtained in literature ([Lis], [Roc73], [Win83], [Ken50]). Of the cross-sections graphs are made and three sorts of fit-functions are used to approximate these. The first fit function is the Hartgers fit (3.40). The cross-sections in table 3.7 are approximated by the Hartgers fit.

\[ \sigma(E) = \begin{cases} 
0 & E < E_{th} \\
\sigma_0 \left( \frac{E - E_{th}}{E_1 - E_m} \right)^p & E_{th} < E < E_1 \\
\sigma_0 & E_1 < E < E_2 \\
\sigma_0 \left( \frac{E_3 - E_2}{E_3 - E} \right) & E > E_2 
\end{cases} \tag{3.40} \]

with \( E_1, E_2, E_3 \) [eV] and \( \sigma_0 \) [m\(^2\)] fit parameters, and \( E_{th} \) the threshold energy for the transition from one level to the other [eV].

Table 3.7: Excitation cross-sections using the Hartgers fit.

<table>
<thead>
<tr>
<th>Transition</th>
<th>( \sigma_0 ) [10(^{-20}) m(^2)]</th>
<th>( E_{th} ) [eV]</th>
<th>( E_1 ) [eV]</th>
<th>( E_2 ) [eV]</th>
<th>( E_3 ) [eV]</th>
<th>Exp. p</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 6^1S_0 \rightarrow 6^3P_0 )</td>
<td>0.3551</td>
<td>4.67</td>
<td>6.46</td>
<td>7.17</td>
<td>4.86</td>
<td>0.54</td>
</tr>
<tr>
<td>( 6^1S_0 \rightarrow 6^3P_1 )</td>
<td>1.741</td>
<td>4.89</td>
<td>5.10</td>
<td>5.94</td>
<td>5.00</td>
<td>3.33</td>
</tr>
<tr>
<td>( 6^1S_0 \rightarrow 6^3P_2 )</td>
<td>3.234</td>
<td>5.46</td>
<td>6.03</td>
<td>6.09</td>
<td>2.09</td>
<td>2.60</td>
</tr>
<tr>
<td>( 6^1S_0 \rightarrow 6^3P_1 )</td>
<td>3.000</td>
<td>6.70</td>
<td>10.0</td>
<td>16.1</td>
<td>-11.7</td>
<td>1.00</td>
</tr>
</tbody>
</table>

An other fit function that is used is the ‘ion’ fit (3.41). The cross-sections approximated by this fit are written down in table 3.8.

\[ \sigma(E) = \sigma_0 \left( 1 - \frac{E_{th}}{E} \right)^p \tag{3.41} \]
Table 3.8: Excitation cross-sections using the ion fit.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$E_{th}$ [eV]</th>
<th>$\sigma_0 [10^{-20} \text{ m}^2]$</th>
<th>exp. p</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6^3P_0 \rightarrow 6^3P_1$</td>
<td>2.03</td>
<td>5.17</td>
<td>1.26</td>
</tr>
<tr>
<td>$6^3P_0 \rightarrow 7S$</td>
<td>3.06</td>
<td>4.90</td>
<td>0.951</td>
</tr>
<tr>
<td>$6^3P_0 \rightarrow \text{ion}$</td>
<td>5.77</td>
<td>13.4</td>
<td>1.16</td>
</tr>
<tr>
<td>$6^3P_1 \rightarrow 6^1P_1$</td>
<td>1.81</td>
<td>6.57</td>
<td>1.53</td>
</tr>
<tr>
<td>$6^3P_1 \rightarrow 7S$</td>
<td>2.84</td>
<td>5.94</td>
<td>1.16</td>
</tr>
<tr>
<td>$6^3P_1 \rightarrow \text{ion}$</td>
<td>5.55</td>
<td>22.5</td>
<td>1.33</td>
</tr>
<tr>
<td>$6^3P_2 \rightarrow 6^1P_1$</td>
<td>1.24</td>
<td>5.79</td>
<td>1.76</td>
</tr>
<tr>
<td>$6^3P_2 \rightarrow 7S$</td>
<td>2.27</td>
<td>6.95</td>
<td>1.33</td>
</tr>
<tr>
<td>$6^3P_2 \rightarrow \text{ion}$</td>
<td>4.98</td>
<td>1.30</td>
<td>0.906</td>
</tr>
<tr>
<td>$6^1P_1 \rightarrow \text{ion}$</td>
<td>3.74</td>
<td>2.40</td>
<td>1.09</td>
</tr>
<tr>
<td>$6^1S_0 \rightarrow \text{ion}$</td>
<td>10.44</td>
<td>8.16</td>
<td>1.28</td>
</tr>
</tbody>
</table>

The last fit function that is used is the ‘step’ fit (3.42). The cross-sections approximated by this fit are written down in table 3.9.

$$\sigma(E) = \begin{cases} 0 & E < E_{th} \\ \sigma_0 & E > E_{th} \end{cases}$$ \hspace{1cm} (3.42)

Table 3.9: Excitation cross-sections using the step fit.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$E_{th}$ [eV]</th>
<th>$\sigma_0 [10^{-20} \text{ m}^2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6^3P_0 \rightarrow 6^3P_1$</td>
<td>0.22</td>
<td>9.60</td>
</tr>
<tr>
<td>$6^3P_1 \rightarrow 6^3P_2$</td>
<td>0.57</td>
<td>1.26</td>
</tr>
</tbody>
</table>

There remain a lot of transition for which no excitation cross-sections are known. For these transitions semi-theoretical expressions for hydrogen like atoms, based on both theoretical ideas and experimental data ([Har96] and [Vri80]), are determined. The equations for the calculation of the rates are now given. For a more precise derivation see [Vri80].

The ionisation rate from a level $p$ is given by:

$$K_{ps} = \frac{9.56 \times 10^{-6} e^{-\epsilon_{ps}}}{T_e^{1.5} \left( \epsilon_{ps}^{2.33} + 4.38 \epsilon_{ps}^{1.72} + 1.32 \epsilon_{ps} \right)} \quad [\text{cm}^3\text{s}^{-1}]$$ \hspace{1cm} (3.43)

with $T_e$ in eV and $\epsilon_{ps}$ defined by:

$$\epsilon_{ps} = \frac{E_s - E_p}{T_e}$$ \hspace{1cm} (3.44)
The excitation rate from a level \( p \) to a level \( q \) is given by:

\[
K_{pq} = \frac{1.6 \times 10^{-7} \sqrt{T_e} e^{-\Delta_{pq}}}{T_e + \Gamma_{pq}} \left[ A_{pq} \ln \left( \frac{0.3T_e}{R} + \Delta_{pq} \right) + b_{pq} \right] \quad [\text{cm}^3\text{s}^{-1}]
\]  \quad (3.45)

with:

\[
\Delta_{pq} = e^{(-b_{pq}/A_{pq})} + \frac{0.06 s^2}{qp^2}
\]  \quad (3.46)

and:

\[
\Gamma_{pq} = R \ln \left( 1 + \frac{p^3 T_e}{R} \right) \left( 3 + 11 \left( \frac{s}{p} \right)^2 \right) \left( 6 + 1.6qs + \frac{0.3}{s^2} + 0.8 \frac{q^{1.5}}{s^{0.5}} |s - 0.6| \right)^{-1}
\]  \quad (3.47)

with \( s = |q - p|^2 \) and \( R \) the Rydberg constant [13.6 eV]. In equations 3.45 and 3.46 \( A_{pq} \) and \( B_{pq} \) are defined by:

\[
A_{pq} = \frac{2R_f}{E_{pq}^2};
\]  \quad (3.48)

\[
B_{pq} = \frac{4R^2}{q^2} \left( \frac{1}{E_{pq}^2} + \frac{4E_{pq}}{3E_{pq}^2} + \frac{E_{pq}^2}{E_{pq}^2} \right)
\]  \quad (3.49)

with \( E_{pq} = E_q - E_p [\text{eV}] \), \( f \) the absorption oscillator strength and \( b_p \):

\[
b_p = \frac{1.4 \ln p - 0.7}{p} - \frac{0.51}{p^2} + \frac{1.16}{p^3} - \frac{0.55}{p^4}
\]  \quad (3.50)

For step processes \( f_{pq} \) becomes [Mul86]:

\[
f_{pq} = \frac{1.52}{p^3 q^3} \left( \frac{1}{p^2 - q^{-2}} \right)^3
\]  \quad (3.51)

For jump processes \( f_{pq} \) becomes [Mul86]:

\[
f_{pq} = \frac{1.95}{p^3 q^3} \left( \frac{1}{p^2 - q^{-2}} \right)^3
\]  \quad (3.52)
3.7 RADIATIVE TRANSITION PROBABILITIES

Once all the different energy levels and the allowed transitions are known, we can now begin determining the transition probabilities. In the paper of Mosberg et al. [Mos78] a number of transition probabilities is given. The problem is that these transition probabilities are given for specific energy levels, and not for grouped levels. So we will have to transform these specific transition probabilities to transition probabilities for grouped levels. An other problem is that certain transitions are forbidden, according to the selection rules. However, some of these forbidden transitions do occur.

First we take a look at the forbidden transitions. As mentioned before, each energy level is described by the notation $p^{2s+1}L_J$. There are three selection rules applicable (although they are not absolute):

- $\Delta S = 0$.
- $\Delta L = 0, \pm 1$.
- $\Delta J = 0, \pm 1$.

For instance, the transition $7^1P_1 \rightarrow 6^1S_0$ is allowed, $\Delta S = 0, \Delta L = 1$ and $\Delta J = 1$. On the other hand, the transition $7^3P_2 \rightarrow 6^1S_0$ is not allowed, $\Delta S = 1, \Delta L = 1$ and $\Delta J = 2$. However, we can see in the data from [Mos78] that the transition rule $\Delta S = 0$ is frequently broken. For mercury, the transition $\Delta S = \pm 1$ is allowed. The transition $6^3P_1 \rightarrow 6^1S_0$ for example, which is responsible for 85% of the light coming from the lamp, does occur, although $\Delta S = 1$. For mercury, the selection rule $\Delta S = 0$ will be extended with $\Delta S = \pm 1$.

The other problem is determining the transition probabilities between energy groups, in stead of energy levels. From one energy group to a lower situated energy group there are several transitions possible. What we want is an averaged $A$-value (transition probability) for a transition between two groups. For this we need the transition probabilities of all allowed transitions between two certain energy groups.

The difficulty is that not every transition probability between energy levels is known. If we are missing a certain transition probability, there are two options. First, the missing transition is forbidden. In this case we can use equation 3.53, and for the forbidden $A$-value we just fill in zero. Second, the missing transition is allowed, but unknown. In this case the average of the known transition probabilities is taken.

$$A = \frac{\sum \sum g_i \cdot A_{if}}{\sum g_i}, i = \text{initial}, f = \text{final}$$

(3.53)
As we can see in table 3.10 it is not complete. A lot of transition probabilities are not known. If we know that a transition is possible, but we do not know what the value of the transition probability is, the hydrogen approximation is used [Sch96]:

\[
A_{pq}^{qw} = \frac{4\pi m_e v|\mathbf{r}_{pq}|^2}{3\hbar} \cdot \frac{2\pi e^2 v^2}{\varepsilon_0 m_e c^3}
\]  

(3.54)

with \(m_e\) the electron mass \([9.1\times10^{-31} \text{ kg}]\), \(\hbar\) Dirac’s constant \([1.055\times10^{-34} \text{ Js}]\), \(e\) the elementary charge \([1.6\times10^{-19} \text{ C}]\), \(\varepsilon_0\) the electric constant \([8.85\times10^{-12} \text{ C/Nm}]\), \(v\) the frequency for the transition and \(r_{pq}\) the matrix element for the dipole transition.

Using equation 3.54 can create a problem. If the values of the transition probabilities of the levels of two grouped levels are not known the CRM will calculate the average value, assuming all transitions between the individual levels are possible, even if certain transitions are forbidden. However, totally neglecting all unknown radiative transitions proves to have very little effect on the ionisation and recombination coefficients.
For a transition to the ground state $6^1S_0$ the probability that a photon will be reabsorbed is significant. This means that the probabilities for the transition to the $6^1S_0$-level have to be corrected with an escape factor $\Lambda$, $\Lambda_{pq}^{\text{eff}} = \Lambda_{pq} \times \Lambda_{pq}$ ($p > q$). The escape factor is the probability that the photons are not reabsorbed. If all photons escape, the escape factor $\Lambda_{pq} = 1$. In the opposite case $\Lambda_{pq} = 0$. For highly excited levels, the escape factor of an optical transition towards an excited state equals unity, since the densities of the excited states are too low for any significant reabsorption. However, if $q$ is the ground state, $n_q$ is large, so a great part of the emitted photons are reabsorbed. For the transition from the first resonant level $6^3P_1$ to the ground state $\Lambda$ is about $2 \times 10^{-3}$, and for the transition from the second resonant level $6^1P_1$ to the ground state $\Lambda$ is about $1 \times 10^{-4}$ [Qin95]. The optical depth $kR$ of a plasma is given by:

$$kR = \frac{\sqrt{\ln 2}}{4\sqrt{\pi^3}} \frac{\lambda_{pq}}{c\Delta \lambda_{pq}} \cdot \frac{g_p}{g_q} \cdot n_p \Lambda_{pq} \mu R$$

(3.55)

with $k$ the absorption coefficient, $\lambda_{pq}$ the wavelength of the transition from $p$ to $q$ ($\lambda_{pq} = 254$ nm for the transition from the first resonant state $6^3P_1$ to the ground state and $\lambda_{pq} = 185$ nm for the transition from the second resonant state $6^1P_1$ to the ground state), $n_q$ the lower density state (in our case, the ground state), $R$ the effective radius of the plasma beam ($R = 1$ cm), $\Delta \lambda_{pq}$ the half width of the emission profile ($\Delta \lambda_{pq} = 0.2$ nm [Jon97]) and $\mu$ a geometrical factor ($\mu = 1.2$). With the optical depth known, the escape factor can be determined from figure 5.5 of [Qin95]. For increasing optical depth the escape factor decreases.
3.8 VALIDITY OF THE MAXWELLIAN EEDF

The current CRM assumes a Maxwellian EEDF. This assumption is now subject of investigation. It is known that the EEDF in a TL-lamp may deviate from Maxwell [Vri73]. However for higher \( n_e \) values which might be expected in the QL-lamp [Jon96] these deviations become smaller [Win83]. The aim of the study is to investigate how the departure of the EEDF from Maxwell depends on the electron density and temperature.

The study on the validity of the presence of Maxwellian equilibrium can be done by dividing the distribution in two parts, the bulk and the tail. The boundary energy \( E_1 \) between those two parts is determined by the energy of the transition from the ground state \( 6^1S_0 \) to the first excited state \( 6^3P_0 \). For mercury, this energy \( E_1 \) is 4.67 eV. Next, the temperatures for both parts (bulk temperature \( T_b \) and tail temperature \( T_t \)) are calculated, using the Two Electron Group Model (2EGM, [Vri73] and [Lig80]). Unfortunately in the literature the use of units in their calculations was not consistent. This means that their equations have to be adjusted.

To calculate the tail temperature as a function of the bulk temperature, the energy conservation law for the tail electrons in a gas discharge is used:

\[
P_c' + P_F' + P_{sel}' = P_{el}' + P_{exc}' + P_{ion}'
\]

where the subscript c refers to coulomb relaxation (e-e collisions), F to the electric field, sel to superelastic collisions, el to elastic collisions, exc to the excitation and ion to the ionisation. On the left hand side the energy gain terms and on the right hand side the energy loss terms are written down. For the bulk temperature we take the average electron temperature. The recombination and diffusion components are neglected [Lig80].

An important part of the treatment we follow [Lig80] is that the tail has a quasi-Maxwell structure with a tail temperature \( T_t < T_b \).

\[
\Gamma_r(E_e) = 2n_e S \frac{E_e}{\pi (k_B T_t)^2} \times e^{-E_e/k_B T_t} \quad [m^3 J^{-1}] \quad (3.57)
\]

with \( n_e \) the electron density \([m^3]\), \( E_e \) the electron energy \([J]\) and \( k_B T_t \) the tail temperature \([J]\). The value of \( S \) is such that the bulk-tail transition is continuous. However, since \( S \) appears in each term of equation 3.56, the value has no influence on the calculation of the tail temperature as a function of the bulk temperature.

First the net Coulomb power density related to the energy gain for the tail electrons due to the coulomb interactions is calculated. For a given form of \( \Gamma_r \) the net energy flow per unit of time and per unit of volume into the tail \( (E_e > E_1) \) can be found from [Lig80]:

\[
P_c' = \int_{E_1}^{E_e} \left( \frac{\partial \Gamma_e}{\partial E_e} \right) E_e dE_e \quad [W/m^3] \quad (3.58)
\]

This is a standard form for most terms of equation 3.56, with \( \Gamma_e \) the electron flux (in this case due to Coulomb interactions).
This equation can be rewritten to equation 3.59. This is done in [Lig80], and it represents the net contribution of the Coulomb interactions. The Coulomb power density equals 0 if the tail and bulk temperature have the same value.

\[
P_c' = \left(1 - \frac{T_b}{T_i}\right) \times (E_i + k_B T_i) \times f'(E_i) \times \left(\frac{dE_e}{dt}\right)_{E_e=E_i} \quad \text{[W/m}^3]\]  

(3.59)

with \(dE_e/dt\)

\[
\left(\frac{dE_e}{dt}\right)_{E_e=E_i} = -2n_e \pi \times \frac{e^4}{(4\pi\varepsilon_0)^2} \times \sqrt{\frac{2}{m_e E_i}} \times \ln \Lambda \quad \text{[J/s]} \]

(3.60)

the average energy gain rate per unit time of an electron with energy \(E_e\) due to coulomb collisions, with \(\ln \Lambda\) the coulomb logarithm, which value is about 9 [Sch96], \(m_e\) the electron mass \(9.1 \times 10^{-31}\) kg, \(e\) the elementary charge \(1.6 \times 10^{-19}\) C, \(\varepsilon_0\) the electric constant \(8.85 \times 10^{-12}\) C/Vm, \(E\) the energy [J] and \(E_i\) the threshold energy \(7.47 \times 10^{-19}\) J.

