

MASTER

Hedging interest rate risk with interest rate derivatives

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9 June, 2015

**Hedging interest rate risk with
interest rate derivatives**

by

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in partial fulfillment of the requirements for the degree of

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Abstract

This thesis considers the interest rate risk faced by a bank and proposes a hedging strategy using plain vanilla interest rate swaps. It was conducted at bank A, a bank in the Netherlands. A model which works out hedging strategies, given a firm's specific assets and liabilities with their unique characteristics and cash flow pattern, will be formulated. Ensuring that the value of the hedges fluctuate in the same fashion but in opposite direction as the value of the assets and liabilities such that shareholder value would be protected against interest rate shocks. A gap analysis was used and iteratively the swap positions will be adjusted until the desired risk profile is achieved. Leaving enough flexibility such that an interest rate vision can be incorporated. The results suggest that the model proposes swap portfolios which hedge interest rate risk very effectively.

Management Summary

Recent financial developments introduced new products and tools which can aid practitioners in their risk management practice. Interest rate derivatives can be used to reduce interest rate risk. Due to this novelty in practice, how these products can be used by practitioners is not well documented. Effective ways of interest rate risk management are probably more relevant today than they ever were. Interest rates are at an all time low and several eminent interest rates have dropped below 0.0%. Various institutions are known for getting into trouble as a result of inadequate interest rate risk management. This research proposes a model which can be used by practitioners in order to reduce their interest rate risk using plain vanilla interest rate swaps.

The research was conducted at bank A. The research was conducted in two phases; in the first phase qualitative data was acquired at three other banks with regard to their derivative usage. These face-to-face meetings revolved around two main themes; how derivatives were used and when derivatives did not have the desired outcomes. The second phase had a quantitative nature in which a model was formulated which proposes a swap portfolio which achieves a desired risk profile.

The first phase's outcomes were conclusive; the other banks used interest rate derivatives in virtually the same manner. A gap analysis was the tool which was used in the decision making process. The risk measure which was being considered were the DV01s of the underlying buckets. These DV01s had to be reduced in such a way that the desired risk profile is achieved. Duration of equity was the risk metric used for targeting a desired risk profile. The target durations did not differ much among the different banks and were all mildly positive, ranging from 2-5 years. About the use of an interest vision the results were not conclusive, one bank did incorporate it when it had a clear-cut interest vision, whereas the others while having an interest vision did not incorporate it in the decision making process. No bank took non-parallel shifts in the yield curve into account. The products which were being used were also aligned; the vast majority of derivatives were plain vanilla interest rate swaps were used, a small portion were other derivatives such as futures and swaptions. For each of these banks, swaps had never not had the desired outcome. It had been the case that swaps were purchased and shortly after hedging, certain assets or liabilities were no longer present, then offsetting swaps would be taken to offset the initial hedge.

The quantitative model was at the centre of attention for the second phase. The model determines DV01 of individual buckets, within 10 iterations it sets the DV01 for all buckets greater than the first bucket, equal to zero. Once this procedure is done, the model has determined the swap portfolio which reduces the DV01 of all the underlying buckets to a minimum. The model will then determine how each swap needs to be altered such that the desired risk profile is met. Duration of equity is the risk metric used for the desired risk profile.

The results suggest the model is able to determine an optimal swap portfolio accurately. For a numerical example, the duration of equity was reduced from 27.1 years to only -0.1 years. The results also indicate that a target duration can be reached very accurately. Several scenarios in which the yield curves moved in a non-parallel fashion have also been examined; the results suggest the swaps perform well under these scenarios as well. Especially when targeting a duration of equity close to 0, volatility of the equity was reduced tremendously as can be seen in figure 1.

