MASTER

The Fresnel zone plate antenna: design and analysis

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Award date:
1992

Link to publication
The Fresnel Zone Plate Antenna: Design and Analysis

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The faculty of Electrical Engineering of the Eindhoven University of Technology disclaims all responsibility for the contents of training and graduation reports.
AS OFTEN AS A STUDY IS CULTIVATED BY NARROW MINDS,
THEY WILL DRAW FROM IT NARROW CONCLUSIONS.

by J.S. Mill

Voor Sonja

1From Auguste Comte and positivism (1865).
Summary

Due to the advent of satellites that transmit high-power direct-broadcast signals, the interest for applications of the Fresnel antenna has been revived, mainly because this antenna is cheaper to manufacture than a parabolic reflector antenna (PRA). The Fresnel antenna is a lens-like device and functions on the basis of a combination of refraction and interference or a combination of diffraction and interference. The type of antenna considered here belongs to the last category and is called a *Fresnel zone plate* antenna (FZPA) because it is composed of *flat* annular zones called Fresnel zones. These zones can be alternately absorbing/transparent or phase-correcting.

Designing an optimal FZPA-system is a complex task because the gain function of such a system is influenced by more parameters than that of a PRA-system. Yet, it is possible to develop a design procedure which requires only the frequency, diameter and the desired gain of the antenna as input parameters. A major problem is that some design rules conflict, which implies that optimizing a FZPA-system always results in a trade-off between different demands imposed by the manufacturer. Applying the same design procedure to a PRA-system reveals that this is easier to optimize than a FZPA-system and that there exist less conflicting design rules. Both antenna systems can also be used for scanning, which implies that the feed is displaced in order to receive signals transmitted by satellites at different positions. A scalar analysis reveals that the scan performances of the FZPA- and PRA-systems in general show the same trends.

The fact that the FZPA functions on the basis of diffraction and interference, complicates the mathematical model of the FZPA with absorbing/transparent zones, because there exists no satisfying model for diffraction at absorbing material, which hampers the investigation of the field behaviour in the aperture of the antenna. This behaviour is important because the width $d$ of a number of Fresnel zones can become small in terms of wavelengths, implying that no longer can be assumed that the aperture field in Kirchhoff's integral can be replaced by the incident field. Therefore, the absorbing material is replaced by perfectly conducting material which is allowed because it does not affect the proper functioning of the FZPA. Using the asymptotic techniques UTD (Uniform Theory of Diffraction) and GTD (Geometrical Theory of Diffraction), it is possible to calculate gain patterns which are expected to be valid for the whole FZPA-geometry (except boresight and vicinity), provided that single and multiple diffraction are included. Because of the fact that it is unknown whether UTD/GTD applies for very small values of $d$, a new approximation method has been developed in order to investigate this aspect.
List of symbols

\( \vec{E} \)  Electric field [V/m]. The unit vector is denoted by \( \hat{E} \). A superscript \( i \) denotes the incident field, \( d \) the diffracted field.

\( \vec{H} \)  Magnetic field [A/m]. The unit vector is denoted by \( \hat{H} \). A superscript \( i \) denotes the incident field, \( d \) the diffracted field.

\( \varepsilon_0 \)  Permittivity of vacuum, \( \varepsilon_0 = 8.85419 \cdot 10^{-12} \) F/m.

\( \mu_0 \)  Permeability of vacuum, \( \mu_0 = 1.25664 \cdot 10^{-6} \) H/m.

\( \pi \)  Constant, \( \pi = 3.141592654 \).

\( e \)  Constant, \( e = 2.718281828 \).

\( \gamma \)  Euler’s constant, \( \gamma = 0.5772156649 \).

\( j \)  Imaginary unit, \( j = \sqrt{-1} \).

\( \lambda \)  Wave length [m].

\( f \)  Frequency [Hz].

\( \omega \)  Angular frequency [rad/s], \( \omega = 2\pi f \).

\( k \)  Wave number [m\(^{-1}\)], \( k = 2\pi/\lambda \). The propagation direction of a wave is indicated by the unit vector \( \hat{k} \).

\( Z_0 \)  Characteristic wave impedance of vacuum (air) [V/A], \( Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \).

\( P_t \)  Total power radiated by the feed [W].

\( G(\hat{r}) \)  Antenna gain function [dBi].

\( G_0 \)  Maximum of the antenna gain function [dBi].

\( G_{\text{feed}} \)  Gain function of the feed [dB].

\( G_{\text{aid}} \)  Gain aided by the feed to the antenna system [dB].

\( G_{\text{edge}} \)  Edge illumination [dB].

\( n \)  Real number which characterizes the gain pattern of the feed.

\( CP(\hat{r}) \)  Co-polarization radiation pattern of the antenna [dBi].

\( XP(\hat{r}) \)  Cross-polarization radiation pattern of the antenna [dBi].

\( \eta_s \)  Spillover efficiency factor.

\( \eta_i \)  Illumination efficiency factor.

\( \eta_p \)  Phase efficiency factor.

\( \eta \)  Overall efficiency factor.

\( D \)  Diameter of a circular antenna aperture [m].

\( F \)  Distance between the origin and the focal point of the antenna [m].

\( F' \)  Distance between the origin and the feed during defocusing [m].

\( F_{\text{max}} \)  Focal distance at which \( \eta \) is maximum [m].
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$F_{\text{min}}$</td>
<td>Focal distance at which $\eta$ is minimum [m].</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>The r.m.s reflector surface tolerance [m].</td>
</tr>
<tr>
<td>$\psi_0$</td>
<td>Subtended angle of an antenna [rad].</td>
</tr>
<tr>
<td>$m$</td>
<td>Integer denoting the $m$th circle or $m$th Fresnel zone of the Fresnel zone plate antenna (starting with $m = 0$).</td>
</tr>
<tr>
<td>$b_m$</td>
<td>Radius of the $m$th circle of the Fresnel zone plate antenna [m].</td>
</tr>
<tr>
<td>$\psi_m$</td>
<td>Opening angle of the $m$th circle of the Fresnel zone plate antenna.</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Distance between the $m$th circle of a Fresnel zone plate antenna and the focal point.</td>
</tr>
<tr>
<td>$M$</td>
<td>Integer denoting the number of zones of a Fresnel zone plate antenna.</td>
</tr>
<tr>
<td>$M_t$</td>
<td>Integer denoting the number of the transparent zone closest to the edge of the Fresnel zone plate antenna.</td>
</tr>
<tr>
<td>$M_p$</td>
<td>Integer denoting the number of the phase-correcting zone closest to the edge of a Fresnel zone plate antenna system.</td>
</tr>
<tr>
<td>$P$</td>
<td>Integer denoting the measure of phase-correction ($P \geq 2$).</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Angle of phase-correction [rad], $\delta = 2\pi/P$.</td>
</tr>
<tr>
<td>$d_m$</td>
<td>Half the width of a Fresnel zone with number $m$ [m], $d_m = \frac{b_{m+1} - b_m}{2}$. In case the subscript $m$ is omitted $d$ represents half the width of an infinite long slit or the radius of a hole in a perfectly conducting screen.</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Feed displacement angle [rad].</td>
</tr>
<tr>
<td>$\theta_{\text{scan}}$</td>
<td>Scan angle [rad].</td>
</tr>
<tr>
<td>$\vec{S}$</td>
<td>Time-averaged Poynting vector [W/m²] of a focused antenna system.</td>
</tr>
<tr>
<td>$\vec{S'}$</td>
<td>The unit vector in denoted by $\vec{S}$.</td>
</tr>
<tr>
<td>$\vec{Q}_i$</td>
<td>Diffraction point with number $i$ ($i = 1, 2$).</td>
</tr>
<tr>
<td>$s_i^d$</td>
<td>Distance between a field point and the diffraction point with subscript $i$, $s_i^d$ is the unit vector pointing from the diffraction point $Q_i$ to the field point.</td>
</tr>
<tr>
<td>$s_i^f$</td>
<td>Distance between the diffraction point with subscript $i$ and the feed, $s_i^f$ is the unit vector pointing from the feed to the diffraction point $Q_i$.</td>
</tr>
<tr>
<td>$\vec{D}$</td>
<td>Dyadic diffraction coefficient.</td>
</tr>
<tr>
<td>$D_s$</td>
<td>Scalar soft and hard diffraction coefficients in case of the Uniform Theory of Diffraction.</td>
</tr>
<tr>
<td>$A(s_i^f, s_i^d)$</td>
<td>Caustic divergence factor.</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>Angle between the incident ray at point $Q_i$ and the antenna surface tangent, which is perpendicular to the plane of incidence [rad].</td>
</tr>
<tr>
<td>$\omega^d$</td>
<td>Angle between the diffracted ray at point $Q_i$ and the antenna surface tangent, which is perpendicular to the plane of diffraction [rad].</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>The radius of the curvature of the edge at the diffraction point $Q_i$ [m].</td>
</tr>
</tbody>
</table>
\( \rho^i_e \)  
Radius of curvature of the incident wavefront in the plane of incidence at point \( Q_i \) [m].

\( \rho_c \)  
Distance between the caustic at the edge of the antenna aperture and the second caustic of the diffracted ray [m].

\( Fr(z) \)  
Transition function involving a Fresnel integral.

\( L^i, L' \)  
Distance parameters [m].

\( N_i, M_i, \mathcal{H}_i \)  
Millar's series coefficients \( (i = 0, \ldots, 6) \).

\( \alpha, \Psi, v, w \)  
Variables of Millar's series coefficients \( (0 \leq v \leq \pi) \).

\( p \)  
Complex number involving \( w \) and Euler's constant and needed for the Millar's series coefficients.

\( \tilde{E}_{te} \)  
The electric component \([\mathrm{V/m}]\) of a wave incident at a slit (Millar's configuration).

\( \tilde{E}_{im} \)  
The electric component \([\mathrm{V/m}]\) of a wave incident at a slit (Millar's configuration) and normal to \( \tilde{E}_{te} \).

\( \tilde{H}_{im} \)  
The magnetic component \([\mathrm{V/m}]\) of a wave (with electric component \( \tilde{E}_{im} \)) incident at a slit (Millar's configuration).

\( \tilde{E}_{cp} \)  
The co-polarization component \([\mathrm{V/m}]\) of the electric field in the focal point when considering the receiving mode.

\( \tilde{E}_{xp} \)  
The cross-polarization component \([\mathrm{V/m}]\) of the electric field in the focal point when considering the receiving mode.

\( A_{x'}^E \)  
Length of the \( x' \)-component of \( \tilde{E}_{im}^E \) in Millar's slit configuration.

\( A_{x'}^H \)  
Length of the \( x' \)-component of \( \tilde{H}_{te}^H \) in Millar's slit configuration.

\( \tilde{q}_{ix}, \tilde{q}_{iv} \)  
Special vectors \([\mathrm{V/m}]\) with \( i = 1, 2 \) which are used to decompose the incident electric field into two orthogonal components in order to satisfy Millar's geometrical configuration.

\( A \)  
Surface of the antenna aperture \([\mathrm{m}^2]\).

\( A_{pseudo} \)  
Projection of the antenna surface \( A \) of a parabolic reflector antenna on the xy-plane \([\mathrm{m}^2]\).

\( S \)  
Closed surface \([\mathrm{m}^2]\) containing the surface \( A \).

\( \epsilon_x \)  
Distance in \( x \)-direction between focal point and the defocused feed [m].

\( \epsilon_z \)  
Distance in \( z \)-direction between focal point and the defocused feed [m].

\( \epsilon \)  
Distance between focal point and the defocused feed [m], \( \epsilon = \sqrt{\epsilon_x^2 + \epsilon_z^2} \).

\( G \)  
Green's function.

\( J_i \)  
Bessel function of the first kind and order \( i \).

\( U \)  
Unity step function.

\( \text{sign} \)  
Sign function.

\( h(\theta, \phi) \)  
Phase function [rad].

\( T_i \)  
Values for which the first derivative of the phase function is zero.

\( \hat{n} \)  
Unit vector normal to the surface \( A \) or to the rim of \( A \).

\( \Gamma \)  
Symbol denoting the rim of \( A \).

\( \hat{T} \)  
Unit vector tangent at the rim \( \Gamma \).

\( a, b \)  
The outer and inner radius \([\mathrm{m}, \mathrm{m}]\) of an annulus in perfectly absorbing material.
$x, y, z$ Rectangular coordinates of the cartesian reference frame [m,m,m]. Unit vector are denoted by $\hat{x}, \hat{y}, \hat{z}$.

$r, \theta, \phi$ Spherical coordinates [m,rad,rad]. Unit vectors are denoted by $\hat{r}, \hat{\theta}, \hat{\phi}$.

$\rho, \xi, \psi$ Spherical coordinates used to describe the feed pattern [m,rad,rad]. Unit vectors are denoted by $\hat{\rho}, \hat{\xi}, \hat{\psi}$.

$r', \phi'$ Cylindrical coordinates used to describe the surface $A$ [m,rad].

$l, \phi'$ Cylindrical coordinates used to describe the surface $A_{pseudo}$ [m,rad].

$\rho'$ Distance between the an arbitrary point of the antenna aperture and the defocused feed [m].

$R$ Distance between an arbitrary point of the surface $A$ and an observation point [m].

$\beta, \varphi$ Edge fixed coordinates used for diffraction problems involving the Geometrical or Uniform Theory of diffraction [rad,rad]. A subscript $i$ denotes the plane of incidence and a subscript $d$ denotes the plane of diffraction.

$x', y', z'$ Rectangular coordinates used for Millar's geometrical configuration of a slit [m,m,m].
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Chapter 1

General introduction

Telecommunication via satellites is becoming more and more important in present-day society. Thus, it is not surprising that the market for small satellite receivers used at home grows rapidly. Due to satellites that transmit high-power direct-broadcast signals it is nowadays commercially interesting to use an antenna for such receiving systems other than the conventional parabolic reflector antenna, for instance the Fresnel antenna. Figure 1.1 shows a Fresnel antenna of a commercial indoor satellite receiving system. Usually, this type of antenna is used for high-frequency applications such as X-ray microscopy. The basic principles of the Fresnel antenna are well known for a long time: the first papers on related subjects go back to the days of Lord Rayleigh (circa 1900 AD) but applications in the mm- and cm-wave region are relatively new.

The main advantages for using such an antenna are the relatively low production costs and the fact that the lens is very flat (some types are made of a kind of foil) which makes it easy to install. For instance, the lens of the antenna can be attached to a window or on the top of an apartment building as shown in figure 1.2. A disadvantage of the Fresnel antenna is its lower gain in comparison with the parabolic reflector antenna, which makes it only suitable for receiving signals from high-power communication systems. Because the Fresnel antenna originates from optics, as mentioned before, some problems arise when using it for frequencies other than light-wave frequencies:

- The theory mostly applied for calculating far-field radiation patterns of the Fresnel antenna is Kirchhoff's diffraction theory in combination with the assumption that the aperture field can be replaced by the incident field. This is surely satisfactory for optical wave lengths but becomes questionable for frequencies in the GHz-range. This is caused by the fact that the focal distance in optical applications is large compared to the wavelength used. For the GHz-region this does not apply.

- The cheapest version of a Fresnel antenna is composed partly of absorbing material which has to be as thin as possible. For optical applications the absorbing material is usually replaced by metal. Absorbing material is an idealized material which is mathematically or physically very difficult to describe.
Chapter 1. General introduction

Figure 1.1: Photo of a Fresnel antenna used for a commercial indoor satellite receiver (source: Revox).

Figure 1.2: Two different applications for Fresnel antennae (source: Mawzones).
Another interesting aspect is the design of an optimum antenna system. A major problem which arises when optimizing a Fresnel antenna system, is that the performance of such a system is influenced by more factors than in case of a parabolic reflector antenna system. Maybe for that reason a design procedure for optimizing the performance of a Fresnel antenna system does not exist yet.

The Fresnel antenna considered in this report does not consist of elliptical zones (figure 1.2) but of circular zones, just as the one depicted in figure 1.1. Of this antenna first the basic principles and the geometrical configuration are explained and the commonly used notations and conventions are introduced (Chapter 2). Further, different types of this Fresnel antenna are elucidated. Next, in Chapter 3, antenna efficiencies are calculated which are required for designing an optimum antenna system. In chapter 4 is investigated how different design parameters influence each other. Results of this investigation are used to set up a design procedure (Chapter 5). After this, the efficiencies of the Fresnel antenna system and the parabolic reflector antenna system are compared in Chapter 6. As already indicated at the beginning of this introduction, the reception of signals from direct-broadcast satellites is one of the main applications of the Fresnel antenna. For that reason, in Chapter 7 the scan properties are examined: the Fresnel and parabolic reflector antenna systems are compared concerning these properties. Next, some models are described and discussed with respect to their usability to analyze a Fresnel antenna consisting of transparent and perfectly absorbing rings (Chapter 8). Further, problems arising from the fact that the widths of these rings may become small are considered in Chapter 9. Different field calculation methods are examined and the far-field radiation patterns of a particular Fresnel antenna are calculated, using different diffraction calculation techniques. Finally, conclusions concerning the design procedure, scan properties and the validity of the different diffraction calculation methods are drawn and discussed.
Chapter 2

The basic principles of the Fresnel antenna

2.1 Introduction

A Fresnel antenna is a device which has lens-like properties and can be used for focusing and imaging electromagnetic waves. These functions are produced by a combination of refraction and interference or a combination of diffraction and interference. In principle, a Fresnel antenna can be considered as a discrete version of a conventional hyperboloid lens which converts a plane wave into a spherical wave (and vice versa) by means of refraction and interference. The Fresnel antenna consists of a set of concentric annular zones which are called Fresnel zones. The successive radii of these zones are chosen so that the distances from a selected (focal) point on the central axis increase by a value $\lambda / P$ when going from the inner to the outer radius of any zone. The variable $\lambda$ is the wavelength [m] of the frequency $f$ [Hz] and the factor $P$ is an integer ($P \geq 2$) related to the phase difference between spherical waves incident on the inner and outer boundary of a Fresnel zone. The phase difference $\delta$ [rad] is therefore defined as:

$$\delta = \frac{2\pi}{P} \quad (2.1)$$

According to the geometry of a Fresnel zone and the material this zone is made of, we can basically distinguish two types of antennae: Fresnel lenses and Fresnel zone plates. The first type functions in exactly the same way as the the hyperboloid lens (refraction and interference). As shown in figure 2.1 the zones each have a smoothly curved surface on the right side of the lens and the contours of these zones together define a hyperboloid.

The Fresnel lens is often used instead of the hyperboloid lens because it has the advantage of having lower absorption loss, thickness and weight. Because of the fact that calculations of radiation patterns of these Fresnel lenses are identical to those of hyperboloid lenses, the Fresnel antenna based on refraction will not be considered in this report.

We are interested in this report in the Fresnel antenna whose operation is based on the combination of diffraction and interference. This antenna has the property to direct not all
of the energy of a normally incident plane wave to one (focal) point but to remove or correct (partially) those parts from the incident wave which have a destructive effect compared to some reference, because of their differences in phase. The antenna consists therefore only of flat surfaces (plate, see figure 2.1). Due to these flat surfaces the antenna never functions on bases of refraction when the incident plane wave is normal to the antenna surface. The (partial) conversion of the incident wave into a spherical wave has to be the result of diffraction. If the incident wave is not normal to the lens surface and the lens is not infinitely thin, then also refraction occurs but in that specific case it is considered as an unwanted effect and will not be included in the calculations. Fresnel zone plates are used instead of Fresnel lenses because of the 'flatness' of the Fresnel zone surfaces. This leads to lower production costs because of the much simpler construction.

Before examining the behaviour of various characteristics of the Fresnel zone plate antenna (for convenience called Fresnel antenna from now on) first some characteristics, variables, commonly used notations and conventions are introduced in the next sections. Further, we will also discuss the material of which the antenna is constructed.

2.2 Configuration of the Fresnel antenna

The geometrical configuration used throughout this report is displayed in figure 2.2. This figure contains the variables which are important for further calculations. In cases where these calculations have a vectorial character an arrow denotes a vector, a caret a unit vector and when both signs are omitted the length of a vector is meant.
Chapter 2. The basic principles of the Fresnel antenna

The reference frame used to describe a far-field point is given by the spherical coordinates \( r, \theta \) and \( \phi \) and the variables \( \rho, \psi \) and \( \xi \) define the spherical coordinate system used to describe the field of the feed (the feed specifications are considered in the next chapter).

The most important geometrical parameters of the Fresnel antenna are:

- Radii \( b_m \).
  
The variable \( b_m \) is the radius of a boundary of a Fresnel zone. The subscript \( m \) (an integer) denotes the \( m \)th circle (boundary) of the antenna. According to figure 2.3 the radii \( b_m \) are given by:

\[
\text{path length} = \frac{m\lambda}{P} = \sqrt{F^2 + b_m^2} \Rightarrow b_m = \sqrt{\frac{2m\lambda}{P} \left( F + \frac{m\lambda}{2P} \right)} \tag{2.2}
\]

The equation above indicates that there exists a lower limit for the difference between two successive radii: \( \lambda/P \ (F=0) \). This property is needed in Chapter 9.
2.2. Configuration of the Fresnel antenna

- **Focal distance** $F$.
  
  In point $z = -F$ the spherical waves from the lens converge. According to equation (2.2) the antenna possesses several focal distances, all given by:
  
  $$F = \frac{b_m^2 P}{2m\lambda} - \frac{m\lambda}{2P}$$

  (2.3)

  Note that the distance $F$ is frequency dependent. Obviously, it can be arranged that different frequencies focus at different points, thus producing focal isolation (a quasi-optical technique for separating different frequencies) [60]. Hence, the antenna is a narrowband antenna in contrast to the parabolic reflector antenna, which is a broadband antenna.

- **Fresnel circles.**
  
  These are circles with radii $b_m$ and they are numbered as shown in figure 2.2. Each radius $b_m$ corresponds with an angle $\psi_m$ and a distance $\rho_m$ (the distance between the focal point and the circle $b_m$). The configuration of all the circles together is called the Fresnel zone plate.

- **Fresnel zones.**
  
  A Fresnel zone is the annular ring bounded by two successive Fresnel circles. Each zone has its own number as indicated in figure 2.2. The total number of zones is $M$. The phase difference $\delta$ is the same for all the zones but it should be noted that it could happen that part of the zone at the edge of the antenna is outside the antenna aperture for a certain combination of $D$ [$m$] (the antenna diameter) and the focal distance $F$. Because the part of the zone outside the antenna aperture is not considered the phase difference of the part of the zone inside the antenna aperture is smaller than $\delta$.

  ![Figure 2.3: Derivation of the radii $b_m$.](image)
2.3 Fresnel zone plates

The materials used for the Fresnel zones are very important for proper functioning of the antenna. Regarding the materials we can distinguish two types of Fresnel antennae:

Antenna with absorbing/transparent zones

This is the simplest Fresnel antenna and consists of a set of plane Fresnel zones which are alternately transparent and absorbing. The number of the transparent zone closest to the antenna edge, i.e. the last transparent zone before reaching the edge of the antenna, is $M$. For this type of antenna it is required that $P = 2$, which means that if a plane wave is normally incident to the zone plate, the portions of the radiation which pass through the various transparent zones all reach by diffraction the focal point with mutual phases which differ less than 180°. In other words, the zone plate acts like a lens, producing a focusing action on the radiation it receives (or transmits).

A disadvantage of absorbing zones is that the material used is not infinitely thin. This causes an effect called shadowing [5]. For the antenna systems considered in this report it is assumed that the absorbing material is infinitely thin and that the odd-numbered zones are covered with absorbing material.

Antenna with phase-correcting zones

If the absorbing/transparent zones of the previous antenna are replaced by zones which transfer radiation and introduce a certain amount of phase shift relative to the portions of the plane wave transferred by the adjacent zones, a phase-correcting zone plate arises. It is not required that $P = 2$ as in case of absorbing/transparent zones, but $P$ can have any value (within the given range of $P$) and the number of the last phase-correcting zone is $M_p$ which equals $M - 1$ (the numbering of the zones starts at zero!).

The altering of phase can be accomplished by cutting circular grooves of correct dimensions into a dielectric plate. These zone plates are also called stepped lenses because of the difference in thickness of the Fresnel zones. A different version of the phase-correcting zone plate can be constructed when two different materials are used for alternate zones. By proper choice of the two dielectrics the zones can be made equal in thickness. This implies that the front and back surfaces of the antenna are flat. Therefore, this type of antenna is called a planar lens [60]. The materials used are mainly low-loss dielectrics such as polystyrene[59].

Phase-correcting zones have the advantage of using the antenna surface more efficiently than antennae with absorbing/transparent zones. Antennae of the second type therefore have a much larger gain than those of the first type. Planar lenses have also the advantage of being flat on both sides, so there is no accumulation of material (such as dirt and snow) on the antenna surface. The gain can be enlarged by increasing $P$, which leads to smaller zones. Still smaller phase increments further increase the gain, but the relative
improvement is not substantial. In practice, the maximum value for $P$ is four [59]. For $P \to \infty$ the Fresnel lens becomes a hyperboloid lens.

However, there are also disadvantages attached to this type of antenna. The four main disadvantages are absorption loss, shadowing [5] due to the grooves, surface errors which cause phase disturbances and, in comparison with antennae with absorbing/transparent zones, higher production costs.

For the analysis in this report the Fresnel antennae with phase-correcting zones will be considered free of absorption, surface errors and shadowing.
Chapter 3

The antenna efficiencies

3.1 Introduction

Before we can optimize the antenna efficiency of a Fresnel antenna system, as will be done in the next chapter, we must first focus on the different types of efficiencies which play a role in the optimization process. These factors can be used to examine the antenna system and to evaluate the antenna efficiency. For the Fresnel antenna system four efficiency factors are important:

1. Spillover efficiency $\eta_s$.
2. Phase efficiency $\eta_p$.
3. Illumination efficiency $\eta_i$.
4. Antenna or overall efficiency $\eta = \eta_s \cdot \eta_p \cdot \eta_i$.

How these efficiencies are defined and calculated for the antenna geometry under consideration, will be explained in the next sections. First we have to consider three quantities which are required for the calculation of the efficiency factors: the electric field produced by the feed, the electric field in the far-field zone of the antenna and the gain function of the antenna.

The field of the feed is very important because it influences all other variables. The gain function $G_{\text{feed}}$ of the feed is modeled by:

$$G_{\text{feed}}(\psi, n) = \begin{cases} 2(n + 1)\cos^n(\psi) & 0 \leq \psi < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq \psi \leq \pi \end{cases}$$ (3.1)

Many feed patterns can be represented by a member of this class over a sizable portion of the main lobe.
3.1. Introduction

The variable $n$ (a real) is used to characterize the gain pattern of the feed. Using equation (3.1), the field radiated by the feed can be expressed as [32]:

$$
\hat{E}_{\text{feed}}(\rho, \psi, \xi, n) = \sqrt{G_{\text{feed}}(\psi, n)} \cdot \frac{e^{-jk\rho}}{\rho} \cdot \sqrt{\frac{2P_i Z_0}{4\pi}} \cdot ( - \cos(\xi) \cdot \hat{\psi} + \sin(\xi) \cdot \hat{\xi} )
$$

(3.2)

The total power radiated by the feed is denoted by $P_i [\text{W}]$. $Z_0 [\text{V/A}]$ is the characteristic wave impedance of vacuum and $k [\text{m}^{-1}]$ is the wave number (being $2\pi/\lambda$).

The polarization of the feed is chosen to be the polarization of a so called Huygens-source. The Huygens-source is a combination of two orthogonal infinitesimal dipoles: an electric and magnetic dipole. If the electric dipole is parallel to the $x$-axis and the magnetic dipole is parallel to the $y$-axis, the polarization of the combined field is as presumed in equation (3.2).

This type of feed will be used throughout this thesis and is also used in the derivation by Leyten [32] of the far-field equations for an antenna system with absorbing/transparent zones. These formulae can easily be adjusted for antenna systems with phase-correcting zones by introducing an additional phase factor $jm2\pi/P$. This factor accounts for the phase correction caused by each Fresnel zone. The extra loss of power caused by each zone needs not to be included because it is assumed in the previous chapter that the materials used for the phase-correcting zones introduce only a phase shift and not an extra loss of power. So, multiplying the original equations of [32] with $e^{jm2\pi/P}$ gives the new far-field equations:

$$
\hat{E}(r, n) = E_{\theta}(r, n) \cdot \hat{\theta} + E_{\phi}(r, n) \cdot \hat{\phi}
$$

(3.3)

with

$$
E_{\theta}(r, n) = \sum_{m} \pi \cos(\phi) \ C(r) \ e^{Q(m, p)} \int_{\phi_m}^{\phi_{m+1}} O(\psi, n) e^{M(\psi)} \ ... \ [-(\cos(\psi) + 1)J_0(N(\theta, \psi)) + (\cos(\psi) - 1)J_2(N(\theta, \psi))]d\psi
$$

(3.4)

$$
E_{\phi}(r, n) = \sum_{m} \pi \cos(\phi) \sin(\phi) \ C(r) \ e^{Q(m, p)} \int_{\phi_m}^{\phi_{m+1}} O(\psi, n) e^{M(\psi)} \ ... \ [(\cos(\psi) + 1)J_0(N(\theta, \psi)) + (\cos(\psi) - 1)J_2(N(\theta, \psi))]d\psi
$$

(3.5)
Chapter 3. The antenna efficiencies

\[ C(r) = \frac{je^{-jkr}}{2\pi r} \sqrt{\frac{2P_z Z_0}{4\pi}} \quad \text{M(\psi)} = -\frac{jkF}{\cos(\psi)} \]

\[ N(\theta, \psi) = kF \sin(\theta) \tan(\psi) \quad \text{Q}(m, p) = \frac{i2m\pi}{p} \]

\[ O(\psi, n) = \sqrt{G_{\text{feed}}(\psi, n)} \frac{F \tan(\psi)}{\cos(\psi)} \]

For the variable \( m \) applies:

- absorbing/transparent zones \( (P = 2) \): \( m = 0, 2, 4, \ldots M_t \) \hfill (3.6)
- phase-correcting zones \( (P \geq 2) \): \( m = 0, 1, 2, \ldots M_p \)

If \( m \) reaches the value \( M_t \) or \( M_p \) then \( \psi_{m+1} = \psi_0 \) [rad], which is the subtended angle of the antenna. Further, \( J_i \) is a Bessel function of the first kind and order \( i \).