The power density associated with the energy gain for the tail electrons due to the electric field can be written as [Lig80]:

\[
P_F' = -\int_{E_i}^{E_1} \left(\frac{d\Gamma_F'}{dE_e}\right)_{E_e=E_i} E_e dE_e \quad \text{[W/m}^3]\]  

(3.61)

with \(\Gamma_F'\)

\[
\Gamma_F' = f'(E_e) \left(\frac{dE_e}{dt}\right)_{E_e=E_i} \quad \text{[m}^3\text{s}^{-1}] \]

(3.62)

the electron flux, in which \(dE_e/dt\)

\[
\left(\frac{dE_e}{dt}\right)_{E_e=E_i} = \sqrt{\frac{2E_i}{m_e}} \times e^2 \times F^2 \times \frac{1}{2N_A \sigma_m k_B T_i} \quad \text{[J/s]} \]

(3.63)

is again the average energy gain per tail electron per unit time, but now as a consequence of the acceleration by the electric field, \(F\) is the electric field [V/m], \(N_A\) is the argon density \([10^{22} \text{ m}^{-3}]\) and \(\sigma_m\) is the momentum transfer cross-section \([10^{-19} \text{ m}^2]\). With equations 3.62 and 3.63 \(P_F\) can be rewritten as:

\[
P_F' = (E_1 + k_B T_i) \times f'(E_i) \times \left(\frac{dE_e}{dt}\right)_{E_e=E_i} \quad \text{[W/m}^3]\]  

(3.64)
We can make an estimation of the electric field in the QL-lamp. The typical voltage $V_p$ across the loop embedded in the active plasma zone and surrounding the antenna inside the QL-lamp is about:

$$V_p = k \frac{N_p}{N_{\text{win}}} V_{\text{lamp}} = 0.75 \frac{1}{15} \times 75 = 4V$$  \hspace{1cm} (3.65)$$

with $k$ the coupling factor (0.75), $N_p$ the number of plasma "windings" inside the lamp (1), $N_{\text{win}}$ the number of windings on the coil (15) and $V_{\text{lamp}}$ the measured voltage over the coil (75 V) [Heu95]. The active zone has a radius of about 2 cm, which corresponds to a length of about $2\pi \times 2 \times 10^{-2} m = 0.12 m$. This means that the electric field is about $4V / 0.12 m = 33.3 V/m$.

The first of the loss terms $P_{el}$, which describes the energy transfer rate due to elastic collisions from electrons to the heavy particles can be derived from [Lig80]:

$$P_{el}^1 = - \int_{E_i}^E \left( \frac{\partial \Gamma_{el}^1}{\partial E_e} \right) E_e dE_e$$  \hspace{1cm} [W/m$^3$] (3.66)$$

with $\Gamma_{el}^1$

$$\Gamma_{el}^1 = f(E_e) \left( \frac{dE_e}{dt} \right)_{el}$$  \hspace{1cm} [m$^3$s$^{-1}$] (3.67)$$

the electron energy flux of the elastic transfer of energy from tail electrons to heavy particles, and $dE_e/dt$

$$\left( \frac{dE_e}{dt} \right)_{el} = 2 \frac{m_e}{m_{Ar}} E_1 N_{Ar} \sigma_{m} \left( \frac{2E_1}{m_e} \right)$$  \hspace{1cm} [J/s] (3.68)$$

the average energy loss rate per unit time due to elastic collisions, with $m_{Ar}$ the argon mass [6.7x10$^{-26}$ kg]. Together, this leads to equation 3.69:

$$P_{el}^1 = (E_1 + k_B T_i) \times f(E_i) \times \left( \frac{dE_e}{dt} \right)_{el}$$  \hspace{1cm} [W/m$^3$] (3.69)$$
The excitation and ionisation terms are combined in equation 3.70 [Vri73]:

\[ p_{\text{exc,ion}} = \sum_{xy} n_x n_y < v_e \sigma_{xy} E_{xy} > \]

(3.70)

with \( E_{xy} \) the energy for the transition from the lower state \( x \) to the higher state \( y \), and \( n_x \) the density of the first state \([\text{m}^{-3}]\). For the first state \( x \) we only take the ground state. This is done because the term scales with \( n_x \). The density of the ground state is always at least two orders larger than the other excited states, so the contributions from these states can be ignored. The cross section as a function of the energy is simplified to:

\[ \sigma_{xy} = 0 \quad (E < E_{xy}) \]
\[ \sigma_{xy} = 0.4 \sigma_{xy}^{\text{max}} \quad (E > E_{xy}) \]

The factor 0.4 is an estimation. This is done because else it extremely difficult to solve equation 3.70. The transitions from the ground state \( 6^1\text{S}_0 \) to the first excited states \( (6^3\text{P}_0, \ 6^3\text{P}_1, \ 6^3\text{P}_2, \ 6^1\text{P}_1 \text{ and } 7^3\text{S}_1) \) and the ion ground state are taken into account. The energy for the transition \( 6^1\text{S}_0 \to 6^3\text{P}_0 \) \( E_{12} \) is the same as the boundary energy \( E_1 \). The values of \( \sigma_{xy}^{\text{max}} \) are written in table 3.11, and were obtained in § 3.6.

All determined power density (3.59, 3.64, 3.69) terms all have the same form:

\[ P' = X \left( \frac{dE_e}{dt} \right) \times (E_i + k_B T_i) \times f'(E_i) \]

(3.71)

with \( X \) equals unity for the electric field and elastic collisions terms, and \((1-T_i/T_e)\) for the Coulomb term. Now an attempt is being made to describe equation 3.70 in the same form as equation 3.71. This is done because then the calculation of the tail temperature as a function of the bulk temperature will become easier. A lot of similar terms can then be crossed off.

\[ P_{\text{exc,ion}} = \sum_{xy} n_x n_y E_{xy} 0.4 \sigma_{xy}^{\text{max}} \frac{1}{n_e} \int_{E_{xy}}^\infty \left( \frac{2E}{m_e} \right)^{1/2} f'(E) \ dE \]

\[ = \sum_{xy} n_x 0.4 \sigma_{xy}^{\text{max}} E_{xy} \sqrt{\frac{2}{m_e}} \frac{2 n_e}{\pi(k_B T_i) \sqrt{m_e}} \int_{E_{xy}}^\infty \left( \frac{1}{\pi(k_B T_i) \sqrt{m_e}} \right)^{1/2} E e^{-E/k_B T_i} \ dE \]

\[ = \sum_{xy} 0.8 n_x n_y \sigma_{xy}^{\text{max}} E_{xy} S \sqrt{\frac{2}{m_e \pi(k_B T_i) \sqrt{m_e}}} \int_{E_{xy}}^\infty E e^{-E/k_B T_i} \ dE \]  \[ \text{[W/m}^3\text{]} \]

(3.72)
The integral in the previous equation can be written as:

\[
\int_{E_{\nu}} E e^{-E/k_B T} dE = \left[ -E - k_B T \cdot e^{-E/k_B T} \right]_{E_{\nu}} - \int_{E_{\nu}} -k_B T e^{-E/k_B T} dE
\]

\[
= \left[ -E - k_B T \cdot e^{-E/k_B T} \right]_{E_{\nu}} - \left[ (k_B T) e^{-E/k_B T} \right]_{E_{\nu}}
\]

\[
= 0 + E_{xy} \cdot k_B T_1 e^{-E_{xy}/k_B T_1} - 0 + (k_B T_1)^2 \cdot e^{-E_{xy}/k_B T_1}
\]

\[
= k_B T_1 (E_{xy} + k_B T_1) e^{-E_{xy}/k_B T_1}
\]

This leads to:

\[
P_{\text{exc,ion}} = \sum_{x \leq y} n_x n_y 0.8 \sigma_{xy}^\text{max} E_{xy} \sqrt{\frac{2}{\pi n_x (k_B T_1)}} S k_B T_1 (E_{xy} + k_B T_1) e^{-E_{xy}/k_B T_1}
\]

\[
= \sum_{x \leq y} 0.4 \sigma_{xy}^\text{max} n_x E_{xy} \sqrt{\frac{2}{m_e E_1}} k_B T_1 (E_{xy} + k_B T_1)
\]

\[
\times 2 n_x S \frac{E_1}{\pi (k_B T_1)^3} e^{(-E_{xy} + E_{xy})/k_B T_1} e^{-E_{xy}/k_B T_1}
\]

(3.73)

(3.74)

Using equation 3.57 this leads to:

\[
P_{\text{exc,ion}} = \sum_{x \leq y} 0.4 \sigma_{xy}^\text{max} n_x E_{xy} \sqrt{\frac{2}{m_e E_1}} k_B T_1 \left( \frac{E_{xy} + k_B T_1}{E_1 + k_B T_1} \right) e^{(-E_{xy} + E_1)/k_B T_1}
\]

\[
\times \left( E_1 + k_B T_1 \right) f'(E_1)
\]

(3.75)

In order to calculate the losses of fast electrons in the tail due to excitation and ionisation the superelastic collisions from equation 3.56 have to be taken into account. The calculation of the superelastic collision term is done different than for the other terms.

When fast electrons are used for excitation or ionisation, a fraction of the excited atoms fall back to the ground state, and thus again creating fast electrons. This means that a part of the excitation/ionisation losses are compensated by superelastic collisions. To calculate this compensation factor we have to compare the excitation rate \( n_x R_e \) [m\(^3\)s\(^{-1}\)] from the ground state to a certain level \( y \) with the deexcitation rate \( n_y R_d \) [m\(^3\)s\(^{-1}\)] from that level \( y \) to the ground state. \( R_e \) and \( R_d \) are the excitation and deexcitation frequencies [s\(^{-1}\)]. These values are calculated by the CRM.
Table 3.11: The cross-sections and averaged superelastic collisions compensation factor for the excitation and ionisation processes from the ground state.

<table>
<thead>
<tr>
<th>Transition from $6^3S_0$ to:</th>
<th>$E_{ly}$ [J]</th>
<th>Cross section $\sigma_{xy}^{\text{max}}$ [10^{-20} m^2]</th>
<th>Averaged compensation factor $(n_iR_i/n_eR_e)/n_iR_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6^3P_0$ ($y = 2$)</td>
<td>7.47x10^{-19}</td>
<td>0.443</td>
<td>0.20</td>
</tr>
<tr>
<td>$6^3P_1$ ($y = 3$)</td>
<td>7.82x10^{-19}</td>
<td>18.2</td>
<td>0.20</td>
</tr>
<tr>
<td>$6^3P_2$ ($y = 4$)</td>
<td>8.74x10^{-19}</td>
<td>3.13</td>
<td>0.20</td>
</tr>
<tr>
<td>$6^3P_1$ ($y = 5$)</td>
<td>1.07x10^{-18}</td>
<td>2.83</td>
<td>0.75</td>
</tr>
<tr>
<td>$7^3S_1$ ($y = 6$)</td>
<td>1.24x10^{-18}</td>
<td>1.74</td>
<td>0.99</td>
</tr>
<tr>
<td>ion ($y = +$)</td>
<td>1.67x10^{-18}</td>
<td>5.15</td>
<td>1.00</td>
</tr>
</tbody>
</table>

As mentioned before in equation 3.71 all power density terms have the same form. The only difference in the four terms is the energy gain or loss per electron per unit of time [J/s]. To compare the four power density terms only $dE_p/dt$ has to be compared. This is done in table 3.12, for an electron density of $10^{19}$ m$^{-3}$, the compensation factor as described in table 3.11 is used for the excitation/ionisation term.

Table 3.12: The energy gain or loss per electron per unit of time for the different terms, with compensation factor for excitation terms, columns 3 and 4.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_i=0.75$ eV</td>
<td>$T_i=1.50$ eV</td>
<td>$T_i=1.50$ eV</td>
</tr>
<tr>
<td>Coulomb interaction</td>
<td>-5.10x10^{-30} $\times n_e \times (1-T_f/T_i)$</td>
<td>5.1x10^{-11}</td>
<td>5.1x10^{-11}</td>
</tr>
<tr>
<td>Electric field</td>
<td>$1.48x10^{-32} \times 1/k_B T_i$</td>
<td>1.23x10^{-13}</td>
<td>6.17x10^{-14}</td>
</tr>
<tr>
<td>Elastic collisions</td>
<td>2.64x10^{-14}</td>
<td>2.64x10^{-14}</td>
<td>2.64x10^{-14}</td>
</tr>
<tr>
<td>Excitation to $6^3P_0$</td>
<td>9.08x10^{4} $k_B T_i$</td>
<td>1.09x10^{-14}</td>
<td>2.18x10^{-14}</td>
</tr>
</tbody>
</table>
| Excitation to $6^3P_1$    | $3.91x10^{6} k_B T_i \times 
\left(\frac{(E_{15}+k_B T_i)}{(E_1+k_B T_i)}\right) \exp\left(-\frac{E_{15}}{k_B T_i}\right)$ | 3.64x10^{-13} | 8.37x10^{-13} |
| Excitation to $6^3P_2$    | $7.50x10^{5} k_B T_i \times 
\left(\frac{(E_{16}+k_B T_i)}{(E_1+k_B T_i)}\right) \exp\left(-\frac{E_{16}}{k_B T_i}\right)$ | 3.60x10^{-14} | 1.20x10^{-13} |
| Excitation to $6^1P_1$    | $3.12x10^{6} k_B T_i \times 
\left(\frac{(E_{15}+k_B T_i)}{(E_1+k_B T_i)}\right) \exp\left(-\frac{E_{15}}{k_B T_i}\right)$ | 3.44x10^{-14} | 2.57x10^{-13} |
| Excitation to 7$S_1$      | $2.95x10^{6} k_B T_i \times 
\left(\frac{(E_{16}+k_B T_i)}{(E_1+k_B T_i)}\right) \exp\left(-\frac{E_{16}}{k_B T_i}\right)$ | 9.36x10^{-15} | 1.38x10^{-13} |
| Ionisation                | $1.18x10^{7} k_B T_i \times 
\left(\frac{(E_{15}+k_B T_i)}{(E_1+k_B T_i)}\right) \exp\left(-\frac{E_{15}}{k_B T_i}\right)$ | 1.33x10^{-15} | 1.17x10^{-13} |

The energy $E$ and temperature $k_B T_i$ are again given in J. By using equation 3.56, and inserting the Coulomb, electric field, elastic collisions and excitation/ionisation terms from table 3.12 for different tail temperatures and electron densities the bulk temperature can be calculated. With this, the bulk temperature as a function of the tail temperature can be determined. A
numerical approach (Newton's method) has been used to calculate the tail temperature as a function of the bulk temperature.

The results can be seen in figure 3.4. For increasing values of $n_e$ the difference between bulk and tail temperature decreases, which is in correspondence with [Win83]. All terms are proportional with $n_e$, except $P_e$, which is proportional to $n_e^2$. For a higher value of $n_e$ the EEDF will become more like Maxwell, because the electron-electron collisions cause a more Maxwellian EEDF. For increasing $T_e (= T_b)$ the relative deviations from Maxwell increase as well. This can be explained by the fact that the excitation/ionisation increases exponentially with $T_e$, and so the tail temperature decreases.

![Fig. 3.4: Tail temperature as a function of the bulk temperature.](image)

In the most active zone of the QL-lamp (fig. 2.3, § 2.2.1) we expect a Maxwellian EEDF ($n_e \approx 10^{19} \text{ m}^{-3}$, $T_e \approx 1 \text{ eV}$, chapter 5). In the less active zones in the lamp the electron density is probably lower, so deviations of the Maxwellian EEDF have to be taken into account.

In the article of [Lig80] several tail temperatures were calculated for different electron (= bulk) temperatures. The results can be seen in table 3.13.

<table>
<thead>
<tr>
<th>$T_b$ [eV]</th>
<th>$n_e$ [m$^{-3}$]</th>
<th>$F$ [V/m]</th>
<th>$T_t$ [eV] [Lig80]</th>
<th>$T_t$ [eV] (This calculation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.14</td>
<td>$1.2\times10^{17}$</td>
<td>97</td>
<td>0.59</td>
<td>0.69</td>
</tr>
<tr>
<td>1.07</td>
<td>$2.5\times10^{17}$</td>
<td>91</td>
<td>0.66</td>
<td>0.71</td>
</tr>
<tr>
<td>0.98</td>
<td>$4.9\times10^{17}$</td>
<td>81</td>
<td>0.70</td>
<td>0.72</td>
</tr>
<tr>
<td>0.95</td>
<td>$9.\times10^{17}$</td>
<td>69</td>
<td>0.76</td>
<td>0.75</td>
</tr>
</tbody>
</table>

The trends [Lig80] and this calculation predict are the same, although [Lig80] predicts a faster increase in the tail temperature as a function of the electron density. This can be explained by the fact that in this case no superelastic collisions were taken into account. To calculate the compensation factor the CRM has to be used, which is not valid for these low electron densities.
An other important part that has to be investigated is the influence of the depletion of the Maxwellian tail on the ionisation coefficient. This is done by calculating the ionisation coefficient \( S_{CR} \) for a small difference in the electron temperature. Although this is not the same as the tail temperature, it is an indication of the error in the ionisation coefficient. The maximal allowed difference in \( S_{CR} \) is taken to be about 15-20% (table 3.14).

Table 3.14: The difference in \( S_{CR} \) as a function of the difference in \( T_e \).

<table>
<thead>
<tr>
<th>( n_e \text{[m}^3\text{]} )</th>
<th>( T_e = 1 \text{ eV} )</th>
<th>( T_e = 1 \text{ eV} )</th>
<th>( T_e = 1.5 \text{ eV} )</th>
<th>( T_e = 1.5 \text{ eV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta T_e \text{[eV]} )</td>
<td>( \Delta S_{CR} % )</td>
<td>( \Delta T_e \text{[eV]} )</td>
<td>( \Delta S_{CR} % )</td>
<td>( \Delta T_e \text{[eV]} )</td>
</tr>
<tr>
<td>( 1 \times 10^{18} )</td>
<td>0.061</td>
<td>50</td>
<td>0.19</td>
<td>64</td>
</tr>
<tr>
<td>( 1 \times 10^{18} )</td>
<td>0.015</td>
<td>13</td>
<td>0.05</td>
<td>20</td>
</tr>
<tr>
<td>( 1 \times 10^{18} )</td>
<td>0.008</td>
<td>8.1</td>
<td>0.03</td>
<td>13</td>
</tr>
<tr>
<td>( 1 \times 10^{18} )</td>
<td>0.001</td>
<td>0.9</td>
<td>0.004</td>
<td>1.0</td>
</tr>
</tbody>
</table>

It should be realised that in this case both the bulk and tail temperature have been decreased. If only the tail temperature were decreased, the difference in the ionisation coefficient might not be so large. As we compare the results from table 3.14 and figure 3.4 it can be seen that the assumption the EEDF is Maxwellian is valid for an electron density of about \( 5 \times 10^{18} \text{ m}^3 \).