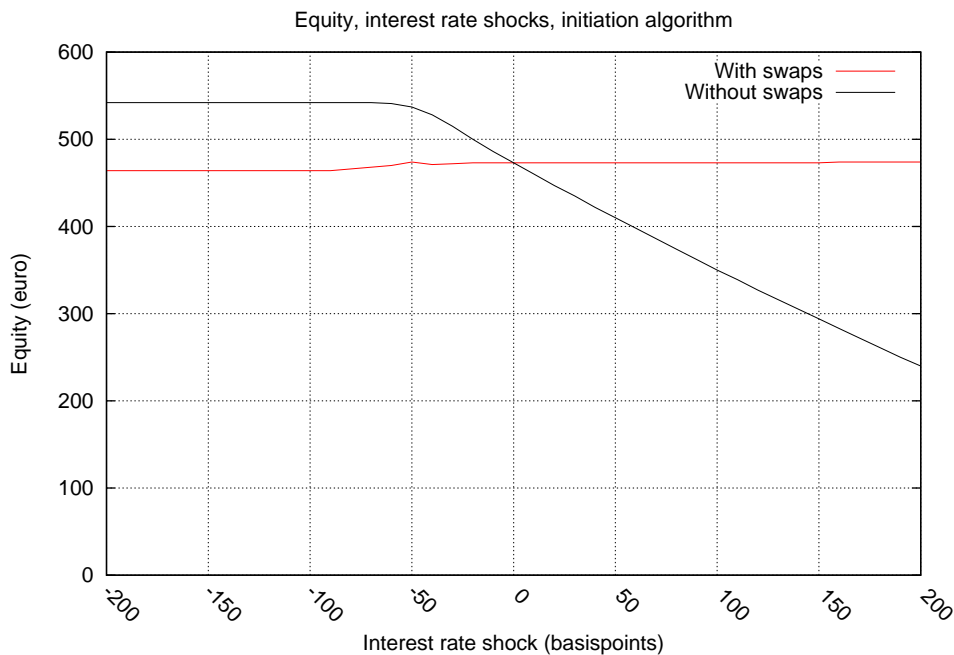


Figure 1: Equity in different interest rate shocks, initiation algorithm

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1 Introduction

A bank is exposed to many kinds of risk, for example credit risk, operational risk and market risk (Rachev et al., 2008). Market risk arises from the fact that a bank's assets and liabilities can be exposed to moves of certain market variables (Rachev et al., 2008). Interest rates are one of the four market risk drivers (Rachev et al., 2008). (Basel Committee, 2004) state that interest rate risk as the exposure of a bank's financial condition to fluctuations in the interest rates. As risk and return go hand in hand, accepting risk is a normal part of banking and can be an important source of profits. Excessive interest rate risk however, can pose a significant threat to a bank's earnings and capital base (Basel Committee, 2004). Changes in the interest levels directly affect the value of certain parts of the bank's trading book and directly affect the net interest income (Hull, 2012). Interest rates deeply influence the performance of both financial and non-financial firms (Gyntelberg and Upper, 2013). A key risk management activity for a bank is the management of the interest rate risk (Hull, 2012). Which is in line with (Nawalkha and Soto, 2005) who claim that the importance of interest rate risk management cannot be overstated. For financial institutions, the sources of risk have to be recognised, managed and controlled (Rachev et al., 2008).

Banks usually have a risk appetite statement (RAS). It is a statement in which banks state the amount of risk they are willing to accept. This implies that certain limits will be set to certain risk measures. Bank A has recently shifted to a market value balance sheet. This has also caused their balance sheet to become a lot more volatile with regard to interest rates. Increasing the probability of exceeding certain RAS limits, with regard to interest rate risk. Therefore ways to control these fluctuations more effectively are desired. Bank executives are interested in proper ways of managing and controlling interest rate risk. The financial world has been developing a lot recently and has come up with many different kinds of products - such as derivatives - which can be used to manage risks. Due to this novelty, how to use derivatives to reduce interest rate risk is not well documented. This thesis will focus on how plain vanilla interest rate swaps can be used to reduce interest rate risk. Interest rate swaps are chosen as hedging instruments as literature states that these products facilitate perfect hedging (Heckinger and Mengle, 2013) and are probably the most successful financial innovation ever (Hull, 2006).

The relevant literature will be described in section 2. The results from the peer bank analysis can be found in the appendix. The individual components on which the model will be based are discussed in section 3. The

solution method will be handled in section 4 in more detail. Finally, the numerical results which also discuss the hedging strategy will be dealt with in section 5.

1.1 Problem Description

The research will be conducted at *Bank A*, a bank in the Netherlands. A bank interested in interest rate swaps as they desire more efficient ways of managing interest rate risk. The research will look into static hedging, without incorporating an estimation for the interest rate evolution. A reason why the estimated interest rate evolution is not incorporated has to do partially with the vision of the bank. Management interprets using an interest vision as a kind of speculation; something they do not pursue. Also Bank A has to be able to justify why certain positions are taken to its supervisor, De Nederlandsche Bank (DNB).