The far-field formulae are not only required for the derivation of the efficiency factors but also for the gain function \( G(r) \) [dBi] of the antenna which is defined as [32]:

\[ G(r) = 10\log \left( \frac{2\pi r^2}{Z_0 P_t} |\vec{E}(r, n)|^2 \right) \] \hfill (3.7)

The maximum of the gain function is usually called the gain, indicated by \( G_0 \) [dBi].

After the three quantities which are required for the calculation of the efficiencies are determined, we can start with the derivation of the efficiency factors.

### 3.2 Spillover efficiency \( \eta_s \)

The spillover efficiency is defined as the ratio of the power radiated by the feed that reaches the antenna surface, and the total power radiated by the feed [1]:

\[ \eta_s = \frac{\int_0^\psi G_{\text{feed}}(\psi) d\psi}{\int_0^\psi G_{\text{feed}}(\psi) d\psi} \] \hfill (3.8)
3.3 Phase efficiency $\eta_p$

For our feed configuration equation (3.8) becomes:

$$\eta_s = 1 - \cos^{n+1}(\psi_0) \quad (3.9)$$

Formula (3.9) shows clearly that this efficiency factor is only dependent of $n$ and $\psi_0$ which means that, besides the gain pattern of the feed, only the parameters $D$ and $F$ of the antenna influence $\eta_s$.

3.3 Phase efficiency $\eta_p$

This efficiency is the most important one because it is very characteristic for a Fresnel antenna and is therefore mostly encountered in literature. It is often involved in discussions about the performance of Fresnel antenna systems.

The factor $\eta_p$ can be defined as the ratio of the radiated power in forward direction ($\theta=0$) and the radiated power in forward direction on condition that the phase of the field in the antenna aperture is constant [41]. The radiated power in forward direction can be derived from equations (3.4), (3.5) and (3.7) by substituting $\theta=0$.

The radiated power of the antenna with uniform phase distribution can be calculated in the same way and with the aid of the same formulae. There is only one difference: all phase terms ($e^{j\phi}$) are replaced by 1. So, $\eta_p$ can then be expressed as:

$$\eta_p = \frac{\sum_{m=0}^{\frac{\pi}{2}} \int (\cos(\psi)+1)\sqrt{2(n+1)} \cos^n(\psi) \sin(\phi) F e^{-j\phi F} \sin(kF_m y - m\lambda) d\phi}{\sum_{m=0}^{\frac{\pi}{2}} \int (\cos(\psi)+1)\sqrt{2(n+1)} \cos^n(\psi) \sin(\phi) F d\phi} \quad (3.10)$$

The range of $m$ is given by equation (3.6). The expression derived for $\eta_p$ is quite different from the one given (without any prove) by Wiltse et al. [59] :

$$\eta_p = \left( \sin \left( \frac{\pi}{2} \right) \right)^2 \quad (3.11)$$

which has its origin in optics. This equation is much simpler, but can it be used in our case? Given the restrictions that $n$ becomes not too large and that a Fresnel circle coincides with the antenna edge, it is possible to prove that both equations are indeed identical. The proof is given in Appendix D.

The 'large' value of $n$ for which equation (3.11) is no longer valid can be determined via the diameter of the first Fresnel zone ($m = 0$). Namely, a large $n$ causes a very tight beam which resembles almost a plane wave. In that situation the feed illuminates only zone number $m = 0$, as is depicted in figure 3.1. This results in a phase efficiency of approximately 1. Computer simulations confirm this theory.
3.4 Illumination efficiency $\eta_i$

This efficiency is determined by the distribution of the field over the antenna surface [1]:

$$\eta_i = \frac{\frac{1}{D} \sum_{m}^\psi^{m+1} \int_{\psi_m} (\cos(\psi) + 1) \sqrt{2(n + 1) \cos^n(\psi) \frac{\sin(\psi)}{\cos^2(\psi)} F d\psi}}{1 - \cos^{n+1}(\psi_0)}$$  \hspace{1cm} (3.12)

Here, just as in the previous section, equation (3.6) defines the range of $m$.

3.5 Efficiency $\eta$

Applying the definition for the overall efficiency $\eta$ given in the beginning of this chapter results in:

$$\eta = \left| \frac{1}{D} \sum_{m}^\psi^{m+1} \int_{\psi_m} (\cos(\psi) + 1) \sqrt{2(n + 1) \cos^n(\psi) \frac{\sin(\psi)}{\cos^2(\psi)} F e^{-i(\frac{p}{2} R - \frac{m^2}{2} \pi)} d\psi} \right|^2$$  \hspace{1cm} (3.13)

Equation (3.6) still defines the range of $m$. 

---

Figure 3.1: Radiation pattern of a feed with a large $n$, resulting in $\eta_p = 1$. 

---
Chapter 4
Optimizing the antenna system

4.1 Introduction

The goal of this chapter is to optimize a Fresnel antenna system, which means optimizing the overall efficiency $\eta$ and the sidelobe envelope. Optimizing the sidelobe envelope implies that the sidelobe levels must be chosen as low as possible. This in order to prevent signals from other satellites operating at the same frequency and polarization to interfere with the wanted signal. In principle, both optimizations are independent of each other but it is possible that design rules for maximum efficiency and the required sidelobe levels conflict and thus result in contradictions in the design procedure. In that case we have to search for a compromise.

Both optimizations will therefore first be treated separately in the next sections. We will investigate how to optimize $\eta$ and which influence the optimization has on the different efficiency factors and other system characteristics such as edge illumination and additional gain. Additional gain in dB is the difference between the gain function of the feed ($G_{feed}$) and the gain function of the antenna system ($G$).

Secondly, the sidelobe envelope is considered and in the next chapter the results of the optimization process will be evaluated in order to develop a design procedure, distilled out of the results of both optimizations.

4.2 Efficiency optimization

4.2.1 How to optimize?

The efficiency $\eta$ is dependent on a number of factors such as $F$, $f$, $D$ and $n$ (parameter of $G_{feed}$) which each have their own specific influence on $\eta$. This means that if we want to determine a realizable design procedure, first we must assume a number of factors constant. Usually the gain $G_0$ and $f$ of an antenna system are the constant parameters in a design procedure, but in case of a Fresnel antenna system the applications of such systems have to be considered.
Chapter 4. Optimizing the antenna system

For example, a low cost Fresnel antenna consists of a lens made of a special foil which can be attached to a window, and a receiver placed on a desk at some distance from the window. The dimensions of the lens are then of importance because of the limited proportions of the window. A current value for the diameter of such an antenna is about one meter. Therefore, \(D\) and \(f\) are chosen as constant factors and \(F\) and \(n\) as variable factors. The desired value of \(G_0\) has also to be given in advance.

The search of a maximum value of \(\eta\) starts with assuming \(F\) constant and \(n\) variable. Computer simulations which use a numerical integration routine \(^1\), show that there exists an \(n\) at which \(\eta\) is maximum. The same applies for other values of \(F\). So, with the aid of a simple iterative search routine it is possible to determine \(\eta\) as function of \(F\) of which on forehand is known that the overall efficiency \(\phi\) is maximum with respect to \(n\).

A question which now arises is how to display the resulting curves, because \(\eta\) is a function of \(F\) and \(n\) which both vary. A three-dimensional plot is confusing, thus is chosen for a two-dimensional plot which has the following properties: \(\eta\) is depicted as function of the focal distance \(F\) on condition that the \(\eta\)-value at any \(F\) is the maximum of the \(\eta - n\) curve for that \(F\)-value.

Figure 4.1 shows the four different efficiency factors of the previous chapter in the way just described. The overall antenna efficiency is separately depicted in figure 4.2 together with the matching \(n\)-values at the \(F\)'s where the \(\eta-F\) curve possesses a local maximum.

Not only the efficiencies are important for the design procedure but also, besides \(G_0\), the additional gain \(G_{\text{aid}}\) [dB] produced by the lens:

\[
G_{\text{aid}} = 10 \log \left( \frac{G_0}{G_{\text{feed}}} \right) \tag{4.1}
\]

\(G_{\text{aid}}, G_0\) and \(G_{\text{feed}}\) are plotted together in figure 4.3 in the same way as the efficiencies. Furthermore, the edge illumination \(G_{\text{edge}}\) [dB] is an important design parameter which is denoted by [32]:

\[
G_{\text{edge}}(\psi_0, n) = n \cdot 10 \log (\cos(\psi_0)) + 20 \log (\cos(\psi_0)) \tag{4.2}
\]

\(G_{\text{edge}}\) is dependent of two factors. The first factor is the free space loss, given by the ratio of the distance between the focal point and the edge of the antenna and the distance between the center of the lens (the point \((x = 0, y = 0, z = 0)\)) and the focal point. The other factor is the feed radiation pattern \(G_{\text{feed}}\). Figure 4.4 shows \(G_{\text{edge}}\) as function of \(F\) and is derived in exactly the same way as the previous curves.

After the efficiency curves and other characteristics are determined, we can examine them in order to obtain more information about their behaviour. This information could appear valuable for the design procedure and gives a better insight in how the antenna exactly functions.

\(^1\)The numerical integration routine is necessary because the formulae (3.10), (3.12) and (3.13) are not closed expressions.
4.2. Efficiency optimization

Figure 4.1: Efficiency curves of a Fresnel antenna with absorbing/transparent zones and phase-correcting zones ($P=2$).
Figure 4.2: Overall efficiency curve of a Fresnel antenna with absorbing/transparent zones and phase-correcting zones ($P=2$).
4.2. Efficiency optimization

\[ f = 11.1 \text{ GHz}, \quad D = 1 \text{ m}, \quad P = 2, \text{ absorbing/transparent zones} \]

Figure 4.3: The additional gain, the gain of the feed and the antenna system gain of a Fresnel antenna with absorbing/transparent zones and phase-correcting zones (\( P = 2 \)).
Chapter 4. Optimizing the antenna system

Figure 4.4: Edge illumination of a Fresnel antenna with absorbing/transparent zones and phase-correcting zones ($P=2$).

\[ f = 11.1 \text{ GHz}, D = 1 \text{ m}, P = 2, \text{ absorbing/transparent zones} \]

\[ f = 11.1 \text{ GHz}, D = 1 \text{ m}, P = 2, \text{ phase-correcting zones} \]
4.2. Efficiency optimization

4.2.2 Efficiency evaluation

What immediately attracts the attention when looking at the figures 4.1 and 4.2 are the local extremes of the \( \eta - F \) curve. These points are of great interest when optimizing an antenna system and are therefore investigated more closely.

In case of an antenna with absorbing/transparent zones, the local extremes of the \( \eta - F \) curve occur only if the zone on the outside, i.e. the last zone before reaching the edge of the antenna, is transparent and partly outside the antenna aperture. An explanation for this can be found in the way the efficiencies are calculated.

Namely, the model, used by [32] for the calculation of the formulae (3.4) and (3.5) and thus for the calculation of the efficiencies, considers everything outside the antenna aperture as absorbing material. This results in a phase contribution of the part of the spherical wave incident on the zone on the outside, which is no longer 180° but smaller (as already mentioned in Chapter 2). This influences the \( \eta_p - F \) curve which contains as many local extremes as the \( \eta - F \) curve but the \( F \)-values at which these local extremes occur do not match those of the \( \eta - F \) curve yielding that the phase disturbance is not the only cause.

The \( \eta_i - F \) curve appears to have a local optimum when the outer Fresnel circle of a transparent zone coincides with the antenna edge. This is not surprisingly because in that configuration the effective use of the antenna surface is optimum (in that case the antenna consists of more transparent zones than absorbing zones), which results in an optimum \( \eta_i \). The local extremes of the \( \eta_p - F \) and \( \eta_i - F \) curves occur at different \( F \)-values as is illustrated in figure 4.5. This explains why the \( F \)-values belonging to the extremes of the \( \eta - F \) curve do not match those of the \( \eta_p - F \) or \( \eta_i - F \) curves.

The \( \eta - F \) curve of an antenna with phase-correcting zones possesses twice as many extremes as the one of an antenna with absorbing/transparent zones (considering the same \( F \)-region). This is the result of the fact that the antenna has no longer absorbing zones. Thereby, every zone at the edge of the antenna causes a phase disturbance if a Fresnel circle does not coincide with the antenna edge. The local extremes of the \( \eta - F \) curve are mainly determined by the extremes of the \( \eta_p \)-curve because the effective use of the antenna surface is now always optimum. In this case the \( F \)-values of the \( \eta_p \)-curve at which the local extremes occur match exactly those of the \( \eta - F \) curve as is shown in figure 4.6.

Further, computer simulations show that with increasing \( P \) (the phase correction parameter) the 'oscillating' character of \( \eta \) decreases drastically, as can be seen in figure 4.7. This means that the extremes become less significant. Also, the gain increases drastically until \( P = 4 \). Increasing \( P \) further delivers only very little extra gain. This is conform the expectations of Chapter 2.
Figure 4.5: The $\eta_p - F$, $\eta_t - F$ and $\eta - F$ curves in case of absorbing/transparent zones.

Figure 4.6: The $\eta_p - F$, $\eta_t - F$ and $\eta - F$ curves in case of phase-correcting zones.
4.2. Efficiency optimization

Due to all these observations it is possible to give an estimation of the values of $F$ at which the extremes occur. These values can be calculated with the aid of the experimentally determined equations:

\[
\begin{align*}
\text{Absorbing/transparent zones} & \quad \text{Phase-correcting zones \((P=2, 3, 4)\)} \\
\frac{F_2 - F_1}{8} + F_1 &= F_{\text{max}} & \frac{F_2 - F_1}{4} + F_1 &= F_{\text{max}} \\
F_2 - \frac{F_2 - F_1}{8} &= F_{\text{min}} & F_2 - \frac{F_2 - F_1}{4} &= F_{\text{min}}
\end{align*}
\]

where $F_1$ and $F_2$ are the focal distances at which the edge of the antenna and a Fresnel circle coincide. At $F_1$ the number of zones must be $m$ and at $F_2$ it must be $m-1$. For antennae with absorbing/transparent zones the restriction applies that zone number $m$ must be transparent.

$F_1$ and $F_2$ can easily be calculated according to equation (2.3), so the points which are the most important for the design procedure can be determined without calculating the whole $\eta - F$ curve. The formulae (4.3) and (4.4) are tested for different values of $D$ and $f$ and give very good results as we have already seen in figure 4.2. The $n$-values of these plots are calculated with the aid of the equations (4.3) and (4.4).

Finally, it should be noted that the $\eta - F$ curve is scalable, which means that halving both $D$ and $F$ and doubling the frequency $f$ results in the same $\eta - F$ curve as in the original case. This is a property which also the parabolic reflector system possesses.

Figure 4.7: The $\eta - F$ curve of antennae with absorbing/transparent zones and phase-correcting zones \((P=2, 3, 4)\).
4.3 Sidelobe envelope

Because of the fact that the far-field formulae are not given in a closed form, it is not possible to determine the sidelobe envelope in a simple way. That is why is chosen for just plain calculation of the gain patterns and determining the sidelobe envelope from these patterns: a simple and effective approach.

In figure 4.8 the gain patterns of a Fresnel antenna (absorbing/transparent zones) at different $F$‘s (and corresponding $n$‘s) are displayed. The $F$‘s are not chosen arbitrarily but are calculated according to equations (4.3). The values of $F$ and $n$ used for the calculation of the gain patterns can be found in table 4.1. So the gain patterns in figure 4.8 represent the systems with an optimum $n$. Figure 4.8 shows that the sidelobe levels increase with increasing $F$.

Table 4.1: Values of $F$ and $n$ used for the calculation of the gain patterns.

<table>
<thead>
<tr>
<th>$F,n$</th>
<th>absorbing/transparent zones</th>
<th>phase-correcting zones $P = 2$</th>
<th>phase-correcting zones $P = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F1$</td>
<td>0.522 m</td>
<td>0.581 m</td>
<td>0.521 m</td>
</tr>
<tr>
<td>$n$</td>
<td>6.40</td>
<td>7.40</td>
<td>6.10</td>
</tr>
<tr>
<td>$F2$</td>
<td>0.778 m</td>
<td>0.709 m</td>
<td>0.777 m</td>
</tr>
<tr>
<td>$n$</td>
<td>14.00</td>
<td>11.00</td>
<td>13.00</td>
</tr>
<tr>
<td>$F3$</td>
<td>0.984 m</td>
<td>0.885 m</td>
<td>0.983 m</td>
</tr>
<tr>
<td>$n$</td>
<td>22.00</td>
<td>16.40</td>
<td>20.20</td>
</tr>
<tr>
<td>$F4$</td>
<td>1.303 m</td>
<td>1.331 m</td>
<td>1.300 m</td>
</tr>
<tr>
<td>$n$</td>
<td>38.42</td>
<td>36.10</td>
<td>35.00</td>
</tr>
<tr>
<td>$F5$</td>
<td>1.875 m</td>
<td>1.934 m</td>
<td>1.868 m</td>
</tr>
<tr>
<td>$n$</td>
<td>82.00</td>
<td>75.30</td>
<td>71.00</td>
</tr>
</tbody>
</table>

In the figures 4.9 and 4.10 the same gain patterns are displayed in the case of phase-correcting zones ($P=2$ and $P=4$). The $F$ and $n$ values used for the calculations can also be found in table 4.1 and are derived with the aid of equations (4.4). As can be seen in figure 4.9 here the sidelobe levels also increase with increasing $F$.

Comparing figures 4.8 and 4.9 we can conclude that for low sidelobe levels it is better to take low values of $F$ and the Fresnel antenna with phase-correcting zones appears to be the most promising.

It is also clearly visible that in case of $P=4$ the sidelobe envelope of the first part of the gain pattern ($0^\circ$ till $10^\circ$) resembles more that of the gain pattern of a parabolic reflector antenna system. This is in conformity with the fact that for $P \rightarrow \infty$ the Fresnel antenna becomes a parabolic reflector antenna (see also Chapter 2).
4.3. Sidelobe envelope

Figure 4.8: Gain patterns of a Fresnel antenna at different $F$'s ($\phi = 0^\circ$, absorbing/transparent zones).

Figure 4.9: Gain patterns of a Fresnel antenna at different $F$'s ($\phi = 0^\circ$, phase-correcting zones, $P=2$).
Figure 4.10: Gain patterns of a Fresnel antenna at different $F$'s ($\phi = 0^\circ$, phase-correcting zones, $P=4$).
Chapter 5

A design procedure

With the aid of the properties found in the previous chapter during the optimization of $\eta$ and the sidelobe envelope it is possible to develop a design procedure for an antenna system which uses a Fresnel antenna.

Here is a list of interesting properties:

- Maximum efficiency occurs when the last zone is transparent (in case of absorbing/transparent zones).
- The overall efficiency increases with $P$ in case of phase-correcting zones.
- The antenna system is scalable.
- Antennae with phase-correcting zones have $\eta - F$ curves which contain twice as many optima as those of antennae with absorbing/transparent zones (considering the same $F$-region).
- The optima of the $\eta - F$ curve can quickly be determined according to the formulae (4.3) and (4.4).
- The value of $\eta_p$ at which a Fresnel circle coincides with the antenna edge can be calculated according to formula (3.11).
- The additional gain is in general much greater in case of phase-correcting zones.
- For optimizing the sidelobe envelope it is desirable to keep $F$ as low as possible and it is advisable to use antennae with phase-correcting zones.
- Increasing $P$ by more than four produces very little extra gain.

On the base of these properties it is possible to develop a procedure which applies to antennae with absorbing/transparent zones and phase-correcting zones. This results in a flow chart which is shown in figure 5.1.
Chapter 5. A design procedure

Figure 5.1: A design procedure for optimizing a Fresnel antenna system. The arrows in the upper part of the flow chart indicate how variations of $F$ and $P$ influence the additional gain and sidelobe levels.
The terms $f$, $D$, $F$, $n$, sidelobe levels, $G_{aid}$ and $G_0$ cannot be chosen arbitrarily because these values are restricted by all kind of practical rules, as for instance the legally available frequency band. That is why this chart also includes the most common restrictions of the values of all the variables and constants used, indicated with the letters A ... H:

A. Frequency limitations. The operating frequency is determined by the application and can therefore not be chosen arbitrarily.

B. Geometrical configuration of usable space. The diameter of the antenna has to be chosen in agreement with its applications, for instance an indoor satellite receiver. In that case the dimensions of the window are the restriction, because the lens has to be attached to the window or put on a tripod behind the window.

C. Link budget. The link budget [1] prescribes the required gain of the receiving antenna.

D. Scanning angle. This is the angle at which a satellite signal is received. It determines the required level of the sidelobes of the gain pattern in order to suppress as good as possible unwanted signals from other satellites, stationed at different scanning angles (the scan process will be considered in more detail in Chapter 7).

E. Geometrical configuration of usable space. $F$ cannot be made too large because of the restriction given by the available space (see also item B).

F. Small Fresnel zones. If $F$ decreases, it is possible that a number of zones become too small for the calculation methods used which means that the calculations become less reliable. This aspect will be investigated in Chapter 9.

G. Manageability of the feed. If $n$ becomes large, the aperture of the feed becomes large which can cause problems while constructing and/or mounting the antenna.

H. Used frequency. The aperture of the feed cannot become smaller than that of a rectangular or circular waveguide which dimensions are determined by the frequency.

With the previous design procedure and properties it is possible to design an optimum system. The only disadvantage is the fact that $D$ and $f$ must be given in advance. Yet it is possible to determine $D$ and $f$ of a system with as constant factors $F$ and $n$, at which $\eta$ is maximum.

We can apply the design procedure a number of times, each time varying $D$ and $f$ within the specified boundaries and choose out of these values the most suitable solution. This can be time-consuming and not practical, but gives a good insight into $\eta$ as function of $D$ and $f$. 
Chapter 6

Comparing Fresnel and parabolic reflector antenna systems, concerning efficiencies and sidelobe envelope

6.1 Introduction

Having derived the efficiencies and sidelobe envelope of a Fresnel antenna system in Chapter 4, it is interesting to compare them with those of a parabolic reflector antenna system and consider similarities and differences between them. For this purpose the efficiencies of the parabolic reflector antenna are given in Appendix C together with the formulae for $G_{aid}$ and $G_{edge}$. The vectorial far-field equations required for the calculation of these efficiencies are derived by Leyten [32] and can be found in Appendix A. They are also used to calculate the sidelobe envelope of the gain patterns of the parabolic reflector antenna. It should be noted that the derivations of the far-field formulae for the parabolic reflector antenna are identical to those of the Fresnel antenna. This makes a fair comparison between the two antenna systems possible.

In the next sections the efficiency factors and curves of the parabolic reflector antenna system are introduced. Further, the sidelobe envelope is determined and the differences between both systems are evaluated.

6.2 Efficiencies and sidelobe envelope

A parabolic reflector antenna is usually characterized by four efficiency factors:

1. Spillover efficiency $\eta_s$.
2. Illumination efficiency $\eta_i$.
3. Phase efficiency $\eta_p$.
4. Overall efficiency $\eta$ ($= \eta_s \cdot \eta_i \cdot \eta_p$).
6.2. **Efficiencies and sidelobe envelope**

The definitions of these efficiencies are the same as those given in Chapter 3, just as the definitions of the additional gain $G_{\text{aid}}$ and edge illumination $G_{\text{edge}}$. Yet, in this case the phase efficiency has a different cause, namely surface errors and is equal to [12]:

$$\eta_p = e^{-\left(\frac{4\pi}{\lambda}\right)^2} \quad (6.1)$$

where $\kappa$ [m] is the r.m.s. reflector surface tolerance. Equation 6.1 clearly shows that $\eta_p$ only depends on the wavelength $\lambda$. Due to the fact that the frequency is a constant parameter during the optimization of $\eta$, the phase efficiency $\eta_p$ is also constant yielding that it has no influence on the optimization process. Therefore, $\eta_p$ will not further be considered in this chapter and is not included in the calculation of $\eta$.

The efficiency curves are calculated in the same way as those of the Fresnel antenna system: for each $F$-value an $n$-value is determined at which $\eta$ is maximum. These curves are depicted in the figures 6.1 and 6.2. The corresponding $G_0$, $G_{\text{feed}}$, $G_{\text{aid}}$ and $G_{\text{edge}}$ are shown in figures 6.3 and 6.4. In figure 6.2 at some $F$-values the matching $n$-values are placed. These $F$-values are not arbitrarily chosen, but are the same as those of table 4.1 (absorbing/transparent zones). In this way the efficiencies of both antennae can easily be compared. The values of $\eta$ ($\sim 0.82$) and the edge illumination ($\sim -10$ dB) at the low $F$-values ($F \approx 0.5$ m) are in good agreement with the values derived in [12].

![Figure 6.1: Efficiency curves of a parabolic reflector antenna.](image-url)
Figure 6.2: Overall efficiency with some corresponding $n$-values of a parabolic reflector antenna.

Figure 6.3: The additional gain, the gain of the feed and the antenna system gain of a parabolic reflector antenna.
The sidelobe envelope needs not to be calculated because simulations show that the radiation patterns of a parabolic reflector antenna at arbitrary \( F \)-values are almost identical. So the sidelobe envelope does almost not change for different \( F \)-values, as can be seen in figure 6.5. In this figure the gain patterns of antenna systems with different \( F \)-values are shown. Also, in the figures 6.6 and 6.7 the gain patterns of different Fresnel antenna systems are displayed. In that way we can compare the sidelobe envelope of both antenna systems.

Figure 6.4: Edge illumination of a parabolic reflector antenna.

Figure 6.5: Gain patterns at two \( F \)-values of a parabolic reflector antenna (\( \phi = 0^\circ \)).
Figure 6.6: Gain patterns of optimized Fresnel antenna systems ($F \approx 0.5$ m, $\phi = 0^\circ$). See for the specifications of the systems the table of section 4.3.

Figure 6.7: Gain patterns of optimized Fresnel antenna systems ($F \approx 1.8$ m, $\phi = 0^\circ$). See for the specifications of the systems the table of section 4.3.
6.3 Efficiency evaluation

According to the behaviour of the efficiencies it appears that it is much simpler to design a parabolic reflector antenna system than of a Fresnel antenna system, because for each arbitrary $F$ a parabolic antenna system can be optimized due to the lack of local maxima of the $\eta - F$ curve. Also the overall efficiency and additional gain are much larger than those of a Fresnel antenna with absorbing/transparent zones and they are always larger than those of a Fresnel antenna with phase-correcting zones (the difference depends on $P$: the larger $P$, the smaller the difference). One disadvantage of the parabolic antenna is the manufacturing cost, which is higher than that of a Fresnel antenna with absorbing/transparent zones and probably also of an antenna with phase-correcting zones.

Another disadvantage is the blockage of the feed. Namely, in situations where $F$ becomes relative large, the dimensions of the feed can become large in comparison to the dimensions of the parabolic reflector, which results in a loss of gain. This does not happen in the case of the Fresnel antenna because this antenna does not suffer from feed blockage.

Concluding we can say that an elaborate design procedure as in the case of the Fresnel antenna is absolutely not necessary for a parabolic reflector antenna system and such system is, concerning optimal efficiency and additional gain, always superior to that of the Fresnel antenna. To illustrate this the $\eta - F$ curves of both systems are depicted (for different $P$-values) in figure 6.8.

![Graph showing overall efficiency $\eta$ of Fresnel antenna systems and a parabolic reflector antenna system.](image)

Figure 6.8: Overall efficiency $\eta$ of Fresnel antenna systems and a parabolic reflector antenna system.
Chapter 7

Scan performances of the Fresnel antenna and the parabolic reflector antenna

7.1 Introduction

In this chapter we focus on the scan performances of Fresnel and parabolic reflector antenna systems. The goal of the analysis of the scan performances of both systems is to compare them in order to obtain a first impression of differences and similarities between them. With a first impression is meant that the comparison has a global character. This influences the choice of the calculation methods used to analyze the performances. Namely, comparing both antenna systems necessitates the gain patterns of the defocused Fresnel and parabolic reflector antennae which are derived from the far-field equations. These equations can be vectorial or scalar. Due to the global character a scalar analysis satisfies for this moment and therefore scalar far-field formulae are chosen here.

Figure 7.1 shows the general geometrical configuration required for the analysis of the scan performances. Scanning implies in our case: receiving different satellite signals by displacing the feed instead of the lens of the antenna. The feed is not displaced arbitrarily but according to a prescribed pattern called the scan surface which describes usually those positions of the feed at which the gain of the antenna for certain scan angles is as large as possible. We assume that the feed points always to the top of the paraboloid or to the center of the Fresnel antenna during defocusing.

The most important scan variables and characteristics are:

- $F'$: the distance [m] between the top of the paraboloid or the center of the Fresnel antenna and the displaced feed.

- $\vec{S}'$: the vector of Pointing [W/m²] in case of defocused antenna systems.
7.1. Introduction

- \( \varepsilon \): the distance [m] between the focal point and the displaced feed. The direction of the displacement is denoted by \( \varepsilon \).

- \( \theta_{\text{scan}} \): the angle between the symmetry axis and the line drawn from the center of the antenna to the satellite. This is also called the scan angle [rad].

- \( \sigma \): the angle between the symmetry axis and the line drawn from the center of the antenna to the displaced feed. This angle is also called the feed displacement angle [rad].

- Scan loss: this is the gain \( G_0 \) as function of the feed displacement angle.