Using the results from table 3.12 an attempt can be made to analytically describe the tail temperature as a function of the bulk temperature. Van der Mullen [Mul97] gave an analytical expression for the ratio of the bulk and tail temperature:

\[
\frac{T_h}{T_t} = 1 + \frac{v_t}{v_b}
\]  
(3.76)

with \( v_t \) the 'decay frequency’ of a tail electron due to ground state excitation or ionisation whereas \( v_b \) is related to the Coulomb cooling. This equation is valid for the assumption that the electric field and elastic collisions term can be neglected. This result can also be obtained using the energy balance (3.56), again neglecting the electric field and elastic collisions term. Equation 3.75 can be rewritten to:

\[
P_{exc,ion} = n_i \hat{\sigma} v_e k_b T_t (E_i + k_b T_t) f'(E_i)
\]  
(3.77)

Combining this equation with equation 3.59 we get:

\[
\left(1 - \frac{T_h}{T_t}\right) \times (E_i + k_b T_t) \times f'(E_i) \times \left(\frac{dE}{dt}\right)_e = n_i \hat{\sigma} v_i \times (E_i + k_b T_t) \times f'(E_i)
\]  
(3.78)

This equation can be rewritten as:

\[
\left(1 - \frac{T_h}{T_t}\right) \times \frac{1}{E_i} \times \left(\frac{dE}{dt}\right)_e = n_i \hat{\sigma} v_i \times \left(\frac{k_b T_t}{E_i}\right)
\]  
(3.79)
Using $v_{inel} = n_1 \sigma_{average} v_t$ we get:

\[
\left(1 - \frac{T_b}{T_i}\right) v_b = v_{exc,ion} = v_{inel} = v_t \tag{3.80}
\]

\[
\frac{T_b}{T_i} = \frac{v_t}{v_b} + 1 \tag{3.81}
\]

This equation is the same as equation 3.76. It can now be seen that the tail temperature will deviate further from the bulk temperature as the disturbance frequency increases as compared to the Coulomb frequency. In principle, it is now possible to analytically determine the tail temperature as a function of the bulk temperature, but until now this has proven to be very difficult.
3.9 RESULTS FROM THE CRM

After the results from the previous chapters have been introduced into the CRM, we can run the model to calculate the ASDF and the coefficients of ionisation and recombination, $S_{CR}$ and $\alpha_{CR}$ [m$^{-3}$s$^{-1}$], and the radiation losses $\varepsilon_1$ (related to the ground state) and $\varepsilon_2$ (related to the ion ground state) [eV m$^{-3}$s$^{-1}$]. We introduce into the CRM the estimated values of the electron temperature $T_e$, the electron density $n_e$, the ion density $n_i$, the heavy particles density $n_1$ and the cut-off level, which is determined by the calculation of the boundary between CB and ESB. We know the boundary to be at an ionisation energy of about 0.6 eV, so all energy levels or grouped levels with an ionisation energy of more than 0.6 eV (in our case: 13) have to be taken into account.

A typical ASDF is depicted in figure 3.5. In this case $T_e = 1.0$ eV, $n_e = 10^{19}$ m$^{-3}$, $n_i = 2 \times 10^{20}$ m$^{-3}$ and $n_1 = 10^{19}$ m$^{-3}$. Also the Saha and Boltzmann lines are depicted, calculated with equations 3.5 and 3.9. It can be seen that the first three excited levels ($6^3P_{0,1,2}$) are in Boltzmann equilibrium with the ground state. This situation is described in [Mul86] as Corona case B. In this case all radiation for the first excited states will be reabsorbed while the plasma is transparent for all other bound-bound transitions. So in this special situation the transitions $6^1S_0 \rightarrow 6^3P_{0,1,2}$ are collisional while transitions between other (lower) excited states are of both collisional and radiative character. Since the excitation to the first excited states is not compensated by (effective) radiative decay, the level densities will increase, i.e. $n(6^3P_0)$, $n(6^3P_1)$ and $n(6^3P_2)$ move towards the Boltzmann value according to equation 3.5. If $T_e$ is high enough, stepwise excitation will dominate over deexcitation, leading to the so-called ladder climbing process flow in the excitation flow.

![Fig. 3.5: ASDF of mercury.](image)

The fact that the $6^3P_{0,1,2}$ levels are in Boltzmann equilibrium can be used to determine the electron temperature. By measuring the densities per statistical weight of these levels the Boltzmann line, as depicted in figure 3.4, can be determined (3.82). This equation is the same as Boltzmann’s law (3.5), given in § 3.2. The derivative of this equation (-1/$T_e$) equals the gradient of the Boltzmann line, and so the electron temperature can be determined.

$$\ln \left( \frac{n_e}{g_p} \right) = \ln \left( \frac{n_i}{g_1} \right) - \frac{E_p}{T_e}$$

(3.82)
An other ASDF is depicted in figure 3.6. Now \( n_e = n_+ = 10^{20} \text{ m}^{-3} \), \( n_1 = 2 \times 10^{20} \text{ m}^{-3} \) an \( T_e = 1.5 \text{ eV} \). Again the Boltzmann and Saha lines are depicted. It can now be seen that the \( 6^3\text{P}_{0,1,2} \) levels, which were in Boltzmann equilibrium in fig. 3.5, are now no longer in Boltzmann equilibrium. This can be expected, because for increasing electron temperature, the excitation increases in favour of deexcitation.

![Boltzmann and Saha lines](image)

**Fig. 3.6: ASDF of mercury.**

It can be seen in figures 3.5 and 3.6 that some energy levels, with an energy of about 9.5 eV, have a small deviation from the expected, smooth, curve. It is not known if this is an error in the model, or that it is a correct prediction of reality. To determine whether the unknown \( A \)-values from § 3.7 have any effect on the ASDF and the ionisation, recombination and radiation losses coefficients, all unknown transition probabilities are deleted from the CRM. It turns out that it has a negligible effect. For now we assume that the model fairly represents the reality. To examine this assumption an electron temperature and density are chosen such that the levels which deviate from the smooth curve are situated in ESB. This is done by choosing a high electron density \( n_e = 10^{20} \text{ m}^{-3} \) and temperature \( T_e = 1.5 \text{ eV} \). Now the ionisation coefficient is calculated as a function of the cut-off level (fig. 3.7).

![Ionisation coefficient](image)

**Fig. 3.7: The influence of the cut-off level on the ionisation coefficient.**
It can be seen in figure 3.7 that changing the cut-off level has little effect on the ionisation coefficient. This implies that including the aforementioned levels, which deviate from the smooth curve, has little effect on the ionisation coefficient.

The next step is to calculate the ionisation and recombination coefficients and the radiation losses coefficients for different $T_e$ and $n_e$. The idea behind this is that we will have to implement the ionisation and recombination coefficients and the radiation losses coefficients into the PLASIMO-model, as a function of $n_e$ and $T_e$. What we will do is calculate $S_{CR}$, $\alpha_{CR}$, $\varepsilon_1$ and $\varepsilon_+$ as a function of the electron temperature ($T_e$ from 0.75 eV to 2.0 eV), for different electron densities ($n_e = 10^{18}$, $10^{19}$ and $10^{20}$ m$^{-3}$). Once we have a good fit of the different coefficients as a function of $T_e$, for certain $n_e$, we will determine a suitable interpolation function to determine the value of the coefficients for intermediate values of $n_e$.

In figures 3.8 through 3.11 the ionisation coefficient $S_{CR}$ [m$^3$s$^{-1}$], the recombination coefficient $\alpha_{CR}$ [m$^3$s$^{-1}$], the radiation losses coefficient related to the ground state $\varepsilon_1$ [eVm$^3$s$^{-1}$] and the radiation losses coefficient related to the ion ground state $\varepsilon_+$ [eVm$^3$s$^{-1}$] are depicted, as a function of the electron temperature $T_e$ in eV, for different values of the electron density $n_e$ in m$^{-3}$.

![Graph showing ionisation coefficient as a function of $T_e$.](image)

Fig. 3.8: $S_{CR}$ as a function of $T_e$.

The ionisation coefficient (fig. 3.8) increases with increasing $T_e$, because the average energy of the electrons increases, and so the number of fast electrons, which can be used for ionisation processes, increases as well. The ionisation coefficient also increases with increasing $n_e$. This can be explained by the decreasing role of the radiation with increasing $n_e$ (figure 3.10). For increasing electron density more levels will be situated in ESB, so more ionisation will take place. If we look at a simple three level model, $S_{CR}$ can be determined by:

$$S_{CR} = K_{21} \frac{n_e K_{12}}{n_e K_{21} + \Lambda_{21} A_{21}} \quad (3.83)$$

With equation 3.83 it is possible to determine the $n_e$-dependency of $S_{CR}$. For increasing $n_e$ the radiative decay will become less important, and so $S_{CR}$ increases.
The recombination coefficient (fig. 3.9) decreases with increasing $T_e$, because with increasing average velocity of the electrons the chance of recombination decreases. However, the recombination coefficient increases with increasing $n_e$. This is because the most important recombination process three-particle recombination scales with $n_e^2 n_x$.

The radiation losses coefficient related to the ground state $\varepsilon_1$ (fig. 3.10) decreases for increasing electron density. This is because for increasing $n_e$ the radiating levels will not be situated in a pure Corona Balance, so the radiation term will decrease (3.83). The density of the main radiating level, $6^3P_1$, remains almost constant for increasing $n_e$ (fig. 3.12). For increasing electron temperature, the density of the $6^3P_1$ level increases (fig. 3.13), and so the radiation losses coefficient $\varepsilon_1$ increases (3.31).
The radiation losses coefficient related to the ion ground state $\varepsilon_+$ (fig. 3.11) decreases with increasing $T_e$, because the recombination decreases with increasing $T_e$. The recombination increases with increasing $n_e$, so the radiation losses increase as well with increasing $n_e$. It should be realised that the power density related to the radiation losses equals $n_e n_i \varepsilon_1 - n_e n_+ \varepsilon_+ [\text{eVm}^{-3}\text{s}^{-1}] \ (3.2)$.

Fig. 3.12: The density of the $6^3P_1$-level as a function of $n_e$. 

---

**Fig. 3.11:** $\varepsilon_+$ as a function of $T_e$. 

---

**Fig. 3.12:** The density of the $6^3P_1$-level as a function of $n_e$. 

---
Fig. 3.13: The density of the $6^3P_1$-level as a function of $T_e$.

The density of the first resonant state remains almost constant for increasing $n_e$ because the density is in Boltzmann balance with the ground state (3.6), as discussed earlier in this section. This means that Boltzmann's law (3.5) is valid, and the density of the resonant level does not depend on $n_e$. However, since excitation strongly depends on the electron temperature the density of the excited state increases for increasing $T_e$. 
3.10 ESCAPE FACTOR OF RESONANT RADIATION

For the transitions from a certain energy level to the ground state we have to take the escape factor into account, \( \Lambda_{pl}^{\text{eff}} = \Lambda_{pl} \times \Lambda_{pi} \), with \( \Lambda_{pi} \) is the escape factor from a state \( p \neq 1 \) to the ground state, as discussed in § 3.7. For transitions from a state \( p \) to a state \( q \) (\( p, q \neq 1 \)) the escape factor \( \Lambda_{pq} \) equals unity, so it can be ignored. We assume that the escape factor \( \Lambda_{pl} \) is \( 2 \times 10^{-3} \) for the transition from the first resonant state \( 6^3P_1 \) to the ground state, and \( 10^{-4} \) for the transition from the second resonant state \( 6^1P_1 \) to the ground state, so the transition values to the ground state \( 6^1S_0 \) have to be multiplied by the escape factor.

To see what the effect of altering the escape factor \( \Lambda_{pl} \) is the ionisation coefficient \( S_{CR} \) and the recombination coefficient \( \alpha_{CR} \) are calculated for two realistic values of \( \Lambda_{pl} \), namely \( 10^{-3} \) and \( 10^{-4} \). For both cases the escape factor is constant for all radiative transitions to the ground state.

As we can see in table 3.15 \( S_{CR} \) has a maximal difference of 57% for a low electron density \( 10^{18} \text{ m}^{-3} \) and an electron temperature \( T_e \) of 2.0 eV. For higher electron densities the difference becomes much less. The difference for an electron density of \( 5 \times 10^{18} \text{ m}^{-3} \) and an electron temperature of 1.0 eV is about 10% (fig. 3.14). The difference for \( \alpha_{CR} \) has a maximum of 10%, as we can see in table 3.15. In figure 3.15 the \( \Lambda \)-dependency is depicted for \( T_e = 2.0 \text{ eV} \), which has the maximum difference.

As we take the results into account we can say that for the most electron densities, \( 5 \times 10^{18} \) to \( 1 \times 10^{20} \text{ m}^{-3} \), the value of the ionisation coefficient \( S_{CR} \) and the recombination coefficient \( \alpha_{CR} \) will not be influenced greatly (i.e. the difference is less than 10%) by the value of the escape factor, so we still assume the value of the escape factor to be the same as mentioned above.

Table 3.15: \( \Lambda \)-dependency of \( S_{CR} \) and \( \alpha_{CR} \).

<table>
<thead>
<tr>
<th>( n_e ) [m(^{-3})]</th>
<th>( S_{CR} ) [m(^3) s(^{-1})], ( T_e = 1 \text{ eV} )</th>
<th>( \alpha_{CR} ) [m(^3) s(^{-1})], ( T_e = 1 \text{ eV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda_{pl} = 10^{-3} )</td>
<td>( \Lambda_{pl} = 10^{-4} )</td>
<td>( \Lambda_{pl} = 10^{-3} )</td>
</tr>
<tr>
<td>( 1 \times 10^{18} )</td>
<td>( 9.21 \times 10^{-18} )</td>
<td>( 1.25 \times 10^{-17} )</td>
</tr>
<tr>
<td>( 3 \times 10^{18} )</td>
<td>( 1.17 \times 10^{-17} )</td>
<td>( 1.38 \times 10^{-17} )</td>
</tr>
<tr>
<td>( 1 \times 10^{19} )</td>
<td>( 1.65 \times 10^{-17} )</td>
<td>( 1.78 \times 10^{-17} )</td>
</tr>
<tr>
<td>( 3 \times 10^{19} )</td>
<td>( 2.97 \times 10^{-17} )</td>
<td>( 3.08 \times 10^{-17} )</td>
</tr>
<tr>
<td>( 7 \times 10^{19} )</td>
<td>( 5.46 \times 10^{-17} )</td>
<td>( 5.57 \times 10^{-17} )</td>
</tr>
<tr>
<td>( 1 \times 10^{20} )</td>
<td>( 7.06 \times 10^{-17} )</td>
<td>( 7.17 \times 10^{-17} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( n_e ) [m(^{-3})]</th>
<th>( S_{CR} ) [m(^3) s(^{-1})], ( T_e = 2 \text{ eV} )</th>
<th>( \alpha_{CR} ) [m(^3) s(^{-1})], ( T_e = 2 \text{ eV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda_{pl} = 10^{-3} )</td>
<td>( \Lambda_{pl} = 10^{-4} )</td>
<td>( \Lambda_{pl} = 10^{-3} )</td>
</tr>
<tr>
<td>( 1 \times 10^{18} )</td>
<td>( 1.92 \times 10^{-15} )</td>
<td>( 3.02 \times 10^{-15} )</td>
</tr>
<tr>
<td>( 3 \times 10^{18} )</td>
<td>( 2.60 \times 10^{-15} )</td>
<td>( 3.32 \times 10^{-15} )</td>
</tr>
<tr>
<td>( 1 \times 10^{19} )</td>
<td>( 3.47 \times 10^{-15} )</td>
<td>( 3.82 \times 10^{-15} )</td>
</tr>
<tr>
<td>( 3 \times 10^{19} )</td>
<td>( 4.71 \times 10^{-15} )</td>
<td>( 4.89 \times 10^{-15} )</td>
</tr>
<tr>
<td>( 7 \times 10^{19} )</td>
<td>( 6.04 \times 10^{-15} )</td>
<td>( 6.14 \times 10^{-15} )</td>
</tr>
<tr>
<td>( 1 \times 10^{20} )</td>
<td>( 6.60 \times 10^{-15} )</td>
<td>( 6.67 \times 10^{-15} )</td>
</tr>
</tbody>
</table>
Fig. 3.14: $\Lambda$-dependency $S_{CR}$.

Fig. 3.15: $\Lambda$-dependency $\alpha_{CR}$. 
3.11 CRM COMPARED TO THE KEIJSER-SMEETS MODEL

3.11.1 INTRODUCTION

To see if our model is in agreement with other models, we compare the results with those obtained with the Keijser-Smeets model [Vos97]. This model was developed at Philips, and it assumes 3 levels, the ground state, a combined radiating and metastable level and the ion ground state. Ionisation occurs directly from the ground state, or via the metastable state. Furthermore, the model assumes a non-Maxwellian EEDF, the tail of the distribution is depopulated.

The ionisation coefficient $S_{CR}$ obtained by the Keijser-Smeets model is now compared with the results from our CRM. The problem is that the Keijser-Smeets model is valid for electron densities $< 10^{18}$ m$^{-3}$, whereas our model is valid for electron densities $> 5 \times 10^{18}$ m$^{-3}$, because we assume a Maxwellian EEDF. The results of the comparison can be seen in figure 3.16.

![Figure 3.16: Comparison of our CRM with the Keijser-Smeets model.](image)

It can be seen that for low electron densities ($n_e = 10^{17}$ m$^{-3}$) our model predicts a higher value of the ionisation coefficient $S_{CR}$. This can be explained by the fact that, as mentioned above, our CRM assumes a Maxwellian EEDF, in contrast to Keijser and Smeets. This means that for these low electron densities, we overestimate the number of fast electrons in the tail of the distribution which are important for ionisation, so we will overestimate the ionisation coefficient.

If we look at the higher electron densities, we see that the Keijser-Smeets model predicts higher values of the ionisation coefficient. This is strange, because they do not assume a Maxwellian EEDF. This could mean that our model predicts values which are too low, or that the Keijser-Smeets model is just not valid for these high electron densities.

In the next sections we are going to examine what the influence of several modifications of the CRM is on the value of the ionisation coefficient $S_{CR}$. The idea behind this is that we want to see if it is possible for our CRM to give larger values of the ionisation coefficient than the Keijser-Smeets model. That is what we expect, because the CRM assumes a Maxwellian EEDF, in contrast to Keijser-Smeets.
3.11.2 INFLUENCE OF THE ESCAPE FACTOR $\Lambda$

The CRM calculates the value of the ionisation coefficient, for several implemented values of the escape factor. A large deviation in the escape factor is used, just to see what the effect is on the ionisation coefficient. The calculations are done for an electron density of $10^{19}$ m$^{-3}$, and a neutral density of $2 \times 10^{20}$ m$^{-3}$. The results can be seen in table 3.16.

Table 3.16: Influence of the escape factor $\Lambda$ on the ionisation coefficient.

<table>
<thead>
<tr>
<th>$T_e$ [eV]</th>
<th>$S_{CR}$ [m$^3$ s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda = 10^2$</td>
<td>$\Lambda = 10^4$</td>
</tr>
<tr>
<td>0.75</td>
<td>$3.3 \times 10^{-19}$</td>
</tr>
<tr>
<td>1.0</td>
<td>$1.2 \times 10^{-17}$</td>
</tr>
<tr>
<td>1.5</td>
<td>$3.9 \times 10^{-16}$</td>
</tr>
<tr>
<td>2.0</td>
<td>$2.2 \times 10^{-15}$</td>
</tr>
</tbody>
</table>

As we can see the ionisation coefficient increases only slightly with decreasing escape factor (for $\Lambda = 10^4 \rightarrow 10^6$ an increase of 1%). Apparently, below a certain critical escape factor the radiative decay becomes negligible in comparison to excitation (table 3.17, § 3.9). If, on the other hand, the value of the escape factor is increased ($10^2 \rightarrow 10^4$), the value of the ionisation coefficient decreases with a factor of about 0.4. This is to be expected, because in that case the radiation losses increase as compared to the (de)excitation.