Currently the economic environment is unique in the sense that the interest rates are at an all time low. The recent buying of government paper by the European Central Bank (ECB) has caused rates to drop even further. The president of the European central bank also tells people to be aware of the low interest rate. (Serra, 2015) thinks interest rates will remain at all time lows for the foreseeable future. One might argue how likely it will be for rates to drop even further as they are already so low.

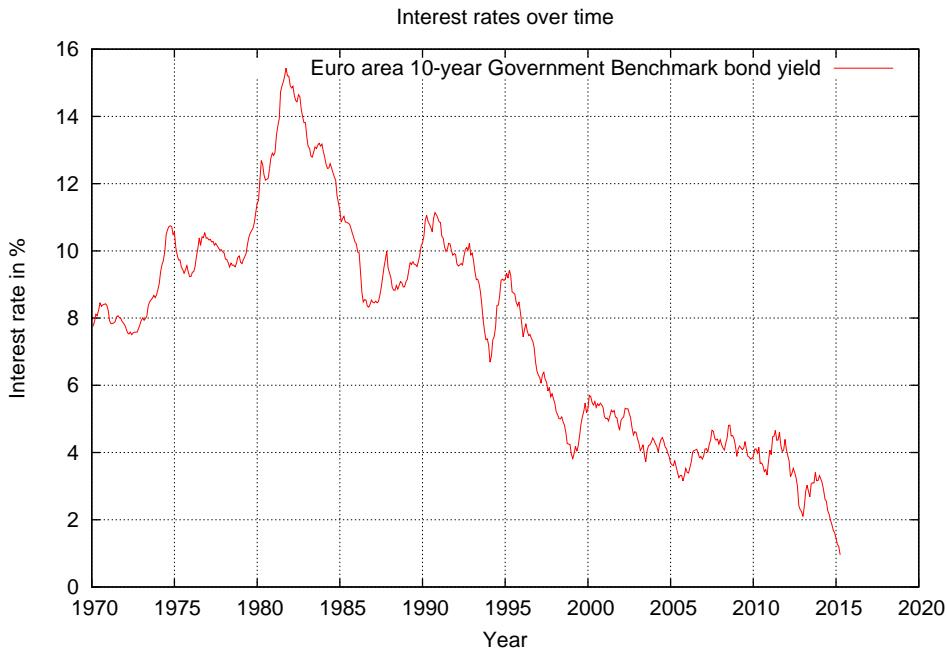


Figure 2: Euro area 10-year Government Benchmark bond yield - Yield

There are three major ways in which interest rate risk management can be done. In the first method one leaves the position of the firm exposed; an argument for doing so is by having a clear cut interest rate vision. In the second method one tries to match the repricing¹ of its assets and liabilities in a procedure called asset and liability matching. In this procedure one would do interest rate risk management by buying and selling assets. The last method would be to use interest rate derivatives. The first method is considered not to be a sound way of managing interest rate risk and can be perceived as a kind of speculation as one is actually betting on the direction the interest rates will shift in. The second way is not flexible in the sense that it restricts one to use assets of certain duration. Some of the assets required might not be available in the quantities required or would require large transaction costs. Derivatives - especially over the counter (OTC) derivatives - are said to give its users the flexibility which banks desire, as these can be tailored to meet the specific needs of the user. Thus, bank A wants to know how interest rate derivatives could be used to hedge interest rate risk.

As mentioned earlier, the thesis will formulate a model which looks into static hedging, without incorporating an interest rate vision and customer behavior. This is done as incorporating an interest rate vision can be con-

¹repricing referring to the point in time when the interest rates are changed for assets and liabilities

sidered a kind of speculation. The position of a bank changes constantly; existing assets can be sold, new assets can be bought, new customer deposits can be made and old deposits can be withdrawn. Therefore, customer behavior will not be incorporated, as the swaps will need to be changed continuously, in line with the position of a firm. As hedging with swaps is not a “hedge and forget” situation - the swaps will need to be adjusted regularly - incorporating customer behavior does not add much value to the model. The model considers a point in time and based on that position hedge the interest rate risk as effectively as possible.

1.1.1 Research question

As interest rates deeply influence the banks financial position, an effective interest rate risk management practice is desired. Due to the novelty of these products in practice, how these could be used in practice is not well documented. The main research question will become:

- Given firm specific assets and liabilities what swaps could be used to reduce interest rate risk?