- Beam Deviation Factor (BDF): this factor is defined as the ratio of the scan angle and the feed displacement angle \( \sigma \).

In the next sections scan surfaces are determined for both systems, first. Further the required scalar far-field formulae and gain functions for the defocused Fresnel and parabolic reflector antenna are derived. Subsequently it will be investigated whether the gain functions can be simplified to reduce computing time and finally the results of simulations (scan loss, BDF and sidelobe envelope of both antenna systems) will be discussed.

Figure 7.1: Geometrical configuration of a defocused antenna system used for scan purposes (the scan surface depicted is arbitrarily chosen).
7.2 Scan surfaces

In our specific case the scan surfaces reduce to scan curves because it is assumed that the feed is only displaced in a plane perpendicular to the antenna aperture. The chosen plane is the $xz$-plane which is the most convenient one for the calculations, because it implies that the variable $\phi$ of the far-field equations, derived in the next section, can be set to zero (this simplifies the equations slightly).

The scan curve of the Fresnel antenna was determined by Jeronimus [20] and appears to be a circle, the center of which coincides with the center of the antenna geometry. The radius is approximately $F$ and the circle curves toward the antenna surface as is displayed in figure 7.2. Thus, the scan curve is described by the equation:

$$x^2 + z^2 = F^2 \quad (7.1)$$

For the parabolic reflector antenna the Petzval curve [29] is chosen. This scan curve, originating from optics, is a paraboloid of which the radius of the curvature is one half of the curvature of the parabolic antenna and curves away from the antenna surface (see figure 7.2):

$$-z + F = \frac{x^2}{2F} \quad (7.2)$$

The Petzval curve is a well-known scan curve used for many scan problems when considering parabolic reflector antenna systems.

![Figure 7.2: Scan curves of the Fresnel antenna and the parabolic reflector antenna.](image)

It is possible that the two scan curves chosen are not the ideal ones because the place of the feed probably depends slightly on the parameter $n$. This was already indicated by
7.3. Gain functions of the defocused antenna systems

Rusch et al. [50] in case of the parabolic reflector antenna. For the Fresnel antenna this has not yet been examined for so far known, but it is most likely that the scan curve of this antenna shows also a small $n$-dependence. However, because of the global character of the analysis of the scan performances, as mentioned in the previous section, the two chosen scan curves suffice and will therefore be used for the calculation of the gain functions.

7.3 Gain functions of the defocused antenna systems

Determining the gain patterns requires the scalar far-field equations. For both antenna systems these equations must be derived in the same way because we strive to a comparison of the two systems which is as fair as possible. Different calculation methods can lead to discrepancies between the gain functions of the Fresnel antenna and parabolic reflector antenna.

To continue we note that far-field formulae have already been derived for both antenna systems. Ruze [51] derived the far-field equations for the defocused parabolic reflector antenna system and Jeronimus [20] for Fresnel antenna systems with absorbing/transparent zones. Yet, these formulae are restricted to applications where the displacement $\epsilon$ is small compared to $F$ ($\epsilon/F < 1$). In our case the displacement $\epsilon$ of the feed will not only be chosen small but also large compared to $F$, which means that these formulae cannot be used.

Fortunately Jeronimus has also derived far-field equations for the defocused Fresnel antenna with absorbing/transparent zones for feed displacements which are not restricted by this condition. These equations are adapted for phase-correcting zones in the same way as the vectorial equivalents of Chapter 3 and they are used to derive the following gain function:

$$G(\hat{r}, n) = 10 \log \left[ \frac{n + 1}{2\lambda^2} \sum_{m=0}^{2^m b_m+1} \int_0^{2\pi} \int_0^\infty \left| E'_{\text{feed}}(\cos(\theta) + (\hat{n} \cdot \hat{S}')) e^{ik \cdot r' d\phi'} \right|^2 \right]$$  \hspace{1cm} (7.3)

with

$$h = j k \frac{-\sqrt{(F')^2 + (r')^2 - 2r'F' \sin(\sigma) \cos(\phi')} + r' \sin(\theta) \cos(\phi - \phi')} + j \frac{m^2 \pi}{P}$$

$$E'_{\text{feed}} = \sqrt{\left( \frac{(F' - r' \sin(\sigma) \cos(\phi'))^n}{((F')^2 + (r')^2 - 2r'F' \sin(\sigma) \cos(\phi'))^{\frac{3}{2}} + 1 \right)}$$

$$(\hat{n} \cdot \hat{S}') = \frac{F' \cos(\sigma)}{\sqrt{(F')^2 + (r')^2 - 2r'F' \sin(\sigma) \cos(\phi')}}$$
For the parameter \( m \) applies:

\[
\text{absorbing/transparent zones } (P = 2): \quad m = 0, 2, 4, \ldots, M_t \tag{7.4}
\]

\[
\text{phase-correcting zones } (P \geq 2): \quad m = 0, 1, 2, \ldots, M_p
\]

The reference frame used to describe the aperture of the antenna is denoted by the cylindrical coordinates \( r' \) and \( \phi' \) shown in figure 7.3.

![Figure 7.3: Geometrical configuration of the defocused Fresnel antenna.](image)

For a defocused parabolic reflector antenna no far-field equations which can be applied to the reflector geometry under consideration exist. Therefore, in Appendix B these formulae are derived. They are based on the Kirchhoff scalar diffraction integral just as in case of the Fresnel antenna. To make the comparison as fair as possible even the aperture which is integrated over is the same for both antennae. The gain function resulting from the derivation in Appendix B is:

\[
G(\tilde{\mathbf{r}}, n) = 10 \log \left[ \frac{n + 1}{2\lambda^2} \int_0^{2\pi} D/2 \int_0^\pi E'_\text{feed} \cdot \left[ \left( \hat{\mathbf{n}} \cdot \tilde{\mathbf{r}} \right) + \left( \hat{\mathbf{n}} \cdot \hat{\mathbf{S}}' \right) \right] \cdot \sqrt{1 + \left( \frac{1}{2F} \right)^2} \cdot e^{i\lambda d\text{id}\phi'} \right]^2 \tag{7.5}
\]

with

\[
\epsilon = \sqrt{\epsilon_x^2 + \epsilon_y^2}
\]

\[
\rho = F + \frac{r^2}{4F}
\]
7.4 Simplification of the gain functions

The gain functions derived in the previous section consist of double integral equations which cannot be expressed in a closed form. Therefore, these integrals have to be numerically evaluated.

The geometrical configuration is displayed in figure 7.4 and the antenna aperture (pseudo aperture, see Appendix B) is defined by the cylindrical coordinates \( l \) and \( \phi' \).

![Figure 7.4: Geometrical configuration of the defocused parabolic reflector antenna.](image)

Equations (7.3) and (7.5) constitute the basis for all further calculations concerning the scan performances.

7.4 Simplification of the gain functions

The gain functions derived in the previous section consist of double integral equations which cannot be expressed in a closed form. Therefore, these integrals have to be numerically evaluated.
approximated. The integrand of this type of equations has, considering both integration ranges, an 'oscillating' character of which the oscillations increase with the place of the sidelobe calculated: the more the sidelobe deviates from boresight, the more rapidly the integrand oscillates.

Numerical integration routines have great difficulty with this oscillating character, meaning that the numerical approximation of the integral is not always successful or often requires much computing time (which also increases with the oscillation rate). Thus, calculating sidelobes far away from the main lobe requires much computing time. Because we need a lot of gain patterns it is worthwhile to try to eliminate at least one integral equation.

Due to the complexity of the gain functions two approximate ways are considered in order to solve this problem:

1. We assume that during defocusing the aperture illumination (the modulus of the integrand) remains the same as in the focused situation and we approximate the phase term (the argument of the integrand) with the aid of a finite number of terms of a series expansion. One of the two integrals could then result into a standard integral for which the solution is known. This method is used by Ruze [51] and Jeronimus [20], leading to Bessel functions of the first kind which replace the integrand as a function of \( \phi' \).

2. We use an approximation method which profits from the oscillating character of the integrand. Such a method is Asymptotic Physical Optics (APO), for example.

Both options will now be considered.

The first option is worked out in Appendix E and appears not to be the right choice. The phase term is approximated with the first three terms of the series expansion of \( \sqrt{1 - z^2} \). Three terms are used because it appears from simulations, done by Jeronimus [20] and the author, that the first two terms do not suffice for 'large' displacements of the feed, and a third term is required. Further, the integrand as a function of \( \phi' \) is rearranged and solved with the aid of a series expansion of the function \( e^{i\phi} \). This results in an infinite double series which converges quickly for certain values of the two integration variables and \( \theta \). Computer simulations show that these specific values represent only a small fraction of the points of the gain pattern, yielding that the method is not suitable for this type of problem.

The second method is based on the following principle: if \( q \) is a large positive constant, \( h(\theta, o) \) a rapidly varying real function and \( g(\theta, o) \) a relatively slow varying function with regard to \( \theta \) then the integral equation (7.6) can be approximated by the sum of the contributions of the stationary points for which applies that \( \partial h(\theta, o)/\partial \theta = 0 \) with \( \theta = T_i \) \((a \leq T_i \leq b)\):

\[
\int_a^b g(\theta, o)e^{i\phi h(\theta, o)}d\theta
\]  

(7.6)
7.4. Simplification of the gain functions

\[ f(q) \simeq \sum_{i} \sqrt{\frac{2\pi}{k|h''(T_i, o)|}} g(T_i, o) e^{j k h(T_i, o) + j \text{sign}[u] h''(T_i, o) \cdot \frac{\xi}{\pi}} \]  

(7.7)

with

\[ h''(\vartheta, o) = \frac{\partial^2 h(\vartheta, o)}{\partial \vartheta^2} \]  

(7.8)

\[ \text{sign}[u] = \begin{cases} 1 & u \geq 0 \\ -1 & u < 0 \end{cases} \]  

(7.9)

The integral as function of \( \varphi' \) of the gain function of both antennae meets the previously mentioned demands with \( q \) being the wave number \( k \) and \( \varphi' = \vartheta \). This implies that APO can be used. Yet, we must note that APO fails when determining the gain pattern in the vicinity of the main lobe because the integrand of equation (7.6) oscillates hardly in that region. This is a well known restriction of APO \[38\].

In order to apply APO to equations (7.3) and (7.5) first the functions \( g(\vartheta, o) \) and \( h(\vartheta, o) \) must be derived for the Fresnel antenna and parabolic reflector antenna.

In case of the Fresnel antenna \( \vartheta = \varphi' \) and \( o = r' \), so \( g(\vartheta, o) \) equals the function \( g(\varphi', r') \) which is deduced from formula (7.3):

\[ g(\varphi', r') = E'\text{foed} \cdot [\cos(\theta) + (\hat{n} \cdot \hat{S})] \cdot r' \]  

(7.10)

The phase function \( h(\varphi', r) \) of the Fresnel antenna is equal to:

\[ h(\varphi', r') = -\sqrt{(F')^2 + (r')^2 - 2r' F' \sin(\sigma) \cos(\varphi') + r' \sin(\vartheta) \cos(\varphi')} + \frac{j m 2\pi}{P} \]

\[ \leftrightarrow h(\varphi', r') = -N \sqrt{1 - \frac{U \cos(\varphi')}{U_1} + \frac{U \cos(\varphi') + U_1}{P}} \]  

(7.11)

with

\[ N = \sqrt{(F')^2 + (r')^2} \]  

(7.12)

\[ W = \frac{2r' F' \sin(\sigma)}{(F')^2 + (r')^2} \]  

(7.13)

\[ U = r' \sin(\vartheta) \]  

(7.14)

\[ U_1 = \frac{j m 2\pi}{P} \]  

(7.15)
Chapter 7. Scan performances of the Fresnel antenna ....

Note that $\phi = 0^\circ$ applies for the previous equations and those to come. This was assumed in section 7.2 because the feed is displaced in a plane normal to the antenna aperture. The illumination and phase function of the parabolic reflector antenna are determined in a similar way. For this antenna system $\vartheta = \phi'$ and $s = l$ apply. The function $g(\phi', l)$ is according to formula (7.5) denoted by:

$$g(\phi', l) = E'_{\text{feed}} \cdot [(\hat{n} \cdot \hat{\delta}') + (\hat{n} \cdot \hat{\delta})] \cdot \sqrt{1 + \frac{l}{2F} \cdot l} \quad (7.16)$$

and $h(\phi', l)$ is:

$$h(\phi', l) = -\rho \sqrt{1 + \frac{2\epsilon_x \cos(\psi)}{\rho} + \left(\frac{\epsilon}{\rho}\right)^2} \sqrt{1 - \frac{2\epsilon_x \sin(\psi)}{N'\rho} \cos(\phi') + l \sin(\theta) \cos(\phi') + \frac{r^2 \cos(\theta)}{4F}} \quad (7.17)$$

with

$$N' = 1 + \frac{2\epsilon_x \cos(\psi)}{\rho} + \left(\frac{\epsilon}{\rho}\right)^2 \quad (7.18)$$

$$N = -\rho \sqrt{N'} \quad (7.19)$$

$$W = \frac{2\epsilon_x \sin(\psi)}{N'\rho} \quad (7.20)$$

$$U = l \sin(\theta) \quad (7.21)$$

$$U_1 = \frac{r^2 \cos(\theta)}{4F} \quad (7.22)$$

After determining the functions $h(\vartheta, s)$ and $g(\vartheta, s)$ we can calculate the first and second derivative of the phase function. Because $h(\phi', r')$ and $h(\phi', l)$ are of exactly the same form, only the derivatives of $h(\phi', l)$ are calculated:

$$h'(\phi', l) = -U \sin(\phi') - \frac{NW \sin(\phi')}{2\sqrt{1 - W \cos(\phi')}} \quad (7.23)$$

$$h''(\phi', l) = -U \cos(\phi') - \frac{NW \cos(\phi')}{2\sqrt{1 - W \cos(\phi')}} + \frac{NW^2 \sin^2(\phi')}{4(1 - W \cos(\phi'))^{3/2}} \quad (7.24)$$
7.4. Simplification of the gain functions

According to the previous equations the zeros of the first derivative are:

\[ h'(\phi', l) = 0 \Rightarrow -\sin(\phi') \left( U + \frac{NW}{2\sqrt{1 - W \cos(\phi')}} \right) = 0 \]

\[ \Rightarrow \phi' = 0, \pi \quad \forall \quad \cos(\phi', t) = \frac{1}{W} \left( 1 - \frac{N^2W^2}{4U^2} \right) \quad (7.25) \]

Four values appear to exist for which the first derivative is zero. The first two, 0 and \( \pi \), apply also in case of the focused antenna. The second two are the result of the defocusing and seem to have no influence on the results, which is shown by computer simulations and also by Rusch [49]. Rusch has proved for a similar situation that the two zeros only contribute to the final result for a very small area of \( \theta \). Furthermore, computer simulations show that this area is located in the vicinity of the main lobe. Due to the fact that this is just the region where APO fails, the second two points are neglected in further calculations.

A special case occurs when \( W = 0 \) and \( l \neq 0 \). These conditions indicate that the antenna is only defocused in the axial direction, which does not happen in our case because of the scan curves chosen. If it would occur in future applications then, in case of the Fresnel antenna, the integral over \( \phi' \) changes into a Bessel function \( J_0 \) because \( g(\phi', r') \) is no longer a function of \( \phi' \). In case of the parabolic reflector antenna, \( g(\phi', l) \) is still a function of \( \phi' \), but nevertheless the integral over \( \phi' \) can then be transformed into two Bessel functions \( J_0 \) and \( J_1 \) according to the relation:

\[ \int_0^\pi \cos(\alpha \alpha) e^{i\theta \cos(\alpha)} d\alpha = j_\alpha \pi J_\alpha (b) \quad (7.26) \]

Thus APO makes it possible to approximate the gain functions of both antenna systems, with \( T_1 = 0 \) and \( T_2 = \pi \), by

Fresnel antenna: \( G(\hat{r}, n) \approx \)

\[ 10 \log \left[ \frac{n + 1}{2\lambda^2} \sum_{m}^{b_{m+1}} \sum_{i=1}^{2} \left\| \frac{2\pi}{k|h''(T_i, r')} \right\| g(T_i, r') e^{j k h(T_i, r')} + j \text{sign}[h''(T_i, r')] \hat{r} d r' \right\|^2 \]  

(7.27)

Parabolic reflector antenna: \( G(\hat{r}, n) \approx \)

\[ 10 \log \left[ \frac{n + 1}{2\lambda^2} \int_0^{D/2} \sum_{i=1}^{2} \left\| \frac{2\pi}{k|h''(T_i, l)|} \right\| g(T_i, l) e^{j k h(T_i, l)} + j \text{sign}[h''(T_i, l)] \hat{l} d l \right\|^2 \]  

(7.28)
The only remaining problem is that equations (7.27) and (7.28) cannot be used in the vicinity of the main lobe. An aspect which also complicates matters is that it is impossible to determine the exact boundaries of this region. Therefore, this problem is solved with the aid of a simple iterative algorithm which will be explained in the following.

The algorithm benefits of the fact that it is known that APO only fails in the vicinity of boresight. Therefore, we begin with the calculation of the gain pattern as a function of $\theta$ at the angle $\theta = \sigma$ (the feed displacement angle) which is approximately in the middle of the main lobe. The calculation starts with the numerical approximation of the double integral equation (formula (7.3) or (7.5)) and moves to the left of the $\theta$-scale. After a number of calculated gain values the difference between the value just calculated and the value calculated with APO at the same $\theta$ is considered. If the difference is smaller than a prescribed value (for example 0.1 dB) then the routine switches to APO (formula (7.27) or (7.28)). If not, then the routine continues with the double integral and repeats after a number of calculated gain values the same procedure. An identical routine can be applied to the remaining right-hand side of the gain pattern. Figure 7.5 shows a gain pattern calculated with the previous algorithm and the same pattern calculated with only APO. The differences are clearly visible, especially the failure of APO in the vicinity of boresight.

Figure 7.5: Two gain patterns: one calculated using only APO (solid line) and the other using a combination of APO and a double integral (broken line).

Yet, before the scan loss and other characteristics can be calculated, the validity of the program using the algorithm has to be checked. For these tests we use four different programs which all calculate gain patterns:

1. Program of [32] to check the focused gain patterns of the Fresnel antenna with absorbing/transparent zones and the parabolic reflector antenna.
7.5. Numerical results

2. Program of [20] to check the lateral defocused gain patterns of the Fresnel antenna with absorbing/transparent zones (small displacements).

3. Program of [31] to check lateral defocused gain patterns of the parabolic reflector antenna (small displacements).

4. Program of [14] to check the axial defocused gain patterns of the parabolic reflector antenna (small displacements).

All these gain patterns agree very well with the patterns calculated according to the algorithm. Also the amount of computing time has substantially been reduced. So the algorithm seems to be reliable and practical which means that we can use it to calculate the gain patterns.

7.5 Numerical results

7.5.1 Antenna systems used

For the simulations, systems are used with optimized efficiencies as described in Chapter 4 and 6. In Table 7.1 the different antenna systems are summarized together with their optimized values of $F$ and $n$, and the systems of this table with almost the same $F$ will be compared concerning their scan performances.

<table>
<thead>
<tr>
<th>Antenna System</th>
<th>$n$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fresnel antenna (absorbing/transparent zones)</td>
<td>82.0</td>
<td>1.87 m</td>
</tr>
<tr>
<td>Fresnel antenna (phase-correcting zones $P = 2$)</td>
<td>75.3</td>
<td>1.93 m</td>
</tr>
<tr>
<td>Parabolic reflector antenna</td>
<td>69.60</td>
<td>1.87 m</td>
</tr>
</tbody>
</table>

7.5.2 Scan performances

In the figures 7.6 and 7.7 the scan losses of the six systems are displayed. The systems with $F \simeq 0.5$ m have the greatest scan loss and the decrease of the gains of the different systems is almost identical. Among the systems with $F \simeq 1.9$ m, the parabolic reflector antenna shows the greatest decrease in gain and the system with the Fresnel antenna with phase-correcting zones seems to have the least trouble with the feed displacement. The BDF can be found in Appendix F. For the Fresnel antenna system with the small $F$ BDF $\sim 0.84$. 
and for the systems with the relative large $F$ BDF ~ 0.98. The parabolic reflector antenna systems have BDF's of ~0.92 and ~0.99, respectively.

**Figure 7.6**: Scan losses of the systems with $F$ approximately 0.5 m.

**Figure 7.7**: Scan losses of the systems with $F$ approximately 1.9 m.
7.5. Numerical results

7.5.3 Sidelobe envelope

In Chapter 4 the importance of the sidelobe envelope was at issue. In the case of scanning, this envelope is also of importance, because it can cause interference due to other satellites. Therefore, in the figures 7.8 until 7.13 the gain functions of all six systems are plotted at feed displacement angles 0° (plot F1), 6° (plot F2), 12° (plot F3) and 18° (plot F4). With the aid of these figures it is possible to determine the influence of unwanted signals of satellites stationed at these positions on the wanted signal.

Note that the gain functions of a single Fresnel antenna system contain sidelobes which levels can become relatively high for certain $\sigma$. Also, the behaviour of these patterns is more unpredictable than that of the gain patterns of parabolic reflector antennae. This aspect could complicate the use of a Fresnel antenna for scan purposes.

Considering different Fresnel antenna systems reveals that the behaviour is more predictable. They all show the same trend.
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Figure 7.8: Gain functions of a Fresnel antenna system with $F$ approximately 0.5 m, at different $\sigma$'s.

Figure 7.9: Gain functions of a Fresnel antenna system with $F$ approximately 0.5 m, at different $\sigma$'s.
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Figure 7.10: Gain functions of a Fresnel antenna system with $F$ approximately 1.9 m, at different $\sigma$'s.

Figure 7.11: Gain functions of a Fresnel antenna system with $F$ approximately 1.9 m, at different $\sigma$'s.
Chapter 7. Scan performances of the Fresnel antenna....

Figure 7.12: Gain functions of a parabolic reflector antenna system with $F$ approximately 0.5 m, at different $\sigma$'s.

Figure 7.13: Gain functions of a parabolic reflector antenna system with $F$ approximately 1.8 m, at different $\sigma$'s.
Chapter 8

Absorbing material

8.1 Introduction

This chapter and the next one will be dedicated to a closer investigation of some problems arising from the special geometrical configuration of the Fresnel antenna and the material used for the construction of this type of antenna. Namely, due to the geometrical configuration, the Fresnel zones become smaller with increasing radius \( (b_m) \). It is doubtful whether the Kirchhoff diffraction integral may be applied to these zones. This problem will extensively be treated in the next chapter. Now the material used for the construction of the absorbing zones, the absorbing material, will be considered.

Absorbing material is mathematically or physically very difficult to describe, because such material has to absorb electromagnetic waves incident to it, regardless of their polarization, amplitude, angle of incidence and frequency \(^1\). In our case the material has to be infinitely thin, which is physically impossible because infinitely thin implies that the material cannot absorb any energy. In spite of all this, still some models of absorbing material exist. These models will be discussed in the next sections. It is not the aim to derive equations which describe the model considered, but to give a common description of the theory behind the model. For this purpose only diffraction at the edge of an absorbing screen, also called a black screen (a term originating from optics), will be considered.

8.2 Voigt’s screen

When a scalar electromagnetic wave is incident to the edge of a perfectly conducting screen, two other waves arise: a reflected and a diffracted wave. Sommerfeld derived an equation which describes these waves (for the two-dimensional case) \([7]\). For this purpose he used the so called Riemann surfaces which enabled him to create special functions with period \(4\pi, 6\pi\) etc. For a detailed description of this, one is referred to \([3]\) or \([30]\). Now Voigt suggested that, if the screen is black, the term of the equation of Sommerfeld describing

\(^1\)This is the author's 'idealized' definition of absorber. In the literature there exists as far as known no general accepted definition of absorber.
Chapter 8. Absorbing material

the reflected wave has to be omitted. There is no theoretical reason for this but when
applied, it gives results which resemble those of Kirchhoff's/Kottler's black screen (this
type of screen will be treated in the next section).

However, it appears that Voigt's screen is not 'black' enough. The screen can be made
'more black' by increasing the number of Riemann surfaces but even the 'blackest' screen of
Voigt cannot compete with the one described in the next section. Another serious problem
is that the electric and magnetic vectors do not satisfy any prescribed boundary condition
[3]. All this does not make it attractive to use Voigt's screen as a model for a black screen.

8.3 Kirchhoff's/Kottler's screen

According to Kirchhoff the effect on a scalar wave $u_0$ incident to a screen $S$ containing
an aperture $A$ is specified by the wave function $u$, satisfying the well-known boundary
conditions:

\[
\begin{align*}
  u &= u_0 & \frac{\partial u}{\partial n} &= \frac{\partial u_0}{\partial n} & \text{on } S_+ \text{ (illuminated side of } S) \quad (8.1) \\
  u &= 0 & \frac{\partial u}{\partial n} &= 0 & \text{on } S_- \text{ (shadow side of } S) \quad (8.2)
\end{align*}
\]

These conditions imply already that the screen used has to be black, because there are
no reflected rays and the incident waves on the screen vanish. Kottler defined blackness in
the following way: a screen is black if $u$ and $\frac{\partial u}{\partial n}$ are discontinuous across $S$. He showed
that the Kirchhoff solution of diffraction by a black screen is not the solution of a boundary
value problem but a rigorous solution of a saltus problem in which the Latin word saltus
signifies a discontinuity or a jump [56]. A restriction added by Kottler to this solution is
that the screen has to be infinitely thin [3].

All this leads to the conclusion that, if we use Kirchhoff's approach to calculate the
diffracted field, the material around the aperture has to be absorbing. This aspect is
often forgotten! A disadvantage of Kirchhoff's approach is that it is scalar. Because we
are interested in the vectorial solution we have to consider the Larmor-Tedone formulæ
modified by Kottler:

\[
\begin{align*}
  \vec{E}^d &= -\nabla \Phi + j\omega \mu_0 \sqrt{\varepsilon_0} \vec{A}_1 + \nabla \times \sqrt{\varepsilon_0} \vec{A}_2 & (8.3) \\
  \vec{H}^d &= -\nabla \Psi - j\omega \varepsilon_0 \sqrt{\mu_0} \vec{A}_2 + \nabla \times \varepsilon_0 \vec{A}_1 & (8.4)
\end{align*}
\]

\[
\begin{align*}
  \Phi &= \frac{1}{4\pi} \int \int_A (\hat{n} \cdot \vec{E}) \cdot GdA + \frac{1}{4\pi j\omega \varepsilon_0} \oint (\vec{H} \cdot \hat{T}) \cdot Gd\tau & (8.5) \\
  \vec{A}_1 &= \frac{1}{4\pi \sqrt{\varepsilon_0}} \int \int_A (\hat{n} \times \vec{H}) \cdot GdA & (8.6)
\end{align*}
\]
8.3. Kirchhoff's/Kottler's screen

\[ \psi = \frac{1}{4\pi} \int \int_A (\hat{n} \cdot \vec{H}^i) \cdot G dA = \frac{1}{4\pi j \omega \mu_0} \int \int_{\Gamma} (\vec{E}^i \cdot \hat{T}) \cdot G d\tau \]  \hspace{1cm} (8.7)

\[ \vec{A}_2 = \frac{1}{4\pi \sqrt{\mu_0}} \int \int_A (\hat{n} \times \vec{E}^i) \cdot G dA \]  \hspace{1cm} (8.8)

with \( G = \frac{e^{jkR}}{R} \) \hspace{1cm} (8.9)

Figure 8.1: Geometrical configuration belonging to the modified Larmor-Tedone equations.

The superscripts \( i \) and \( d \) denote the incident and diffracted field. The Larmor-Tedone equations describe the field inside a volume bounded by a closed surface. They are in principle the required analytical expressions of the Huygens' principle in an electromagnetic field. If the surface \( S \) contains an aperture \( A \) like in our case (see figure 8.1), then the Larmor-Tedone equations fail because the secondary waves emitted by each finite surface element do not satisfy the Maxwell equations [3] in this case. This was recognized by Kottler. He introduced the contour integrals of equations (8.5) and (8.7) \(^2\) and proved that equations (8.3) and (8.4) are the solution of diffraction of electromagnetic waves by a black screen. Looking closely at them it appears that they are familiar formulae. According to Bouwkamp [7] these equations can namely be written as:

\[ \vec{E}^d = \nabla \times \frac{1}{4\pi} \int \int_A (\hat{n} \times \vec{E}^i) \cdot G^* dA + \nabla \times \nabla \times \frac{1}{4\pi j \omega \epsilon_0} \int \int_A (\hat{n} \times \vec{H}^i) \cdot G^* dA \]  \hspace{1cm} (8.10)

\[ \vec{H}^d = \nabla \times \frac{1}{4\pi} \int \int_A (\hat{n} \times \vec{H}^i) \cdot G^* dA - \nabla \times \nabla \times \frac{1}{4\pi j \omega \mu_0} \int \int_A (\hat{n} \times \vec{E}^i) \cdot G^* dA \]  \hspace{1cm} (8.11)

\(^2\)The original formulae of (8.3) and (8.4), given by Copson and Baker [3], were expressed in quantities of the Gaussian system of the Maxwell equations which is more suitable for microscopic problems involving the electrodynamics of individual charged particles. The terms of the original formulae are transformed to those of the more often used rationalized M.K.S.A. system.
which represent the Lorentz-Larmor theorem \( G^* = e^{-i\omega R} \). Formulae (8.10) and (8.11) correspond also with those given by Silver [53]. Thus applying Lorentz-Larmor implies automatically the use of absorbing material. If the material is perfectly conducting, then the equation for the electric field becomes simpler:

\[
\vec{E}^d = \nabla \times \frac{1}{2\pi} \int \int_A (\hat{n} \times \vec{E}^t) \cdot G^* \, dA \tag{8.12}
\]

This formula is derived amongst others by Jackson [18] and Johnson [22] and is used for the calculation of the far-field equations given in Chapters 3 and 7 and Appendix A. The basis for formula (8.12) is Kirchhoff’s theory and some additional information concerning the behaviour of the electric and magnetic component of the waves in interaction with the conducting screen. Namely, in the aperture the tangential magnetic field and the normal electric field are not disturbed by the presence of the screen [7].