Table 3.17: The radiative decay versus the (de)excitation, $T_e = 1$ eV.

<table>
<thead>
<tr>
<th>$\Lambda$</th>
<th>$\Lambda = 10^2$</th>
<th>$\Lambda = 10^4$</th>
<th>$\Lambda = 10^4$</th>
<th>$\Lambda = 10^4$</th>
<th>$\Lambda = 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6^1P_1$</td>
<td>$6.8 \times 10^2$</td>
<td>$4.5 \times 10^6$</td>
<td>$6.8 \times 10^4$</td>
<td>$6.8 \times 10^4$</td>
<td>$1.1 \times 10^7$</td>
</tr>
<tr>
<td>$6^3P_1$</td>
<td>$8.0 \times 10^5$</td>
<td>$4.2 \times 10^5$</td>
<td>$4.2 \times 10^5$</td>
<td>$8.0 \times 10^5$</td>
<td>$5.0 \times 10^5$</td>
</tr>
</tbody>
</table>

3.11.3 THREE LEVEL MODEL

The next step is altering the CRM, to see what the effect is on the ionisation coefficient. Two simplified forms of the existing CRM are being constructed, and one CRM extended with jump-excitation (see § 3.11.5). All calculations were done for an electron and ion density of $10^{19}$ m$^{-3}$ and a neutral density of $2 \times 10^{20}$ m$^{-3}$.

First a three level model is being constructed, which means that the model consists of only three levels, the ground state ($6^1S_0$, $I_p = 10.44$ eV) and the ion ground state ($I_p = 0$ eV). Furthermore there is one intermediate level ($6^3P_1$, $I_p = 5.0$ eV, $g = 3$). Excitation occurs from the ground state directly to the ion ground state ($6^1S_0 \rightarrow \text{ion}$), from the ground state to the metastable state ($6^1S_0 \rightarrow 6^3P$) and from the metastable state to the ion ground state ($6P \rightarrow \text{ion}$). The results of all alterations to the CRM can be seen in figure 3.18. It can be seen that the values of the ionisation coefficient are lower, compared to original CRM, as can be expected, because only electrons with an energy of more than 5 eV can excite an atom or ionise an excited atom.
3.11.4 SIX LEVEL MODEL

Next we are going to extend the three level model to a simple six level model. The model consists of the ground state $6^1S_0$ ($I_p = 10.44 \text{ eV}$), the metastable states $6^3P_0$ ($I_p = 5.77 \text{ eV}$) and $6^3P_2$ ($I_p = 4.98 \text{ eV}$), the resonant states $6^3P_1$ ($I_p = 5.55 \text{ eV}$) and $6^1P_1$ ($I_p = 3.74 \text{ eV}$) and the ion ground state. The allowed transitions are now the transitions to all above lying levels, $6^1S_0 \rightarrow 6^3P_0$, $6^1S_0 \rightarrow 6^3P_2$, and the ion ground state, $6^3P_0 \rightarrow 6^3P_1$, $6^3P_2$, $6^1P_1$ and the ion ground state, etceteras. Again the results can be seen in figure 3.18. Because there are now more ways available for ionisation than for the three level model, the ionisation coefficient is larger.

3.11.5 CRM EXTENDED WITH JUMP-EXCITATION

As can be seen in the previous sections, all reductions of the number of levels have resulted in a decrease of the value of the ionisation coefficient. Up until now, all modifications implied simplifying the CRM. Now we are going to do the opposite, we are going to extend the CRM. The CRM only has stepwise excitation. The CRM will now be extended with jump excitation, excitations from each level or group to all levels or groups above (fig. 3.17). The selection rules of § 3.7 still apply. For the transition for which no cross-sections are known, the Vriens rates are implemented. This can create an error for the transitions from the ground state to high excited levels, because the hydrogen-approximation may not be valid.

In this way we expect the ionisation rate to increase. The values of the ionisation and coefficient are now larger than for the first CRM (figure 3.18), as was expected. Still, these values of the ionisation coefficient are lower than the Keijser-Smeets values.

![Fig. 3.17: Original vs. extended CRM.](image)
3.11.6 INFLUENCE HEAVY-HEAVY COLLISIONS

The next step is examining the influence of the heavy-heavy collisions (3.84). A mercury atom with an excited electron in a metastable or resonant level \( (6^1\text{P}_{0,1,2}) \) collides with an other mercury atom with an excited electron in a metastable or resonant level. This collision usually produces a highly excited atom \( \text{Hg}^* \) which is situated in ESB, so eventually a free electron will be produced.

\[
\text{Hg}(6^1\text{P}_{0,1,2}) + \text{Hg}(6^1\text{P}_{0,1,2}) \rightarrow \text{Hg}^* + \text{Hg}(6^1\text{S}_0)
\]

\[
\text{Hg}^* + e \rightarrow \text{Hg}^+ + e + e
\]  

(3.84)

There are also some collisions which directly produce a free electron (processes 1 and 2 of table 3.19). Until now all heavy-heavy collisions in which free electrons are created have been neglected. What we are going to do is comparing the production rate calculated by the CRM with the production rates of several heavy-heavy collisions [Vri78], to see if neglecting these collisions is justified. In order to do this we need the electron density and ground state density, as well as the densities of the 6P-levels (table 3.18). Furthermore we need the cross-sections for heavy-heavy collisions (table 3.19).

Table 3.18: Electron, ground state and 6P-densities, \( T_e = 1 \text{ eV} \).

<table>
<thead>
<tr>
<th>State</th>
<th>Density per statistical weight (n/g) [m(^{-3})]</th>
<th>Statistical weight g</th>
<th>Density [m(^{-3})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_e )</td>
<td>-</td>
<td>-</td>
<td>1.00x10(^{19})</td>
</tr>
<tr>
<td>( 6^1\text{S}_0 )</td>
<td>2.00x10(^{20})</td>
<td>1</td>
<td>2.00x10(^{20})</td>
</tr>
<tr>
<td>( 6^1\text{P}_0 )</td>
<td>1.91x10(^{18})</td>
<td>1</td>
<td>1.91x10(^{18})</td>
</tr>
<tr>
<td>( 6^1\text{P}_1 )</td>
<td>1.48x10(^{18})</td>
<td>3</td>
<td>4.44x10(^{18})</td>
</tr>
<tr>
<td>( 6^1\text{P}_2 )</td>
<td>8.29x10(^{17})</td>
<td>5</td>
<td>4.15x10(^{18})</td>
</tr>
<tr>
<td>( 6^1\text{P}_1 )</td>
<td>5.01x10(^{16})</td>
<td>3</td>
<td>1.50x10(^{17})</td>
</tr>
</tbody>
</table>
Table 3.19: Various heavy-heavy collisions according to [Vri78].

<table>
<thead>
<tr>
<th>Number</th>
<th>Process</th>
<th>Cross-section $\sigma$ [$10^{-20}$ m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hg(6$^3$P$_0$) + Hg(6$^3$P$_1$) → Hg$_2^*$ + e</td>
<td>460</td>
</tr>
<tr>
<td>2</td>
<td>Hg(6$^3$P$_2$) + Hg(6$^3$P$_2$) → Hg$_2^*$ + Hg(6$^1$S$_0$) + e</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>Hg(6$^3$P$_0$) + Hg(6$^3$P$_1$) → Hg$^*$ + Hg(6$^1$S$_0$)</td>
<td>80</td>
</tr>
<tr>
<td>4</td>
<td>Hg(6$^3$P$_0$) + Hg(6$^3$P$_2$) → Hg$^*$ + Hg(6$^1$S$_0$)</td>
<td>170</td>
</tr>
<tr>
<td>5</td>
<td>Hg(6$^3$P$_1$) + Hg(6$^3$P$_1$) → Hg$^*$ + Hg(6$^1$S$_0$)</td>
<td>350</td>
</tr>
<tr>
<td>6</td>
<td>Hg(6$^3$P$_1$) + Hg(6$^3$P$_2$) → Hg$^*$ + Hg(6$^1$S$_0$)</td>
<td>100</td>
</tr>
</tbody>
</table>

In the first process a Hg$_2^*$-ion is created ($I_p = 0.9$ eV). The inverse process, dissociative recombination (3.85), becomes increasingly important with increasing current [Fra72].

$$\text{Hg}_2^* + e \rightarrow \text{Hg}(6P) + \text{Hg}(6P) \quad (1)$$

$$\rightarrow \text{Hg}(6^1S_0) + \text{Hg}^* \quad (2)$$

If process 1 dominates, the net ionisation flow is zero. However, if process 2 dominates, a highly excited electron which is situated in or near ESB is created. Probably this process plays a less significant role than suggested in table 3.20. As can be seen in table 3.20, neglecting this process decreases the total heavy-heavy collisions rate by about 20%.

The process rate for ionisation for the first and jump-extended CRM and the heavy-heavy collisions will now be compared, in order to see if neglecting these collisions is justified. The ionisation process rate $K$ for the CRM can be calculated with:

$$K = n_e n_1 S_{CR} \quad [m^3 s^{-1}] \quad (3.86)$$

For the different heavy-heavy collisions the ionisation process rate $K$ can be calculated with:

$$K = n_p n_q \sigma < v > \quad [m^3 s^{-1}] \quad (3.87)$$

With $<v> = \left(8k_b T_e / \pi \mu \right)^{1/2} = 313$ m/s, with $\mu$ the reduced mass of both colliding particles, $k_b$ the Boltzmann constant and $T_e$ the heavy particles temperature [K]. In table 3.20 we can see the ionisation process rates for the current CRM, the jump-extended CRM and the various heavy-heavy particles.
Table 3.20: Ionisation process rates, \( n_1 = 2 \times 10^{20} \text{ m}^{-3} \), \( n_e = 10^{19} \text{ m}^{-3} \) and \( T_e = 1 \text{ eV} \).

<table>
<thead>
<tr>
<th>Process</th>
<th>Value ionisation process rate ([10^{22} \text{ m}^{-3} \text{s}^{-1}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRM</td>
<td>3.56</td>
</tr>
<tr>
<td>CRM (jump-extended)</td>
<td>5.58</td>
</tr>
<tr>
<td>1</td>
<td>1.22</td>
</tr>
<tr>
<td>2</td>
<td>0.539</td>
</tr>
<tr>
<td>3</td>
<td>0.212</td>
</tr>
<tr>
<td>4</td>
<td>0.422</td>
</tr>
<tr>
<td>5</td>
<td>2.16</td>
</tr>
<tr>
<td>6</td>
<td>0.617</td>
</tr>
<tr>
<td>7</td>
<td>0.577</td>
</tr>
<tr>
<td>Total rate for all heavy-heavy collisions (process 1 included)</td>
<td>5.75</td>
</tr>
<tr>
<td>Total rate for all heavy-heavy collisions (process 1 excluded)</td>
<td>4.53</td>
</tr>
<tr>
<td>Keijser-Smeets</td>
<td>20.0</td>
</tr>
</tbody>
</table>

The total ionisation rate for the heavy-heavy collisions is in the same order of magnitude as the ionisation rate of the jump-extended CRM. This means that we can not neglect these collisions. However, if we compare the total production rate for the extended CRM and the heavy-heavy collisions the values for both processes are still smaller than for the Keijser-Smeets model. It seams fair to conclude that the Keijser-Smeets model is just not valid for these high electron densities.

We want to include the heavy-heavy collisions in the CRM. The only way to do this is to make the 6P-levels (which are responsible for the heavy-heavy collisions) independent. This means that the CRM does not calculate the densities of these levels any more. PLASIMO will now calculate these densities, including the effect of the heavy-heavy collisions.

The method of creating a CRM with six independent levels (atom and ion ground state and the 6P-levels) is described in appendix 2. An other advantage of making the 6P-levels independent is that the non-local treatment of resonant radiation can now be included as well. However, this will not be done in this report.
The coefficients calculated by the jump-extended CRM are implemented in PLASIMO. With this information, the program can calculate all plasma parameters. One of these parameters is the current-voltage characteristic of a TL-lamp. The main advantage of these simulations is that they can be measured, and compared with the results of the simulation. By doing this, the validity of the CRM can be verified. The measurements are done at 2.65 MHz, because this is the same frequency as the operating frequency of the QL-lamp. Furthermore, for this high frequency the electrons can not follow the changes of the electric field, and so the electron density and temperature are time-independent ([Pol75] and [Way71]).

4.1 TL MEASUREMENTS

4.1.1 EXPERIMENTAL SETUP

The setup is depicted in figure 4.1. A function generator presents a 2.65 MHz alternating voltage to the RF-amplifier. This amplifier generates the desired power, dependent on the voltage. Because the amplifier is only able to cope with a very low voltage, and the function generator has a very poor resolution for those low voltages, two attenuators have been placed between the function generator and the amplifier (not depicted). Next, the generated power goes through the lamp. A matching network is placed between the lamp and the amplifier. In this way, the total impedance remains 50 Ω, to prevent the reflection of a large part of the power.

![Diagram of experimental setup](image)

Fig. 4.1: The experimental setup for TL measurements.
The electrodes in the TL-lamp are SOX-electrodes, specially designed to withstand the high currents. They are the same for all lamps. The amalgam is the same as in the QL-lamp. The measurements are automatically regulated by a Windows program. With the voltage and power known, the current can be calculated. The TL lamps are placed in a water bath. This is done to keep the temperature stable. The temperature is a measure for the mercury pressure (fig. 4.2). If the temperature is stable, so is the mercury pressure. A water heater in combination with a temperature controller and a water pump is used to keep the water at a constant temperature. Earlier test measurements in open air have shown that the temperature at the glass wall can increase to about 200°C. For those high temperatures the mercury pressure is undetermined.

Fig. 4.2: Mercury pressure as a function of the temperature.

4.1.2 MEASUREMENT APPROACH

Measurements are done for three configurations, as outlined in table 4.1. The temperature of the water bath is 67°C, which corresponds with a mercury pressure of 0.6 Pa.

Table 4.1: Configurations of the used TL-lamps.

<table>
<thead>
<tr>
<th>Tube nr.</th>
<th>External Diameter [mm]</th>
<th>Internal Diameter [mm]</th>
<th>Total length [mm]</th>
<th>Electrode-electrode distance [mm]</th>
<th>Argon filling pressure [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22.2</td>
<td>19.5</td>
<td>340</td>
<td>250</td>
<td>205</td>
</tr>
<tr>
<td>2</td>
<td>22.2</td>
<td>19.5</td>
<td>590</td>
<td>500</td>
<td>205</td>
</tr>
<tr>
<td>3</td>
<td>13.75</td>
<td>12.0</td>
<td>310</td>
<td>250</td>
<td>333</td>
</tr>
<tr>
<td>4</td>
<td>51.0</td>
<td>49.0</td>
<td>350</td>
<td>250</td>
<td>82</td>
</tr>
</tbody>
</table>
First the voltage over all tubes is measured. The problem with this high frequency is that a number of parasitic effects can occur. The first is the presence of leak currents from the lamp via the water and the metal container to the earth. Measurements have shown that this leak current can be reduced by not grounding the container and the water pump. Furthermore it is possible that the lamp itself acts as a capacitor and an inductive effect arises. To measure whether these effects actually occur, voltage measurements are done at different frequencies. This is done because both effects are frequency-dependent. Because the measured voltage directly across the lamp (so not including the cables from the matching network to the lamp) does not change, the inductive and capacitive effects can be neglected, and the lamp can be seen as a pure resistor. Also the inductive effect of the cable from the matching network to the lamp itself has to be accounted for. This can be done by correcting the measured voltage $V_{\text{meas}}$ over the lamp and the cables:

$$V_{\text{cor}} = V_{\text{meas}} \left( \frac{R}{R + j\omega L} \right) = V_{\text{meas}} \left( \frac{1}{1 + j\omega L / R} \right) = V_{\text{meas}} \left( \frac{1}{1 + j \tan \varphi} \right)$$

(4.1)

with $\varphi$ the phase angle, $R$ the resistance and $L$ the coefficient of self induction. Using figure 4.3 it can be seen that the corrected real voltage over the tube $V_{\text{cor}}$ equals the cosine of the phase angle $\varphi$. This correction is valid, because the lamp is a pure resistor, and the wires have a negligible capacity.

![Fig. 4.3: Determination of $V_{\text{cor}}$ from $V_{\text{meas}}$.](image)

Second the electric field [V/m] in tubes 1 and 2 is determined. The only difference between these tubes is the difference in length $\Delta l$, the cathode-anode fall is the same, so the electric field is the same for both tubes. By measuring the difference in corrected voltage $\Delta V$ the electric field can be calculated [Val97]:

$$E = \frac{\Delta V}{\Delta l}$$

(4.2)

Next the cathode-anode fall $V_{\text{ca}}$ [V] is determined for these two tubes. We assume that $V_{\text{ca}}$ is the same for all tubes, because the same electrodes are used for all tubes, and the measurements are done for the same currents. With the calculated electric field $E$ inside the tubes the cathode-anode fall can now be calculated, using equation 4.3:

$$V_{\text{ca}} = V_{\text{cor}} - E \times l_{\text{el-el}}$$

(4.3)
with \( l_{\text{el-el}} \) the electrode-electrode distance, which equals the length of the positive column. The last step is determining the electric field for all tubes. \( V_{\text{cor}} \), \( V_{\text{ca}} \) and \( l_{\text{el-el}} \) are known, so with equation 4.3 the electric field for tubes 3 and 4 can be calculated. These calculations are done under the assumption that the cathode-anode fall is independent of the configuration.

### 4.1.3 RESULTS

In figures 4.4 the total voltage over the tube \( V_{\text{cor}} \) is depicted, as a function of the current \( I \). The error for the corrected voltage has a maximum of 0.5V. However, tube 3 exhibits fluctuations with an amplitude larger than 0.5V. The cause for this is unknown.

**Fig. 4.4: Total corrected voltage over the tube as a function of the current, all tubes.**

It can be seen that for tubes 1, 2 and 4 the corrected voltage over the tube decreases for increasing current, but for tube 3 the corrected voltage increases, after a decrease as a function of the current. With corrected voltage for tubes 1 and 2 known, the electric field for both tubes can be determined, using equation 2.2.

**Fig. 4.5: The electric field as a function of the current, tubes 1 and 2.**
Since the maximal error in the corrected voltage is about 0.5V, the maximal error in the difference of the corrected voltages (4.2) is about 1V. Because this has to be divided by the difference in length (0.25 m), the total maximal error is about 4V.

It can be seen that the electric field decreases as a function of the current. This is explained in § 2.1.3 (fig. 2.2). The lamp is operated in the normal TL-regime, which means that in stead of just direct ionisation stepwise excitation and ionisation is becoming more important for increasing current densities. Because the required average electron energy is now lower, the electron temperature and so the voltage and the electric field can drop.

If the electric field for tubes 1 and 2 is known, the cathode-anode fall as a function of the current can be calculated, using equation 4.3.

![Graph: Cathode-anode fall as a function of current](image)

Fig. 4.6: The cathode-anode fall as a function of the current.