In order to answer the main research question, three sub-questions will be addressed. Each of these sub-questions belongs to a different phase. As other banks are already using interest rate derivatives, these could provide valuable information regarding derivative usage. Information regarding what risk measures could be used; how one can determine what swaps are required and what could be possible hiccups in using derivatives to manage interest rate risk. The first sub-question will be as follows:

- *How are other banks hedging, using interest rate derivatives?*

Partially based on the previous section - which will be referred to as the peer bank analysis - the following phase will need to identify what hedging strategy should be pursued and how this can be achieved. This phase will involve the formulation of a model which determines a hedging strategy given a firms specific assets, liabilities and unique characteristics. The sub-question for this phase is defined as:

- *What hedging strategy should a bank use in a static environment given its characteristics?*

Once the model determines what swap portfolio achieves a certain risk profile, one has to determine how well this portfolio performs. Therefore the last sub-question will become.

- *How does one know how well the hedges are performing?*

2 Relevant Literature

Interest rate risk has different sources; (Basel Committee, 2004) has divided interest rate risk into four categories. The first category and most important source of interest rate risk is repricing risk (Wright and Houpt, 1996). (Basel Committee, 2004) agrees and claims that repricing risk is the primary form of interest rate risk, it arises from timing mismatches between the moment of repricing of assets and liabilities. This is consistent with the view of (Hull, 2012) who states that one of the major causes of interest rate risk is the mismatch in maturity between the assets and liabilities. (Kaufman, 1984) claims that institutions expose themselves to interest rate risk whenever there is a mismatch between the sensitivity of the assets and liabilities. Embedded option risk is the second category (Basel Committee, 2004) and arises whenever a client has to ability to affect the timing and size of a cash flow. (Nawalkha and Soto, 2005) state that when home-owners refinance their mortgages at record low interest rates, they will not be eager to prepay their mortgages when interest rates rise. This is turn will increase the duration of the mortgages significantly, this consequently exposes the banks to a high level of interest rate risk (Nawalkha and Soto, 2005). Yield curve risk is the third type of interest rate risk and occurs when the yield curve does not move in a parallel fashion (Basel Committee, 2004). The last form of interest rate risk is basis risk (Basel Committee, 2004), which occurs when a hedged item and instrument do not fluctuate in exactly the same fashion, OTC derivatives are argued to minimize basis risk as they facilitate perfect hedging (Heckinger and Mengle, 2013).

A lot has been written with regard to the hedging instruments; interest rate swaps. (Hull, 2006) states that swaps are arguably one of the most successful innovations in financial markets ever. An interest rate swap can be considered the primary tool for hedging fixed-rate assets such as residential mortgages (Trinder, 2000). An important characteristic of swaps is that they protect financial assets and liabilities from interest rate fluctuations (Allen et al., 2012). They have proven to be very flexible instruments of managing risk (Hull, 2006). (Schröder and Dunbar, 2011) state that swaps enable its users to synthetically increase the duration of its portfolio to match liabilities. Swaps have played an important role in the success of the OTC derivatives market (Hull, 2006). The swaps market has emerged because swaps escape many of the limitations inherent in futures and exchange traded options markets (Kolb and Overdahl, 2003).

The most important indicator of interest rate sensitivity is duration (Hull, 2006), this consistent with the view of (Haugen, 1990). Many prac-

titioners use duration to estimate the interest risk exposure (Cohen, 1993). The duration model is a widely used measure of a portfolio's exposure to yield curve movements (Hull, 2012). Changes in a bank's financial position due to interest rate changes can be analysed by looking at the duration analysis (Kaufman, 1984). A gap analysis is a tool which indicates where the repricing mismatches are. (Stulz, 2003) argues that the best known approach measure the exposure of net interest income to interest rate changes, is gap measurement. (Bank, 2005) state that a gap analysis is probably the most used methods to determine interest rate risk exposure and in particular, repricing risk, this is in line with the view of (Kocken, 1997). (Coleman, 2011) argues that DV01 to be the correct method of hedging using swaps.