In first instance it would have been better to use the Lorentz-Larmor equations for the far-field calculations of the radiation pattern of the Fresnel antenna, but the differences between the formulae (8.10) and (8.12) are presumably very small because in both cases we make the same wrong assumption; the unperturbed incident field is assumed to be the aperture field \( (\vec{E}^i, \vec{H}^i) \). This is not true because the absorbing and perfectly conducting screen respectively influence the aperture field, a problem which makes these aperture integration methods approximate solutions and not exact ones! Because of the presumed small differences between the formulae (8.10) and (8.12) we can therefore maintain the simple formula (8.12).

### 8.4 A non-ideal screen

This model represents a more practical approach. The absorbing material can be assumed less ideal, so that it can be replaced by a substance of which the attenuation and phase shift are known through measurements. During the aperture integration over the absorbing zones the field is then not zero but equal to the incident field multiplied by an attenuation and phase-shift factor. This can easily be implemented mathematically. An advantage of this model is that it is far more realistic than the other two. The main disadvantage is that for each antenna configuration the characteristics of the material have to be measured for each frequency. In most cases this requires high-precision measurement equipment and/or elaborate measurement methods, which are very expensive.

The model discussed here has been applied to a Fresnel antenna system with optimized \( \eta \). The absorbing zones are assumed to cause only a constant attenuation of -10 dB and no phase shift.
8.5. An evaluation

This results into figure 8.2 in which we can clearly see the difference between an antenna with absorbing zones and one with semi-absorbing zones.

![Graph showing gain patterns of antennae with absorbing/transparent and semi-absorbing/transparent zones](image)

Figure 8.2: Gain patterns of antennae with absorbing/transparent and semi-absorbing/transparent zones ($F = 1.87 \text{ m}, n = 82.0, \phi = 0^\circ$).

8.5 An evaluation

None of the three models above satisfy completely the demands of absorbing material. Kirchhoff's/Kottler's method provides the most accurate description in case of a infinitely thin black screen, but it has no sound theoretical basis. This does not imply that Voigt’s screen cannot be used. It is best if each black screen diffraction problem is discussed independently on its own merits. The real difficulty lies in the fact that absorbing material is an idealization which cannot be attained experimentally and which has no precise definition in electromagnetic theory [3].
Chapter 9

Small Fresnel zones

9.1 Introduction

We will now consider the influence on the radiation pattern of the Fresnel zones of which the dimension $d_m$ is in the order of a few wavelengths or even smaller. The parameter $d_m$ is called the width of a zone, and is defined as:

$$d_m = \frac{b_{m+1} - b_m}{2}$$  \hspace{1cm} (9.1)

To illustrate the relation between the width of a Fresnel zone in the vicinity of the edge and the focal distance $F$, in figure 9.1 the distance $b[M - 1] - b[M - 2]$ is depicted (in terms of $\lambda$) as a function of $F$. Also in figure 9.2 the front views of two antennae are shown with different focal lengths $F$.

It is clearly visible from both figures that not only the number of zones decreases with $F$ but also $d_m$ decreases with the radius $b_m$. This implies that it is questionable whether Kirchhoff’s diffraction theory may be applied or not, because Kirchhoff’s diffraction integral (equation (8.12)) works the best for $\lambda / d_m \ll 1$ assuming that the aperture field can be substituted by the incident field.

The goal of the analysis in this chapter is to investigate if it is possible to calculate the far-field patterns of a Fresnel antenna, with the aid of a technique or a combination of techniques which is also valid for the 'smaller' zones.

In the following sections, first the annulus (a single Fresnel zone) in absorbing material will be considered. Next, the absorbing material will be replaced by perfectly conducting material and far-field patterns of this new antenna will be calculated using different diffraction techniques.
9.1. Introduction

The distance \( (b[M - 1] - b[M - 2]) / \lambda \) at different \( F \)’s.

Figure 9.2: Front views of two Fresnel antennae designed for different \( F \)'s at a frequency of 11.1 GHz. The drawing is on a scale of 1:15.
9.2 Annulus in absorbing material

It is very difficult to predict the influence of absorbing material on the incident field, because no boundary conditions are known. Therefore, we do not know the interaction between the absorber and the incident field coming from the feed. In the previous chapter we already indicated that there exists no satisfying mathematical or physical model for absorbing material. That is the reason why is chosen for another configuration of the antenna system, namely an antenna with perfectly conducting zones instead of absorbing zones. This type of antenna functions in exactly the same way as the one with absorbing/transparent zones, with only one difference: the destructive parts of the incident spherical wave are not absorbed but reflected.

An advantage of this kind of antenna is that it can very easily be fabricated. From measurements performed by Huder et al. [17] it appears that an antenna system with metallic/transparent zones functions properly.

9.3 Fresnel antenna with metallic/transparent zones

9.3.1 Introduction

Now that the absorbing zones are substituted for metallic zones it is possible to examine a small annulus. From apertures in perfectly conducting screens is known that in the aperture the tangential magnetic field and the normal electric field are not disturbed by the screen and that the tangential electric field and normal magnetic field are zero on the screen [7, 18]. Also it is known how the field behaves in the neighbourhood of a sharp edge (edge condition of Meixner [36]).

Thus, it is not surprising that there exists extensive literature on diffraction problems involving slits and holes in perfectly conducting material. For example, a report of Bouwkamp [7], published in 1954, contains a list of over 500 references! Therefore, in the next sections first some important techniques will be examined and there will be investigated if they are suitable for our kind of problem. Secondly, two of those methods will be applied to the configuration of the Fresnel antenna with metallic/transparent zones in order to calculate the far-field patterns.

9.3.2 Slits and holes: a review

Usually diffraction problems can be divided into two categories: problems concerning apertures small in comparison with $\lambda$ and problems concerning apertures large in comparison with $\lambda$. Small and large appear to be very flexible terms in diffraction theory, which implies that it is sometimes very difficult (even impossible) to determine the exact region of validity of certain techniques in terms of $d_m$. This will be a point of interest. Another problem is that most techniques give only scalar solutions and we are interested in the vectorial situation (our incident wave is spherical and linear polarized).
Some calculation methods (on slits and holes) will be examined and properties of these solutions will be discussed (some more extensively than others). An important parameter which will be used during the review is \( k \cdot d \), where \( d \) is half the slit width or the radius of the hole and \( k \) the wave number of vacuum. The validity of calculation methods, considering diffraction problems, is usually given in terms of \( k \cdot d \).

Mathieu functions

Morse and Rubenstein [42] have solved the (two-dimensional) diffraction problem of a slit (and ribbon) excited by a scalar plane wave at arbitrary angle of incidence. They separated the wave equation in elliptic cylinder coordinates and computed the values of the resulting solutions using Mathieu functions. In principle these solutions are valid for each value of \( k \cdot d \), but the convergence becomes bad for probably \( k \cdot d > 10 \) \((d > 5\lambda/\pi)[7]\). Another problem is that Mathieu functions are difficult to implement.

In the context of the Mathieu functions it is worthwhile to mention Skavlem [54] who calculated the eigenvalues and eigenfunctions of the slit problem (for normal incidence) without mentioning the term 'Mathieu functions', though his solutions closely resemble these functions.

Spheroidal wave functions

What Mathieu functions are for slit problems, spheroidal wave functions are for hole problems. Meixner and Andrejewski [37] attained the exact solution for this problem (two-dimensional plane wave, linear polarized and at arbitrary angle of incidence) by separating the wave equation in oblate and prolate spherical coordinates, which lead to spherical wave functions. For this kind of solutions the same restrictions apply as in the previous case. They are even more difficult to implement.

Discontinue Weber-Schafheitlin integral

Nomura and Katsura are among the few who have solved the slit and hole problem [45, 44] for small \( k \cdot d \) values without using Mathieu or spheroidal wave functions. They used the special properties of the Weber-Schafheitlin integral (for the properties of this type of integral see Watson [58]) which employs Bessel functions of fractional order. The diffracted field is described in terms of Weber-Schafheitlin integrals with unknown coefficients which can be determined with the aid of hypergeometric polynomial expansion. An advantage of their method over the previous two is that the incident plane wave can have arbitrary polarization. Beside this, another advantage is that the resulting solution is relative easier to implement. Yet, a disadvantage is that convergence becomes bad for \( k \cdot d > 5 \).

The Weber-Schafheitlin integral is also used by Otsuki [46] who solved the slit problem (normal incidence) with the aid of Fourier-orthogonal functions transformation. His solution resembles closely that of Nomura and Katsura, and Otsuki gives solutions up to \( k \cdot d = 16 \) \((d = 8\lambda/\pi)\).
Chapter 9. Small Fresnel zones

Differential-integral equations method

The solution for the boundary-value problem of diffraction at a slit and a hole can be determined according to a system of differential-integral equations. A rigorous solution of this complicated system seems to be impossible, but it can be approximated with a power series in $k \cdot d$. In this way many have solved diffraction problems, as for instance Bouwkamp [7] and Millar [39]. Some solutions are only scalar, others vectorial. Jain et al. [19] have even solved the problem of diffraction of a linear polarized plane wave normal incident to an annulus, yet the results may be false because the two authors derive different edge conditions for the inner and outer ring (edge conditions of Meixner), which is very questionable 1.

This type of solutions has the advantage of relative easy implementation but has as disadvantage that it is only valid for $k \cdot d < 1$ ($d < \lambda/2\pi$), because otherwise the power series in $k \cdot d$ does not converge.

GTD, UTD and UAT

The Geometrical Theory of Diffraction (GTD) was, in general form, introduced by Keller [28] who compared his diffraction expressions with Sommerfeld's exact solution for several canonical problems. It is an approximate method of solving diffraction problems, which combines the principles of Geometrical Optics (GO) with asymptotic diffraction theory and accounts for the influence of the edges of the aperture on the incident field. Because GTD fails at shadow boundaries 2 and caustics 3 several procedures have evolved to correct these problems. The most common one is the Uniform Theory of Diffraction (UTD). Globally it is the same method as GTD, but it corrects the failure of GTD at shadow boundaries. The Uniform Asymptotic Theory of diffraction (UAT) provides the correct asymptotic solution for an arbitrary incident field on a half-plane and has been developed by Lewis, Boersma and Ahluwalia [23]. UAT can be regarded as a generalization of Keller's GTD, just as Keller's theory may be regarded as a generalization of GO.

It is difficult to define for which values of $k \cdot d$ these three techniques are valid. Surely, they give excellent results if $k \cdot d$ is very large, but how do they behave when applied to small apertures? According to Jull [23] slit and hole problems can be solved until $k \cdot d = 2$ if we include multiple diffractions at the edges of slit and hole. Suedan et al. [57] suggest a radius of $1.5 \lambda$ and Kapany et al. [25] talk about $2 \lambda$, with a good prediction down to $1 \lambda$.

Advantages of GTD and UTD are that they can very easily be implemented, they require almost no computing time and the solutions are vectorial (this includes plane, spherical and cylindrical waves!). The big disadvantage is that all methods fail in the neighbourhood of boresight. This means that part of the main lobe cannot be determined.

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1The author wants to thank prof. Boersma for his helpful suggestions concerning the article of Jain and Kanwal [19].

2A shadow boundary is the transition region between the lighted area (GO+GTD) and the shadow area (GTD).

3Caustics are points where two or more rays cross.
Maggi-Rubinowicz theory of the BDW

To avoid misunderstandings, this method is included in the review because one is tempted to assume that this is a technique which has no relation with Kirchhoff's theory. Nothing is less true! Namely, Miyamoto and Wolf [40] proved that Kirchhoff's diffraction integral can be written as the sum of the contribution of the rays coming directly from the source (if not blocked by the screen) and the contribution of the radiating rim of the aperture, which can be calculated with the aid of a contour integral [16]. The contribution of the rim is then called the Boundary Diffraction Wave (BDW).

Because the principles of BDW are based on Kirchhoff's theory, the same restrictions for aperture dimensions apply as for Kirchhoff's theory and it is very difficult to give a lower limit of $k \cdot d$ for which Kirchhoff's theory fails. In literature very seldom such lower limit is given. Kapany et al. [25] give as boundary for $d$ three to four $\lambda$, for example.

Now that some important techniques have been treated, still the question remains which technique to use. We need a method which covers the region of $d > \lambda/4$ ($k \cdot d > \pi/2$) because the width of a Fresnel zone cannot become smaller than $\lambda/2$ (this boundary value was already derived in Chapter 2).

It is difficult to find a method which covers the total region $d > \lambda/4$. Another aspect is that most techniques are mathematically very complicated which makes them not easy accessible. Furthermore, the vectorial diffraction problem of an annulus has hardly been treated in the literature (as far as known), the only solution provided by Jain et al. [19] is probably false.

Due to the previous considerations in the next subsections first UTD and GTD will be applied to derive the far-field equations because these methods are easy to implement and, more important, they are assumed to give good predictions for $k \cdot d > 2$, which is very close to our required boundary. For boresight probably Kirchhoff can be used.

Secondly, a new approximation method for calculating the radiation pattern of a 'very small' annulus will be developed using the results of Millar [39]. These patterns will be compared with those calculated according to UTD and GTD. In that way we hope to attain more insight in the behaviour of a very small annulus and to check the usability of UTD/GTD for very small apertures.

9.3.3 UTD/GTD

GTD and UTD describe the diffraction phenomena by introducing various kinds of diffracted rays, such as single diffracted rays and multiple diffracted rays. This is illustrated in figure 9.3. This figure shows an incident ray $\vec{E}_i$ diffracting at a point $Q$, resulting in a diffracted field $\vec{E}_d$. 
The field at point $K$ can be expressed as [48]:

$$
\vec{E}_d(K) = \vec{d} \cdot A(s^i, s^d) \cdot e^{-jks^d} \cdot \vec{E}^i(Q)
$$

(9.2)

where $\vec{d}$ is the dyadic diffraction coefficient and $A$ the caustic divergence factor. The distance from the diffraction point $Q$ to the field point $K$ is denoted by $s^d$, $s^i$ is the distance between the source and $Q$, and $\vec{E}^i(Q)$ is the incident field at point $Q$.

![Figure 9.3: General diffraction configuration.](image)

Before starting with the calculations, first the notational conventions are explained, because we have to handle many parameters which need to be properly categorized. Due to our antenna configuration we only need to derive the formulae for the contributions of the diffracted field caused by one Fresnel zone. These formulae, applied to all the zones of a Fresnel lens, result into a complete radiation pattern of the antenna.

As we will see later on, one zone possesses four diffraction points: two at the outer circle of the zone, with radius $b_{m+1}$, and two at the inner circle, with radius $b_m$. When a parameter is used, it generally contains a subscript indicating to which diffraction points is referred. A $b$ refers to the inner circle, an $a$ to the outer circle of a zone.

Further, the same convention of notation is applied when indicating which pair of diffraction points is meant. The diffraction points the closest together form a pair which is numbered 1 and 2 respectively. To distinguish between the incident and diffracted fields, we have introduced the superscripts $i$ (incident) and $d$ (diffracted), just as in the previous chapter. When the indexes $a$, $b$, 1 or 2 are omitted then the parameter refers to the general case as given in figure 9.3. For example, $s^d_{2a}$ is the distance between the diffraction point $Q_{2a}$ and a field point $K$. The unit vector $\hat{s}^d_{2a}$ denotes the direction of the diffracted ray, arriving at $K$. In case the length of a vector is meant, the vector sign is omitted and a caret denotes a unit vector.

According to UTD/GTD, in case of our configuration the contributions to the field in an observation point $K(r)$ originate mainly from four points [28], as mentioned before.
These points can easily be found by considering the shortest and longest route between the feed and the field point \( K \) via the edge of the aperture. Figure 9.4 indicates that this are the points \( Q_{1a}, Q_{1b}, Q_{2a} \) and \( Q_{2b} \) (also called stationary points), which are the intersection points of the plane containing the lines from \( Q_{1a,1b} \) to \( Q_{2a,2b} \) and from 0 to the point \( z = -F \). The coordinates of these points are given by \((\rho, \psi, \phi)\) (pair 1) and \((\rho, \psi, \phi + 180^\circ)\) (pair 2).

![Diagram of General geometry of diffraction at a disc or hole.](image)

The diffraction points \( Q \), the point \( K \) and the feed lie in the same \( \phi \)-plane, so we can consider the case as a two-dimensional problem and we only take a single \( \phi \)-plane to analyze the diffracted fields arriving in point \( K \). Note that therefore applies \( \phi = \xi \). This results in the figures 9.5 and 9.6 where already some unit vectors \((\hat{\beta}, \hat{\phi})\) are included which are necessary for definitions further on.

The diffracted field is composed of single diffracted rays and multiple diffracted rays. Multiple diffraction (see figure 9.7) is very important in our case, because the width of some Fresnel zones can become small in terms of wavelengths. This means that we cannot neglect multiple diffracted rays between the edges of the zone. The contributions of these rays will be calculated in the same way as those of single diffracted rays, with the only difference that the diffraction 'geometry' involved is more complex. Therefore, we will start with the calculations of the contributions of the single diffracted rays.
Figure 9.5: Two-dimensional geometry to analyze the diffracted field due to the outer edge of a transparent Fresnel zone.
Figure 9.6: Two-dimensional geometry to analyze the diffracted field due to the inner edge of a transparent Fresnel zone.
Chapter 9. Small Fresnel zones

Figure 9.7: Multiple diffracted rays.

To be able to calculate the diffracted field first the caustic divergence factor is determined, followed by the derivation of the dyadic diffraction coefficient.

The caustic divergence factor

For diffraction of an incident spherical wave at a curved edge, the caustic divergence factor takes the following form [48]:

\[ A(s^i, s^d) = \sqrt{\frac{\rho_c}{\rho_c + \rho_d}} \]  \hspace{1cm} (9.3)

where

\[ \frac{1}{\rho_c} = \frac{1}{\rho_c^e} + \frac{\hat{n} \cdot (\hat{s}^i - \hat{s}^d)}{\rho_g} \]  \hspace{1cm} (9.4)

with

\( \hat{s}^i \) \hspace{1cm} the unit vector in the direction of the incident ray.

\( \hat{s}^d \) \hspace{1cm} the unit vector in the direction of the diffracted ray.

\( \hat{n} \) \hspace{1cm} the unit vector normal to the edge at the diffraction point \( Q \) (see figure 9.4) and directed away from the center of the curvature.
\[ \rho_e \]

the radius of the curvature of the incident wavefront at the edge-fixed plane of incidence, which contains the unit vector \( \hat{s}^i \) and the unit vector \( \hat{T} \) tangent to the edge at the diffraction point. This variable can have a positive or negative value depending on whether the incident ray is converging \( (\rho_e^i < 0) \) or diverging \( (\rho_e^i > 0) \).

\[ \rho_c \]

the distance between the caustic at the edge and the second caustic of the diffracted ray.

According to the figures 9.5 and 9.6, and equation (9.3) it is easy to find that for the upper diffraction points \((Q_{1a}, Q_{1b})\) applies:

\[
A_{1a} = \sqrt{\frac{\rho_c}{s_1^a(\rho_{c1} + s_1^a)}} \quad \text{with} \quad \rho_{c1} = \frac{\rho_m \sin(\psi_m)}{\sin(\theta_{1a})} 
\]

\[
A_{1b} = \sqrt{\frac{\rho_c}{s_1^b(\rho_{c1} + s_1^b)}} \quad \text{with} \quad \rho_{c1} = \frac{\rho_{m+1} \sin(\psi_{m+1})}{\sin(\theta_{1b})} 
\]

For the lower diffraction points \((Q_{2a}, Q_{2b})\) applies:

\[
A_{2a} = \sqrt{\frac{\rho_c}{s_1^a(\rho_{c2} + s_2^a)}} \quad \text{with} \quad \rho_{c2} = \frac{-\rho_m \sin(\psi_m)}{\sin(\theta_{2a})} 
\]

\[
\begin{align*}
\rho_{c2} &< 0, \\
\rho_{c2} + s_2^d &> 0 \} \Rightarrow A_{2a} = e^{i\frac{\pi}{2}} \cdot |A_{2a}| \quad (9.8)
\end{align*}
\]

\[
A_{2b} = \sqrt{\frac{\rho_c}{s_1^b(\rho_{c2} + s_2^b)}} \quad \text{with} \quad \rho_{c2} = \frac{-\rho_{m+1} \sin(\psi_{m+1})}{\sin(\theta_{2b})} 
\]

\[
\begin{align*}
\rho_{c2} &< 0, \\
\rho_{c2} + s_2^d &> 0 \} \Rightarrow A_{2b} = e^{i\frac{\pi}{2}} \cdot |A_{2b}| \quad (9.10)
\end{align*}
\]

From the previous equations it appears that the caustic divergence factors for the lower points are complex. This is not surprising because diffracted rays from these points have to cross the z-axis in order to reach the field point \( K \). The z-axis is a caustic for all the diffracted rays \[28\], and causes an extra phase shift of \( 90^\circ \).

The dyadic diffraction coefficient

According to Kouyoumjian and Pathak \[48\], the dyadic diffraction coefficient can be divided into two separate diffraction coefficients \( D_s \) and \( D_h \).
These coefficients can be written, in case of our geometrical configuration, in the following form:

\[
D_k^s = \frac{-e^{-j\frac{\pi}{2}}}{2\sqrt{2\pi k}} \left\{ \frac{Fr[kL_i^2 \cos \left( \frac{\omega_i - \omega_t}{2} \right)]}{\cos \left( \frac{\omega_i - \omega_t}{2} \right)} \mp \frac{Fr[kL_r^2 \cos \left( \frac{\omega_d - \omega_t}{2} \right)]}{\cos \left( \frac{\omega_d + \omega_t}{2} \right)} \right\} \tag{9.11}
\]

with

\begin{align*}
\omega_i & \quad \text{the angle between the incident ray and the antenna surface tangent, which is perpendicular to the plane of incidence.} \\
\omega_d & \quad \text{the angle between the diffracted ray and the antenna surface tangent, which is perpendicular to the plane of diffraction.} \\
Fr(x) & \quad \text{this is called the transition function which involves a Fresnel integral and is given by the following equation} \ [48]: \\
& \quad \text{\quad } Fr(x) = 2j \cdot \sqrt{2} \cdot e^{jx} \cdot \int_{\sqrt{2}}^{\infty} e^{-jv^2} dv \tag{9.12}
\end{align*}

This function validates UTD uniformly across a shadow boundary. Setting \( Fr(x) = 1 \) results in a reduction of UTD in GTD which is not uniformly valid across a shadow boundary.

\( L_i, L_r \) \quad \text{these are the distance parameters defined in case of spherical waves as (see figure 9.8):}

\[
L_i = L_r = \frac{s^d \cdot \rho}{s^d + \rho} \tag{9.13}
\]

with \( \rho \) being the distance between the feed and the diffraction point \( Q \). It should be noted that the parameter \( k \cdot L_i \) or \( k \cdot L_r \) should be larger than one, for the UTD to be valid [13]!

\( D_s \) \quad \text{the scalar diffraction coefficient for the soft boundary conditions. These conditions imply that the tangential component of the incident E-field is zero at the diffraction point.}

\( D_h \) \quad \text{the scalar diffraction coefficient for the hard boundary conditions. These conditions imply that the tangential component of the incident H-field is zero at the diffraction point.}

Now that the formulae for the dyadic diffraction coefficient have been presented, the hard and soft diffraction coefficients for a Fresnel zone can be derived.
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Figure 9.8: Definitions of the distance parameters $L^i$ and $L^r$ in case of a circular aperture.

For the upper diffraction points figures 9.5 and 9.6 show that

$$
\cos\left(\frac{\omega_d^i - \omega_{1a}^i}{2}\right) = \sin\left(\frac{\theta_{1a} - \psi_m}{2}\right) \quad (9.14)
$$

$$
\cos\left(\frac{\omega_d^i + \omega_{1a}^i}{2}\right) = -\cos\left(\frac{\theta_{1a} + \psi_m}{2}\right) \quad (9.15)
$$

$$
\cos\left(\frac{\omega_d^b - \omega_{1b}^i}{2}\right) = \sin\left(-\frac{\theta_{1b} + \psi_{m+1}}{2}\right) \quad (9.16)
$$

$$
\cos\left(\frac{\omega_d^b + \omega_{1b}^i}{2}\right) = -\cos\left(\frac{\theta_{1b} + \psi_{m+1}}{2}\right) \quad (9.17)
$$

and according to figure 9.8 and formula (9.13) $L^i$ and $L^r$ can be expressed as:

$$
L^i_{1a} = \frac{s_{1a}^d \cdot \rho_m}{s_{1a}^d + \rho_m} \approx \rho_m \approx L^r_{1a} \quad \text{(far-field approximation)} \quad (9.18)
$$

$$
L^i_{1b} = \frac{s_{1b}^d \cdot \rho_{m+1}}{s_{1b}^d + \rho_{m+1}} \approx \rho_{m+1} \approx L^r_{1b} \quad \text{(far-field approximation)} \quad (9.19)
$$

This yields for the diffraction coefficients for the upper points:

$$
D^i_{1a} = \frac{-e^{-j\pi}}{2\sqrt{2\pi k}} \left\{ \frac{Fr \left[ 2k \rho_m \sin^2 \left( \frac{\theta_{1a} - \psi_m}{2} \right) \right]}{\sin \left( \frac{\theta_{1a} - \psi_m}{2} \right)} \mp \frac{Fr \left[ 2k \rho_m \cos^2 \left( \frac{\theta_{1a} + \psi_m}{2} \right) \right]}{-\cos \left( \frac{\theta_{1a} + \psi_m}{2} \right)} \right\} \quad (9.20)
$$

$$
D^i_{1b} = \frac{-e^{-j\pi}}{2\sqrt{2\pi k}} \left\{ \frac{Fr \left[ 2k \rho_{m+1} \sin^2 \left( -\frac{\theta_{1b} + \psi_{m+1}}{2} \right) \right]}{\sin \left( -\frac{\theta_{1b} + \psi_{m+1}}{2} \right)} \mp \frac{Fr \left[ 2k \rho_{m+1} \cos^2 \left( \frac{\theta_{1b} + \psi_{m+1}}{2} \right) \right]}{-\cos \left( \frac{\theta_{1b} + \psi_{m+1}}{2} \right)} \right\} \quad (9.21)
$$
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The calculations of the diffraction coefficients for the lower points are similar. According to figures 9.5 and 9.6 the following relations exist:

\[
\begin{align*}
\cos\left(\frac{\omega_{2a} - \omega_{1a}}{2}\right) &= -\sin\left(\frac{\theta_{2a} + \psi_m}{2}\right) \quad (9.22) \\
\cos\left(\frac{\omega_{2a} + \omega_{1a}}{2}\right) &= -\cos\left(\frac{\theta_{2a} - \psi_m}{2}\right) \quad (9.23) \\
\cos\left(\frac{\omega_{2b} - \omega_{1b}}{2}\right) &= \sin\left(\frac{\theta_{2b} + \psi_{m+1}}{2}\right) \quad (9.24) \\
\cos\left(\frac{\omega_{2b} + \omega_{1b}}{2}\right) &= -\cos\left(-\frac{\theta_{2b} + \psi_{m+1}}{2}\right) \quad (9.25)
\end{align*}
\]

and for the distance parameters applies:

\[
\begin{align*}
L_{2a}^i &= \frac{s_{2a}^d \cdot \rho_m}{s_{2a}^d + \rho_m} \simeq \rho_m \simeq L_{2a}^r \quad \text{(far-field approximation)} \quad (9.26) \\
L_{2b}^i &= \frac{s_{2b}^d \cdot \rho_{m+1}}{s_{2b}^d + \rho_{m+1}} \simeq \rho_{m+1} \simeq L_{2b}^r \quad \text{(far-field approximation)} \quad (9.27)
\end{align*}
\]

This leads to the following diffraction coefficients for the lower points:

\[
\begin{align*}
D_{2a}^a &= -e^{-j\frac{\pi}{8}} \cdot \left\{ \frac{F_r \left[ 2k\rho_m \sin^2 \left(\frac{\theta_{2a} + \psi_m}{2}\right) \right]}{-\sin \left(\frac{\theta_{2a} + \psi_m}{2}\right)} \mp F_r \left[ 2k\rho_m \cos^2 \left(\frac{\theta_{2a} + \psi_m}{2}\right) \right] \right\} \quad (9.28) \\
D_{2b}^a &= -e^{-j\frac{\pi}{8}} \cdot \left\{ \frac{F_r \left( 2k\rho_{m+1} \sin^2 \left(\frac{\theta_{2b} + \psi_{m+1}}{2}\right) \right)}{\sin \left(\frac{\theta_{2b} + \psi_{m+1}}{2}\right)} \mp F_r \left( 2k\rho_{m+1} \cos^2 \left(\frac{\theta_{2b} + \psi_{m+1}}{2}\right) \right) \right\} \quad (9.29)
\end{align*}
\]

Now the scalar diffraction coefficients have been obtained. But the vector properties of the fields have to be considered. This can be done by expressing the incident fields (due to the feed) and diffracted fields (due to the Fresnel zone) in terms of two components, according to the orthogonal directions defined in figure 9.9:

\[
\begin{align*}
\vec{E}^i &= E^i_\beta \cdot \hat{\beta} + E^i_\varphi \cdot \hat{\varphi} \quad (9.30) \\
\vec{E}^d &= E^d_\beta \cdot \hat{\beta} + E^d_\varphi \cdot \hat{\varphi} \quad (9.31)
\end{align*}
\]