The maximal error in the cathode-anode fall is about 1.5V. The cathode-anode fall drops gradually for increasing current. This can be explained by the increasing electrode temperature when the current increases. For increasing electrode temperature the creation of electrons rises exponentially. This means that if the current is doubled, the number of created electrons is more than doubled, and so the velocity of the electrons decreases. Because of this the positive space charge and thus the cathode fall is decreased (§ 2.1.2). With the cathode-anode fall known, it is possible to determine the electric field of tubes 3 and 4 (equation 4.3).
Since the electric field is calculated from the difference between corrected voltage and the cathode-anode fall, and divided by the electrode-electrode distance (0.25 m) the maximal error is about 8V. The electric field for tube 3 first decreases for increasing current. The reason for the decrease is already explained in § 2.1.3. However, eventually the electric field increases for increasing current. This can be explained by the occurrence of depletion. The diameter of tube 3 is much smaller than the diameters of tubes 1, 2 and 4. Because of this the current density is much larger. For large currents, depletion can become important and noble gas (in our case: argon) particles have to be ionised. This requires more energy, and so the electron temperature and the electric field rise.

Tube 4 exhibits the same type of behaviour as tube 3. This is strange, because the diameter of this tube is much smaller, so the current density is much smaller. We expect that depletion is negligible, and so this lamp is operated in the same region as tubes 1 and 2. A possible cause for this behaviour could be the fact that the fact that the error is relatively large (8V) and that the cathode-anode fall is not constant for all tubes. If the cathode-anode fall remains constant for increasing current, the electric field decreases as a function of the current.

Fig. 4.7 The electric field as a function of the current, tubes 3 and 4.
4.2 RESULTS FROM PLASIMO

4.2.1 INTRODUCTION

The coefficients of ionisation ($S_{CR}$), recombination ($\alpha_{CR}$) and the radiation losses coefficients $\varepsilon_1$ (related to the ground state) and $\varepsilon_\pi$ (related to the ion ground state) are now implemented in PLASIMO. These coefficients are calculated by the CRM. In stead of the CRM with six independent levels (see appendix 2) the jump-extended CRM with two independent levels is used (§ 3.11.5). This is done because PLASIMO turns out to be numerically unstable, if the J-coefficients and radiation losses of the six-independent levels model are implemented in PLASIMO. This means that the influence of the heavy-heavy collisions can not be included.

As mentioned in § 3.9 the coefficients of ionisation, recombination and radiation losses are calculated as a function of the electron temperature, for several values of the electron density. To calculate the value of these coefficients for an intermediate value of the electron density, a linear interpolation function is used for the logarithm of the coefficients:

$$\log \text{Coeff}(n, T_e) = \log \text{Coeff}(n_{low}, T_e) + \left( \frac{n_{high}}{n_{low}} \right) \times \left( \log \text{Coeff}(n_{high}, T_e) - \log (n_{low}, T_e) \right)$$

(4.4)

The error using this interpolation function is less than 5%. Next PLASIMO calculates the electric field (as thus the voltage over the positive column), the electron density and temperature as a function of the current. Also the ground state densities of the mercury and argon particles as a function of the current and the radius are calculated.

With the last data it is possible to determine whether the plasma is becoming depleted. This means that the density of the mercury neutral particles is dropping very rapidly in the centre of the tube, as a function of the current (§ 2.1.3). Because there are not enough mercury particles left to be excited and ionised, argon neutral particles are being excited and ionised.

The calculations are done for tubes 1, 2 and 4 (§ 4.1.2, table 4.1). The only difference between tubes 1 and 2 is the length of the positive column. The length of the tube does not matter, since the lamp is symmetrical in that direction. Tube 3 is not simulated, because this simulation turns out to be numerically unstable. Also the argon excitation has to be turned off for all simulations, because else PLASIMO would again become unstable.

4.2.2 THE ELECTRIC FIELD

First the electric field [V/m] is calculated, for the tube configurations 1, 2 and 4. The results can be seen in figure 4.8. It can be seen that the trend for tubes 1 and 2 is not the same as the measured trend for those tubes. As we look to the measurements the electric field drops as a function of the current. We expect that the electric field will increase for even higher currents, but the electrodes can not handle the high currents. The electric field PLASIMO calculates drops in the beginning and then rises, as a function of the current. The average difference in value between the simulations and measurements is about 15 V/m.
As we look at tube 4 the trends are the opposite. PLASIMO calculates a decreasing electric field as a function of the current, whereas an increasing electric field is measured, after a small decrease in the beginning. If we want to compare the measurements and simulations for this tube, we will have to take into account the fact that the random error in the measurements is relatively large (about 8V), as discussed in the previous section. This could be a cause for the difference in trends.

An other possible cause for the difference between the measurements and the simulations is the absence of the non-local treatment of resonant radiation and the heavy-heavy collisions. Up to now, an escape factor is used to take the absorption of resonance radiation into account. As mentioned in appendix 2, creating a CRM six independent levels can be used to take the non-local treatment of resonant radiation and the heavy-heavy collisions into account, because the densities of the two resonant levels can then be calculated by PLASIMO.

![Electric field vs current](image)

Fig. 4.8: The electric field as a function of the current, measurements (tubes 1, 2, 3 and 4) and simulations (tubes 1, 2 and 4).

### 4.2.3 THE ELECTRON DENSITY AND TEMPERATURE

PLASIMO can also calculate the electron density $n_e$ and the electron temperature $T_e$ for several configurations and currents. The results can be seen in figures 4.9 and 4.10. The electron density increases for increasing current, which is to be expected because the current density is proportional to $n_e$ (2.4). It can be seen in figures 4.8 and 4.10 that an increasing electric field corresponds with an increasing electron temperature, and vice versa.
The problem is that the E-I characteristic for tubes 1 and 2 is worse as compared to the measurements than for tube 4. The electron density for tubes 1 and 2 is about $10^{19} \text{ m}^{-3}$ (so a Maxwellian EEDF, § 3.8) whereas the electron density for tube 4 is about $10^{18} \text{ m}^{-3}$ (so no Maxwellian EEDF). This would suggest that the simulations for tubes 1 and 2 have to better as compared to the measurements than for tube 4.

A possible cause for this could be the fact that the heavy-heavy collisions and the non-local treatment of the resonant radiation have not been taken into account. For instance, if less resonant radiation appears to be reabsorbed (so a larger escape factor), less ionisation will occur and less free electrons will be created. Now the conductivity will decrease, and so the electric field will increase. Also the fact that these are the first simulations have to be taken into account. A lot of numerical problems have to be solved, before judgement can be passed about the validity of the CRM and PLASIMO.
4.2.4 ARGON AND MERCURY GROUND STATE DENSITIES, TUBES 1 AND 2

Now the argon and mercury ground state densities are calculated for tubes 1 and 2 as a function of the radius, for different values of the current. The radius is 9.75 mm, the argon filling pressure is 205 Pa and the mercury pressure is 0.6 Pa. The ground state density of the argon particles is depicted in figure 4.11.

Fig. 4.11: The argon ground state density as a function of the radius, for several currents, tubes 1 and 2.

It can be seen that for higher currents the density falls in the centre of the tube, and rises at the wall. This can be explained using the Gas Law, \( p = nkT \). When we increase the current, the temperature in the centre increases as well, whereas the temperature at the wall remains constant (assumption made by PLASIMO). Since the argon pressure is almost constant in the lamp, the number of argon neutral particles decreases in the centre (\( T \) increases, so \( n \) decreases). When we assume that the total number of argon neutral particles remains constant, the number of argon neutral particles increases at the wall with increasing current.

In figure 4.12 the mercury neutral particles density as a function of the radius is depicted. It can be seen that for increasing current the density remains almost constant at the wall. This can be explained by the fact that we assume a constant wall temperature in PLASIMO. The mercury pressure \( p_{\text{Hg}} \) is determined by the coldest spot. Again looking at the Gas Law, the density will remain constant when the temperature and the pressure remain constant. The small decrease in density can be explained by the ionisation of some mercury neutral particles.

In the centre we see a much larger decrease in the density. This can be partially explained by the ionisation of neutral particles, and again by the Gas Law, because in the centre the temperature increases with increasing current (\( p \) is constant, \( T \) increases, so \( n \) decreases).
Fig. 4.12: Mercury ground state density as a function of the radius, for several currents, tubes 1 and 2.

In fig 4.13 the mercury ground state density divided by the argon ground state density is depicted as a function of the radius, to cancel out the decrease in density due to heating. It can be seen that the mercury ground state density decreases faster than the argon ground state density. This is a sign of depletion of mercury. The ratio \( n_{Hg}/n_{Ar} \) is about \( 10^5 \) in the centre for a current of 5A.
4.2.5 ARGON AND MERCURY GROUND STATE DENSITIES, TUBE 4

Now the argon and mercury ground state densities are calculated for tube 4 as a function of the radius, for different values of the current. The radius is 24.5 mm, the argon filling pressure is 82 Pa and the mercury pressure is 0.6 Pa. The ground state density of the argon particles is depicted in figure 4.14.

![Fig. 4.14: The argon ground state density as a function of the radius, for several currents.](image)

The argon ground state density for tube 4 as a function of the current and the radius shows the same behaviour as tubes 1 and 2. The density decreases for increasing current in the centre, and increases at the wall. However, the decrease of the density in the centre is smaller for tube 4. This is because the current density is much less, because the radius is larger. Now the increase of the gas temperature in the centre is less, and so the decrease of the density.

![Fig. 4.15: The mercury ground state density as a function of the radius, for several currents.](image)
Also the mercury ground state density for tube 4 as a function of the current and the radius (fig. 4.15) shows the same behaviour as tubes 1 and 2. The decrease of the density in the centre is now much less. This can also be seen in figure 4.16. Almost no depletion occurs. This can again be explained by the fact that the current density is much less than for tubes 1 and 2. The apparent maximum of the relation \( n_{\text{Hg}}/n_{\text{Ar}} \) is caused by the error in the simulations (1%).

![Graph showing the relation of the mercury and argon ground state densities.](image)

Fig. 4.16: The relation of the mercury and argon ground state densities.

### 4.2.6 CRM INCLUDING HEAVY-HEAVY COLLISIONS

As mentioned before, PLASIMO becomes unstable if the heavy-heavy collisions are included. However, if the mercury pressure is increased to 6.5 Pa (in contrast to 0.6 Pa from the previous simulations) PLASIMO is numerically stable. Because the mercury pressure is so high, no comparison can be made with the previous high current TL measurements and simulations.

To see what the effect is of including heavy-heavy collisions the electron density is calculated with and without these collisions. The results are depicted in fig. 4.17. It can be seen that by including the heavy-heavy collisions the electron density increases. Because the first excited levels are in Boltzmann equilibrium with the ground state the excitation from the ground state to the these levels is no longer the bottleneck, but stepwise excitation ionisation is. By including heavy-heavy collisions there will be more ways available for ionisation.

It should be noticed that these are the first simulations including heavy-heavy collisions. The unstability of PLASIMO when including these collisions for low mercury pressures has to be solved.
Fig. 4.17. The influence of heavy-heavy collisions.
5 THOMSON SCATTERING

5.1 INTRODUCTION

As can be read in the report of Marco Bakker [Bak96], several methods have been used to measure the electron density $n_e$ and the electron temperature $T_e$ of the Philips QL-lamp. The problem is that the results of these measurements do not yield consistent results. This can be attributed to the fact that all these techniques are based on assumptions, which might be wrong. A better way is to measure $n_e$ and $T_e$ directly with Thomson scattering.

The experimental setup is depicted in figure 5.1 [Reg96]. A frequency doubled Nd:YAG-laser (GCR 3 Quanta Ray, $E_{\text{pulse}} = 0.4 \text{ J}$, $\tau_{\text{pulse}} = 7 \text{ ns}$, $f_{\text{rep}} = 10 \text{ Hz}$) at 532 nm is used. Two prisms lead the beam into the detection volume. The beam is focused by a lens ($f = 1000 \text{ mm}$). Finally, the laser beam is absorbed in a laser dump. The detection branch starts with two lenses ($f = 600 \text{ mm}$) and an aperture that image the detection volume onto the entrance slit ($0.3 \text{ mm}$ for the PDA and CCD setup, $10 \text{ mm}$ for the Camac setup). The dispersive element is the concave holographic grating (Jobin Yvon, 2000 lines/mm, $100 \times 100 \text{ mm}$ and $f = 1 \text{ m}$).

There are three detection units available. The first is a Photo Diode Array (PDA, 1024 pixels of $25 \mu\text{m} \times 2.5 \text{ mm}$, total length $25.6 \text{ mm}$, apparatus profile $1/e$ width of $0.14 \text{ nm}$, quantum efficiency of 0.3) plus intensifier (amplification $10^4$, $\tau_{\text{gate}} = 100 \text{ ns}$). The second is a Couple Charged Device (CCD, total length $9.1 \text{ mm}$, $750 \times 242$ pixels, pixelsize $11.5 \mu\text{m} \times 27 \mu\text{m}$, saturation charge $2.0 \times 10^5$ electrons) [Tas94]. The data from these detection units is transferred to a PC via a 16-bit ADC. The last detection unit is the combination monochromator ($\Delta\lambda = 0.2 \text{ nm}$, attenuation factor of $10^{-5}$) - PMT (type R268, amplification factor of $2 \times 10^6$, $V_{\text{set}} = -1000\text{V}$) - Camac (12 channels, 10-bit ADC, maximum charge of 256 pC).

In the Thomson scattering setup a laser pulse is shot through the plasma in the QL-lamp. A Thomson profile (not typical for the QL-lamp) is depicted in figure 5.2. The wavelength of the laser photons is 532 nm. The photons in that laser pulse can scatter on the free and the bound electrons. Since the velocity of all these electrons is not the same, the scattered signal will be Doppler-broadened, i.e. the wavelength will vary around 532 nm. The full width at half maximum (FWHM) of the measured signal is a measure for the velocity of the electrons, and the area under the curve is a measure for the electron density. The signal which originates from scattering on the bound electrons, is known as the Rayleigh signal. The signal which originates from scattering on the free electrons, is known as the Thomson signal.

We are only interested in the free electrons, so we only want to measure the Thomson signal. To prevent the detectors (PDA and CCD) from blooming during the measurements due to the intense Rayleigh signal the PDA is physically darkened for about 50 pixels in the centre. If the CCD setup is used, it is not possible to measure the entire Thomson-signal in one go, so to prevent blooming due to the Rayleigh signal the measurements are done on the right and left hand side of the Rayleigh peak.
Fig. 5.1: The setup of the Thomson scattering experiment.

Fig. 5.2: A typical Thomson profile in an atmospheric high $n_e$ plasma.
5.2 THEORY

5.2.1 ELECTRON TEMPERATURE DETERMINATION

With the detection unit the Doppler broadened Thomson signal is measured, as a function of the wavelength. For an expected electron density of about $5 \times 10^{18} \text{ m}^{-3}$ the collective effect can be ignored, so that the electron temperature can be determined by equation (5.1) [Gro95]:

$$T_e = \frac{m_e e^2}{32 \sin^2(\theta/2) k_\text{B} \lambda_0^2} \times (\Delta \lambda_{\text{ine}})^3 = 1312 \times (\Delta \lambda_{\text{ine}})^2 \quad (5.1)$$

with $T_e$ the electron temperature [K], $m_e$ the electron mass [$9.1 \times 10^{-31} \text{ kg}$], $\theta$ the angle between the laser beam and the detector line of sight [90°], $\lambda_0$ the central wavelength [532 nm] and $\Delta \lambda_{\text{ine}}$ the FWHM in nm.

5.2.2 ELECTRON DENSITY DETERMINATION

First the Doppler broadened Thomson signal is measured, as a function of the wavelength. The area $A$ under the curve is a measure for the electron density [Gro95]:

$$n_e = \frac{A}{C \cdot \sigma_{\text{th}}} \quad (5.2)$$

with $n_e$ the electron density [m$^{-3}$], $C$ the calibration factor and $\sigma_{\text{th}}$ the cross section for Thomson scattering [m$^2$]. If we want to determine the electron density we first have to perform a Rayleigh calibration measurement. This measurement is done on argon. If the area $A_0$ under the Rayleigh peak is known, the calibration factor $C$ can be determined.

$$C = \frac{A_0}{n_\text{Ar} \cdot \sigma_{\text{Ar}}} \quad (5.3)$$

with $n_\text{Ar}$ the argon density [m$^{-3}$] and $\sigma_{\text{Ar}}$ the cross section for Rayleigh scattering [m$^2$]. The argon density can be determined with the ideal gas law, $p = n kT$. With equations 5.2 and 5.3 the electron density can be determined:

$$n_e = \frac{A \cdot \sigma_{\text{Ar}}}{A_0 \cdot \sigma_{\text{th}} \cdot n_\text{Ar}} \quad (5.4)$$

To measure the area under the Rayleigh peak a Rayleigh scattering measurement can be performed on argon. An other option is Raman scattering on nitrogen. This calibration technique is worthwhile to use when calibration using Rayleigh scattering is laborious to implement. This is the case in our Thomson-setup. The central part of the PDA is physically darkened for Rayleigh scattering wavelengths to prevent the detector from blooming due to the strong Rayleigh signal. With the CCD setup it is possible to measure the Rayleigh signal, although it is very difficult, because of the stray light and again blooming of the detector.
To perform a Raman calibration measurement a QL-lamp is constructed, which can be flushed with nitrogen gas. A typical rotational Raman scattered profile, measured on N\textsubscript{2} at 1 atm, 295 K, is shown in figure 5.3. The following procedure is used for calibration [Gro95]: the area under certain Raman peaks is measured. In the graph, the transitions $J = 6 \rightarrow 8$ and $J = 8 \rightarrow 6$ are indicated. The intensity of each transition peak is related to the Rayleigh signal. The intensity of the Rayleigh signal can now be reconstructed, and so the electron density can be determined. The involved theory is covered in [Reg96].

![A typical Raman scattered profile.](image)

Fig. 5.3: A typical Raman scattered profile.