Not much literature is publicly available regarding hedging interest rate risk using derivatives. Information is available regarding the individual components, such as what risk measures exist, how derivatives work and metrics can be used by practitioners, but a general framework or model which proposes hedging strategies is lacking. It is well documented how interest rate risk can be measured. (Nawalkha and Soto, 2005) propose several methods which can be used to quantify interest rate risk, they do not mention however, how interest rate swaps can be used once the risk has been quantified. With regard to the hedging instruments - interest rate swaps - (Hull, 2012) describes in an introductory way how derivatives can be used to hedge individual bonds; he doesn't however, mention how interest rate swaps could be used by financial institutions like banks which hold large portfolios of assets and liabilities. (Jaffal and Yassine, 2013) propose a model which determines, given a list of swaps, the optimal amount of swaps to be used to hedge a small portfolio of bonds. The model does however not determine what the swaps should look like, something practitioners might be very interested in, it only determines amounts. Thus it is well documented what a swap is and how it works, how risks can be quantified, but the bridge is lacking. This will be the main contribution to interest rate risk management literature.

3 Interest rate risk hedging

In this section the individual building blocks on which the model is established will be discussed. These include interest rate sensitivity determination, the gap analysis, the hedging instruments and a method of interpolating the swap curve. These individual components are chosen based on the literature study and the peer bank analysis. Interest rate sensitivity will be handled in section 3.1. The tool which will be used is the gap analysis and will be discussed in section 3.2. The hedging instruments; swaps, will be discussed briefly in subsection 3.3. How one can determine the value of a swap will be tackled in section 3.4. As the swap curve is only given for a few specific points; the other swap rates for the other maturities will have to be interpolated; the Nelson Siegel Svensson method is used and can be found in section 3.5. Finally, how the hedge performance will be analysed will be discussed in section 3.6.

3.1 Interest rate sensitivity

Before one can define a proper hedging strategy one has to determine how its position is affected by interest rates. The interest rate sensitivity has to be determined. A concept which is at the heart of interest rate risk management practice is duration. Duration is a measure of interest rate sensitivity. It can be computed using formula 1. For asset a for example its duration can be computed hence. Where $PV_{a,+1bps}$ implies the present value of asset a given a 1 basis point shock increase of the yield curve.

$$DUR_a = \left(\frac{PV_{a,0bps} - PV_{a,+1bps}}{PV_{a,0bps}} \right) \cdot 10^4 \quad (1)$$

Duration is a metric which measures the percentage change in value of an asset given a change in the underlying yield curve. To illustrate; if a bond has a duration of 2 year, this means that its value will reduce by approximately 2% if interest rates rise with 1%. This methodology is also used to determine the duration of the legs of a swap. This will need to be done as one is interested in how swaps affect the duration of the equity; when hedging one tries to reduce the volatility of the equity. Duration of the equity will be used as a metric to attain a certain risk profile, as practitioners often aim for a target duration of the equity. The duration of the equity is determined via the duration of the assets and the duration of the liabilities.

DV01 is a concept very closely related to duration, but doesn't describe the percentage change but dollar change, therefore this metric is also called dollar duration. This metric will be used to determine how sensitive an asset

or liability is to changes in the interest rates, it can be computed using the following formula:

$$\begin{aligned} DV01 &= \frac{\partial PV}{\partial y} \\ &= \frac{PV_{0bps} - PV_{+1bps}}{0.0001} \\ &= (PV_{0bps} - PV_{+1bps}) \cdot 10000 \end{aligned} \tag{2}$$

Where y is the change in interest rates, for DV01 this change is 1 basis point. PV is the present value of an asset or liability. Thus DV01 is the change in value of an asset or liability in case of a 1 basis point increase in interest rates.

Interest rates affect the value of a mortgage for example, the value of a mortgage increases if rates drop. This is the case because a new mortgage with the same maturity as before will now have a lower interest rate. Therefore the old mortgage is worth more than the new issued mortgages; its value increases. The opposite is also true, as soon rates increase the value of a mortgage decreases. Due to specific product characteristics this change can be higher or lower for different kinds of products.

3.2 Gap Analysis

To determine what swaps hedge the interest rate properly, one can use a gap analysis. A gap analysis provides a visual representation of where the “gap mismatches” are. In a gap analysis interest sensitive cash flows from the assets and liabilities are positioned in time buckets. One positions all the cash flows which reprice in their respective buckets. Repricing refers to the point in time where the interest rate is changed for assets and liabilities which have a variable rate. For assets and liabilities which have a fixed rate, the moment of repricing refers to the point in time where it matures or where it turns variable. These interest sensitive cash flows will be determined for both the assets and the liabilities. The bigger the mismatch, the bigger the interest rate risk of that particular bucket. In section 3.1 the concept of DV01 was introduced. The total of the DV01 of the underlying buckets is equal to the duration of equity as to be found on the balance sheet. Therefore adjusting the DV01 of the underlying buckets on the gap-analysis allows one to alter the interest rate risk a firm is exposed to. A bucket is simply a time interval, which can be determined by the user individually.