The variables $\beta$ and $\varphi$ are used to describe the 'edge fixed' coordinate systems (plane of incidence and plane of diffraction).
Equations (9.30) and (9.31) lead to the following relation between the incident and diffracted field [48]:

\[
\begin{pmatrix}
E_p^d \\
E_\phi^d
\end{pmatrix} = \begin{pmatrix}
-D_s & 0 \\
0 & -D_h
\end{pmatrix} \begin{pmatrix}
E_p^i \\
E_\phi^i
\end{pmatrix} \cdot \mathcal{A}(s^i, s^d) \cdot e^{-jks^d}
\] (9.32)

The field components $E_p^i$ and $E_\phi^i$ are provided by the incident field from the feed, which is specified in Chapter 3 in the the coordinate system $(\psi, \xi, \rho)$. As mentioned before the variable $\xi$ equals $\phi$ due to the diffraction 'geometry' of the Fresnel zone. Thus, equation (3.2) becomes in that case:

\[
\vec{E}(\psi, \phi, n) = \vec{E}_{\text{feed}}(\psi, \phi, n) = \\
\sqrt{G_{\text{feed}}(\psi)} \cdot \frac{e^{-jkr}}{\rho} \cdot \sqrt{\frac{2\pi a}{4\pi}} \cdot \left( -\cos(\phi) \cdot \hat{\psi} + \sin(\phi) \cdot \hat{\phi} \right)
\] (9.33)

If the lower diffraction points are considered then equation (3.2) changes into:

\[
\vec{E}(\psi, \phi, n) = \vec{E}_{\text{feed}}(\psi, \phi + 180^\circ, n) = \\
\sqrt{G_{\text{feed}}(\psi)} \cdot \frac{e^{-jkr}}{\rho} \cdot \sqrt{\frac{2\pi a}{4\pi}} \cdot \left( \cos(\phi) \cdot \hat{\psi} - \sin(\phi) \cdot \hat{\phi} \right)
\] (9.34)
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Considering the diffraction points at the inner and outer circle of a zone the following relations are found for the vector components of the incident and diffracted fields:

\[
Q_{1a} \rightarrow \begin{cases} E_{1\beta}^i = E_{1\beta}^i \\ E_{1\varphi}^i = E_{1\varphi}^i \\ E_{1\beta}^d = -E_{1\beta}^d \\ E_{1\varphi}^d = -E_{1\varphi}^d \end{cases} \quad Q_{2a} \rightarrow \begin{cases} E_{1\beta}^i = -E_{1\beta}^i \\ E_{1\varphi}^i = -E_{1\varphi}^i \\ E_{1\beta}^d = -E_{1\beta}^d \\ E_{1\varphi}^d = -E_{1\varphi}^d \end{cases}
\]

\[
Q_{1b} \rightarrow \begin{cases} E_{1\beta}^i = -E_{1\beta}^i \\ E_{1\varphi}^i = -E_{1\varphi}^i \\ E_{1\beta}^d = E_{1\beta}^d \\ E_{1\varphi}^d = E_{1\varphi}^d \end{cases} \quad Q_{2b} \rightarrow \begin{cases} E_{1\beta}^i = E_{1\beta}^i \\ E_{1\varphi}^i = E_{1\varphi}^i \\ E_{1\beta}^d = E_{1\beta}^d \\ E_{1\varphi}^d = E_{1\varphi}^d \end{cases}
\]

Using the far-field approximations:

\[
\theta_{1a,1b,2a,2b} \simeq \theta \quad \text{(9.35)}
\]
\[
s_{1a,1b,2a,2b}^d \simeq r \quad \text{(amplitude)} \quad \text{(9.36)}
\]
\[
s_{1a,1b,2a,2b}^d \simeq r - \rho_m \cos(\psi_m - \theta) \quad \text{(phase)} \quad \text{(9.37)}
\]
\[
s_{1a,1b,2a,2b}^d \simeq r - \rho_m \cos(\psi_m + \theta) \quad \text{(phase)} \quad \text{(9.38)}
\]

the field in \( K \), diffracted at the points \( Q_{1a}, Q_{1b}, Q_{2a} \) and \( Q_{2b} \), yields:

\[
E_{1a\beta}^d = -E_{1\beta}^d = D_s^a \cdot e^{-jk(r-\rho_m \cos(\psi_m - \theta))} \cdot A_{1a} \cdot E_{1\beta}^i \quad \text{(9.39)}
\]
\[
E_{1a\varphi}^d = -E_{1\varphi}^d = D_h^a \cdot e^{-jk(r-\rho_m \cos(\psi_m - \theta))} \cdot A_{1a} \cdot E_{1\varphi}^i \quad \text{(9.40)}
\]

with

\[
\begin{pmatrix} E_{1\beta}^i \\ E_{1\varphi}^i \end{pmatrix} = \sqrt{\frac{P_0 Z_0}{2\pi}} \cdot \sqrt{G_{feed}(\psi_m)} \cdot \frac{e^{-jk\rho_m}}{\rho_m} \cdot \begin{pmatrix} \sin(\phi) \\ -\cos(\phi) \end{pmatrix} \quad \text{(9.41)}
\]

\[
E_{1b\beta}^d = E_{1\beta}^d = -D_s^b \cdot e^{-jk(r-\rho_{m+1} \cos(\psi_{m+1} + \theta))} \cdot A_{1b} \cdot E_{1\beta}^i \quad \text{(9.42)}
\]
\[
E_{1b\varphi}^d = E_{1\varphi}^d = -D_h^b \cdot e^{-jk(r-\rho_{m+1} \cos(\psi_{m+1} + \theta))} \cdot A_{1b} \cdot E_{1\varphi}^i \quad \text{(9.43)}
\]

with

\[
\begin{pmatrix} E_{1\beta}^i \\ E_{1\varphi}^i \end{pmatrix} = \sqrt{\frac{P_0 Z_0}{2\pi}} \cdot \sqrt{G_{feed}(\psi_{m+1})} \cdot \frac{e^{-jk\rho_{m+1}}}{\rho_{m+1}} \cdot \begin{pmatrix} -\sin(\phi) \\ \cos(\phi) \end{pmatrix} \quad \text{(9.44)}
\]

\[
E_{2a\beta}^d = E_{2\beta}^d = -D_s^a \cdot e^{-jk(r-\rho_m \cos(\psi_m + \theta))} \cdot A_{2a} \cdot E_{2\beta}^i \quad \text{(9.45)}
\]
\[
E_{2a\varphi}^d = E_{2\varphi}^d = -D_h^a \cdot e^{-jk(r-\rho_m \cos(\psi_m + \theta))} \cdot A_{2a} \cdot E_{2\varphi}^i \quad \text{(9.46)}
\]

with

\[
\begin{pmatrix} E_{2\beta}^i \\ E_{2\varphi}^i \end{pmatrix} = \sqrt{\frac{P_0 Z_0}{2\pi}} \cdot \sqrt{G_{feed}(\psi_m)} \cdot \frac{e^{-jk\rho_m}}{\rho_m} \cdot \begin{pmatrix} -\sin(\phi) \\ \cos(\phi) \end{pmatrix} \quad \text{(9.47)}
\]
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\[
\begin{align*}
E_{2b}^d &= -E_{2b}^s = D_{b}^{2b} \cdot e^{-jk(r-\rho_{m+1} \cos(\psi_{m+1}+\theta))} \cdot A_{2b} \cdot E_{2b}^i \\
E_{3b}^d &= -E_{3b}^s = D_{b}^{3b} \cdot e^{-jk(r-\rho_{m+1} \cos(\psi_{m+1}+\theta))} \cdot A_{3b} \cdot E_{3b}^i
\end{align*}
\]

(9.48) (9.49)

with \[
\left( \frac{E_{2b}^i}{E_{3b}^i} \right) = \sqrt{\frac{P_0 Z_0}{2\pi}} \cdot \frac{e^{-jk\rho_{m+1}}}{\rho_{m+1}} \cdot \frac{\sin(\phi)}{-\cos(\phi)}
\]

(9.50)

The total field due to single diffracted rays is the sum of the fields originating from the four diffraction points:

\[
\vec{E}_{\text{single}}^d = \left( \frac{E_{1a}^d}{E_{1b}^d} \right) + \left( \frac{E_{2a}^d}{E_{2b}^d} \right) + \left( \frac{E_{1a}^d}{E_{1b}^d} \right) + \left( \frac{E_{2a}^d}{E_{2b}^d} \right)
\]

(9.51)

After the single diffracted fields are calculated, we can restrict our attention to the multiple diffracted rays. The calculations of the contribution of the multiple diffracted rays proceed in the same way as those of single diffracted rays. There are only some modifications of the used parameters. Namely, it is not necessary and not possible to use UTD. UTD was applied to prevent discontinuities of the diffracted field at shadow boundaries, caused by GTD. In case of single diffracted rays these discontinuities can occur at several \(\theta\)-angles of the radiation pattern. Yet, for multiple diffraction these shadow boundaries are given by \(\theta = -90^\circ\) and \(\theta = 90^\circ\), which means that they coincide with the antenna aperture. At the shadow boundaries GTD is not uniformly valid but since we are not interested in \(\theta\)-angles larger than \(90^\circ\) GTD satisfies. Also the parameters \(k \cdot L^{hr}\) are smaller than 1 which means that it is not allowed to use UTD [13]. GTD has the advantage over UTD that it requires less computing time and is easier to implement. So, instead of formula (9.11) the formula (9.52) is used for the soft and hard diffraction coefficients:

\[
G_h(\omega^l, \omega^s) = \frac{-e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi k}} \cdot \left\{ \frac{1}{\cos(\frac{\omega^l-\omega^s}{2})} \mp \frac{1}{\cos(\frac{\omega^l+\omega^s}{2})} \right\}
\]

(9.52)

The caustic divergence factors remain unchanged.

In order to calculate the multiple diffraction contributions first, it is necessary to analyze the different travel routes of the diffracted rays. Figure 9.10 shows the four possible situations for the upper and the lower points. All these situations will not be calculated in this subsection. One example will be treated and a review of all the different situations is given in Appendix G.

As example situation I of the upper diffraction points is considered. The single diffracted ray from \(Q_{1a}\) in the direction of \(Q_{1b}\) strikes the upper ring of the Fresnel zone in the point \(Q_{1b}\) and is diffracted again in the direction of point \(K\), yielding for the multiple diffracted field in \(K\):

\[
\left( \frac{E_{2a}^d}{E_{2b}^d} \right) = \left[ D_{1a}^{2a} \right]_h \cdot \left[ D_{1b}^{2b} \right]_h \cdot \left( \frac{E_{1b}^i}{E_{2b}^i} \right)
\]

(9.53)
Double diffracted rays in the direction of $Q_{1a}$ produce triple diffracted fields, and so on, which results in:

$$
\begin{align*}
\left( \frac{E_{d \beta}}{E_{d \phi}} \right) &= \left( [D_{1a}^1]_s \cdot [D_{1b}^1]_s \cdot [D_{1a}^2]_s \cdot [D_{1b}^2]_s \cdot [D^3]_s \cdot [D^4]_s + \ldots \right) \cdot \left( \frac{E_i^\beta}{E_i^\phi} \right) \\
\text{with } [D^2]_s &= -G_s(\omega^i = \pi, \omega^d = \pi) \cdot \frac{e^{-jkd}}{\sqrt{2d(1 + 2d/b_m)}} \\
[D^3]_s &= -G_s(\omega^i = \pi, \omega^d = 3\pi/2 + \theta) \cdot \frac{e^{-jkd}}{\sin(\theta)} \cdot \frac{e^{-jkd}}{\sqrt{d}} \\
\text{(9.54)}
\end{align*}
$$

The contributions in the former expression can be written as a power series in $[D^2]_s \cdot [D^3]_s$ and this results in a geometric series which can be summed:

$$
\begin{align*}
\left( \frac{E_{d \beta}}{E_{d \phi}} \right) &= \frac{[D_{1a}^1]_s \cdot [D_{1b}^1]_s \cdot [D_{1a}^2]_s \cdot [D_{1b}^2]_s \cdot [D^3]_s \cdot [D^4]_s + \ldots}{1 - [D^2]_s \cdot [D^3]_s} \cdot \left( \frac{E_i^\beta}{E_i^\phi} \right) \\
\text{(9.56)}
\end{align*}
$$

Applying the geometric series implies that $|[D^2]_s \cdot [D^3]_s| < 1$. This condition is always satisfied due to our geometrical configuration which causes the factors $d/b_m$ and $d/b_{m+1}$ to be much smaller than 1. From a physical point of view the previous restriction is logical because the diffracted rays have to fade out in the end (principle of energy conservation).

The total multiple diffraction contribution has been calculated in Appendix G, which results in a field $E_{\text{multiple}}^d$. The first Fresnel zone (number 0) is not included in the calculations because this zone has a radius of at least several wavelengths, so the influence of these multiple diffracted rays is negligible.

Summarizing, the total diffracted field in point $K$ is composed of two fields:

$$
E_d^d = E_{\text{single}}^d + E_{\text{multiple}}^d \\
(9.57)
$$

The total field which is necessary for calculation of the radiation pattern is the sum of the field due to diffraction, and the direct field of the feed:

$$
E_{\text{total}} = E_d^d + E_{\text{feed}} \cdot [U(\psi_{m+1}) - U(\psi_m)] \\
(9.58)
$$
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Figure 9.10: Multiple diffraction configurations. For upper points $k = 1a$ and $l = 1b$, and for the lower points $k = 2b$ and $l = 2a$. 
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$U(z)$ symbolizes the unity step function. Equation (9.58) gives the complete field in point $K$ due to one Fresnel zone, so we are now able to calculate the far-field radiation patterns of a Fresnel antenna in the region $-90^\circ \leq \theta \leq 90^\circ$, excluding the vicinity of $\theta = 0$ (as earlier mentioned, UTD and GTD fail in the vicinity of boresight).

Radiation patterns have been calculated for two different Fresnel antenna systems. Figures 9.11 till 9.14 show antenna patterns calculated with UTD/GTD and Kirchhoff's formula (equation (8.12)). It appears that patterns according to UTD/GTD and Kirchhoff of the systems with small $F$ differ the most. This is not surprising because these systems possess almost twice as many zones as those with a relative large $F$. Also, the parameter $d_m$ of many of these zones is small in terms of wavelength thus multiple diffraction has a much larger influence. This is also confirmed by computer simulations which show that patterns calculated with UTD/GTD and Kirchhoff match better when the multiple diffraction term is omitted. This implies that the influence of the edges of the aperture is important and that the Kirchhoff diffraction integral in combination with the assumption that the aperture field is the incident field, does not account for the currents at the edges.

Also, the difference between Kirchhoff's integral and UTD/GTD is smaller in case of $\phi = 0^\circ$. This is the result of the following.

The plane $\phi = 0^\circ$ is the plane of polarization of the feed which means that the vector $\vec{E}_{feed}$ lies in this plane. Thus, $\vec{E}_{feed}$ is normal to the edge of the aperture at the diffraction points implying that the influence of the multiple reflections is negligible. The incident field is in this case a reasonable approximation for the aperture field (due to symmetry properties) which implies that Kirchhoff's diffraction integral in combination with the 'normal' assumptions gives also a better result.

In figure 9.15 two wide-angle radiation patterns are depicted, both using UTD/GTD, but one is calculated without the multiple diffraction contribution. We can see clearly that the contribution of the multiple diffracted rays is significant.

![Figure 9.11: Gain patterns calculated with UTD/GTD and Kirchhoff's theory for $\phi = 0^\circ$, $F = 0.522$ m and $n=6.4$.](image)
Figure 9.12: Gain patterns calculated with UTD/GTD and Kirchhoff’s theory for $\phi = 90^\circ$, $F = 0.522$ m and $n=6.4$.

Figure 9.13: Gain patterns calculated with UTD/GTD and Kirchhoff’s theory for $\phi = 0^\circ$, $F = 1.875$ m and $n=82$. 

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Figure 9.14: Gain patterns calculated with UTD/GTD and Kirchhoff's theory for $\phi = 90^\circ$, $F = 1.875$ m and $n=82$.

Figure 9.15: Wide-angle radiation patterns calculated with UTD/GTD, with and without multiple diffraction contribution ($F = 0.522$ m, $n=6.4$ and $\phi = 90^\circ$).
Let us now focus on the polarization of the electric field, which is important in most antenna systems. Two orthogonal polarizations can be used to transmit or receive two different signals using the same frequency. Therefore the radiation pattern is split into two patterns. One pattern is the reference or co-polarization pattern, which is the radiation pattern of the desired component. The other pattern is the cross-polarization pattern, which is the radiation pattern of the component of the electric field that is orthogonal to the desired component.

The antenna systems analyzed in this report have the $x$-component of the electric field as the desired component. Hence the $y$-component is the cross-polarization component. According to [32] the co- and cross-polarization patterns $CP$ and $XP$ are:

$$CP(\hat{r}) = 10 \log \left[ \frac{2\pi r^2}{Z_0 P_t} \right] \left| \cos(\phi) \cdot E_\theta - \sin(\phi) \cdot E_\phi \right|^2$$  \hspace{1em} (9.59)

$$XP(\hat{r}) = 10 \log \left[ \frac{2\pi r^2}{Z_0 P_t} \right] \left| -\sin(\phi) \cdot E_\theta - \cos(\phi) \cdot E_\phi \right|^2$$  \hspace{1em} (9.60)

For the antenna system with $F = 0.522$ m and $n = 6.4$ the $CP$ and $XP$ patterns are calculated for $\phi = 45^\circ$. The resulting patterns according to UTD/GTD and Kirchhoff's theory are displayed in figures 9.16 and 9.17. These figures show clearly the differences between UTD/GTD and Kirchhoff's theory. The influence of $XP$ is in case of UTD/GTD larger than in case of Kirchhoff's theory. This is once more an indication that influence of the edges of the zones on the field in the aperture is not negligible implying that multiple diffraction must be included in the UTD/GTD model.

The assumption that the aperture field can be substituted by the incident field in Kirchhoff's diffraction integral is no longer valid for the $XP$-component. For the $CP$-component Kirchhoff's integral with as aperture field the incident field reasonably satisfies.
Chapter 9. Small Fresnel zones

Figure 9.16: Co-polarization component calculated with UTD/GTD and Kirchhoff ($\phi = 45^\circ$).

Figure 9.17: Cross-polarization component calculated with UTD/GTD and Kirchhoff ($\phi = 45^\circ$).
Millar’s method

In this subsection we attempt to determine the radiation pattern of a 'very small' annulus in a perfectly conducting screen. For this purpose the method of Millar [39] will be used in a slightly modified form. In principle each solution for the slit problem can be applied, but the solution provided by Millar is the most convenient one to implement and it is also very well documented [43, 8].

The strategy which will be employed to tackle the diffraction problem of a 'small' annulus is the following: first Millar’s method is introduced and we investigate how it can be applied to our situation. Next, the results of the investigation will be used to convert Millar’s solution to the geometry of the annulus. Finally the radiation patterns of the annulus are calculated according to the resulting equations and compared with patterns obtained with UTD/GTD.

Millar has solved the two-dimensional (infinitely long) slit problem by making use of the properties of the tangent and normal components of the electric and magnetic fields in the aperture, which are mentioned before. Namely, it is known that the tangential magnetic field and the normal electric field are not disturbed by a perfectly conducting screen. This makes it possible to derive an integral-differential equation due to behaviour of these specific components of the fields on either side of the conducting screen [43].

This integral-differential equation represents the solution of one field component of the aperture field in the slit. In principle, he reduced the vectorial problem to a scalar problem and in order to solve it, Millar has expanded functions on the left and right-hand side of the integral-differential equation in series, thus creating an infinite system of equations. By choosing special functions which contain infinite series of cosine and sine terms, a solution can be obtained in the form of a power series in \( k \cdot d \). From this series only the first seven terms are taken and higher-order terms are omitted. These seven terms represent the approximate solution of the two-dimensional slit problem. It should be noted that this method only applies for \( k \cdot d < 1 \) because otherwise the resulting power series does not converge.

The geometrical configuration used by Millar in order to derive his solution is composed of two separate incident waves which are displayed in figure 9.18. This figure shows Transversal Electromagnetic (TEM) waves incident to infinitely long slits. Note that these are two-dimensional configurations. In the first configuration (I) the \( \vec{E}_{\text{im}} \) is parallel to the screen and in the second configuration (II) \( \vec{H}_{\text{im}} \) is parallel to the screen. Configuration I gives \( x' \)- and \( y' \)-components of the aperture field and configuration II only a \( y' \)-component. Combining both incident waves results in a TEM-wave with arbitrary polarization which is given by configuration III.

The only remaining aspect is that we consider the three-dimensional situation and not the two-dimensional situation like Millar. This is not a problem because it is possible to adapt this type of configuration to the three-dimensional situation as indicated by [8]. Neelen [43] used this in order to expand the formulae of Millar to the three-dimensional situation, by introducing the angle \( \Psi \).
Figure 9.18: Geometrical configurations of the incident plane waves, used by Millar.

Using figure 9.19 he derived the following formulae:

\[ E_{x'}^{im}(x', y') = A_{x'} e^{-j k x' \cos(\Psi)} \cdot w \cdot \sum_{i=0}^{6} \mathcal{M}_i \cdot w^i + O(w^6) \] (9.61)

\[ E_{y'}^{im}(x', y') = A_{x'} e^{-j k x' \cos(\Psi)} \cdot \frac{j \cos(\Psi)}{\sin(\Psi) \sin(v)} \cdot \sum_{i=0}^{6} \mathcal{H}_i \cdot w^i + O(w^6) \] (9.62)

\[ E_{y}^{te}(x', y') = A_{y'} e^{-j k x' \cos(\Psi)} \cdot \frac{1}{j \omega \varepsilon_0 \sin(v) \sin^2(\Psi)} \cdot \sum_{i=0}^{6} \mathcal{N}_i \cdot w^i + O(w^6) \] (9.63)

with

\[ y' = d \cdot \cos(v) \quad (z' = 0) \] (9.64)

\[ w = k \cdot d \cdot \sin(\Psi) \] (9.65)

\[ p = \log \left( \frac{w}{4} \right) + \gamma + j \cdot \frac{\pi}{2} \quad (\gamma: \text{constant of Euler}) \] (9.66)

\[ A_{x'}^E = \text{length of the } x'-\text{component of } \hat{E}_{im} \] (9.67)

\[ A_{x'}^H = \text{length of the } x'-\text{component of } \hat{H}_{te} \] (9.68)

\[ \hat{k} = \sin(\Psi) \cdot \hat{z}' + \cos(\Psi) \cos(\alpha) \cdot \hat{y}' + \cos(\Psi) \sin(\alpha) \cdot \hat{z}' \] (9.69)

\[ \hat{E}_{im}^{i} = [\sin(\Psi) \cdot \hat{z}' - \cos(\Psi) \cos(\alpha) \cdot \hat{y}' - \cos(\Psi) \sin(\alpha) \cdot \hat{z}'] \cdot e^{-j(k \cdot r)} \] (9.70)

\[ \hat{E}_{te}^{i} = [-\sin(\alpha) \cdot \hat{y}' + \cos(\alpha) \cdot \hat{z}] \cdot e^{-j(k \cdot r)} \] (9.71)

The expressions for the coefficients \( N, H \) and \( M \) are given in Appendix H and the reference frame used to describe the slit is denoted by \( x', y' \) and \( z' \). With the aid of previous equations it is possible to calculate the fields in the aperture of the slit.
How can the previous equations solve our diffraction problem? The solution of Millar can be used to approximate the aperture field of the annulus. This implies that in each two opposite points of the rims of the annulus, one at the outer rim and the other at the inner rim, a slit is drawn tangent to these two points, followed by the assumption that the field in the annulus at the line between the two points can be approximated by the corresponding field in the tangent slit. Once the aperture field is known, it can be substituted in Kirchhoff's diffraction integral (equation (8.12)) and the field pattern can be calculated. Note that in this case not the incident field from the feed is assumed to be the aperture field.

Yet there still remains a problem. The incident wave is plane in contrary to the spherical wave produced by the feed used. In order to solve this problem the receiving mode is considered instead of the transmitting mode. This means calculating the field in the focal point due to TEM-waves incident on the annulus at different $\theta$- and $\phi$-angles. Figure 9.20 shows the geometrical configuration of the annulus in combination with the tangent slit and the incident TEM-wave. The reference frame used to describe the diffraction problem.

---

**Figure 9.19:** Geometrical configuration of the incident plane waves.
of the annulus is denoted by the rectangular coordinates $x$, $y$ and $z$. Note that $z$ is the same as $z'$ of the reference frame used by Millar.

![Diagram of geometrical configuration of the annulus and tangent slit]

**Figure 9.20:** Geometrical configuration of the annulus and tangent slit.

Because we are interested in the vectorial behaviour of the received field, only the co- and cross-polarization components $\vec{B}_c^d$ and $\vec{B}_x^d$ are considered. In this case the polarization of the feed is important. In Chapter 3 the polarization chosen was linear ($-x$)-polarization which means that the vector $\vec{E}_{feed}$ has no $y$-component. In this subsection we maintain the same polarization, only the $x$-direction of $\vec{E}_{feed}$ is changed from $-x$ to $+x$ because this direction suits better our geometrical configuration.

In the receiving mode the polarization of the feed and the incident TEM-wave, denoted by $\vec{E}_i^d$, must be exactly the same because we want to compare the radiation patterns (in different $\phi$-planes) of the co- and cross-polarization components calculated according to Millar with those calculated according to UTD/GTD. Analyzing the transmitting mode, depicted in figure 9.21, and comparing it with the configuration of figure 9.20 in combination with the formulae (9.59) and (9.60) we come to the conclusion that instead of turning the $\phi$-plane over an angle $\phi$, we can consider only the plane $\phi = 0$ on condition that we turn the feed over the same angle in opposite direction of $\phi$. For $\vec{E}_i^d$ must apply that it has always to point in the co-polarization direction. This procedure has the advantage of
having only to change the co- and cross-polarization vectors of the feed and the direction of $\vec{E}^i$, which is easy to accomplish.

![Diagram of a feed in transmitting mode](image)

Figure 9.21: The feed in transmitting mode, observed at the angles $\theta$ and $\phi$.

The new unit vectors, describing the feed polarization in the receiving mode, can be found from the relations of the two polarization components of the feed in transmitting mode ($\xi$ and $\psi$ are part of the reference frame used to describe the field of the feed, Chapter 3):

$$
\hat{E}_{cp}(\xi, \psi) = \cos(\xi) \cdot \hat{\psi} - \sin(\xi) \cdot \hat{\xi}
$$

(9.72)

$$
\hat{E}_{zp}(\xi, \psi) = \sin(\xi) \cdot \hat{\psi} + \cos(\xi) \cdot \hat{\xi}
$$

(9.73)

Previous equations, combined with the substitution of $\xi$ by $\xi - \phi$ lead to the polarization components of the feed in the receiving mode:

$$
\hat{E}_{cp} = [\cos(\phi) \cos(\xi) + \sin(\phi) \sin(\xi)] \cdot \hat{\psi} + [-\cos(\phi) \sin(\xi) + \sin(\phi) \cos(\xi)] \cdot \hat{\xi}
$$

(9.74)

$$
\hat{E}_{zp} = [-\sin(\phi) \cos(\xi) + \cos(\phi) \sin(\xi)] \cdot \hat{\psi} + [\sin(\phi) \sin(\xi) + \cos(\phi) \cos(\xi)] \cdot \hat{\xi}
$$

(9.75)

Also, the following relations between $\vec{E}_{xx}^i$ and $\vec{E}_y^i$ must be introduced in order to satisfy the equations (9.59) and (9.60):

$$
\vec{E}_{xx}^i + \vec{E}_y^i = \vec{E}^i \quad \land \quad E^i \cdot \cos(\phi) = E_{xx}^i \quad \land \quad E^i \cdot \sin(\phi) = E_y^i
$$

(9.76)

It should be noted that the amplitude of $\vec{E}^i$ is constant and can be arbitrarily chosen.
Before we begin with expressing the aperture field of the annulus in the components \(E_x, E_y, E_z\), first the Kirchhoff diffraction integral is worked out for the geometry under consideration in order to investigate which components of the aperture field are required.