For a $\Delta J = +2$ transition, we get:

$$\frac{I_{\text{Rayleigh}}}{I_{\text{Raman}}} = \frac{A_{\text{Rayleigh}}}{A_{\text{Raman}}} \cdot S \cdot \frac{2(2J + 3)}{(J + 1)(J + 2)} \cdot \frac{1}{g_J} \cdot e^{(E(J)/k_B T)}$$

(5.5)

For a $\Delta J = -2$ transition, we get:

$$\frac{I_{\text{Rayleigh}}}{I_{\text{Raman}}} = \frac{A_{\text{Rayleigh}}}{A_{\text{Raman}}} \cdot S \cdot \frac{2(2J - 1)}{J(J - 2)} \cdot \frac{1}{g_J} \cdot e^{(E(J)/k_B T)}$$

(5.6)

with $I_{\text{Rayleigh}}$, the intensity of the Rayleigh signal, $I_{\text{Raman}}$, the intensity of the Raman signal, $g_J$, the nuclear degeneracy ($g_J = 2$ for $J$ even, $g_J = 1$ for $J$ odd) and $E(J)$ the energy separation between the Rayleigh peak and the $j^{th}$ Raman peak. The $A$ values are dependent on the instrumental factors and the gas that is used for the calibration. For our setup, using N\textsubscript{2}, $A_{\text{Rayleigh}} = 24.84$, $A_{\text{Raman}} = 0.2362$. For N\textsubscript{2} and $T = 300$ K $S = 38.43$. The area of a Raman peak is determined by fitting the peak with the apparatus profile of known width.
5.2.3 THOMSON SIGNAL

The next step is determining the detection limit of the setup. Each laser pulse has an energy of 0.4 J, which at a wavelength of 532 nm corresponds with $1.1 \times 10^{18}$ photons. Of these photons only a fraction will be scattered in the solid angle in which they can be detected. The scattered power $P_s$ as compared to the incoming power $P_0$ can be calculated with equation 5.7 [Gro95]:

$$P_s d\omega = P_0 n_e L g(\theta, \varphi) \frac{d\sigma_{th}}{d\Omega} \Delta \Omega \times S(k, \omega) d\omega$$

with $L$ the length of the detection interval [0.3 mm], $g(\theta, \varphi)$ the shape of the scattered radiation, $\Delta \Omega$ the solid angle of detection [sr], $d\sigma_{th}/d\Omega$ the differential cross section for Thomson scattering [$7.95 \times 10^{-30}$ m$^2$/sr], $\omega = \omega_s - \omega_{YAG}$ the shift in frequency, $\omega_s$ the frequency of the scattered photons, $\omega_{YAG}$ the frequency of the laser and $S(k, \omega)$ the spectral density function. The shape of the scattered radiation field is given by $g(\theta, \varphi) = 1 - \sin^2 \theta \cos^2 \varphi$, where $\theta$ is the angle between the laser beam and the detector line of sight, and $\varphi$ is the angle between the polarisation direction of the laser beam and the detector line of sight. In our setup, both angles are $90^\circ$, and consequently $g$ equals 1. The electron density $n_e$ as calculated in [Bak96] and [Jon96] lies between $5 \times 10^{18}$ and $5 \times 10^{19}$ m$^{-3}$. To be on the safe side $n_e$ is taken to be in the order of $5 \times 10^{18}$ m$^{-3}$.

With equations 5.8, 5.9 and 5.10 the scattered power can be calculated as a function of the incoming power:

$$\int P_s d\omega = P_0 n_e L g(\theta, \varphi) \frac{d\sigma_{th}}{d\Omega} \Delta \Omega \int S(k, \omega) d\omega$$

$$\int S(k, \omega) d\omega = \frac{1}{\sqrt{\pi}} \int e^{-x^2} dx = 1$$

$$P_s = P_0 n_e L \Delta \Omega \frac{d\sigma_{th}}{d\Omega} = P_0 \times 5 \times 10^{18} \times 0.3 \times 10^{-3} \times \frac{0.25 \pi \times 5^2}{60^2} \times 7.95 \times 10^{-30}$$

$$= P_0 \times 6.5 \times 10^{-17}$$

Only a fraction of $6.5 \times 10^{-17}$ of the total of $1.1 \times 10^{18}$ photons is scattered, so per laser pulse only about 70 photons are scattered in the solid angle $\Delta \Omega$. With a grating that only reflects about 40% of the incoming light, that makes about 28 photons.
5.3 THOMSON MEASUREMENT USING THE PDA-SETUP

With the PDA detection unit an attempt is made to detect the Thomson signal. First a Raman spectrum on a nitrogen gas is measured, for calibration (§ 5.2.2). The result can be seen in figure 5.4.

Fig. 5.4: A Raman spectrum for nitrogen.

For the Thomson measurement a total of 86 measurements are done with the lamp on (i.e. mercury/argon plasma), and 15 measurements with the lamp off (i.e. argon gas). Each measurement lasts 150 seconds, which at a repetition rate of 10 Hz corresponds with 1500 laser pulses. The reason for the large number of measurements is the improvement of the signal to noise ratio. The result of the Thomson measurement can be seen in figure 5.5.

Fig. 5.5: Result of the Thomson measurement with the PDA-setup.
This result is achieved by taking the sum of all 86 measurements with the lamp on, and then subtracting the sum of the 15 measurements with the lamp off (corrected with a factor of 86/15). This is done in an attempt to eliminate the dark current and stray light signal, which are present in the Thomson measurement. The dark current is the signal the detector measures, even when it is physically darkened, and the stray light is the scattered light. There is no visible Thomson signal, the only thing we do see is stray light.

An other problem with detecting the Thomson signal is the combination of switching noise, read out noise and dark current noise.

The switching noise is caused by switching the gated intensifier on and off. The read out noise is caused by the fact that the ADC always has a small error when reading out the data. The PDA also has a dark current. By doing a dark current measurement this dark current can be subtracted, but there is always a noise on the signal, which remains.

The total of the switching, read out and dark current noise is depicted in figure 5.6. This noise is determined by calculating the standard deviation of ten measurement of 100 seconds, when the detector is physically darkened. There seems to be an oscillating signal on the noise, but even if this is subtracted, the noise level is still high.

![Fig. 5.6: Total noise of the PDA.](image)

It can be seen that the average noise is about 500 counts per pixel per 100 seconds. The noise per pixel per laser pulse is 0.50 counts. The calculated pure Thomson signal is 28 photons per laser pulse, so with a total of 1024 pixels there are about $2.7\times10^2$ photons per pixel per laser pulse. With a Quantum Efficiency (QE) of 0.3 that makes about $8.2\times10^{-3}$ counts per pixel per laser pulse. This means that we have a poor signal to noise ratio of 0.016.

Summarised this means that it will be very difficult to measure a Thomson profile using this setup. The main problem is the sum of the switching noise, read out noise and dark current noise, which can not be eliminated, or even reduced. In stead of the PDA - intensifier detection unit we are going to use the CCD setup, to examine whether this setup will prove to be more useful. In stead of just trying to measure a Thomson profile, first an extensive study is done to examine the QL-lamp itself, the different noise sources and the possibilities to reduce the noise.
5.4 STABILITY OF THE QL-LAMP

First the QL-lamp itself is under investigation. As mentioned in the previous paragraph the detection unit has been changed to the CCD-setup. The lamp itself is thermostrated, to make sure that the amalgam temperature, and so the mercury pressure is constant (§ 4.1.1). The stability of the lamp is determined by measuring the time-dependency of its spectrum. The spectrum of the QL-lamp on the right hand side of the Rayleigh peak is depicted in figure 5.7. The relatively small peak in the picture is an argon line. Each pixel in the figure represents a certain wavelength interval. To determine which pixel corresponds with which wavelength a calibration measurement has to be done.

The time dependency of two points in the spectrum, namely a point in the continuum (pixel 50 of figure 5.7) and the top of the aforementioned argon peak (pixel 184 of figure 5.7), is determined. The results can be seen in figures 5.8 and 5.9. These measurements are done to determine how long it takes those points to become stable.

Fig. 5.7: Spectrum of the QL-lamp.

The top of the argon peak (fig. 5.8) increases, after a small decrease for about 700 seconds, up until almost two hours, when the value drops in a very short period of time, before remaining stable. The cause for this is still unknown. The continuum (fig. 5.9) drops very rapidly in the beginning, and is stable after almost an hour. The results of these measurements would suggest that we will have to wait for two hours before the spectrum of the lamp becomes stable.
Fig. 5.8: Time dependency argon peak.

Fig. 5.9: Time dependency continuum.
5.5 NOISE SOURCES

5.5.1 MEASUREMENT APPROACH

Only if the magnitude of the noise sources is smaller than the original Thomson signal, a Thomson profile can be detected. In the CCD-setup, there are six noise sources. Three of them originate from unwanted signals, the stray light, the dark current and the spectrum of the lamp. The fourth noise source is the read out noise, the fifth noise is the shot noise and the sixth noise is the instability of the laser itself.

The stray light is the light from the laser that is reflected from the lamp and other obstacles, such as lenses, diaphragms etceteras. The dark current is the signal the CCD-camera measures, even when the camera is darkened, and the spectrum is the light from the lamp itself.

The read out noise is caused by reading out the data from the CCD-camera. The shot noise is the square root of the pure Thomson signal, and the laser noise originates from the fact that the laser does not produce an equal amount of photons, each time a laser pulse is fired. Because this instability is very small (less than 5% per laser pulse), this noise will be neglected.

These signal sources are not negligible, and not a homogeneous contribution. If that were the case, the Thomson signal would only receive an extra offset. We will have to find a way to subtract these effects. In order to do that, we have to do four measurements (table 5.1).

<table>
<thead>
<tr>
<th>Measurement nr.</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Dark current</td>
</tr>
<tr>
<td>2</td>
<td>Thomson scattering</td>
</tr>
<tr>
<td>3</td>
<td>Spectrum</td>
</tr>
<tr>
<td>4</td>
<td>Stray light</td>
</tr>
</tbody>
</table>

The Thomson scattering measurement will have to be done together with a Raman calibration measurement. Because it takes the lamp almost two hours to become stable, first the lamp and laser are turned on, and in the mean time a dark current measurement is performed, with a darkened camera. After the spectrum of the lamp becomes stable, we can do the Thomson measurement (lamp on, laser on), which contains a dark current, a spectral signal and a stray light signal. After that the laser is shut down, or a beam stop is placed in the laser beam, and we can measure the spectrum (lamp on, laser off), and finally the stray light (lamp off, laser on). Both signals contain a dark current (table 5.2).

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Laser</th>
<th>Lamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dark current</td>
<td>off</td>
<td>off</td>
</tr>
<tr>
<td>Thomson</td>
<td>on</td>
<td>on</td>
</tr>
<tr>
<td>Spectrum</td>
<td>off</td>
<td>on</td>
</tr>
<tr>
<td>Stray light</td>
<td>on</td>
<td>off</td>
</tr>
</tbody>
</table>
We should realise that a Thomson measurement is composed of the pure Thomson signal, the spectrum, the stray light and the dark current. If we want to get the pure Thomson component, we add the Thomson measurement and the dark current measurement, and then subtract the spectral and stray light signal. Thomson measurement (pure Thomson signal + dark current + stray light + spectrum) + dark current measurement (dark current) - stray light measurement (stray light + dark current) - spectral measurement (spectral signal + dark current) = pure Thomson signal. In this way the dark current is included four times, which means that the contribution of the noise on the dark current to the total noise is two times the noise in a single dark current measurement. Since also four measurements are taken to obtain the net Thomson scattering spectrum the same applies to the read out noise of the CCD-camera. The spectrum and the stray light contribute only twice, so that their noise has to be multiplied by a factor of \( \sqrt{2} \) in the determination of the total noise.

5.5.2 THE EXPECTED AVERAGE THOMSON SIGNAL

To detect the Thomson signal a CCD-camera is used. This camera can operate in several modes. The mode which we are using is the SL-mode, which means that it uses 250 pixels horizontally by 30 pixels vertically. From the 30 vertical pixels only 6 are used, because the height of our signal is no more than 6 pixels. After each measurement, the 6 vertical pixels are binned off-chip, which means that the signal that is collected on each of the 6 pixels is added by the software. To minimise the dark current (and thus the dark current noise) the CCD-camera is cooled by using a Peltier element and two fans to a temperature of \( -28^\circ \)C. Each mm on the CCD-camera corresponds with a wavelength interval of 0.7 nm, and so with a total length of 9.1 mm the total spectral range of the CCD-camera is 6.4 nm. This is not enough to measure the entire Thomson profile in one go (the \( \Delta \lambda_{1/e} \) of the Thomson spectrum for an electron temperature of 1.5 eV is 3.7 nm (equation 5.1)), so in order to measure the entire Thomson profile two measurements, on the right and left hand side of the central wavelength have to be done.

The maximum detection limit of the CCD-camera is \( 2.0 \times 10^5 \) electrons/pixel. The camera has a 16-bit ADC, so \( 2.0 \times 10^3 \) electrons correspond with \( 2^{16} = 65535 \) counts, so 3.05 electrons / count. With a QE of the CCD-camera of 40%, that makes 7.6 photons / count. Per laser pulse 28 photons are scattered in the solid angle of the detection unit (§ 5.2.3). Because we only look at half the Thomson spectrum, only about 14 photons will be detected in one laser pulse. It takes an average of 7.6 photons to make one count, so 14 photons correspond with 1.8 counts per laser pulse. In total there are 250 horizontal pixels, so the detected Thomson signal is \( 7.4 \times 10^3 \) counts per pixel per laser pulse, which equals 7.4 counts per pixel per 1000 laser pulses.

5.5.3 DETECTED NOISE

As stated before, there are five sources of the noise, the shot noise, the read out noise, the dark current noise, the stray light noise and the spectral noise.

The shot noise = \( \sqrt{(\text{pure Thomson signal})} \). So for a 100 seconds measurement the shot noise is \( \sqrt{7.4} = 2.7 \) counts per pixel. The read out noise can be found in [Tas94]. In order to estimate the other noise sources the standard deviation on ten measurements of 1000 laser pulses each is calculated.
The readout noise is about 25 electrons per pixel, which corresponds with 8 counts. Six vertical pixels are binned, so the noise increases by a factor $\sqrt{6}$. The total readout noise becomes $\sqrt{6} \times 8 = 20$ counts per pixel per measurement (= 1000 laser shots = 100 seconds).

The dark current signal is depicted in figure 5.10, and the noise is depicted in figure 5.11. The dark current noise turns out to be about 60 counts per pixel per 100 seconds. This noise here is composed of the dark current noise and the readout noise. The actual dark current noise is $\sqrt{60^2-20^2}=57$ counts per pixel per 100 seconds.

![Fig. 5.10: Mean dark current.](image)

![Fig. 5.11: Dark current noise.](image)

The stray light and stray light noise are depicted in figures 5.12 and 5.13. The noise is about 90 counts per pixel, that is including the dark current noise and the readout noise. The actual stray light noise is $\sqrt{90^2-57^2-20^2}=67$ counts per pixel per 100 seconds.

![Fig. 5.12: Mean stray light.](image)

![Fig. 5.13: Stray light noise.](image)
The last form of noise is the spectral noise. This is caused by the light of the lamp itself. In figure 5.14 part of the spectrum is depicted. The noise is in the order of 350 counts per pixel (fig. 5.15), but as before it is composed of the actual spectral noise, the dark current noise and the read out noise. The stray light noise is not included here, because the laser is not on. The actual spectral noise is \( \sqrt{350^2 - 57^2 - 20^2} = 345 \) counts per pixel per 100 seconds.

\[350^2 - 57^2 - 20^2 = 345\]

![Fig. 5.14: Mean spectrum.](image)

![Fig. 5.15: Spectral noise](image)

### 5.5.4 SIGNAL TO NOISE RATIO

The total amount of noise can now be calculated by adding all the noises quadratically (table 5.3). We have to take into account that the read out noise and the dark current noise both occur four times, and the stray light and spectral noise two times.

<table>
<thead>
<tr>
<th>Noise type</th>
<th>Intensity [counts/pixel/100 sec.]</th>
<th>Occurrence</th>
<th>Total intensity [counts/pixel/100 sec.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shot noise</td>
<td>2.7</td>
<td>1</td>
<td>2.7</td>
</tr>
<tr>
<td>Read out noise</td>
<td>20</td>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>Dark current noise</td>
<td>57</td>
<td>4</td>
<td>114</td>
</tr>
<tr>
<td>Stray light noise</td>
<td>67</td>
<td>2</td>
<td>95</td>
</tr>
<tr>
<td>Spectral noise</td>
<td>345</td>
<td>2</td>
<td>488</td>
</tr>
<tr>
<td>Total noise</td>
<td></td>
<td></td>
<td>512</td>
</tr>
</tbody>
</table>

The total noise is 512 counts per pixel. As can be seen in table 5.3, the shot noise can be neglected. The expected pure Thomson value is 7.4 counts per pixel per 100 second measurement (§ 5.5.2). This means that the signal to noise ratio is approximately 0.014, and that it is impossible to detect the Thomson signal. In order to improve the signal to noise ratio we will have to decrease the total noise.
5.6 IMPROVEMENTS

5.6.1 HORIZONTAL BINNING

As mentioned before in the previous section the signal to noise ratio is much smaller than unity, so it is impossible to detect the Thomson spectrum. Since the signal can not be boosted, the only possibility is decreasing the noise. One option is binning the horizontal pixels off-chip. This means that after the pixels have been read out, the signal of 8 pixels will be added. After the horizontal binning 31 pixels remain. This is a high enough resolution to detect a Thomson spectrum. The reason for binning is that if the signal is increased by a factor of 8, the noise should only be increased by a factor of \( \sqrt{8} \), and so the signal to noise ratio is increased by a factor of \( 8/\sqrt{8} = \sqrt{8} \).

The intensity of the expected average pure Thomson signal per pixel per 100 seconds (= 1000 laser pulses) has been calculated in § 5.5.2. Before binning, the calculated average Thomson signal equals 7.4 counts per pixel per 100 seconds. After horizontal binning, the signal is increased to 59 counts per pixel per 100 seconds.

The read out noise before horizontal binning was calculated in § 5.5.3 to be about 20 counts per pixel per 100 seconds measurement. After 8 horizontal pixels are binned together there should be a read out noise of about \( \sqrt{8} \times 20 \approx 57 \) counts per pixel per 100 seconds measurement.

The measured dark current noise before horizontal binning is about 57 counts per pixel per 100 seconds. We expect the noise to become about \( \sqrt{8} \times 57 \) counts \( \approx 161 \) counts. In order to verify this number the measured dark current signal is binned, and the standard deviation of ten 100 seconds measurements is calculated. The average dark current noise now turns out to be about 300 counts per pixel (fig. 5.16), and after subtracting the read out noise the dark current noise is about 295 counts per pixel per 100 seconds.

The real noise is almost a factor of two larger than the expected noise. This could mean that the pixels are somehow correlated. We can see in figure 5.17 that the mean value of the dark current decreases in time. This means that part of the calculated deviation (which we consider to be the “noise”) is caused by this decrease in time. To cancel this effect we will have to wait until the CCD-array becomes stable (i.e. the mean value of the dark current remains constant). This measurement is done in § 5.6.3.

Fig. 5.16: Dark current noise horizontal binning.

Fig. 5.17: Correlation CCD-array.
The stray light noise is depicted in figure 5.18. As we can see the measured stray light noise after horizontal binning is about $575$ counts per pixel per $100$ seconds. To get the actual stray light noise the dark current noise and the read out noise are subtracted. The actual stray light noise then becomes $\sqrt{(575^2-295^2-57^2)} = 490$ counts per pixel. The expected noise is $\sqrt{8 \times 67}$ counts $= 190$ counts per pixel.

The spectral noise is depicted in figure 5.19. The measured spectral noise after horizontal binning is approximately $2200$ counts. This is again composed of the read out noise and the dark current noise. The actual spectral noise is $\sqrt{(2200^2-295^2-57^2)} = 2180$ counts per pixel, whereas the expected noise is $\sqrt{8 \times 345}$ counts $= 976$ counts per pixel.