3.2.1 Assets and Liabilities

In a gap analysis one needs to determine the interest sensitive cash flow pattern of assets and liabilities, which is determined using the moment of repricing. For products which have a fixed interest rate; for example mortgages, the moment of repricing is either its maturity or the moment at which the fixed rate is changed. For products which have a variable rate, the moment at which this rate is changed is the moment of repricing. As mentioned, all the cash flows for the assets and liabilities which are affected by interest rates have to be taken into account.

The buckets refer to different time intervals. Common buckets are the following: within 1 month; 1 to 3 months; 3-6 months, 6-9 months, 9-12 months, 1-2 year, 2-3 year, 3-5 year, 5-7 year, 7-10 year, 10-15 year, 15-20 year and 20+ years. But the length of these buckets can be adjusted to the specific needs of the user. For these buckets the cash flows of the respective months have to be add together. Once one has the cash flow pattern for the assets and liabilities one can determine what the gaps are by adding the two together.

An example of what a gap analysis might look like can be found in figure 3.

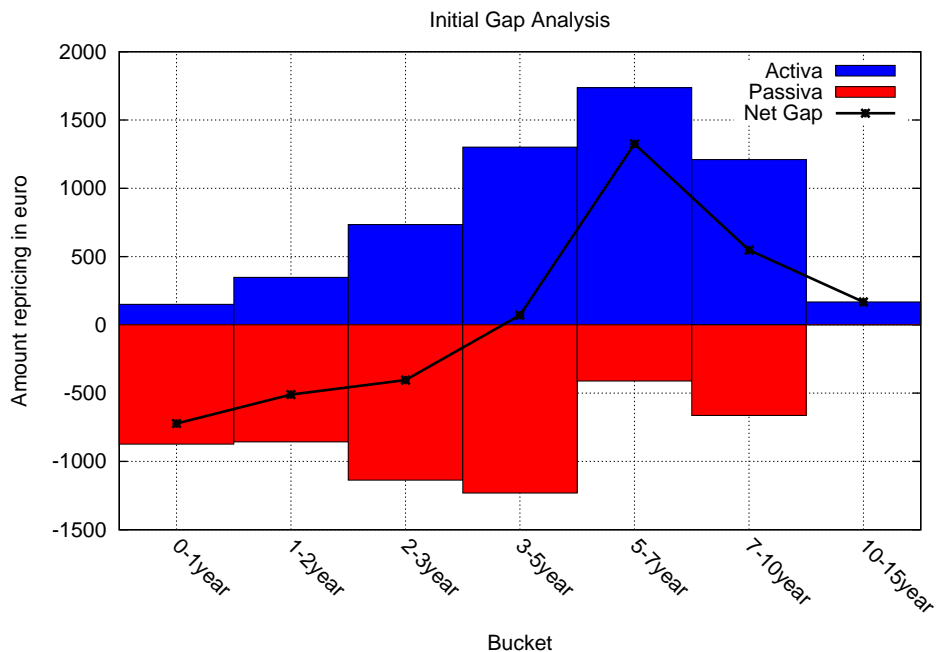


Figure 3: Initial Gap Analysis

3.3 Plain vanilla interest rate swaps

As mentioned in the literature study, interest rate swaps are very effective instruments to hedge interest rate risk. Interest rate swaps can be used to hedge; hedging is done in order to reduce the interest rate sensitivity of equity of a firm. A firm wants to protect shareholder value. Interest rate swaps will be the tools which will be used in order to reduce interest rate risk. These tools have been chosen as they are relatively simple, can hedge very effectively, are in line with what the management wants and are also the same as what the competition is using.

The first swap transaction was done in the early 1980's and has become an enormous market. The swap market has seen tremendous growth over the past 30 years. Swaps are OTC derivatives, which means that these are contracts which are traded without the interference of an exchange. The absence of an exchange enables its users to tailor the swaps to its particular needs; exchange traded contracts are very standardized. Interest rate swaps are said to facilitate perfect hedging. Originally these contracts were bilateral agreements, but since recently these contracts have to be cleared centrally. This has been done in order to reduce counter party credit risk.