According to Kirchhoff (equation (8.12)) the field in point \(z = F\) is given by:

\[
\vec{E}_{f,\text{feed}}(0, 0, F) = \nabla_x \times \frac{1}{2\pi} \int_0^a \int_b (\hat{\mathbf{x}} \times \vec{E}'(r', \phi')) \cdot \frac{e^{-jkR}}{R} r'dr'd\phi' \quad (9.77)
\]

where \(r'\) and \(\phi'\) are the cylindrical coordinates used to describe the geometry of the annulus and \(R\) is the distance between the field point and a surface element of the antenna. The limits \(a\) and \(b\) are the outer and inner radius of the annulus. The vectorial product of the integrand can be replaced by:

\[
\hat{\mathbf{x}} \times \vec{E}'(r', \phi') = -E_y' \cdot \hat{\mathbf{r}}' + E_z' \cdot \hat{\phi}' = \vec{E}'(r', \phi') \quad (9.78)
\]

Because the operator \(\nabla_x\) only works on the coordinates \(x, y\) and \(z\) it is permitted to include \(\nabla_x\) in the integrand, which leads to the following relation:

\[
\nabla_x \times \vec{E}'(r', \phi') e^{-jkR} = \nabla_x \left( \frac{e^{-jkR}}{R} \right) \times \vec{E}'(r', \phi') \quad (9.79)
\]

In order to determine the previous vectorial product \(R\) is required. It is easily found from figure 9.20 that for the variable \(R\) applies:

\[
R = \sqrt{(r')^2 + F^2} \quad (9.80)
\]

which yields for the vectorial product:

\[
\nabla_x \left( \frac{e^{-jkR}}{R} \right) \times \vec{E}'(r', \phi') = \left( -F \left[ -E_y' \sin(\phi') + E_z' \cos(\phi') \right] \cdot \hat{\mathbf{x}} + F \left[ -E_y' \cos(\phi') - E_z' \sin(\phi') \right] \cdot \hat{\mathbf{y}} - r' \cdot E_z' \cdot \hat{\mathbf{z}} \right) \cdot \left( \frac{1}{R} + jk \right) \cdot \frac{e^{-jkR}}{R^2} \quad (9.81)
\]
Hence, the Kirchhoff diffraction integral becomes:

\[
\mathcal{E}_{\text{feed}}(0,0,F) = -\frac{1}{2\pi} \int_{0}^{2\pi} \int_{b} \left( \frac{(k + j\xi)e^{-j\lambda R}}{R^2} \right) \cdot \left( -F \left[ -E'_{\psi} \sin(\phi') + E'_{\psi} \cos(\phi') \right] \cdot \hat{z} - F \left[ E'_{\psi} \cos(\phi) + E'_{\psi} \sin(\phi') \right] \cdot \hat{y} - r' \cdot E'_{r} \cdot \hat{z} \right) \cdot r'dr'd\phi' \quad (9.82)
\]

This equation describes the field in point \( z = F \), but does not possess any polarization dependence of the feed. In order to include this we must take the scalar product of the integrand and \( \hat{E}_{\psi} \) or \( \hat{E}_{\psi} \). In this way the co- or cross-polarization components \( E_{\psi} \) and \( E_{\psi} \) are obtained:

\[
E_{\psi}(\theta,\phi) = -\frac{1}{2\pi} \int_{0}^{2\pi} \int_{b} \left( \left[ \nabla_{r} \left( \frac{e^{-j\lambda R}}{R} \right) \times \vec{E}'(r',\phi') \right] \cdot \hat{E}_{\psi} \right) r'dr'd\phi' \quad (9.83)
\]

\[
E_{\psi}(\theta,\phi) = -\frac{1}{2\pi} \int_{0}^{2\pi} \int_{b} \left( \left[ \nabla_{r} \left( \frac{e^{-j\lambda R}}{R} \right) \times \vec{E}'(r',\phi') \right] \cdot \hat{E}_{\psi} \right) r'dr'd\phi' \quad (9.84)
\]

The fields defined above can be calculated if the aperture fields \( E_{r} \) and \( E_{\psi} \) are known. In cases where the aperture dimensions are large compared to the wavelength, \( E_{r} \) and \( E_{\psi} \) are the components of the incident field. Yet, this is not true in our situation and therefore we approximate the aperture field as described in the beginning of this subsection ('tangent slit' solution).

\( E_{r} \) and \( E_{\psi} \) can be expressed in terms of \( E_{x,y}^{\text{in}} \) and \( E_{y}^{\text{in}} \). For this, we consider the two components \( \vec{E}_{x} \) and \( \vec{E}_{y} \) (which define \( \vec{E} \)) as separate waves which are each composed of two vectors. These vectors are displayed in figure 9.22: \( \vec{q}_{1x} \) and \( \vec{q}_{2x} \) define \( \vec{E}_{x} \), \( \vec{q}_{1y} \) and \( \vec{q}_{2y} \) define \( \vec{E}_{y} \). The vectors \( \vec{q}_{1x} \) and \( \vec{q}_{1y} \) are not arbitrarily chosen but coincide with the vector \( \vec{E}_{x} \), similarly \( \vec{q}_{2x} \) and \( \vec{q}_{2y} \) coincide with \( \vec{E}_{y} \).

First the coefficients \( A_{x}, A_{y}, \alpha \) and \( \Psi \) will be determined for the \( \vec{q} \)-vector in terms of \( \phi' \) and \( \theta \). Next, the directions of the \( \vec{q} \)-vectors relative to the tangent slit are considered and further some variables of the diffraction integral (equation (9.82)) will be converted into more appropriate ones.

The angles \( \Psi \) and \( \alpha \) can easily be expressed in terms of \( \phi' \) by rewriting the vector \( \hat{k} \) in \((x',y',z')\)-coordinates:

\[
\begin{align*}
\hat{x} &= \cos(\phi') \cdot \hat{y}' + \sin(\phi') \cdot \hat{z}' \\
\hat{k} &= -\sin(\theta) \cdot \hat{x} + \cos(\theta) \cdot \hat{z}'
\end{align*}
\]
Figure 9.22: Decomposing vectors $\vec{E}_{zz}^i$ and $\vec{E}_{y}^i$ each in two orthogonal components.

Considering the different components of $\vec{k}$ leads to the following relations:

\[
\alpha = \frac{\pi}{2} + \arctan(\tan(\theta) \cos(\phi')) \\
\Psi = \frac{\pi}{2} + \arctan \left( \frac{\sin(\theta) \sin(\phi')}{\sqrt{\sin^2(\theta) \cos^2(\phi') + \cos^2(\theta)}} \right) 
\]

In order to derive $A_z^H$ and $A_y^E$, the lengths of the $q$-vectors are required. The lengths $q_{1z}$ and $q_{2z}$ can be determined by calculating the scalar product of the previously mentioned $q$-vectors and $\vec{E}_{zz}^i$ and $\vec{E}_{y}^i$ respectively:

\[
\begin{align*}
q_{1z} &= q_{1x} \cdot (\sin(\Psi) \cdot \hat{\hat{z}}' - \cos(\Psi) \cos(\alpha) \cdot \hat{\hat{y}}' - \cos(\Psi) \sin(\alpha) \cdot \hat{\hat{z}}') \cdot e^{-i(k \cdot r)} \\
\vec{E}_{zz}^i &= E_{zz}^i \cdot (\cos(\theta) \sin(\phi') \cdot \hat{\hat{z}}' + \cos(\theta) \cos(\phi') \cdot \hat{\hat{y}}' + \sin(\theta) \cdot \hat{\hat{z}}') \cdot e^{-i(k \cdot r)} \\
q_{1z} &= |\cos(\theta) \sin(\phi') \sin(\Psi) - \cos(\theta) \cos(\phi') \cos(\Psi) \cos(\alpha) - \sin(\theta) \cos(\Psi) \sin(\alpha)| \quad (9.87)
\end{align*}
\]

\[
\begin{align*}
\hat{q}_{1y} &= -\hat{q}_{1x} \\
\vec{E}_{y}^i &= E_{y}^i \cdot (-\cos(\phi') \cdot \hat{\hat{z}}' + \sin(\phi') \cdot \hat{\hat{y}}') \cdot e^{-i(k \cdot r)} \\
q_{1y} &= |\cos(\phi') \sin(\Psi) - \cos(\Psi) \sin(\phi') \cos(\alpha)| \quad (9.88)
\end{align*}
\]
Once \( q_{1x} \) and \( q_{1y} \) are known, \( q_{2x} \) and \( q_{2y} \) can be calculated according to:

\[
q_{2x} = \sqrt{(E_{zz}^i)^2 - (q_{1x})^2} \tag{9.89}
\]
\[
q_{2y} = \sqrt{(E_{yy}^i)^2 - (q_{1y})^2} \tag{9.90}
\]

Now \( A_{x'}^E \) is the \( x' \)-component of \( \vec{q}_{1x} \) and \( \vec{q}_{1y} \) and is according to figure 9.22 given by:

\[
A_{x'}^E = \sin(\Psi) \cdot \vec{q}_{1x} \quad (\vec{E}_{zz}^i - \text{case}) \tag{9.91}
\]
\[
A_{x'}^E = \sin(\Psi) \cdot \vec{q}_{1y} \quad (\vec{E}_{yy}^i - \text{case}) \tag{9.92}
\]

The length \( A_{x'}^H \) requires more attention because this is the modulus of the \( x' \)-component of \( \vec{H} \) and not of \( \vec{E} \) which we have calculated. However the values of \( q_{2x} \) and \( q_{2y} \) can be used because a TEM-wave satisfies the relation:

\[
|\vec{E}^i| = |Z_0 \cdot (\vec{H}^i \times \hat{k})| \tag{9.93}
\]

which makes it possible to express \( A_{x'}^H \) in terms of \( q_{2x} \) and \( q_{2y} \) respectively:

\[
A_{x'}^H = \frac{1}{Z_0} \cdot \sin(\Psi) \cdot \vec{q}_{2x} \quad (\vec{E}_{zz}^i - \text{case}) \tag{9.94}
\]
\[
A_{x'}^H = \frac{1}{Z_0} \cdot \sin(\Psi) \cdot \vec{q}_{2y} \quad (\vec{E}_{yy}^i - \text{case}) \tag{9.95}
\]

Until now we have restricted us to the length of the \( q \)-vectors and not considered their exact direction which changes of sign when the tangent slit moves over the annulus in the direction of \( \phi' \). This direction is very important because the solutions of the slit problem (equations (9.61), (9.62) and (9.63)) depend strongly on it. If compared, the figures 9.19, 9.20 and 9.22 show that the following relations must be included:

\[
\hat{E}_{le}^i = -\text{sign}[\cos(\phi')] \cdot \vec{q}_{2x} \quad \hat{E}_{lm}^i = \text{sign}[\sin(\phi')] \cdot \vec{q}_{1x} \tag{9.96}
\]
\[
\hat{E}_{le}^i = -\text{sign}[\sin(\phi')] \cdot \vec{q}_{2y} \quad \hat{E}_{lm}^i = -\text{sign}[\cos(\phi')] \cdot \vec{q}_{1y} \tag{9.97}
\]

The only remaining aspect before determining the aperture field is the displacement of the reference frame of the tangent slit. As displayed in figure 9.23 the reference frame is first rotated over an angle \( \phi' \) and consequently translated over a distance \((a+b)/2\). The rotation is already included in the expressions for \( \alpha, \Psi \) and the \( q \)-vectors, in contrary to the translation which causes a phase shift because \( y' \) is shifted over a distance \((a+b)/2\). Namely, replacing \( y' \) by \( y' + (a+b)/2 \) implies a phase shift of magnitude \( k \sin(\Psi) \cos(\alpha) \cdot (a+b)/2 \) according to equations (9.70) and (9.71) (which describe the incident waves of Millar's configuration).
This results in the following relations:

\[
E_{\varphi \chi}^{\text{tm}}(x', y' + (a+b)/2) = e^{-jk\sin(\psi)\cos(\alpha)\cdot(a+b)/2} \cdot E_{\varphi \chi}^{\text{tm}}(x', y') \tag{9.98}
\]
\[
E_{\psi \chi}^{\text{tm}}(x', y' + (a+b)/2) = e^{-jk\sin(\psi)\cos(\alpha)\cdot(a+b)/2} \cdot E_{\psi \chi}^{\text{tm}}(x', y') \tag{9.99}
\]
\[
E_{\psi \chi}^{\text{tr}}(x', y' + (a+b)/2) = e^{-jk\sin(\psi)\cos(\alpha)\cdot(a+b)/2} \cdot E_{\psi \chi}^{\text{tr}}(x', y') \tag{9.100}
\]

![Figure 9.23: Translation and rotation of the reference frame denoted by $x'$, $y'$ and $z'$.

Hence, in order to finally substitute $E_{\varphi \chi}'$ and $E_{\psi \chi}'$ by $E_{\varphi \chi}^{\text{tm}}$ and $E_{\psi \chi}^{\text{tr}}$, and adapt Millar's geometry to Kirchhoff's diffraction integral, the following relations must be considered:

\[\begin{align*}
\xi &= \phi' \\
y' + (a+b)/2 &= r' \\
y' &= d \cos(v)
\end{align*}\]

Applying all the previous relations to the diffraction integral results in:

\[
E_{\varphi \psi}(\theta, \phi) = -\frac{d}{2\pi} \int_0^{2\pi} \int_0^{\pi} \left( \nabla_r \left( \frac{e^{-jkR}}{R} \right) \times \tilde{E}'(r', \phi') \right) \cdot \tilde{\hat{e}}_{\varphi \psi} r' \sin(v) dv d\phi' \tag{9.101}
\]
\[
E_{\psi \varphi}(\theta, \phi) = -\frac{d}{2\pi} \int_0^{2\pi} \int_0^{\pi} \left( \nabla_r \left( \frac{e^{-jkR}}{R} \right) \times \tilde{E}'(r', \phi') \right) \cdot \tilde{\hat{e}}_{\psi \varphi} r' \sin(v) dv d\phi' \tag{9.102}
\]

with

\[
\begin{align*}
r' &= d \cdot \cos(v) + (a+b)/2 \\
R &= \sqrt{(d \cos(v) + (a-b)/2)^2 + F'^2}
\end{align*}\]

(9.103)

(9.104)
Equations (9.101) and (9.102) are used to calculate the radiation patterns of an annulus \((d = \lambda/8, a=0.5\ m\ and\ F = 0.5\ m)\) at the angles \(\phi = 0^\circ\) and \(\phi = 45^\circ\). The values for \(d, a\) and \(F\) are chosen in such a way that the configuration considered represents the worst case situation. Namely, the 'smallest' zones of the antenna are near the antenna edge \((D=1.0\ m)\) and at small \(F\)'s \((F=0.5\ m)\).

The radiation patterns are plotted, together with those calculated with UTD/GTD (including multiple diffraction), in figures 9.24 until 9.28. It should be noted that only the shape of the patterns can be compared because the absolute values of the levels of the patterns calculated with Millar are unknown. Therefore, the patterns are all normalized to their maximum value. For the patterns calculated with UTD/GTD this implies that they are normalized to a maximum value which is probably not correct because it is calculated in the vicinity of boresight. This does not matter because we compare only the shape of the different patterns.

For \(\phi = 0^\circ\) it is clearly visible that in both cases only a co-polarization component exists. This is due to symmetry properties of the received/transmitted waves relative to the annulus. The shape of \(E_{cp}\) is for both systems similar, only in case of UTD/GTD the decrease of the signal at larger \(\theta\)-angles is less. For \(\phi = 45^\circ\) both polarization components exist and it appears that both diffraction techniques give similar results. Only the cross-polarization component, obtained via UTD/GTD, has a slightly different envelope which is probably caused by the fact that UTD/GTD considers the outer and inner edge of the annulus as two separate apertures (half planes). This does not apply for very small apertures because in that case the combined influence of the two edges on the incident field becomes significant, thus the model used by UTD/GTD no longer suffices.

Due to these observation we could conclude that the influence of the cross-polarization component is larger and the pattern calculated with Millar has a steeper descent than the one according to UTD/GTD. Yet, we must stress the fact that this could be a preliminary conclusion because the method of Millar, as applied in this specific situation, is not checked for validity because no comparing measurements or simulations are available.
Figure 9.24: $|E_{cp}|$, calculated according to Millar and UTD/GTD. All patterns are normalized to the maximum value of $|E_{cp}| (\phi = 0^\circ, n = 0)$.

Figure 9.25: $|E_{cp}|$, calculated according to Millar and UTD/GTD. All patterns are normalized to the maximum value of $|E_{cp}| (\phi = 45^\circ, n = 0)$. 
Figure 9.26: $|E_{xp}|$, calculated according to Millar and UTD/GTD. All patterns are normalized to the maximum value of $|E_{ep}|$ ($\phi = 45^\circ$, $n = 0$).

Figure 9.27: $|E_{ep}|$ and $|E_{xp}|$, calculated according to UTD/GTD. All patterns are normalized to the maximum value of $|E_{ep}|$ ($\phi = 55^\circ$, $n = 0$).
Figure 9.28: $|E_{\varphi}|$ and $|E_{\varphi'}|$, calculated according to Millar. All patterns are normalized to the maximum value of $|E_{\varphi}| (\phi = 45^\circ)$. 

\textbf{Figure 9.28:} $|E_{\varphi}|$ and $|E_{\varphi'}|$, calculated according to Millar. All patterns are normalized to the maximum value of $|E_{\varphi}| (\phi = 45^\circ)$. 

\begin{center}
\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure9.28.png}
\caption{Normalized electric field pattern [dB].}
\end{figure}
\end{center}

\textbf{Figure 9.28:} $|E_{\varphi}|$ and $|E_{\varphi'}|$, calculated according to Millar. All patterns are normalized to the maximum value of $|E_{\varphi}| (\phi = 45^\circ)$. 

\begin{center}
\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure9.28.png}
\caption{Normalized electric field pattern [dB].}
\end{figure}
\end{center}
Chapter 10

Conclusions and recommendations

10.1 Introduction

In the preceding chapters many characteristics of the Fresnel antenna system with absorbing/transparent or phase-correcting zones were discussed and new theoretical models for a Fresnel antenna with metallic/transparent zones were presented. We now can draw our conclusions concerning the usability of a design procedure developed to optimize the overall efficiency \( \eta \) and the sidelobe envelope of the system and we look at the discrepancies between an existing theoretical model and new ones. Also, we suggest topics for further investigations. We shall describe the conclusions and recommendations in the same order as we have studied the different topics in this report.

10.2 Conclusions

In the first part of this report the optimization process of \( \eta \) and the sidelobe envelope were considered. Further the scan performances were investigated.

The behaviour of certain efficiency factors was studied, which resulted in a design procedure. This procedure requires the frequency \( f \), the diameter \( D \) and the desired gain of the antenna as input parameters and gives as output parameters those combinations of the focal distance \( F \) and feed parameter \( n \) at which the \( \eta - F \) curve has a local optimum. Computer simulations show that the \( \eta - F \) curve contains a number of local extremes which can easily be determined. This curve also demonstrates that both \( \eta \) and the sidelobe levels increase with increasing \( F \) and \( n \).

In order to compare the efficiencies and other characteristics of the Fresnel antenna with those of the parabolic reflector antenna, a similar design procedure was applied to a parabolic reflector antenna system. It was found that \( \eta \) is almost constant as a function of the optimal combinations of \( F \) and \( n \). Further simulations showed that the sidelobe envelope did not change notably for the combinations of \( F \) and \( n \) just mentioned. From calculated values it appeared that the parabolic reflector antenna system has an efficiency of \( \sim 81 \% \) which is very high compared to that of the Fresnel antenna system (\( \sim 11 \% \)) with
absorbing/transparent zones. The Fresnel antenna with phase-correcting zones \( P = 2 \) has a higher \( \eta \) namely \( \sim 32 \% \) and the \( \eta \) of a Fresnel antenna system with \( P = 4 \) is \( \sim 65 \% \) which approximates the overall efficiency of the parabolic reflector antenna conform the expectations. Only for \( P \to \infty \) the \( \eta \) of the Fresnel antenna becomes the \( \eta \) of the parabolic reflector antenna. When the scan performances were considered, the differences between both systems appeared to be smaller.

From computed results we found that the decrease of the gain at increasing feed displacement angle was slightly greater in case of the parabolic reflector antenna \( (F \approx 1.8 \text{ m}) \). For both systems the scan loss became greater in case of a smaller \( F \). The behaviour of the gain patterns at different feed displacement angles was quite different in case of a single Fresnel antenna system. At certain feed displacement angles these patterns often possessed relative 'high' sidelobes. When considering different Fresnel antenna systems they all showed the same trend.

The conclusions from the first part of the report are the following: it is possible to optimize the \( \eta \) and sidelobe envelope of a Fresnel antenna system with the parameters \( f, D \) and desired gain as input and parameters \( F \) and \( n \) as output. Certain design rules such as low sidelobe levels, large overall efficiency and small dimensions of the feed (characterized by \( n \)) conflict. This leads to the conclusion that we always have to compromise when designing an optimal antenna system. Therefore, designing an optimum Fresnel antenna system (in our way) is quite complex.

The parabolic reflector antenna is in almost all aspects superior to the Fresnel antenna except for its scan loss. The antenna efficiency and additional gain are higher and the design of an optimum parabolic reflector antenna system is less complicated. A global analysis of the scan performances indicates that both antenna systems show the same trend in case of the scan loss. The sidelobe envelope of the gain patterns of the Fresnel antenna at different feed displacement angles must receive attention, because its behaviour is quite 'arbitrarily'. This aspect can complicate the use of a Fresnel antenna for scan purposes. Yet, it should be noted that the formulae used for analyzing the scan performances are scalar. This implies that the influence of the polarization properties is not included in the analysis.

The second part of the report deals with some shortcomings of the theoretical models describing the radiation patterns of the Fresnel antenna. The two most important shortcomings are: the lack of a good mathematical or physical model for diffraction at absorbing material (used to cover the odd numbered Fresnel zones) and its possible failure due to the small width of a number of Fresnel zones.

It appeared to be almost impossible to find a model for diffraction at absorbing material which reasonably satisfies the idealized requirements such as infinite thin material and total absorption. In our case the best solution was replacing the absorbing material by a substance which causes a known phase shift and attenuation at the frequency used.

The second shortcoming was caused by the width \( d \) of the Fresnel zones. This width becomes very small for increasing radius \( \bar{b}_m \). The assumption that the aperture field was equal to the incident field of the feed did not longer apply in these situations. In order to
examine the influence of the 'small' zones on the radiation pattern, the absorbing material was replaced by perfectly conducting material. The change of the material of the zones was necessary because there existed no satisfactory model for diffraction at absorbing material. For the Fresnel antenna with metallic/transparent zones, the radiation patterns were calculated with the aid of UTD/GTD and subsequently compared with those calculated according to Kirchhoff's theory. The model of UTD/GTD included single diffracted rays as well as multiple diffracted rays. It was found that the radiation patterns according to UTD/GTD and Kirchhoff were almost the same if multiple diffraction was omitted. If multiple diffraction was included, the patterns differed clearly, especially the cross-polarization component. Because the validity of UTD/GTD had to be checked for very small values of $k \cdot d$ ($k \cdot d < 2$ or $d < \lambda/\pi$) a new method was developed which used Millar's solution for the slit problem. This method was applied to one single zone (annulus), just as UTD/GTD, in order to calculate the co- and cross-polarization components of the electric field. These components have globally the same shape for both diffraction calculation techniques.

From the theoretical analysis and the numerical calculations presented in the second part of this report, it was concluded that it is almost impossible to find a satisfying mathematical model for diffraction at absorbing material. In cases where the absorbing material has been replaced by perfectly conducting material, it is possible to calculate the gain patterns of the Fresnel antenna (excluding the boresight and its vicinity because UTD/GTD fails in that region) more precise and far more quicker with UTD/GTD than with Kirchhoff's diffraction integral. According to the analysis performed with these techniques it can be concluded that the widths of the zones have more influence on the radiation patterns than one would expect according to Kirchhoff's theory. The interaction of the incident field from the feed with the edges of a zone must not be neglected because it leads to an increase of the cross-polarization component as shown by UTD/GTD. In case of the co-polarization component UTD/GTD and Kirchhoff give similar results, so it is maybe possible in this case to substitute the aperture field by the incident field.

For very low $k \cdot d$, the validity of UTD/GTD remains questionable. On the assumption that the new method using Millar's solution is valid, the influence of the 'small' zones on the polarization properties is slightly different from those according to UTD/GTD. Therefore we can conclude that UTD/GTD is expected to give reasonable approximations of the radiation patterns of very small zones.

10.3 Recommendations

A Fresnel antenna should be constructed using the design procedure discussed in the beginning of this chapter, not only in order to check the theoretical models, but also to include all kinds of practical details which have not been considered in the present report. Furthermore, the behaviour of the gain patterns at different feed displacement angles must be investigated. This includes: calculating the gain patterns, using vectorial equations, and checking the validity of the scan surface chosen in case of the Fresnel antenna.
The new method using Millar’s solution should be checked by measurements. The value of the gain function near and in boresight must also be determined in case of an antenna with metallic/transparent zones. This could be done with Kirchhoff’s diffraction integral because this method gives for boresight probably good results as may be expected from the simulations. Another aspect which not has been considered yet, is the interaction of the fields of two adjacent transparent zones. Due to the geometrical configuration of the Fresnel zones, the distances between two successive transparent zones is in the same order of magnitude as the width of the transparent zones. Chou and Adams [10] have shown that the interaction between slits with width $\lambda/2$ at a distance of $0.1\lambda$ -the minimum distance between two transparent zones is in our case $\lambda/2$- is no longer negligible. This interaction could be investigated by calculating the radiation patterns using the multiple diffraction model presented in this report, because this model can be expanded with the multiple diffracted rays between edges of different transparent zones.
References


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Appendix A

Kirchhoff’s vectorial far-field equations for the parabolic reflector antenna

Using figure A.1 and the same feed specifications as in case of the Fresnel antenna, Leyten [32] derived the following equations:

\[ \mathbf{E}(r, n) = E_\theta(r, n) \cdot \hat{\theta} + E_\phi(r, n) \cdot \hat{\phi} \]  
\[ (A.1) \]
\[ \text{with} \]
\[ E_\theta(r, n) = 2\pi \cos(\phi) C(r) e^{ikF \cos(\theta)} \int_0^\psi O(\psi, n) e^{iM(\theta, \psi)} \cdot \cos(\psi/2) J_0(N(\theta, \psi)) d\psi \]  
\[ (A.3) \]
\[ E_\phi(r, n) = -2\pi \sin(\phi) C(r) e^{ikF \cos(\theta)} \int_0^\psi O(\psi, n) e^{iM(\theta, \psi)} \cdot \left[ \cos(\theta) \cos(\psi/2) J_0(N(\theta, \psi)) - j \sin(\theta) \sin(\psi/2) J_1(N(\theta, \psi)) \right] d\psi \]  
\[ (A.4) \]
\[ C(r) = \frac{jke^{-jkr}}{2\pi r} \sqrt{\frac{2P_t}{4\pi \sqrt{\mu_0 \epsilon_0}}} \]  
\[ (A.5) \]
\[ O(\psi, n) = \sqrt{G_{\text{feed}}(\psi, n) \frac{2F \tan(\psi/2)}{\cos(\psi/2)}} \]  
\[ (A.6) \]
\[ M(\theta, \psi) = \frac{-jkF}{\cos^2(\psi/2)} - jkF \cos(\theta) \frac{\cos(\psi)}{\cos^2(\psi/2)} \]  
(A.7)

\[ N(\theta, \psi) = 2kF \sin(\theta) \tan(\psi/2) \]  
(A.8)

Figure A.1: Geometrical configuration of the parabolic reflector antenna.
Appendix B

Kirchhoff's scalar far-field equations for the defocused parabolic reflector antenna

The definitions of the variables which will be used and the geometrical configuration of the defocused parabolic reflector antenna are shown in figure B.1. This is the same as figure 7.4, but is also given here for convenience. Before we can start with the derivation, first the definitions of some important characteristics -they are calculated amongst others in [32] or [51]- are given:

\[ \hat{n} = -\sin\left(\frac{\psi}{2}\right)\cos(\xi) \hat{x} - \sin\left(\frac{\psi}{2}\right)\sin(\xi) \hat{y} + \cos\left(\frac{\psi}{2}\right) \hat{z} \]

\[ \hat{p} = \rho \sin(\psi) \cos(\xi) \hat{x} + \rho \sin(\psi) \sin(\xi) \hat{y} - \rho \cos(\psi) \hat{z} \]

\[ \hat{r}' = \rho \sin(\psi) \cos(\xi) \hat{x} + \rho \sin(\psi) \sin(\xi) \hat{y} + (F - \rho \cos(\psi)) \hat{z} \]

\[ \hat{r} = r \sin(\theta) \cos(\phi) \hat{x} + r \sin(\theta) \sin(\phi) \hat{y} + r \cos(\theta) \hat{z} \]

\[ \epsilon = \sqrt{\epsilon_1^2 + \epsilon_0^2} \]

\[ \rho' = \rho \sqrt{\left(1 + \frac{2\epsilon_2 \sin(\psi) \cos(\xi)}{\rho} + \frac{2\epsilon_2 \cos(\psi)}{\rho} + \left(\frac{\epsilon}{\rho}\right)^2\right)} \]
With the help of the previous definitions, the field pattern of the parabolic antenna will be calculated according to Kirchhoff’s scalar diffraction integral of which the corresponding geometrical configuration is displayed in figure B.2:

\[ E_{\text{feed}} = \sqrt{P_i Z_0 2(n + 1) \cos^n(\psi')} \frac{e^{-jkr'}}{r'} \]  

(B.6)

**Figure B.1:** Geometrical configuration of the defocused parabolic reflector antenna.

This is the same method used by Jeronimus [20] to derive the far-field patterns of the defocused Fresnel antenna. A question that arises now is: which aperture to use?

In first instance the surface of the paraboloid will function as the antenna aperture, which means integrating according to the variables \( \psi \) and \( \xi \) (also used by Leyten [32]). Then expressions involving \( \psi \) and \( \xi \) will be transformed to the cylindrical reference frame, denoted by \( l \) and \( \phi' \), which defines the pseudo aperture that can be seen in figure B.3. This pseudo aperture is the same as the aperture of the Fresnel antenna, which is the purpose of the transformation.
Appendix B. Kirchhoff's scalar far-field equations for the defocused.

Figure B.2: Geometrical configuration required for Kirchhoff's scalar diffraction integral.

Figure B.3: Definition of the pseudo aperture.
The transformation of the surface aperture into the pseudo aperture is performed according to the following relations:

\[ dA = \sqrt{1 + \left( \frac{l}{2F} \right)^2} \, dA_1 \quad \text{(B.8)} \]

\[ \phi' = \xi \quad \text{(B.9)} \]

\[ l_{\max} = \frac{D}{2} \quad \text{(radius)} \quad \text{(B.10)} \]

Some important relations which are needed for further calculations due to the aperture transformation, are:

\[ \cos(\psi) = \frac{F - \frac{l^2}{4F}}{\rho} \quad \Rightarrow \quad \cos(\psi) = \frac{(1 + \cos(\psi)) (F - \frac{l^2}{4F})}{2F} \quad \text{(B.11)} \]

\[ \rho = \frac{2F}{1 + \cos(\psi)} \]

\[ \sin(\psi) = \frac{l}{\rho} \quad \Rightarrow \quad \sin(\psi) = \frac{l(1 + \cos(\psi))}{2F} \quad \text{(B.12)} \]

\[ \cos(\psi) = \frac{1 - \left( \frac{l}{2F} \right)^2}{1 + \left( \frac{l}{2F} \right)^2} \]

\[ \rho = F + \frac{l^2}{4F} \quad \text{(B.13)} \]

\[ \cos \left( \frac{\psi}{2} \right) = \frac{1}{\sqrt{1 + \left( \frac{l}{2F} \right)^2}} \quad \text{(B.14)} \]

\[ \sin \left( \frac{\psi}{2} \right) = \frac{\left( \frac{l}{2F} \right)^2}{\sqrt{1 + \left( \frac{l}{2F} \right)^2}} \quad \text{(B.15)} \]
Appendix B. Kirchhoff's scalar far-field equations for the defocused....