![Fig. 5.18: Stray light noise after horizontal binning.](image1)

![Fig. 5.19: Spectral noise after horizontal binning.](image2)

### 5.6.2 Reducing Spectral Noise

First the option for reducing the main source of noise, the spectral noise, is examined. The actual measuring time for the detection unit is ten times $10$ ns per second, so in total $100$ ns per second. The spectrum however is detected the entire time. A possible way to decrease the spectral signal and thus the spectral noise is placing a shutter in front of the CCD-camera.

There is a shutter available, the Displaytech video shutter. This device has an open shutter transmission of $> 25\%$, a closed shutter transmission of $< 0.03\%$. The shutter is open for about $50$ µs, and closed for about $0.1$ s per laser shot. By using this device, the spectral signal is reduced by a factor of about $\frac{4 \times 0.1}{50 \times 10^{-6}} = 8000$. The factor $4$ comes from the open shutter transmission of $25\%$. This means that the spectral noise will be reduced by a factor of $\sqrt{8000} = 89$ to $\frac{2180}{89} = 24$ counts per pixel per $100$ seconds.

The shutter also has a negative aspect. Because the open shutter transmission is $0.25$, the Thomson signal will also be reduced to about $\frac{59}{4} = 15$ counts per pixel per $100$ seconds.
5.6.3 REDUCING DARK CURRENT

As mentioned in § 5.6.1 we suspect that the CCD-camera exhibits some sort of correlation, that means that the total value of the measured signal displays some sort of time-dependent behavior. To see if this is the case, a dark current measurement is done for 10,000 seconds (100 measurements of 100 seconds) under the same conditions as stated in § 5.5.3 (SL-mode, \( T = -28^\circ C \)). The results can be seen in figure 5.20. We can see that that the dark current is time-dependent, and that after about 6000 seconds it stays fairly stable.

![Figure 5.20: Dark current time-dependency.](image)

To see what the effect of this time dependency is on the dark current noise, we calculated the noise, with horizontal binning to 31 pixels, 9000 seconds after the CCD-array was turned on (see figure 5.21). The average noise turns out to be about 170 counts per pixel. We already calculated an expected noise of 161 counts (§ 5.6.1), so this result corresponds with the theory.

![Figure 5.21: Dark current noise.](image)

A way to further reduce the dark current is using water cooling to reduce the temperature of the CCD-array. Every 6°C drop in temperature results in a signal reduce of a factor of 2 [Tas94]. If we can lower the temperature to -40°C, which means a 12°C extra drop in temperature, the dark current signal will be reduced by a factor of four, and the dark current noise by a factor of \( \sqrt{4} \). The dark current noise can now be reduced to \( 170/2 = 85 \) counts per pixel per 100 seconds.
The last main noise source is the stray light noise. The main cause is the fact that we are shooting on a glass lamp, which causes a lot of scattering. The scattered light has the same wavelength as that of the laser, 532 nm. This would mean that all the stray light would come together at the same point where the Rayleigh peak is situated. However, light that originates from another place than the place we are examining, can cause stray light at other places at the detector. It can be seen in figure 5.12 that the stray light is not located solely at the Rayleigh peak, at 532 nm, but is distributed across the spectrum that is being examined.

A possible way to reduce this noise is the use of a double monochromator. The first monochromator is needed to unravel the incoming light in its spectrum, and the second is needed to put the spectrum back together again. Each monochromator reduces incoming light not at the selected wavelength, or not originating from the same place we are examining, by a factor of $10^3$, so the double monochromator reduces that light by a factor of $10^6$. This means that the stray light noise is reduced by $\sqrt{10^6} = 10^3$.

However, there is downside to this procedure. Each monochromator has a grating, and each grating reflects only about 25% of the incoming light of the right wavelength, so incoming light is reduced by a factor of $4^2 = 16$. This implies an extra reduction factor for all incoming light. In the case of the spectral noise, that is of course no problem. The spectral signal is reduced by an extra factor of 16, and so the spectral noise by a factor of 4. On the other hand, it is a problem for the pure Thomson signal. That signal will also be reduced by a factor of 16, so only $15/16 = 0.9$ counts per pixel per 100 seconds remain.

In paragraph 5.6.1 the stray light noise was determined to be about 490 counts per pixel per 100 seconds. With a reduction factor of $10^3$ the stray light noise will become 0.5 counts per pixels per 100 seconds. However, the pure Thomson signal is reduced as well. This means that other possibilities have to be examined.

An other way to reduce the stray light is altering the position of the lamp. In the present setup, light that is scattered on the entrance glass and reflected off the antenna is scattered towards the detection unit (fig. 5.22). By rotating the lamp $180^\circ$ light that reflects off the antenna is scattered away from the detection unit. It is however not known what the effect of this alteration is.

![Fig. 5.22: Repositioning the QL-lamp.](image-url)
5.6.5 NUMBER OF MEASUREMENTS

We calculated in the previous paragraph that the stray light signal, and the stray light noise would become negligible, using a double monochromator. This means that no separate stray light measurement has to be done. This also means that we will not have to do the extra dark current measurement (§ 5.5.1). The only measurements that have to be done are the Thomson measurement (which contains the pure Thomson signal, the dark current and the spectral signal, and the negligible stray light signal) and the spectral measurement (which contains the spectral signal and the dark current). By subtracting those two we get the pure Thomson signal. We will have to take into account the fact that both the CCD-camera and the QL-lamp are not stable at switch on. It takes both devices about two hours to become stable (see § 5.4 and § 5.6.3).

5.6.6 REPEATED MEASUREMENTS

Another way to increase the signal to noise ratio is to repeat the measurements. As the measurements are repeated N times, the signal is also increased by a factor of N, whereas the noise is only increased by a factor of √N. The signal to noise ratio is then increased by a factor of N/√N = √N.

5.6.7 SIGNAL TO NOISE RATIO AFTER IMPROVEMENTS

Now the results from the previous paragraphs are summarized. In table 5.4 the different reduction methods and reduction factors, and in table 5.5 the Thomson signal and noise sources are written down. This is done in the case of a 100 seconds measurement.

Table 5.4: Improvements to the setup.

<table>
<thead>
<tr>
<th>Improvements</th>
<th>Reduces</th>
<th>Reduction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooling the CCD to -40°C</td>
<td>Dark current noise</td>
<td>2</td>
</tr>
<tr>
<td>Double monochromator</td>
<td>Stray light noise</td>
<td>1000</td>
</tr>
<tr>
<td>Double monochromator</td>
<td>Spectral noise</td>
<td>4</td>
</tr>
<tr>
<td>Double monochromator</td>
<td>Thomson signal</td>
<td>16</td>
</tr>
<tr>
<td>Video shutter</td>
<td>Spectral noise</td>
<td>89</td>
</tr>
<tr>
<td>Video shutter</td>
<td>Thomson signal</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 5.5: Signal and noise values for a 100 second measurement.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Thomson signal</td>
<td>59</td>
<td>0.9</td>
<td>1</td>
<td>0.9</td>
</tr>
<tr>
<td>Read out noise</td>
<td>57</td>
<td>57</td>
<td>2</td>
<td>81</td>
</tr>
<tr>
<td>Dark current noise</td>
<td>170</td>
<td>85</td>
<td>2</td>
<td>120</td>
</tr>
<tr>
<td>Stray light noise</td>
<td>490</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Spectral noise</td>
<td>2180</td>
<td>6.0</td>
<td>2</td>
<td>8.5</td>
</tr>
<tr>
<td>Total noise</td>
<td>2255</td>
<td>103</td>
<td></td>
<td>145</td>
</tr>
</tbody>
</table>
After improvements the expected average pure Thomson signal is reduced to 0.9 counts per pixel per 100 seconds. The total noise is reduced to an average of 145 counts per pixel. This makes a signal to noise ratio of $6.2 \times 10^{-3}$. As mentioned in § 5.6.6 this signal to noise ratio can be increased by repeated measurements. Because two kinds of measurements have to be done, the Thomson measurement and the spectral measurement, and the QL-lamp and CCD-camera have a relaxation time of about 2 hours, each kind of measurement can last for 2 hours, which corresponds with 72 measurements of 100 seconds. In this way, the signal to noise ratio can be increased to $\sqrt{72} \times 6.2 \times 10^{-3} = 0.05$. However, this is not nearly enough to measure the Thomson spectrum. The main problems are the read out noise and dark current noise. Although they have been reduced, their values are about 2 orders of magnitude larger than the expected Thomson signal.

Not using the double monochromators is an option, because then the Thomson signal is about 15 counts per pixels per photon, but the stray light noise then becomes about 490 counts per pixel per 100 seconds. This also means that an extra stray light measurement has to be done (§ 5.5.1). We now have to take into account the fact that the read out and dark current noise both occur four times, and the spectral and stray light noise two times. The expected values of the Thomson signal and noise sources are written down in table 5.6.

Table 5.6: Signal and noise, no stray light reduction.

<table>
<thead>
<tr>
<th>Signal / noise</th>
<th>Value [counts / pixel / 100 seconds]</th>
<th>Occurrence</th>
<th>Value [counts / pixel / 100 seconds]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thomson signal</td>
<td>15</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>Read out noise</td>
<td>57</td>
<td>4</td>
<td>114</td>
</tr>
<tr>
<td>Dark current noise</td>
<td>85</td>
<td>4</td>
<td>170</td>
</tr>
<tr>
<td>Stray light noise</td>
<td>490</td>
<td>2</td>
<td>693</td>
</tr>
<tr>
<td>Spectral noise</td>
<td>6.0</td>
<td>2</td>
<td>8.5</td>
</tr>
<tr>
<td>Total noise</td>
<td></td>
<td></td>
<td>723</td>
</tr>
</tbody>
</table>

In this case the signal to noise ratio becomes $15/723 = 2.1 \times 10^{-2}$, and after 72 100 second measurements the signal to noise ratio is increased to $\sqrt{72} \times 2.1 \times 10^{-2} = 0.18$. This is an improvement, compared to using the double monochromator. However, the signal to noise ratio is still smaller than unity. From this we can conclude that using this setup no Thomson signal can be detected, with the expected electron density and temperature.
5.7 THOMSON SCATTERING WITH PHOTON COUNTING

5.7.1 THEORY

In the previous paragraph the signal to noise ratios for Thomson scattering measurements using the PDA-intensifier and CCD setup were calculated. For both setups the ratios were much smaller than unity, so that no Thomson spectrum can be measured. Now we are going to try to determine the Thomson spectrum with photon counting, using the Camac setup. The idea is to measure the Thomson signal for a number of wavelengths, near the central wavelength of 532 nm, and so determine a Thomson spectrum. The setup is depicted in figure 5.23. The first branch of the setup, from Nd:YAG laser to laser dump is the same as the setup depicted in figure 5.1, only the position of the lamp has been changed to reduce the stray light noise (§ 5.6.4). The major difference between the setups are the detection branches.

In the PDA / CCD branch a horizontal detection volume is imaged on a vertical slit. This means that the length of the detection interval is very small (0.3 mm). In the Camac setup the horizontal detection volume is rotated 90° by a set of four mirrors, so that there now is a vertical detection volume. This volume is imaged on a vertical slit, so now the length of the detection volume is much larger (10 mm).

![Setup Thomson scattering, with the use of a mono - PMT - Camac.](image)

Fig. 5.23: Setup Thomson scattering, with the use of a mono - PMT - Camac.
With a single monochromator ($\Delta \lambda = 0.2$ nm, attenuation factor of $10^{-5}$ [DJ097]) a certain wavelength interval is selected. Next the photons within this wavelength interval fall onto the Photo Multiplier Tube (PMT type R268, $V_{\text{set}} = -1000$ V). Since the PMT has a QE of about 10%, one out of every ten photons is detected. At the set voltage of -1000 volt the amplification factor is $2 \times 10^6$, so each detected photon creates a charge of $2 \times 10^6 \times 1.6 \times 10^{-19}$ C = $3.2 \times 10^{-13}$ C = 0.32 pC. This number can be increased by increasing the voltage over the PMT. The maximal voltage that can applied is about -1500 V. However, it is not clear what the amplification factor for those high voltages is.

The Camac is a charge integrator. This means that within a certain time interval, determined by the triggering, all charge that is presented to the Camac by the PMT is integrated. The Camac has a maximal range of 256 pC. A 10-bit ADC reads out the data, and presents it to a PC. A 10-bit ADC corresponds with $2^{10} = 1024$ units, so that each unit represents a charge of 256 pC / 1024 = 0.25 pC. As calculated, at the set voltage one photon creates a charge in the PMT of 0.32 pC, which corresponds with about one unit on the ADC. This can be improved by increasing the voltage over the PMT, as mentioned before. This means, that if the dark current of the PMT and Camac is in the order of one unit or less, single photon counting is possible.

5.7.2 THOMSON SIGNAL

The total detectable Thomson signal can be calculated by equation 5.10 determined in § 5.2.3. For this setup $\Delta \Omega = 5.5 \times 10^{-3}$ sr, $d\sigma / d\Omega = 7.95 \times 10^{-30}$ m$^2$/sr and the electron density $n_e$ is assumed to be $5 \times 10^{18}$ m$^{-3}$. The major difference is the length of the detection volume, this is now increased by a factor of 30 to about 1 cm.

$$P_s = P_\lambda n_e L \Delta \Omega \frac{d\sigma_{\text{th}}}{d\Omega}$$  \hspace{1cm} (5.10)

This means that in the solid angle $\Delta \Omega$ the scattered power is about $2.2 \times 10^{-15}$ of the incident power. The incident power is now decreased by a factor of 4 to about 0.1 J per laser pulse. This is because the focus of the laser pulse is now located inside the QL-lamp, so if the laser power is increased to its maximum, the glass cover will damage, and this will cause more laser scattering. A laser power of 0.1 J corresponds with $3.0 \times 10^{17}$ photons. In total, 640 photons are scattered in the solid angle per laser pulse. To calculate a signal to noise ratio, the average Thomson signal within the specified wavelength interval of 0.2 nm has to be compared to the average noise calculated with the CCD-setup, for the same wavelength interval.
As can be calculated with equation 5.1, the FWHM of the Thomson spectrum is determined by the electron temperature.

$$T_e = \frac{m_e c^2}{32 \sin^2(\theta/2) k_B \lambda_0^2} \times (\Delta \lambda_{1/e})^2 = 1312 \times (\Delta \lambda_{1/e})^2$$  (5.1)

We estimate the maximal electron temperature to be about 1.5 eV (1.7x10^4 K). For that temperature, the FWHM is about 3.6 nm. This means that 99% of all Thomson photons will be in an area with a width of 11 nm. The average Thomson signal for the selected wavelength interval in one laser pulse is then 640 photons \times (0.2 / 11) = 12 photons. The number of detected photons decreases by the use of the monochromator (about 25%) and the PMT (QE of 10%). This means that the expected average pure Thomson signal in one laser pulse per wavelength interval is about 0.3 photons. For a 100 second measurement (1000 laser pulses), the average Thomson signal per wavelength interval is 3x10^2 photons.

With the CCD-setup, the noise was calculated per horizontal pixel. First the wavelength interval which corresponds to one horizontal pixel is determined. This is necessary if we want to compare the Thomson signal for the Camac-setup, with the noise determined with the data from the CCD-setup. The CCD-camera we use has a total of 750 pixels horizontally each with a width of 12.5 \mu m. The CCD was used in the SL-mode, which means 250 horizontal pixels. Three pixels are binned together on-chip. After reading out the data we also added 8 pixels. In total we binned 8 \times 3 = 24 times. This means each pixel has a width of 24 \times 12.5 \mu m = 300 \mu m. Each mm on the CCD-array corresponds with a wavelength interval of 0.7 nm. 300 \mu m corresponds with 0.3 \times 0.7 nm = 0.21 nm. So each 'super' pixel corresponds with 0.21 nm. The monochromator in the Camac setup also has a \Delta \lambda of about 0.2 nm. This means that we can compare the calculated Thomson signal for this setup with the calculated noise from § 5.6.1. The noise in that paragraph is calculated in counts, which has to be converted to the number of photons, since we are now counting photons. This conversion factor is calculated in § 5.5.2 to be 7.6 photons per count.

5.7.3 NOISE

There remain five forms of noise, the stray light noise, the spectral noise, the dark current of the PMT, the shot noise, and the read out noise of the Camac. From the previous chapter we know the values of the stray light noise and the spectral noise.

The stray light noise is about 490 counts per wavelength interval (which equals one horizontal CCD pixel of 300 \mu m) per 100 seconds. With an average of 7.6 photons per count (§ 5.5.2) this makes a noise of about 3.7x10^3 photons per wavelength interval per 100 seconds. To reduce this noise the single monochromator, with an attenuation factor of 10^5, is used. The grating used in the CCD setup has a attenuation factor of 10^7. This means that the stray light calculated with the CCD setup will now only be reduced by a factor of 10^5/10^3 = 100 and the stray light noise by a factor of \sqrt{100} = 10. The stray light noise will be reduced to 3.7x10^3 photons per wavelength interval per 100 seconds/(10 \times \sqrt{10}) = 117 photons per wavelength interval per 100 seconds.

The mean stray light intensity is about 9x10^4 counts per wavelength interval per 100 seconds, which corresponds with 7.6 \times 9x10^4 = 6.8x10^5 photons per wavelength interval per 100 seconds. The monochromator reduces this signal by a factor of 100, and with a QE of the PMT of 10%, the remaining stray light will be about 680 photons per wavelength interval per 100 seconds.
The spectral noise is about $2.2 \times 10^3$ counts per wavelength interval per 1000 shots, so $1.7 \times 10^4$ photons per wavelength interval per 100 seconds. The mean intensity of the spectrum is about $7 \times 10^3$ counts per wavelength interval per 100 seconds, which corresponds with $5.3 \times 10^6$ photons per wavelength interval per 100 seconds. The monochromator reduces the spectral signal by a factor of 4, so the spectral noise is reduced by a factor of 2. The spectral signal and noise can be strongly reduced by the fact that the Camac acts as a shutter of about 100 ns, because the integrating time is determined by the external triggering. This means that the spectral signal is reduced by a factor of $1.0 \times 10^4$ (i.e. laser repetition time)/ $1.0 \times 10^7 = 10^6$, and the spectral noise is reduced by a factor of $10^3$. The PMT reduces the signal by a factor of 10, and the noise by a factor of $\sqrt{10}$. This means that the spectral noise will be reduced to about $1.7 \times 10^4$ photons per wavelength interval per 100 seconds $\div (\sqrt{10} \times 2 \times 10^3) = 2.7$ photons per wavelength interval per 100 seconds. The mean spectral signal will be reduced to about $5.3 \times 10^6 / (10 \times 4 \times 10^6) = 0.12$ photons per wavelength interval per 100 seconds.

The third form of noise is the dark current of the PMT. The photomultiplier tube creates a pulse as a photon falls on the tube. However, the PMT also creates pulses while no photons fall on it, the dark current. The problem is that we have no means to discriminate between a ‘real’ pulse and a dark current pulse. This means that the dark current has to be much smaller than our signal, if we want to ignore the dark current.