After these expressions are determined, we can focus on the calculation of the gain function. There are four characteristic terms which have to be calculated in order to determine \( G(\hat{r}, \hat{n}) \):

- \( R \)
- \( \cos(\psi') \)
- \( (\hat{n} \cdot \hat{S}') \)
- \( (\hat{n} \cdot \hat{r}) \)

These four terms will now be considered.

1. Calculation of \( R \).

This variable can be expressed in terms of \( r \) and \( r' \). Further, expressions with \( \psi \) and \( \xi \) have to be replaced by terms with \( \lambda \) and \( \phi' \), yielding:

\[
\begin{align*}
|\vec{R}| & = |r - (\vec{r}' \cdot \hat{r}')| \\
|\vec{R}| & = R = r - \rho \sin(\psi) \sin(\theta) \cos(\phi - \xi) - F \cos(\theta) + \rho \cos(\psi) \cos(\theta) \\
\rho \sin(\psi) & = l \\
\rho \cos(\psi) & = F \left(1 - \left(\frac{1}{2F}\right)^2\right) \\
\phi' & = \xi
\end{align*}
\]

\[
R = r - F \cos(\theta) - l \sin(\theta) \cos(\phi - \phi') + F \left(1 - \left(\frac{l}{2F}\right)^2\right) \cos(\theta)
\]

\[
\Rightarrow R = r - l \sin(\theta) \cos(\phi - \phi') - \left(\frac{l^2}{4F}\right) \cos(\theta)
\]  
(B.16)

The far-field approximations can also be applied to the term \( 1/R \), which means that in the denominator \( R \) can be approximated by \( r \). These approximations will be used in the formula (B.7) which will result in the far-field function (B.26) and the gain function (B.27).
2. Calculation of \( \cos(\psi') \).

The term \( \cos(\psi') \) can, according to figure 7.4, be written as a function of \( I, F, \rho', \epsilon_x \) and \( \epsilon_z \):

\[
\cos \psi' = \frac{I^2 + \left( \frac{\rho}{4I} \right)^2 - (F + \epsilon_z)^2 - (\rho')^2}{-2\sqrt{((F + \epsilon_z)^2 + \epsilon_z^2)^2}}
\]

3. Calculation of \( \hat{n} \cdot \hat{S}' \).

If the paraboloid is focused then \( \hat{n} \cdot \hat{S}' \) can easily be calculated:

\[
\hat{n} \cdot \hat{S}' = \left| \hat{n} \right| \cdot \left| \hat{S}' \right| \cdot \cos \left( \frac{\psi}{2} \right)
\]

If that is not the case then with the relations

\[
\hat{n} \cdot \hat{S}' = \frac{1}{\rho'} (\hat{n} \cdot \hat{p}')
\]

and equations (B.6) and (B.1) the term \( \hat{n} \cdot \hat{S}' \) becomes:

\[
\hat{n} \cdot \hat{S}' = \frac{1}{\rho'} (\hat{n} \cdot \hat{p}') = -\rho \cos \left( \frac{\psi}{2} \right) + \epsilon_x \sin \left( \frac{\psi}{2} \right) \cos(\psi') - \epsilon_z \cos \left( \frac{\psi}{2} \right)
\]
4.Calculation of \((\hat{n} \cdot \hat{r})\).

According to the equations (B.1) and (B.6) this term can be expressed as:

\[
(\hat{n} \cdot \hat{r}) = \left( -\sin \left( \frac{\psi}{2} \right) \cos(\xi) \right) (\sin(\theta) \cos(\phi)) + \\
\left( -\sin \left( \frac{\psi}{2} \right) \sin(\xi) \right) (\sin(\theta) \sin(\phi)) + \\
\left( \cos \left( \frac{\psi}{2} \right) \cos(\theta) \right) \]  \hspace{1cm} (B.24)

\[
\phi' = \xi \Rightarrow (\hat{n} \cdot \hat{r}) = -\sin \left( \frac{\psi}{2} \right) \sin(\theta) \cos(\phi - \phi') + \cos \left( \frac{\psi}{2} \right) \cos(\theta) \]  \hspace{1cm} (B.25)

Now that all the characteristics are determined, the formula for the far-field (E-field) can be determined:

\[
E(\hat{r}, n) = \frac{jk e^{-jkr}}{4\pi r} \int_0^{2\pi} \int_0^{D/2} E_{\text{feed}} \cdot \left[ (\hat{n} \cdot \hat{r}) + (\hat{n} \cdot \hat{S}') \right] \cdot \sqrt{1 + \left( \frac{\rho'}{2F} \right)^2} \cdot \\
e^{j\leftnow{1}{0}(\sin(\theta) \cos(\phi - \phi') + \frac{\rho' \cos(\theta)}{2F}) \right) dl d\phi' \]  \hspace{1cm} (B.26)

Substituting the previous formula in equation (3.7) results in the desired gain function:

\[
G(\hat{r}, n) = 10 \log \left[ \frac{\lambda + 1}{2\lambda^2} \int_0^{2\pi} \int_0^{D/2} E'_{\text{feed}} \cdot \left[ (\hat{n} \cdot \hat{r}) + (\hat{n} \cdot \hat{S}') \right] \cdot \sqrt{1 + \left( \frac{l}{2F} \right)^2} \cdot e^{hl d\phi'} \right]^2 \]

with

\[
E'_{\text{feed}} = \frac{1}{\rho'} \cdot \sqrt{\cos^n(\psi')} \]

\[
h = jk \left( -\rho' + l \sin(\theta) \cos(\phi - \phi') + \frac{l^2 \cos(\theta)}{4F} \right) \]
Appendix C

Efficiencies and other characteristics of the parabolic reflector antenna

These formulae are derived using Appendix A and the definitions of Chapter 3.

Spillover efficiency:

\[ \eta_s = 1 - \cos^{n+1}(\psi_0) \quad \text{(C.1)} \]

Illumination efficiency:

\[ \eta_i = \frac{\left| \frac{1}{D} \int_0^{\psi_0} 2F \sqrt{2(n+1)\cos^n(\psi)} \tan(\psi/2)e^{-jKF \frac{1+\cos(\psi)}{\cos^2(\psi/2)}}d\psi \right|^2}{1 - \cos^{n+1}(\psi_0)} \quad \text{(C.2)} \]

Overall antenna efficiency:

\[ \eta = \frac{\left| \frac{1}{D} \int_0^{\psi_0} 2F \sqrt{2(n+1)\cos^n(\psi)} \tan(\psi/2)e^{-jKF \frac{1+\cos(\psi)}{\cos^2(\psi/2)}}d\psi \right|^2}{1 - \cos^{n+1}(\psi_0)} \quad \text{(C.3)} \]

additional gain [dBi]:

\[ G_{aid} = 10 \log \left( \frac{G_0}{G_{feed}} \right) \quad \text{(C.4)} \]

Edge illumination [dB]:

\[ G_{edge} = n \cdot 10 \log (\cos(\psi_0)) + 20 \log \left( \cos^2(\psi_0/2) \right) \quad \text{(C.5)} \]
Appendix D

Derivation of a simple expression for $\eta_p$

Deriving a simple formula for $\eta_p$ involves knowledge of the graphical representation in the complex plane of the E-field of one zone in boresight. According to equations (3.4) and (3.5) this field contains only a component in $x$-direction and can be expressed as (see figure D.1):

$$E_x(\alpha, n) = \int_{\psi_m}^{\alpha} \pi F(\cos(\psi) + 1) \sqrt{2(n + 1) \cos^n(\psi)} \frac{\sin(\psi)}{\cos(\psi)} e^{-j\left(k_F \frac{\cos(\psi)}{P} - \frac{m2\pi}{P}\right)} d\psi \quad (D.1)$$

The graphical representation in the complex plane of formula (D.1) is a curve $A$, with length $K_2$, which is shown in figure D.2. The modulus of formula (D.1) describes a curve $B$ with length $K_1$ and is given by:

$$K_1(\alpha, n) = \left| \int_{\psi_m}^{\alpha} \pi F(\cos(\psi) + 1) \sqrt{2(n + 1) \cos^n(\psi)} \frac{\sin(\psi)}{\cos(\psi)} e^{-j\left(k_F \frac{\cos(\psi)}{P} - \frac{m2\pi}{P}\right)} d\psi \right|$$

$$\Rightarrow \quad K_1(\alpha, n) = |X(\alpha, n) - jY(\alpha, n)| \quad (D.2)$$

with

$$X(\alpha, n) = \int_{\psi_m}^{\alpha} \pi F(\cos(\psi) + 1) \sqrt{2(n + 1) \cos^n(\psi)} \cos \left( \frac{kF}{\cos(\psi)} - \frac{m2\pi}{P} \right) d\psi \quad (D.3)$$

$$Y(\alpha, n) = \int_{\psi_m}^{\psi_{m+1}} \pi F(\cos(\psi) + 1) \sqrt{2(n + 1) \cos^n(\psi)} \sin \left( \frac{kF}{\cos(\psi)} - \frac{m2\pi}{P} \right) d\psi$$

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Consider the case $\alpha = \psi_{m+1}$. In that specific case the argument of $E_z(\alpha, n)$ is equally for all zones which means that all $B$-curves with lengths $K_1$ line up: the modulus of the total field is equal to the sum of the modulo of the fields of the separate zones. This is depicted in figure D.3. Further, the curve $A$ with length $K_2$ can be calculated using

$$K_2 = \int_{\psi_m}^{\psi_{m+1}} \sqrt{\left(\frac{dX}{d\alpha}\right)^2 + \left(\frac{dY}{d\alpha}\right)^2} \, d\alpha$$  \hspace{1cm} (D.4)$$

which results in:

$$K_2 = \int_{\psi_m}^{\psi_{m+1}} \pi F (\cos(\alpha) + 1)\sqrt{2(n + 1)\cos^n(\alpha)\frac{\sin(\alpha)}{\cos^2(\alpha)}} \, d\alpha$$  \hspace{1cm} (D.5)$$
Appendix D. Derivation of a simple expression for $\eta_p$

On the condition that the edge of the antenna and a Fresnel circle coincide which is always the case if $\alpha = \psi_{m+1}$, the sum of $K_2$'s equals the denominator of the formula (3.10). The numerator of this same formula equals in that case the sum of $K_1$'s. The phase efficiency can therefore be written as:

$$\eta_p = \left( \frac{\sum_{m} K_1}{\sum_{m} K_2} \right)^2$$

(D.6)

A question that arises now is: what is the relation between $K_1$ and $K_2$ or between $E_{K1}$ and $E_{K2}$?

Computer simulations show that the value of $\eta_p$ is independent of $n$ (if $n$ is not too large) on condition that a Fresnel circle coincides with the antenna edge. That is why we assume that $n$ has negligible influence on $\eta_p$ and can be set to zero. This yields for equation (D.1):

$$n = 0 \rightarrow E_x(\psi_{m+1}) = C_1 \int_{\psi_m}^{\psi_{m+1}} \frac{\sin(\psi)}{\cos^2(\psi)} \left( \cos(\psi) + 1 \right) F e^{-i \frac{A_p}{\cos(\psi)}} d\psi$$

(D.7)

with $C_1 = \sqrt{2} \pi F e^{i \frac{m \pi}{2}}$
With the help of partial integration, equation (D.7) can be expressed as:

\[
E_x(\psi_{m+1}) = \frac{iC_1}{kF} \left( \frac{1 + \cos(\psi_{m+1}) + \frac{j \cos^2(\psi_{m+1})}{kF}}{Q_1} \right) e^{\frac{-j k F}{\cos(\psi_{m+1})}} - \left( \frac{1 + \cos(\psi_m) + \frac{j \cos^2(\psi_m)}{kF}}{Q_2} \right) e^{\frac{-j k F}{\cos(\psi_m)}} \right) e^{\frac{-j k F}{\cos(\psi_m)}} \right) e^{\frac{-j k F}{\cos(\psi_m)}} d\psi \]

(D.8)

In order to simplify equation (D.8) the expressions \( Q_1, Q_2 \) and \( Q_3 \) are evaluated:

- **\( Q_1 \):**

\[
Q_1 = \sqrt{[1 + \cos(\psi_{m+1})]^2 + \left( \frac{\cos^2(\psi_{m+1})}{kF} \right)^2 \frac{e^{\frac{-j k F}{\cos(\psi_{m+1})}} e^{j \arctan\left( \frac{\cos^2(\psi_{m+1})}{kF[1 + \cos(\psi_{m+1})]} \right)}}} \]

(D.9)

\[
Q_1 \simeq \frac{1 + \cos(\psi_{m+1})}{e^{\frac{-j k F}{\cos(\psi_{m+1})}}} \]

\[
kF \gg \cos^2(\psi_{m+1}) \\
cos(\psi_m) > 0 \\
cos(\psi_{m+1}) > 0 \]

- **\( Q_2 \):**

Applying the same assumptions as in case of \( Q_1 \) lead to:

\[
Q_2 \simeq [1 + \cos(\psi_m)] e^{\frac{-j k F}{\cos(\psi_m)}} \]

(D.10)
Evaluating $Q_3$:
The contribution of $Q_3$ is negligible. This can be proven with help of:

\[
|f(z)| \leq \text{MAX}, \quad \int K ds = L
\]

\[
\left|\int f(z) dz\right| \leq \text{MAX} \cdot L \tag{D.11}
\]

Applying previous condition to $Q_3$ gives:

\[
z = \cos(\psi) \quad \rightarrow \quad Q_3 = \frac{-2j}{kF} \int_{x_m}^{z_{m+1}} z e^{-jkr} dz \quad (z_m = \cos(\psi_m))
\]

\[
\rightarrow \quad f(z) = \frac{2zj}{kF} e^{-jkr}
\]

\[
\rightarrow \quad L = \cos(\psi_m) - \cos(\psi_{m+1})
\]

\[
\rightarrow \quad \text{MAX} = \frac{2}{kF} \cos(\psi_{m+1}) \tag{D.12}
\]

A property of the Fresnel antenna can be used to find an upper limit of $L$. Namely, the property that $\delta$ is constant for all zones of the antenna when a Fresnel circle coincides with the antenna edge:

\[
\left(\frac{1}{\cos(\psi_m)} - \frac{1}{\cos(\psi_{m+1})}\right) = -A_p \quad (A > 0) \tag{D.13}
\]

This property leads to the following estimations:

\[
L = \cos(\psi_m) - \cos(\psi_{m+1}) = A_p \cos(\psi_{m+1}) \cos(\psi_m)
\]

\[
\Rightarrow \quad |Q_3| \leq \text{MAX} L = \frac{2}{kF} \cos(\psi_m)[\cos(\psi_m) - \cos(\psi_{m+1})]
\]

\[
\Rightarrow \quad |Q_3| \leq \frac{2A}{kF} \cos(\psi_{m+1}) \cos^2(\psi_m)
\]

\[
\Rightarrow \quad |Q_3| \leq \frac{2A}{kF} \cos(\psi_1)
\]
The factor \( \frac{2A}{kF} \cos(\psi_1) \) only depends on \( F \) and \( \psi_1 \) and is very small, relative to the modulus of \( Q_1 \) and \( Q_2 \), on condition that \( kF \gg \frac{2A}{kF} \cos(\psi_1) \). In table D.1 some values of this factor are given. These values show that the contribution of \( Q_3 \) is indeed negligible if \( kF \) does not become too small in relation to \( 2A \cos(\psi_1) \).

Table D.1: Estimation of influence of \( Q_3 \) on \( E_{\pi} \) (\( f=11.1 \; \text{GHz}, \; D=1 \; \text{m} \)).

<table>
<thead>
<tr>
<th>( F ) ( m )</th>
<th>( \frac{2A}{kF} A \cos(\psi_1) ) ( (P=2) )</th>
<th>( \frac{2A}{kF} A \cos(\psi_1) ) ( (P=4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.01024</td>
<td>0.00542</td>
</tr>
<tr>
<td>0.5</td>
<td>0.00044</td>
<td>0.00022</td>
</tr>
<tr>
<td>1.0</td>
<td>0.00011</td>
<td>0.00006</td>
</tr>
<tr>
<td>1.5</td>
<td>0.00005</td>
<td>0.00003</td>
</tr>
<tr>
<td>2.0</td>
<td>0.00003</td>
<td>0.00001</td>
</tr>
</tbody>
</table>

Using the approximations of the \( Q \)-factors formula (D.8) becomes:

\[
E_{\pi}(\psi_{m+1}) \approx C_1 \frac{j}{kF} \left( [1 + \cos(\psi_{m+1})]e^{-\frac{j\pi P}{4m+1}} - [1 + \cos(\psi_m)]e^{-\frac{j\pi P}{4m}} \right) \quad (D.14)
\]

Formula (D.14) is simpler than formula (D.7) but does it already give a clue concerning any relation between \( K_1 \) and \( K_2 \)? For an answer we have to examine figure D.4. This figure is basically the same as figure D.2, only some new curves and the angle \( \delta \) (\( =2\pi/P \)) are introduced.

The curves with length \( L_1 \) and \( L_2 \) are part of arcs and the curves with length \( L_3 \) and \( L_4 \) are straight lines. These lengths can be determined according to the geometrical configuration displayed in figure D.5.

\[
K_a^2 = R^2 + R^2 - 2R^2 \cos(\delta) \quad (D.15)
\]

\[
K_a = \sqrt{2R\sqrt{1 - \cos(\delta)}} \quad \Rightarrow \quad K_a = 2R \sin\left(\frac{\delta}{2}\right) \quad (D.16)
\]

\[
K_b = R \delta \quad (D.17)
\]
Appendix D. Derivation of a simple expression for $\eta_p$

Figure D.4: Approximating certain curves by arcs.

Figure D.5: Geometrical configuration used to derive expressions for $L_1$, $L_2$, $L_3$ and $L_2$. 
Formula (D.16) can be applied to $L_3$ and $L_4$, formula (D.17) to $L_1$ and $L_2$ yielding:

$$
\begin{align*}
L_3 &= 2 \sin \left( \frac{\delta}{2} \right) \left[ 1 + \cos(\psi_m) \right] \\
L_4 &= 2 \sin \left( \frac{\delta}{2} \right) \left[ 1 + \cos(\psi_{m+1}) \right]
\end{align*}
\Rightarrow
\Delta L_{34} = L_3 - L_4 \Leftrightarrow
\Delta L_{34} = 2 \sin \left( \frac{\delta}{2} \right) A \cos(\psi_m) \cos(\psi_{m+1})
$$

$$
\begin{align*}
L_1 &= \delta \left[ 1 + \cos(\psi_m) \right] \\
L_2 &= \delta \left[ 1 + \cos(\psi_{m+1}) \right]
\end{align*}
\Rightarrow
\Delta L_{12} = L_1 - L_2 \Leftrightarrow
\Delta L_{12} = \delta \left[ \cos(\psi_m) - \cos(\psi_{m+1}) \right] \Leftrightarrow
\Delta L_{12} = \delta A \cos(\psi_m) \cos(\psi_{m+1})
$$

The previous equations make it possible to approximate $K_1$ and $K_2$:

$$
\begin{align*}
K_1 &\simeq 2 \sin \left( \frac{\delta}{2} \right) \left[ 1 + \cos(\psi_m) + \Delta K_{1m} \right] \quad \text{(D.18)} \\
K_2 &\simeq \delta \left[ 1 + \cos(\psi_m) + \Delta K_{2m} \right] \quad \text{(D.19)}
\end{align*}
$$

with $\Delta K_{1m}, \Delta K_{2m} \leq \Delta_m = A \cos(\psi_m) \cos(\psi_{m+1})$

Substituting (D.18) and (D.19) in equation (D.6) results in equation (D.20):

$$
\sqrt{\eta_p} = \frac{2 \sin \left( \frac{\delta}{2} \right)}{\delta} \cdot \frac{\left( \sum_m \left[ 1 + \cos(\psi_m) + \Delta K_{1m} \right] \right)}{\left( \sum_m \left[ 1 + \cos(\psi_m) + \Delta K_{2m} \right] \right)}
$$

(D.20)

The difference $\Delta_m$ is very small compared to the term $\left[ 1 + \cos(\psi_m) \right]$. Because the same applies for $K_1$ and $K_2$, it is allowed to approximate $\sqrt{\eta_p}$ in the following way:

$$
\sqrt{\eta_p} = \frac{2 \sin \left( \frac{\delta}{2} \right)}{\delta} \frac{1 + \Delta K_1}{1 + \Delta K_2}
$$

(D.21)

with

$$
\begin{align*}
\Delta K_1 &= \frac{\sum_m \Delta K_{1m}}{\sum_m A_m} \\
\Delta K_2 &= \frac{\sum_m \Delta K_{2m}}{\sum_m A_m}
\end{align*}
$$

(D.22) (D.23)
Appendix D. Derivation of a simple expression for $\eta_p$

in which

$$A_m = 1 + \cos(\psi_m)$$  \hspace{1cm} (D.24)

Due to the relations

$$\frac{1 + \Delta K_1}{1 + \Delta K_2} = \frac{1}{1 + \Delta K_2} + \frac{\Delta K_1}{1 + \Delta K_2}$$  \hspace{1cm} (D.25)

and

$$1 - \Delta K_2 + \Delta K_1 \cdot (1 - \Delta K_2) \simeq 1 + \Delta K_1 - \Delta K_2 \simeq 1 \pm \frac{\sum A_m}{\sum A_m}$$

\[ \Leftrightarrow \left( 1 \pm \frac{\sum A_m}{\sum A_m} \right)^2 \simeq 1 \pm 2 \cdot \frac{\sum A_m}{\sum A_m} \]  \hspace{1cm} (D.26)

the phase efficiency can be expressed as:

$$\Rightarrow \eta_p \simeq \left( \frac{2 \sin \left( \frac{\delta}{2} \right)}{\delta} \right)^2 \left( 1 \pm 2 \cdot \frac{\sum A_m}{\sum A_m} \right)$$  \hspace{1cm} (D.27)

If the error $\chi$ is small then it is allowed to approximate the value of $\eta_p$ by:

$$\eta_p \simeq \left( \frac{\sin \left( \frac{\delta}{2} \right)}{\frac{\delta}{2}} \right)^2$$  \hspace{1cm} (D.28)

The error $\chi$ is calculated for two different values of $F$ (0.5 m and 2.0 m) according to table D.2 in case of absorbing/transparent zones. The results are:

- $F=0.5$ m: $\chi = 0.022$
- $F=2.0$ m: $\chi = 0.006$

The error $\chi$ appears to be very small and it is therefore allowed to approximate the phase efficiency by formula (D.28).
Table D.2: Values needed for the calculation of $\chi$ ($f=11.1$ GHz, $D=1$ m).

<table>
<thead>
<tr>
<th>$F$</th>
<th>0.5 m</th>
<th>0.5 m</th>
<th>2.0 m</th>
<th>2.0 m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta_m$</td>
<td>$A_m$</td>
<td>$\Delta_m$</td>
<td>$A_m$</td>
</tr>
<tr>
<td>0</td>
<td>0.026</td>
<td>2.00</td>
<td>0.0067</td>
<td>2.00</td>
</tr>
<tr>
<td>2</td>
<td>0.024</td>
<td>1.95</td>
<td>0.0065</td>
<td>1.97</td>
</tr>
<tr>
<td>4</td>
<td>0.021</td>
<td>1.90</td>
<td>0.0064</td>
<td>1.97</td>
</tr>
<tr>
<td>6</td>
<td>0.020</td>
<td>1.86</td>
<td>0.0062</td>
<td>1.96</td>
</tr>
<tr>
<td>8</td>
<td>0.018</td>
<td>1.82</td>
<td>0.0060</td>
<td>1.95</td>
</tr>
<tr>
<td>10</td>
<td>0.016</td>
<td>1.79</td>
<td>0.0059</td>
<td>1.94</td>
</tr>
</tbody>
</table>
Appendix E

Series expansion of an integral

It is possible to write the integrals (7.3) and (7.5) (of one zone) in the general form:

\[ E \sim \int_a^b \int_0^{2\pi} f(\phi', \theta)e^{jkh(\phi', \theta)} d\phi' d\theta \]  \hspace{1cm} (E.1)

with \( h(\phi', \theta) = -N\sqrt{1 - W\cos(\phi')} + U\cos(\phi') + U_1 \)

where \( a = b_m, b = b_{m+1} \) and \( o = r' \) in case of the Fresnel antenna and \( a = 0, b = D/2 \) and \( o = l \) in case of the parabolic reflector antenna. The phase function is represented by \( h(\phi', \theta) \) which is the same function as defined in Chapter 7. The aperture illumination \( f(\phi', \theta) \) is similar to \( g(\phi', \theta) \) but requires some extra attention. It is assumed that \( f(\phi', \theta) \) remains the same as in the focused situation during the scan process, so that \( f(\phi', \theta) \) becomes \( \phi' \)-independent. This is true in case of the Fresnel antenna but not in case of the parabolic antenna. Namely, the term \([\hat{n} \cdot \hat{r}]\) contains still the expression \([\tan(\psi/2)\sin(\theta)\cos(\phi')]\). However, this expression can be neglected compared to the other terms of \([\hat{n} \cdot \hat{r}]\) and \([\hat{n} \cdot \hat{S}']\).

Hence \( f(\phi', \theta) \) of the paraboloid is also \( \phi' \)-independent when the aperture illumination is assumed to remain unchanged during the scan process. The variables \( N, W, U \) and \( U_1 \) are exactly the same as those of Chapter 7.

The term \( \sqrt{1 - W\cos(\phi')} \) of \( h(\phi', \theta) \) can be expanded into a series according to the following equation (E.2):

\[ \sqrt{1 - x} = 1 - \frac{1}{2} x - \frac{1}{8} x^2 \ldots \quad |x| < 1 \]  \hspace{1cm} (E.2)

Considering only the first three terms of the series yields for the phase function:

\[ h(\phi', \theta) \approx -N \left[ 1 - \frac{W}{2} \cos(\phi') - \frac{W^2}{8} \cos^2(\phi') \right] + U\cos(\phi') + U_1 \]
The phase function is now divided into two parts; one is \( \phi' \)-dependent and the other is \( \phi' \)-independent. Only the part of the integral which is \( \phi' \)-independent, is of importance and will be considered. Using the variables:

\[
C_1 = \sqrt{\frac{NW^2}{8}}
\]

\[
C_2 = \sqrt{\frac{2}{NW^2}} \left( U + \frac{NW}{2} \right)
\]

and the fact that the integral is symmetric relative to the \( xx \)-plane (the plane of feed displacements) as function of \( \phi' \), it becomes:

\[
I = 2 \int_0^{\pi} e^{ik[C_1 \cos(\phi')+C_2]^2} d\phi'
\]

A problem in solving (E.5) is the e-power in combination with the square of the phase function. The only way to solve this equation is to expand the e-power in a series:

\[
I = 2 \int_0^{\pi} \left\{ 1 + \frac{ik}{11}[C_1 \cos(\phi')+C_2]^2 + \frac{(ik)^2}{21}[C_1 \cos(\phi')+C_2]^4 + \frac{(ik)^3}{31}[C_1 \cos(\phi')+C_2]^6 + \frac{(ik)^4}{41}[C_1 \cos(\phi')+C_2]^8 \ldots \right\} d\phi'
\]

This expression can be divided into a real and imaginary part:

\[
I = 2 \int_0^{\pi} \left\{ 1 - \frac{k^2}{31}[C_1 \cos(\phi')+C_2]^4 + \frac{k^4}{41}[C_1 \cos(\phi')+C_2]^8 - \ldots \right\} d\phi'
\]

\[
+ 2j \int_0^{\pi} \left\{ \frac{k}{11}[C_1 \cos(\phi')+C_2]^2 - \frac{k^3}{31}[C_1 \cos(\phi')+C_2]^6 + \ldots \right\} d\phi'
\]

With the aid of two goniometrical relations

\[
(C_1 \cos(\phi')+C_2)^n = (C_2)^n \sum_{j=0}^n \binom{n}{j} \left( \frac{C_1 \cos(\phi')}{C_2} \right)^j
\]
\[
\int_0^\pi \cos^n(\phi) d\phi = \begin{cases} \frac{(n-1)!!}{n!!} \pi & n = 0, 2, 4, 6, \ldots \\ 0 & n = 1, 3, 5, 7, \ldots \end{cases}
\]

it is possible to express formula (E.7) as a double series:

\[
I = 2 \sum_{i=0}^{\infty} \frac{k^{2i}(-1)^i}{(2i)!} \int_0^\pi [C_1 \cos(\phi') + C_2]^{4i} d\phi' +
\]

\[
2j \sum_{i=0}^{\infty} \frac{k^{2i+1}(-1)^i}{(2i+1)!} \int_0^\pi [C_1 \cos(\phi') + C_2]^{2(2i+1)} d\phi' \quad (E.8)
\]

\[
I = 2 \sum_{i=0}^{\infty} \frac{k^{2i}(-1)^i}{(2i)!} \int_0^\pi (C_2)^{4i} \sum_{j=0}^{4i} \binom{4i}{j} \left( \frac{C_1 \cos(\phi')}{C_2} \right)^j d\phi' +
\]

\[
2j \sum_{i=0}^{\infty} \frac{k^{2i+1}(-1)^i}{(2i+1)!} \int_0^\pi (C_2)^{2(2i+1)} \sum_{j=0}^{2(2i+1)} \binom{2(2i+1)}{j} \left( \frac{C_1 \cos(\phi')}{C_2} \right)^j d\phi' \quad (E.9)
\]

\[
I = 2\pi \sum_{i=0}^{\infty} \frac{k^{2i}(-1)^i}{(2i)!} (C_2)^{4i} \left[ \sum_{j=0}^{2i} \binom{4i}{2j} \left( \frac{C_1}{C_2} \right)^{2j} \frac{(2j-1)!!}{(2j)!} \right] +
\]

\[
2\pi j \sum_{i=0}^{\infty} \frac{k^{2i+1}(-1)^i}{(2i+1)!} (C_2)^{2(2i+1)} \left[ \sum_{j=0}^{2(2i+1)} \binom{2(2i+1)}{2j} \left( \frac{C_1}{C_2} \right)^{2j} \frac{(2j-1)!!}{(2j)!} \right] \quad (E.10)
\]

Formula (E.10) gives the solution for the integral as function of \( \phi' \), of which the convergence is very important. This formula indicates that the series converges for \( C_2 \neq 0 \) because the denominator \((2i)!!\) increases eventually more quickly than the numerator \( k^{2i} \) due to the faculty. For \( C_2 = 0 \) the integral converts into a Bessel function of the first kind (see also in Chapter 7 the equivalent case where \( W = 0 \)). The problem is how quickly the series converges. From a logical point of view a quick convergence is always guaranteed when the combinations of \( C_1, C_2 \) and \( k \) are smaller than 1. This results in the following conditions for quick convergence:

\[
C_2 < \frac{1}{\sqrt{k}} \quad \land \quad C_1 < C_2 \quad (E.11)
\]

Under the previous conditions, equation (E.10) is usable. If these conditions are not satisfied then the series converges in general only after hundreds of terms, which requires much computing time.
Appendix F

Beam Deviation Factor

The Beam Deviation Factor (BDF) of the antenna systems in Chapter 6 is shown in the figures F.1 and F.2. We can see clearly the differences between the systems with $F$ approximately 0.5 m and those with $F$ approximately 1.9 m.

The fact that the curves of the figures just mentioned are not so smooth, can be caused by roundoff errors. Namely, the values of BDF are calculated with intervals of 0.4°. This leads to deviations of $\pm 0.4^\circ$ which could cause the discontinuities of the BDF-functions.

![Graph showing beam deviation factor for different systems with $F$ approximately 0.5 m.](image)

Figure F.1: Beam deviation factor of the systems with $F$ approximately 0.5 m.
Figure F.2: Beam deviation factor of the systems with $F$ approximately 1.9 m.
Appendix G

Diffraction coefficients for the multiple diffraction configuration

This Appendix is divided in several parts. The diffracted fields will be calculated separately for the upper and lower diffraction points for each situation. The parameters used for these expressions, such as $[D_{ia}^2]_g$, are explained and also the vector relations based on the specifications of the feed (equations (9.33) and (9.34)) are given.

Upper diffraction points $Q_{ia}$ and $Q_{ib}$

Situation I:

$$
\begin{align*}
\begin{pmatrix} E^d_\beta \\ E^d_\phi \end{pmatrix} &= \vec{E}_1^d = [D_{ia}^2]_g \cdot [D_{ia}^1]_g \cdot \sum_{i=0}^{\infty} ([D^a]_g \cdot [D^b]_g)^i \cdot (E^i_\beta/E^i_\phi) \Rightarrow (G.1) \\
\vec{E}_1^d &= \frac{[D_{ia}^1]_g \cdot [D_{ia}^2]_g}{1 - [D^a]_g \cdot [D^b]_g} \cdot \begin{pmatrix} E^i_\beta \\ E^i_\phi \end{pmatrix} \Rightarrow (G.2)
\end{align*}
$$

Situation II:

$$
\begin{align*}
\begin{pmatrix} E^d_\beta \\ E^d_\phi \end{pmatrix} &= \vec{E}_2^d = [D_{ia}^2]_h \cdot [D_{ia}^1]_h \cdot [D^b]_h \cdot \sum_{i=0}^{\infty} ([D^a]_h \cdot [D^b]_h)^i \cdot (E^i_\beta/E^i_\phi) \Rightarrow (G.3) \\
\vec{E}_2^d &= \frac{[D_{ia}^1]_h \cdot [D_{ia}^2]_h \cdot [D^b]_h}{1 - [D^a]_h \cdot [D^b]_h} \cdot \begin{pmatrix} E^i_\beta \\ E^i_\phi \end{pmatrix} \Rightarrow (G.4)
\end{align*}
$$

Situation III:

$$
\begin{align*}
\begin{pmatrix} E^d_\beta \\ E^d_\phi \end{pmatrix} &= \vec{E}_3^d = [D_{ib}^2]_h \cdot [D_{ia}^1]_h \cdot \sum_{i=0}^{\infty} ([D^a]_h \cdot [D^b]_h)^i \cdot (E^i_\beta/E^i_\phi) \Rightarrow (G.5)
\end{align*}
$$
Appendix G. Diffraction coefficients for the multiple diffraction configuration

\[
\tilde{E}_{3}^{d} = \frac{[D_{1b}^{A}]_h \cdot [D_{1a}^{b}]_h}{1 - [D_{a}]_h \cdot [D_{b}]_h} \cdot (E_{b}^{i}) (E_{\phi}^{i}) \quad (G.6)
\]

Situation IV:

\[
\left( \begin{array}{l}
E_{b}^{d} \\
E_{\phi}^{d}
\end{array} \right) = \tilde{E}_{4}^{d} = [D_{1b}^{A}]_h \cdot [D_{1a}^{b}]_h \cdot [D_{a}]_h \cdot \sum_{i=0}^{\infty} ([D_{a}]_h \cdot [D_{b}]_h)^{i} \cdot (E_{b}^{i}) (E_{\phi}^{i}) \Rightarrow (G.7)
\]

\[
\tilde{E}_{4}^{d} = \frac{[D_{1b}^{A}]_h \cdot [D_{1a}^{b}]_h \cdot [D_{a}]_h}{1 - [D_{a}]_h \cdot [D_{b}]_h} \cdot (E_{b}^{i}) (E_{\phi}^{i}) \quad (G.8)
\]

The total diffracted field due to the upper diffraction points becomes

\[
\tilde{E}_{\text{multiple-1}}^{d} = \tilde{E}_{1}^{d} - \tilde{E}_{2}^{d} - \tilde{E}_{3}^{d} + \tilde{E}_{4}^{d}
\]

Lower diffraction points \(Q_{2a}\) and \(Q_{2b}\)

Situation I:

\[
\left( \begin{array}{l}
E_{b}^{d} \\
E_{\phi}^{d}
\end{array} \right) = \tilde{E}_{1}^{d} = [D_{1b}^{A}]_h \cdot [D_{1a}^{b}]_h \cdot \sum_{i=0}^{\infty} ([D_{a}]_h \cdot [D_{b}]_h)^{i} \cdot (E_{b}^{i}) (E_{\phi}^{i}) \Rightarrow (G.10)
\]

\[
\tilde{E}_{1}^{d} = \frac{[D_{1b}^{A}]_h \cdot [D_{1a}^{b}]_h \cdot [D_{a}]_h}{1 - [D_{a}]_h \cdot [D_{b}]_h} \cdot (E_{b}^{i}) (E_{\phi}^{i}) \quad (G.11)
\]

Situation II:

\[
\left( \begin{array}{l}
E_{b}^{d} \\
E_{\phi}^{d}
\end{array} \right) = \tilde{E}_{2}^{d} = [D_{1b}^{A}]_h \cdot [D_{1a}^{b}]_h \cdot [D_{a}]_h \cdot \sum_{i=0}^{\infty} ([D_{a}]_h \cdot [D_{b}]_h)^{i} \cdot (E_{b}^{i}) (E_{\phi}^{i}) \Rightarrow (G.12)
\]

\[
\tilde{E}_{2}^{d} = \frac{[D_{1b}^{A}]_h \cdot [D_{1a}^{b}]_h \cdot [D_{a}]_h}{1 - [D_{a}]_h \cdot [D_{b}]_h} \cdot (E_{b}^{i}) (E_{\phi}^{i}) \quad (G.13)
\]

Situation III:

\[
\left( \begin{array}{l}
E_{b}^{d} \\
E_{\phi}^{d}
\end{array} \right) = \tilde{E}_{3}^{d} = [D_{1b}^{A}]_h \cdot [D_{1a}^{b}]_h \cdot \sum_{i=0}^{\infty} ([D_{a}]_h \cdot [D_{b}]_h)^{i} \cdot (E_{b}^{i}) (E_{\phi}^{i}) \Rightarrow (G.14)
\]

\[
\tilde{E}_{3}^{d} = \frac{[D_{1b}^{A}]_h \cdot [D_{1a}^{b}]_h}{1 - [D_{a}]_h \cdot [D_{b}]_h} \cdot (E_{b}^{i}) (E_{\phi}^{i}) \quad (G.15)
\]
Situation IV:

\[
\frac{E_d^i}{E_d^\phi} = \sum_{i=0}^{\infty} \left( \frac{[D^i_{2a}]_s [D^i_{2a}]_h}{[D^i_s]_h} \right) \cdot \left( \frac{E_d^i}{E_d^\phi} \right) \Rightarrow (G.16)
\]

\[E_d^4 = \frac{[D^4_{2a}]_s \cdot [D^4_{2a}]_h \cdot [D^4_s]_h}{1 - [D^4_s]_h} \cdot \left( \frac{E_d^i}{E_d^\phi} \right) \Rightarrow (G.17)
\]

The total diffracted field due to the lower diffraction points is:

\[\bar{E}_{\text{multiple-2}} = E_1 - E_2 - E_3 + E_4 \Rightarrow (G.18)\]

This yields for the final total:

\[\bar{E}_{\text{multiple}} = E_{\text{single-a}} + E_{\text{single-b}} \Rightarrow (G.19)\]

Parameters used:

\[
[D_{1a}^2]_h = -G_s \left( \omega^i = \pi/2 - \psi_m, \omega^d = \pi \right) \cdot \frac{e^{-jk^2d}}{\sqrt{2d(1 + 2d/b_m)}} \Rightarrow (G.20)
\]

\[
\left( \begin{array}{c} E_d^i \\ E_d^\phi \end{array} \right) = \sqrt{G_{\text{feed}}(\psi_m)} \cdot \frac{e^{-jk\rho_m}}{\rho_m} \cdot \frac{2P_L Z_0}{4\pi} \cdot \left( \begin{array}{c} \sin(\phi) \\ -\cos(\phi) \end{array} \right) \Rightarrow (G.21)
\]

\[
[D_{1b}^1]_h = -G_s \left( \omega^i = \pi, \omega^d = 3\pi/2 + \theta \right) \cdot \frac{b_{m+1}}{\sin(\theta)} \cdot \frac{e^{-jk^2d}}{\delta^d} \Rightarrow (G.22)
\]

\[
\left( \begin{array}{c} E^d_\theta \\ E^d_\phi \end{array} \right) = \left( \begin{array}{c} E^d_\phi \\ E^d_\theta \end{array} \right) \Rightarrow (G.23)
\]

\[
[D_{1a}^2]_h = -G_s \left( \omega^i = \pi, \omega^d = 3\pi/2 - \theta \right) \cdot \frac{b_m}{\sin(\theta)} \cdot \frac{e^{-jk^2d}}{\delta^d} \Rightarrow (G.24)
\]

\[
\left( \begin{array}{c} E^d_\theta \\ E^d_\phi \end{array} \right) = \left( \begin{array}{c} -E^d_\phi \\ -E^d_\theta \end{array} \right) \Rightarrow (G.25)
\]

\[
[D_{1b}^1]_h = -G_s \left( \omega^i = \pi/2 + \psi_{m+1}, \omega^d = \pi \right) \cdot \frac{e^{-jk^2d}}{\sqrt{2d(1 - 2d/b_{m+1})}} \Rightarrow (G.26)
\]
Appendix G. Diffraction coefficients for the multiple diffraction configuration

\[
\begin{align*}
\left( \frac{E^i_{\beta}}{E^i_{\phi}} \right) &= \sqrt{G_{feed}(\psi_{m+1})} \cdot \frac{e^{-jk_{m+1}}}{\rho_{m+1}} \cdot \sqrt{\frac{2P_1Z_0}{4\pi}} \cdot \left( -\sin(\phi) \right) \\
\left( D_{1a} \right)_{\alpha} &= -G_h(\omega^i = \pi, \omega^d = 3\pi/2 - \theta) \cdot \sqrt{\frac{b_{m+1}}{\sin(\theta)}} \cdot \frac{e^{-jka_d}}{s_d} \\
\left( \frac{E^d_{\phi}}{E^d_{\beta}} \right) &= \left( -E^d_{\phi} \right)
\end{align*}
\]

\[
\begin{align*}
\left( \frac{D_{1b}}{D_{1a}} \right)_{\alpha} &= -G_h(\omega^i = \pi, \omega^d = 3\pi/2 + \theta) \cdot \sqrt{\frac{b_{m+1}}{\sin(\theta)}} \cdot \frac{e^{-jka_d}}{s_d} \\
\left( \frac{E^d_{\phi}}{E^d_{\beta}} \right) &= \left( E^d_{\phi} \right)
\end{align*}
\]

\[
\begin{align*}
\left( \frac{D_{2b}^{1,2}}{D_{2a}} \right)_{\alpha} &= -G_h(\omega^i = \pi/2 + \psi_{m+1}, \omega^d = \pi) \cdot \frac{e^{-jk_{m+1}}}{\sqrt{2d(1 - 2d/b_{m+1})}} \\
\left( \frac{E^d_{\phi}}{E^d_{\beta}} \right) &= \sqrt{G_{feed}(\psi_{m+1})} \cdot \frac{e^{-jk_{m+1}}}{\rho_{m+1}} \cdot \sqrt{\frac{2P_1Z_0}{4\pi}} \cdot \left( -\sin(\phi) \right) \\
\left( \frac{E^d_{\phi}}{E^d_{\beta}} \right) &= \left( -E^d_{\phi} \right)
\end{align*}
\]

\[
\begin{align*}
\left( \frac{D_{2a}^{1,2}}{D_{2a}} \right)_{\alpha} &= -G_h(\omega^i = \pi, \omega^d = 3\pi/2 + \theta) \cdot \sqrt{\frac{b_{m+1}}{\sin(\theta)}} \cdot \frac{e^{-jka_d + j\pi/2}}{s_d} \\
\left( \frac{E^d_{\phi}}{E^d_{\beta}} \right) &= \left( -E^d_{\phi} \right)
\end{align*}
\]

\[
\begin{align*}
\left( \frac{D_{2b}^{1,2}}{D_{2b}} \right)_{\alpha} &= -G_h(\omega^i = \pi/2 + \psi_m, \omega^d = \pi) \cdot \frac{e^{-jk_{m+1}}}{\sqrt{2d(1 + 2d/b_m)}} \\
\left( \frac{E^d_{\phi}}{E^d_{\beta}} \right) &= \sqrt{G_{feed}(\psi_m)} \cdot \frac{e^{-jk_{m+1}}}{\rho_{m+1}} \cdot \sqrt{\frac{2P_1Z_0}{4\pi}} \cdot \left( -\sin(\phi) \right) \\
\left( \frac{E^d_{\phi}}{E^d_{\beta}} \right) &= \left( -E^d_{\phi} \right)
\end{align*}
\]
\[
[D^3_{a b}]_h = -G_h(\omega_i = \pi, \omega_d = 3\pi/2 - \theta) \cdot \sqrt{\frac{b_{m+1}}{\sin(\theta)}} \cdot \frac{e^{-jk\delta + j\pi/2}}{s^d}
\] (G.40)

\[
\begin{pmatrix}
E_\theta^d \\
E_\phi^d
\end{pmatrix} = 
\begin{pmatrix}
-E_\phi^d \\
-E_\theta^d
\end{pmatrix}
\] (G.41)

\[
[D^4_{a s}]_h = -G_h(\omega_i = \pi, \omega_d = 3\pi/2 + \theta) \cdot \sqrt{\frac{b_m}{\sin(\theta)}} \cdot \frac{e^{-jk\delta + j\pi/2}}{s^d}
\] (G.42)

\[
\begin{pmatrix}
E_\theta^d \\
E_\phi^d
\end{pmatrix} = 
\begin{pmatrix}
E_\phi^d \\
E_\theta^d
\end{pmatrix}
\] (G.43)

\[
[D^\ast]_h = -G_h(\omega_i = \pi, \omega_d = \pi) \cdot \frac{e^{-jk\delta}}{\sqrt{2d(1 + 2d/b_m)}}
\] (G.44)

\[
[D^b]_h = -G_h(\omega_i = \pi, \omega_d = \pi) \cdot \frac{e^{-jk\delta}}{\sqrt{2d(1 - 2d/b_{m+1})}}
\] (G.45)
Appendix H

Millar’s series coefficients

The coefficients $N_i$:

\[ N_0 = \frac{1}{p} \quad (H.1) \]

\[ N_1 = j \cdot \cos(\alpha) \cdot \cos(v) \quad (H.2) \]

\[ N_2 = \frac{1}{4p} \sin^2(\alpha) + \left[ \frac{1}{2} \cos^2(\alpha) - \frac{1}{4} \left( 1 + \frac{1}{2p} \right) \right] \cdot \cos(2v) \quad (H.3) \]

\[ N_3 = j \cdot \left[ \frac{-1}{8} \cos^3(\alpha) \cos(v) \right] - j \cdot \left[ \frac{-1}{8} \cos^3(\alpha) \cdot \cos(3v) \right] \quad (H.4) \]

\[ N_4 = \frac{1}{64p} \cos^4(\alpha) + \frac{1}{32} \left( 1 - \frac{3}{2p} \right) \cdot \cos^2(\alpha) + \frac{1}{128} \left( \frac{1}{p^2} + \frac{3}{2p} - 2 \right) \]
\[ + \left[ \frac{-1}{24} \cos^4(\alpha) + \frac{1}{16} \left( \frac{1}{3} + \frac{1}{2p} \right) \cdot \cos^2(\alpha) \cdot \frac{1}{16} \left( \frac{1}{3} - \frac{1}{2p} \right) \right] \cdot \cos(2v) \]
\[ + \left[ \frac{-1}{48} \cos^4(\alpha) + \frac{1}{96} \cos^2(\alpha) + \frac{1}{384} \left( 1 + \frac{3}{4p} \right) \right] \cdot \cos(4v) \quad (H.5) \]

\[ N_5 = j \cdot \left[ \frac{1}{192} \cos^5(\alpha) - \frac{1}{32} \left( p + \frac{1}{3} \right) \cos^3(\alpha) \right] + \frac{1}{16} \left( p^2 - \frac{1}{4} p + \frac{13}{48} \right) \cos(\alpha) \cdot \cos(v) \]
\[ + j \cdot \left[ \frac{1}{128} \cos^5(\alpha) - \frac{1}{256} \cos^3(\alpha) - \frac{1}{128} \left( p + \frac{3}{8} \right) \cos(\alpha) \right] \cdot \cos(3v) \]
\[ + j \cdot \left[ \frac{1}{384} \cos^5(\alpha) - \frac{1}{768} \cos^3(\alpha) - \frac{1}{3072} \cos(\alpha) \right] \cdot \cos(5v) \quad (H.6) \]
\[ N_0 = -\frac{1}{2304p} \cos^6(\alpha) - \frac{1}{384} \left(1 - \frac{1}{p}\right) \cdot \cos^4(\alpha) \]
\[ + \frac{1}{768} \left(1 + \frac{3}{4p} - \frac{3}{2p^2}\right) \cdot \cos^2(\alpha) + \frac{1}{768} \left(1 - \frac{29}{12p} + \frac{3}{2p^2}\right) \]
\[ + \left[ \frac{1}{768} \cos^6(\alpha) - \frac{1}{512} \left(\frac{1}{p} + \frac{1}{3}\right) \cdot \cos^4(\alpha) + \frac{1}{256} \left(p - \frac{49}{24} + \frac{3}{p}\right) \cdot \cos^3(\alpha) \right. \]
\[ - \frac{1}{512} \left(p - \frac{13}{8} + \frac{5}{16p} + \frac{1}{2p^2}\right) \cdot \cos(2v) \]
\[ + \left[ \frac{1}{960} \cos^6(\alpha) - \frac{1}{1920} \cos^4(\alpha) \right. \]
\[ - \frac{1}{2048} \left(\frac{1}{p} + \frac{1}{15}\right) \cdot \cos^2(\alpha) + \frac{1}{2048} \left(\frac{1}{p} - \frac{4}{5}\right) \cdot \cos(4v) \]
\[ + \left[ \frac{1}{3840} \cos^6(\alpha) - \frac{1}{7680} \cos^4(\alpha) \right. \]
\[ - \frac{1}{30720} \cos^2(\alpha) - \frac{1}{73728p} - \frac{1}{61440} \right] \cdot \cos(6v) \]  

(H.7)

The coefficients \( \mathcal{M}_i \):
\[ \mathcal{M}_0 = j \cdot \sin(\alpha) \cdot \sin(v) \]  

(H.8)
\[ \mathcal{M}_1 = \frac{1}{8} \sin(2\alpha) \cdot \sin(2v) \]  

(H.9)
\[ \mathcal{M}_2 = -\frac{1}{4} \cdot j \cdot \sin(\alpha) \cdot \left[ p - \frac{3}{4} + \frac{1}{2} \cos^2(\alpha) \right] \cdot \sin(v) \]
\[ - \frac{1}{24} \cdot j \cdot \sin(\alpha) \cdot \left[ \frac{1}{2} + \cos^2(\alpha) \right] \cdot \sin(3v) \]  

(H.10)
\[ \mathcal{M}_3 = \frac{1}{48} \sin^3(\alpha) \cdot \cos(\alpha) \cdot \sin(2v) - \frac{1}{384} \sin(2\alpha) \cdot \left[ \frac{1}{2} + \cos^2(\alpha) \right] \cdot \sin(4v) \]  

(H.11)
\[ \mathcal{M}_4 = \frac{1}{16} \cdot j \cdot \sin(\alpha) \cdot \left[p^2 - p \left(\frac{5}{4} - \frac{1}{2} \cos^2(\alpha)\right) + \frac{7}{16} \cos^2(\alpha) + \frac{1}{12} \cos^4(\alpha) \right] \cdot \sin(v) \]
\[ + \frac{1}{128} \cdot j \cdot \sin(\alpha) \cdot \left[p - \frac{5}{8} + \frac{1}{6} \cos^2(\alpha) + \frac{1}{3} \cos^4(\alpha) \right] \cdot \sin(3v) \]
\[ + \frac{1}{5120} \cdot j \cdot \sin(\alpha) \cdot \left[1 + \frac{4}{3} \cos^2(\alpha) + \frac{8}{3} \cos^4(\alpha) \right] \cdot \sin(5v) \]  

(H.12)
\[ \mathcal{M}_8 = -\frac{1}{1024} \cdot \sin(2\alpha) \cdot \left[ p - \frac{9}{8} + \frac{5}{6} \cos^2(\alpha) - \frac{1}{3} \cos^4(\alpha) \right] \cdot \sin(2v) \]
\[ - \frac{1}{15360} \sin(2\alpha) \cdot \left[ 3 - \cos^2(\alpha) - 2 \cos^4(\alpha) \right] \cdot \sin(4v) \]
\[ + \frac{1}{46080} \sin(2\alpha) \cdot \left[ \frac{3}{8} + \frac{1}{2} \cos^2(\alpha) + \cos^4(\alpha) \right] \cdot \sin(6v) \]  
(H.13)

\[ \mathcal{M}_6 = -\frac{1}{256} \cdot j \cdot \sin(\alpha) \cdot \left[ 4p^3 - 7p^2 + \frac{109}{24}p - \frac{607}{576} \right] \]
\[ + \left( 2p^2 - \frac{9}{4}p + \frac{23}{32} \right) \cdot \cos^2(\alpha) \]
\[ + \left( \frac{p}{3} - \frac{5}{24} \right) \cdot \cos^4(\alpha) + \frac{1}{36} \cos^6(\alpha) \] \[ \cdot \sin(v) \]
\[ - \frac{1}{512} \cdot j \cdot \sin(\alpha) \left[ p^2 - \frac{9}{8}p + \frac{29}{96} + \left( \frac{1}{2}p - \frac{19}{80} \right) \right] \cdot \cos^2(\alpha) \]
\[ + \frac{1}{60} \cos^4(\alpha) + \frac{1}{30} \cos^6(\alpha) \] \[ \cdot \sin(3v) \]
\[ - \frac{1}{61440} \cdot j \cdot \sin(\alpha) \left[ 5p - \frac{35}{12} + \frac{1}{2} \cos^2(\alpha) \right] \]
\[ + \frac{2}{3} \cos^4(\alpha) + \frac{4}{3} \cos^6(\alpha) \] \[ \cdot \sin(5v) \]
\[ - \frac{1}{322560} \cdot j \cdot \sin(\alpha) \left[ \frac{5}{16} + \frac{3}{8} \cos^2(\alpha) \right] \]
\[ + \frac{1}{2} \cos^4(\alpha) + \cos^6(\alpha) \] \[ \cdot \sin(7v) \]  
(H.14)

The coefficients \( \mathcal{H}_i \):  

\[ \mathcal{H}_0 = j \cdot \sin(\alpha) \cdot \cos(v) \]  
(H.15)

\[ \mathcal{H}_1 = \frac{1}{4} \sin(2\alpha) \cdot \cos(2v) \]  
(H.16)

\[ \mathcal{H}_2 = \frac{1}{4} \cdot j \cdot \sin(\alpha) \cdot \left[ p - \frac{3}{4} + \frac{1}{2} \cos^2(\alpha) \right] \cdot \cos(v) \]
\[ - \frac{1}{8} \cdot j \cdot \sin(\alpha) \cdot \left[ \frac{1}{2} + \cos^2(\alpha) \right] \] \[ \cdot \cos(3v) \]  
(H.17)

\[ \mathcal{H}_3 = \frac{1}{24} \sin^3(\alpha) \cdot \cos(\alpha) \cdot \cos(2v) - \frac{1}{96} \sin(2\alpha) \cdot \left[ \frac{1}{2} + \cos^2(\alpha) \right] \] \[ \cdot \cos(4v) \]  
(H.18)
\[ \mathcal{H}_4 = \frac{1}{16} \cdot j \cdot \sin(\alpha) \cdot \left[ p^2 - p \left( \frac{5}{4} - \frac{1}{2} \cos^2(\alpha) \right) + \frac{7}{16} \right. \\
\left. - \frac{1}{3} \cos^2(\alpha) + \frac{1}{12} \cos^4(\alpha) \right] \cdot \cos(v) \\
+ \frac{3}{128} \cdot j \cdot \sin(\alpha) \cdot \left[ p - \frac{5}{8} + \frac{1}{6} \cos^3(\alpha) + \frac{1}{3} \cos^4(\alpha) \right] \cdot \cos(3v) \\
+ \frac{1}{1024} \cdot j \cdot \sin(\alpha) \cdot \left[ 1 + \frac{4}{3} \cos^2(\alpha) + \frac{8}{3} \cos^4(\alpha) \right] \cdot \cos(5v) \] (H.19)

\[ \mathcal{H}_5 = -\frac{1}{512} \cdot j \cdot \sin(2\alpha) \cdot \left[ p - \frac{9}{8} + \frac{5}{6} \cos^2(\alpha) - \frac{1}{3} \cos^4(\alpha) \right] \cdot \cos(2v) \\
- \frac{1}{3840} \cdot \sin(2\alpha) \cdot \left[ 3 - \cos^2(\alpha) - 2 \cos^4(\alpha) \right] \cdot \cos(4v) \\
+ \frac{1}{7680} \cdot \sin(2\alpha) \cdot \left[ \frac{3}{8} + \frac{1}{2} \cos^2(\alpha) + \cos^4(\alpha) \right] \cdot \cos(6v) \] (H.20)

\[ \mathcal{H}_6 = -\frac{1}{256} \cdot j \cdot \sin(\alpha) \cdot \left[ 4p^3 - 7p^2 + \frac{109}{24} p - \frac{607}{576} \right] \\
+ \left( 2p^2 - \frac{9}{4} p + \frac{23}{32} \right) \cdot \cos^2(\alpha) \\
+ \left( \frac{p}{3} - \frac{5}{24} \right) \cdot \cos^4(\alpha) + \frac{1}{35} \cos^6(\alpha) \right] \cdot \cos(v) \\
- \frac{3}{512} \cdot j \cdot \sin(\alpha) \left[ p^2 - \frac{9}{8} p + \frac{29}{96} + \left( \frac{1}{2} p - \frac{19}{80} \right) \right] \cdot \cos^2(\alpha) \\
+ \frac{1}{60} \cos^4(\alpha) + \frac{1}{30} \cos^6(\alpha) \right] \cdot \cos(3v) \\
- \frac{1}{12288} \cdot j \cdot \sin(\alpha) \cdot \left[ 5p - \frac{35}{12} + \frac{1}{2} \cos^2(\alpha) \\
+ \frac{2}{3} \cos^4(\alpha) + \frac{4}{3} \cos^6(\alpha) \right] \cdot \cos(5v) \\
- \frac{1}{46080} \cdot j \cdot \sin(\alpha) \cdot \left[ \frac{5}{16} + \frac{3}{8} \cos^2(\alpha) \\
+ \frac{1}{2} \cos^4(\alpha) + \cos^6(\alpha) \right] \cdot \cos(7v) \] (H.21)