We know that our signal is about 300 photons per 100 seconds. The PMT has a dark current of about 1000 counts per second (cps). It seems to be that the dark current is larger than the average Thomson signal, but we have to take into account that if we do a 100 seconds measurement, we do not measure 100 seconds at a stretch, but we do 1000 measurements of 100 ns, so 100 $\mu$s in total. In 100 $\mu$s, the averaged dark current equals $10^3$ cps $\times$ $100 \times 10^{-6}$ s = 0.1 counts. For a 100 second measurement the dark current of the PMT equals 0.1 counts for each wavelength interval, so the dark current is negligible.

The shot noise is about the square root of the signal, so $\sqrt{300} = 17$ photons per wavelength interval per 100 seconds. The read out noise of the Camac was measured for all channels of the ADC, and it turned out that a number of channels exhibited no form of read out noise.

All noise can be reduced by a factor of $\sqrt{N}$ when we do $N$ measurements of 100 seconds.
5.7.4 SIGNAL TO NOISE RATIO

Finally, an estimate for the signal to noise ratio for the Camac setup can be calculated. The result can be seen in table 5.7.

Table 5.7: Signal / noise.

<table>
<thead>
<tr>
<th>Signal / Noise type</th>
<th>Mean signal [photons / wavelength interval / 100 sec.]</th>
<th>Noise [photons / wavelength interval / 100 sec.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thomson signal</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>Stray light</td>
<td>680</td>
<td>117</td>
</tr>
<tr>
<td>Spectrum</td>
<td>0.1</td>
<td>2.7</td>
</tr>
<tr>
<td>Dark current</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Shot noise</td>
<td></td>
<td>17</td>
</tr>
<tr>
<td>Read out noise</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The maximal noise is about 117 photons per wavelength interval per 100 seconds. If we perform a 100 second measurement (i.e. 1000 pulses), in a wavelength interval of 0.2 nm, a signal to noise ratio of \( \frac{300}{117} = 2.5 \) can be achieved. This means that a Thomson spectrum can be measured. Of course, the signal is a typical signal, so dependent on the chosen wavelength the signal can be larger or smaller than 300 photons per wavelength interval per 100 seconds. This means that the signal to noise ratio can vary.

5.7.5 TEST MEASUREMENTS

As mentioned before, the Camac has a total of 12 channels. Of each channel the combination of dark current and read out noise is determined. The total noise turns out to be zero for all channels, except channels 5 and 8. All channels do have a certain offset, which is different for each channel. Because channel 11 is the only one who does not have to be reset, each time it is read out, the channel is chosen.

To determine whether the signal to noise ratio of 2.5 can be achieved, test measurements are done, to determine the magnitude of the noise sources. Because we know that the spectral noise can be neglected (table 5.7), we focus our attention on the stray light noise.

The stray light noise is determined by doing measurements on a QL-lamp flushed with argon and helium. This is done because argon has a large Rayleigh cross-section \( \sigma_{\text{Rayleigh}} = 4.66 \times 10^{-31} \text{ m}^2 \) and helium has a very small Rayleigh cross-section \( \sigma_{\text{Rayleigh}} = 7.36 \times 10^{-33} \text{ m}^2 \). By extrapolating the line connecting the two points (fig. 5.24) to the point where \( \sigma_{\text{Rayleigh}} = 0 \text{ m}^2 \), the stray light can be determined. This is done for several wavelengths.

To measure the stray light a filter is placed in front of the PMT, which attenuates the signal by a factor of 1644. This is done because the Rayleigh signal for argon at the central wavelength is very strong. The set voltage of the PMT is -1400 V. The laser focus is set inside the lamp, because then the width of the laser beam in the detection volume is much smaller. This is done because the width of the slit is only 250 \( \mu \text{m} \), so as little as possible light is lost. The laser oscillator is set at 40, the laser amplifier is off (laser power = 0.1 J).
First the stray light level at the centre of the Rayleigh peak is determined. The results can be seen in figure 5.24. The stray light signal is about $4 \times 10^6$ photons per second.

Next the stray light signal is measured away from the Rayleigh peak, at a wavelength of 532.5 nm. This is done for the same settings, only now without the filter. The average stray light signal turns out to be about $3 \times 10^3$ photons per second per wavelength interval. The stray light signal at the Rayleigh peak is about $4 \times 10^6$ photons per second. The true attenuation is about $10^3$, in stead of $10^5$. The average Thomson signal, as calculated in § 5.7.2, was about 3 photons per wavelength interval per second.

The average stray light noise is about $10^2$ photons per wavelength interval per second, so still about two orders larger than the average Thomson signal. A stray light noise of 100 photons per wavelength interval equals about $10^3$ photons per wavelength interval per 100 seconds. This is about ten times as large as the calculated noise (table 5.7) The reason for this is that the attenuation factor is $10^3$ in stead of $10^5$, as mentioned above. This means that the average stray light signal will be about a factor of 100 larger than the calculated signal, and the noise a factor of $\sqrt{100} = 10$. The calculated noise will now be about $1.2 \times 10^3$ photons per wavelength interval per 100 seconds, which is very close to the measured noise.

If a 100 seconds measurement is performed, the average expected Thomson signal will be about $3 \times 10^2$ photons per wavelength interval, and the stray light noise about $10^3$ photons per wavelength interval, which corresponds with a signal to noise ratio of 0.3. This means that it is still impossible to detect a Thomson spectrum. All forms of unwanted signal have been reduced, except for the stray light.

Because the focus is now placed inside the lamp, it turns out that the laser is creating little pits in the entrance glass. This is because the beam width is much smaller, and the glass quality is poor. The poor quality of the glass also causes extra stray light, because laser photons can scatter on impurities in the glass. A possible improvement could be the use of best quality quartz glasses, in stead of this poor quality glass. Furthermore, the new entrance and exit glasses could be placed at Brewster angles, to further reduce the stray light noise. It is however not known if this will reduce the straylight noise enough to be able to detect a Thomson profile.
6 CONCLUSIONS

The final goal is to develop a Plasma Simulation Model (PLASIMO) for the Philips QL-lamp. The main parts of PLASIMO are those which describe the plasma and the inductive energy transfer. The plasma part has been the subject of investigation in this report.

The plasma in both the QL-lamp and the TL-lamp consists of two atomic species, argon and mercury. Mercury is the so-called ‘active medium’ in the plasma. This means that the radiation coming from the lamp and the creation of new free electrons, which is necessary to compensate the loss of electrons due to diffusion, is mainly caused by mercury.

Up until now a three-level model, consisting of the ground state, ion ground state and a radiating ‘metastable’ level, was used to calculate the production of new free electrons and radiation. This model is not valid for high electron densities, because then stepwise excitation becomes more important. To include this effect, a more extensive Collisional Radiative Model (CRM) model has been developed.

The CRM consists of two independent levels and 13 dependent levels. The number of dependent levels is determined by calculating the boundary between the levels in Corona Balance (CB) and the levels in Electron Saturation Balance (ESB). Only the densities of the levels in CB are calculated. The net flow from CB to ESB is determined analytically, and all electrons which cross that border from CB to ESB will eventually become a free electron.

Using the two Electron Group Model (2EGM) the validity of the assumption that the Electron Energy Distribution Function (EEDF) is Maxwellian is checked. It turns out that this assumption is valid ($\Delta S_{CR} < 15\text{-}20\%$) for an electron density larger than $5\times10^{18}$ m$^{-3}$.

With the CRM the Atomic State Distribution Function (ASDF) is calculated. It turns out that for an electron temperature up to about 1.25 eV the metastable levels and the first resonant level are in Boltzmann equilibrium with the ground state. Knowing this, it seems to be possible to determine experimentally the Boltzmann line and thus the electron temperature from the gradient of this line.

Furthermore the influence of the metastable-metastable and metastable-resonant (heavy-heavy) collisions has been estimated. The rate of the production of free electrons due to heavy-heavy collisions is in the same order of magnitude as the rate due to electron induced collisions. Because of this, the heavy-heavy collisions can not be neglected. In order to take these collisions into account, a CRM with six independent levels has been developed. The densities of the metastable and resonant levels are taken to be independent. Now the densities of these levels are calculated by PLASIMO, and the effect of the heavy-heavy collisions can be included. However, up to now it was not possible to get PLASIMO to become numerically stable with this extended CRM. The only simulations that have been done successfully were with a mercury pressure of 6.5 Pa. In this case the heavy-heavy collisions lead to an increase of the electron density of about 30%.

To determine the validity of the CRM measurements on and simulations of high-current TL-lamps have been done. TL-lamps have been used because in this way the inductive energy transfer part does not have to be taken along. It turns out that PLASIMO can not yet precisely simulate the experimental results. The causes for this problem could be that no heavy-heavy interactions have been taken into account, due to the numerical instability of PLASIMO. At this moment an escape factor is used to compensate for the absorption of resonant radiation. The non-local treatment of resonant radiation is not included, which may be important.
The conclusion concerning the development of the CRM is that the heavy-heavy collisions part and the non-local treatment of resonant radiation have to be further examined, and implemented in PLASIMO. Since a CRM has been developed with six independent levels including the non-local treatment of resonant radiation is possible. However, the reason for the numerical instability of PLASIMO is still unknown.

The possibility of using the method of Thomson scattering to determine the electron density and temperature in the QL-lamp has been examined. With this information it should be possible to determine the validity of PLASIMO, if both the plasma part and the inductive energy transfer part have been included. Several detection units have been used to measure a Thomson profile. It appears that the PDA and CCD detection units are troubled by several forms of noise: stray light noise, spectral noise, dark current noise and read out noise. Although these noise sources can be suppressed, their intensity after suppression is still much larger than the pure Thomson signal.

Also the method of photon counting is used. With this method, all forms of noise can be neglected, except for the stray light noise. This stray light appears to be three orders of magnitude larger than the Thomson signal, and the noise about two orders of magnitude. One of the causes for the stray light is the poor quality of the glass, which causes extra divergence of the laser beam. Because of this it is now not possible to measure a Thomson profile. An option to decrease this stray light noise is the use of very high quality quartz, placed at the Brewster angle.

For now the conclusion is that it is not possible to determine the electron density and temperature in the QL-lamp, using the method of Thomson scattering.
APPENDIX 1

ALTERNATIVE SOLUTION CRM

In the CRM, designed by [Har96], the calculation of the ASDF is done slightly different than described in [Mul86]. In § 3.3.2 the populating and depopulating processes are written separately for the direct and indirect (de)population from the independent levels. As described in that section, the population of a dependent level from another dependent levels is nothing other than the population from an independent level, via the dependent levels.

All (de)populating processes from and to the dependent levels are written in one matrix $M$, and the direct population from the independent levels $i$ are written in the vectors $S_i$ (3.19). Now all of those (de)populating processes are placed inside the matrix $M$. In order to do this, first equation 3.19 has to be rewritten to:

$$M N = n_i \Pi^i + n_i \Pi^i$$  \hspace{1cm} (A1.1)

The matrix $M$ and the vectors $N$ and $\Pi$ are N-1 dimensional. The vector $N$ gives the densities of the dependent levels $p$, $n_p$. The vectors $\Pi^i$ give the direct population contribution of a dependent level $p$ from an independent level $i$ (equations A1.2 and A1.3). These equations are the same as equations 3.20 and 3.21 from § 3.3.2, but now the ground state and ion ground state densities are placed outside the vectors $S_i'$, which results in the new vectors $\Pi^i$. The components of matrix $M$ are written down in equations A1.4 and A1.5.

$$\Pi^i_p = n_e D_{\star p} = n_e \left[ n_i K_{\star p} + A_{\star p} \Lambda_{\star p} \right]$$  \hspace{1cm} (A1.2)

$$\Pi^i_i = n_e K_{1p}$$  \hspace{1cm} (A1.3)

$$M_{pq} = -D_{qp} = -n_e K_{qp} - A_{qp} \Lambda_{qp}$$  \hspace{1cm} (A1.5)

$$M_{pp} = D(p) = n_e K(p) + A(p)$$  \hspace{1cm} (A1.6)

Equation A1.1 can also be written as:

$$\begin{bmatrix} m_{22} & \cdots & m_{2N} \\ \vdots & \vdots & \vdots \\ m_{N2} & \cdots & m_{NN} \end{bmatrix} \begin{bmatrix} n_2 \\ \vdots \\ n_N \end{bmatrix} = n_i \Pi^i_2 + n_i \Pi^i_N$$  \hspace{1cm} (A1.7)

This equation can be modified. The $\Pi^i$ vectors are now placed in the matrix $M$, which now has a dimension of N+1. The vector $N$ is modified into a matrix, also N+1 dimensional. The $i^{th}$ column $N^i$ in this matrix represents the densities of all levels $n_{p_i}$, with the contribution from one level $i$, with $p$ from 1 to N+1. The matrix $X$ gives the densities $n_i$ of the independent levels.
The matrix $M$ is written down in the case of two independent levels ($i = 1$ (atom ground state) and $+ \ (ion \ ground \ state)$), and $N-1$ dependent levels. Matrix $N$ can now be solved by calculating the inverse matrix $M^{-1}$.

\[
N = M^{-1} X \Rightarrow N^i = (M^{-1} X)^i,
\]

(A1.9)

With the ASDF known, the $r$-values can now be calculated:

\[
r^i_p = \frac{n^i_p}{n^i_1}
\]

(A1.10)

with $i = 1 \ or \ +$. We can now use these $r$-values to calculate the effective ionisation ($S_{CR}$) and recombination ($\alpha_{CR}$) coefficients:

\[
\alpha_{CR} = n_e K_{ri} + A_{ri} + \sum_{q>i} \frac{n^i_q r^i_q}{n^i_e n^i_+} (n_e K_{q1} + A_{q1} \lambda_{q1}) \quad [m^3 s^{-1}]
\]

(A1.11)

\[
S_{CR} = \sum_{q>i} K_{iq} + S_{i1} - \sum_{q>i} \frac{n^i_q r^i_q}{n^i_e n^i_1} (n_e K_{q1} + A_{q1} \lambda_{q1}) \quad [m^3 s^{-1}]
\]

(A1.12)

Also the radiation losses coefficients can now be calculated:

\[
\varepsilon_i = \frac{1}{n^i_e n^i_1} \sum_p n^i_p \sum_{q>i} A_{pq} \lambda_{pq} \times E_{pq} \quad [eV m^3 s^{-1}]
\]

(A1.13)

with $i$ again $1 \ or \ +$ and $E_{pq}$ the energy difference in eV.
APPENDIX 2

SIX INDEPENDENT LEVELS

As determined in § 3.11.6, the influence of the heavy-heavy collisions for the ionisation process can not be neglected. Up until now, the densities of the metastable and resonant levels are only determined by the processes of equation 3.16 (§ 3.3.2). The heavy-heavy collisions, which are a depopulating process, have not been included. This means that the existing CRM has to be adjusted, to include this process.

In the current CRM, the densities of the excited levels are determined by the ground state and the continuum. The CRM will now be adjusted in such a way, that the density of a dependent level $p$ is determined by six independent levels, namely the ground state and ion ground state, and the four 6P levels (the metastable levels $6^3P_0$ (n=2) and $6^3P_2$ (n=4) and the resonant levels $6^3P_1$ (n=3) and $6^3P_1$ (n=5), equation A2.1).

\[ n_p = n_p^+ + n_p^1 + n_p^2 + n_p^3 + n_p^4 + n_p^5 \quad (A2.1) \]

For a certain value of the electron temperature and density and the densities of the ground state, ion ground state and 6P-levels the CRM can calculate the densities of the dependent levels. The densities of the 6P levels are now calculated by PLASIMO. PLASIMO also calculates the effects of the heavy-heavy collisions on the ASDF and the (de)population of the independent levels by other independent levels.

Equation A2.2 describes the matrix $M$ with two independent levels, the ground state and the ion ground state. The extension of matrix $M$ with two independent levels to a matrix $M$ with three independent levels is written down in equation A2.3.

\[
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & 0 & 0 \\
-\Pi_2^1 & m_{22} & m_{23} & \cdots & m_{2,N-1} & m_{2N} & -\Pi_2^* \\
-\Pi_3^1 & m_{32} & m_{33} & \cdots & m_{3,N-1} & m_{3N} & -\Pi_3^* \\
\vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\
-\Pi_{N-1}^1 & m_{N-1,2} & m_{N-1,3} & \cdots & m_{N-1,N-1} & m_{N-1,N} & -\Pi_{N-1}^* \\
-\Pi_N^1 & m_{N,2} & m_{N,3} & \cdots & m_{N,N-1} & m_{NN} & -\Pi_N^* \\
0 & 0 & 0 & \cdots & 0 & 0 & 1
\end{bmatrix}
\]

\[
\Rightarrow
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 1 & 0 & \cdots & 0 & 0 & 0 \\
-\Pi_3^1 & -\Pi_3^2 & m_{33} & \cdots & \cdots & \cdots & -\Pi_3^* \\
-\Pi_4^1 & -\Pi_4^2 & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-\Pi_N^1 & -\Pi_N^2 & m_{N,3} & \cdots & \cdots & \cdots & -\Pi_N^* \\
0 & 0 & 0 & \cdots & 0 & 0 & 1
\end{bmatrix}
\]

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Each 'zero' in the column of an independent level corresponds with a transition which will be calculated by PLASIMO, and not the CRM. These transitions are the direct transitions from one independent level to another independent level. Extending the matrix $M$ to a total of six independent levels can be done in the same way as described in equation A2.3. To calculate the densities of the dependent levels $N^i$ with a contribution from the independent level $i$ equation A2.4 can be used:

$$N = M^{-1}X \Rightarrow N^i = (M^{-1}X)^i,$$ \hfill (A2.4)

With this the so-called $J$-coefficients $[\text{m}^3\text{s}^{-1}]$ can be calculated. This $J$-coefficient is the population of an independent level $j$ by another independent level $i$, direct or via the dependent levels.

$$J_{ij} = \frac{1}{n_i n_j} \left( \sum_p n_p^i \left[ n_p K_{pj} + A_{pj} \right] + n_i \left[ n_e K_{ji} + A_{ji} \right] \right) \hfill (A2.5)$$

with $n_p^i$ the density of a level $p$ with contributions only from the independent level $i$, $K_{pj}$ the (de)excitation from the dependent level $p$ to the independent level $j$ and $A_{pj}A_{pi}$ the effective radiative decay from that level $p$ to level $j$. $K_{ij}$ is the (de)excitation from an independent level $i$ to an independent level $j$, and $A_{ij}A_{ij}$ the effective radiative decay from level $i$ to level $j$. The term on the left describes the indirect processes (via the dependent levels), whereas the term on the right describes the direct processes.

The $J$-coefficient is equivalent to the $S_{CR}$ and the $\alpha_{CR}$ coefficients. The ionisation coefficient is nothing other than $J_{1+}$, and the recombination coefficient is nothing other than $J_{+1}$. Because there are now six independent levels, in stead of two, the number of coefficients is extended. An other positive effect of creating a CRM with independent 6P-levels is that the non-local treatment of resonant radiation from the resonant states ($6^3\text{P}_1$ and $6^1\text{P}_1$) to the ground state can be determined much easier.
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