Plain vanilla interest rate swaps are contracts in which two counter parties agree to exchange cash flows for a predetermined amount of time. In a plain vanilla interest rate swap one party pays a variable rate whereas the other counter party pays a fixed rate. A very common variable rate is 6 month Euribor (EUROpean Inter Bank Offered Rate). This is the rate at which European banks are willing to lend each other money for a period of 6 months. Each party pays its rate based on a notional. A visual representation of what a swap contract looks like can be seen in figure 4.

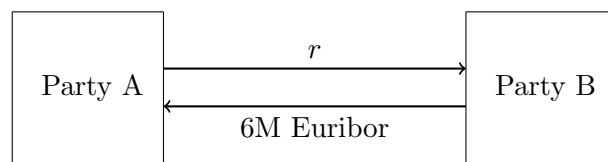


Figure 4: Plain vanilla interest rate swap

For party A the swap is a payer swap; which means it pays a fixed rate r on a notional N . Party A receives a floating rate which is based on 6 month Euribor on the same notional N . For party B this swap contract is called a receiver swap, as it receives the fixed leg and it has to pay a floating leg based on 6M Euribor. The floating leg, is changed periodically, this case every 6 months. Every 6 months the floating leg is said to reprice or reset. On initiation no counter party will be better off in engaging in the swap, each leg is worth the same. This is achieved by setting the fixed rate in

such a way that given prevailing market conditions the value of fixed leg payments are equal to the floating leg payments. The swap curve is used to determine the fixed leg.

On initiation of the swap the value to either counter party is zero. As interest rates start to change however, the value of the swap changes. This is the case because a swap is a chain of future cash flows, interest rates affect the value of these cash flows. The goal is to determine what swaps are required to offset the gains or losses in the value of the assets and liabilities, such that the value of equity is protected against fluctuations in the interest rates.

There are a few risks to swaps; the first risk being misuse. Multiple institutions are known for getting into trouble as a result of swap misuse and speculation. The second risk inherent to swaps is basis risk, this is a source of risk which occurs when the hedging instrument and the hedge item do not fluctuate perfectly in similar fashion, but because OTC derivatives can be tailored to the specific needs of the user, OTC derivatives said are reduce this basis risk (Heckinger and Mengle, 2013). The last risk inherent to swaps is counter party credit risk, the risk that the other party does not live up to its obligations, but as all interest rate swaps have to be centrally cleared, this counter party credit risk is reduced significantly.

3.3.1 How to incorporate a swap in the gap analysis

As for the assets and liabilities, also the swaps will be positioned in the bucket where they reprice. Meaning that the floating leg will be positioned in the bucket where it reprices whereas the fixed leg payments will be positioned in the buckets of the individual payments. One of the legs of the swap will be an asset, the other will be a liability. This will be done for both the coupon payments and the notional payments. The notional payments have to be included, assuming that the notional is exchanged at maturity. This has to be done as each counter party faces interest rate risk on the notional as well.

Essential for the gap analysis is to know what a swap transaction looks like and how the individual legs should be treated in order to use them for in the gap analysis. The floating leg has a feature of being at par at initiation and at every reset date. The value of the floating leg is equal to the notional at every repricing moment and at initiation. As mentioned earlier; both legs have to be incorporated in the gap analysis whenever it reprices; the

floating leg reprices periodically, this case 6 months. The fixed leg reprices at maturity. Table 1 shows the undiscounted cash flows for the two legs of an interest rate swap. This particular swap has a notional of N €, the fixed leg is $a\%$ which is paid annually, the floating leg is based on 6 Month Euribor which is currently at $b\%$. The maturity of the swap it 3 years.

t	Floating leg	Fixed leg
0.5	$(1 + b) \cdot N$	
1		$a \cdot N$
1.5		
2		$a \cdot N$
2.5		
3		$(1 + a) \cdot N$

Table 1: Cash flow pattern at $t = 0$ of individual legs

Table 1 can be transformed such that it can be incorporated in the gap analysis. Assume an 8 year payer swap with a notional of €100 and a fixed rate leg of $\approx 11.55\%^2$. This is how it will be add to the gap analysis, as in figure 5. The fact why the bucket in the third gap is lower than in the second and why the second is lower than the first has to do with discounting.

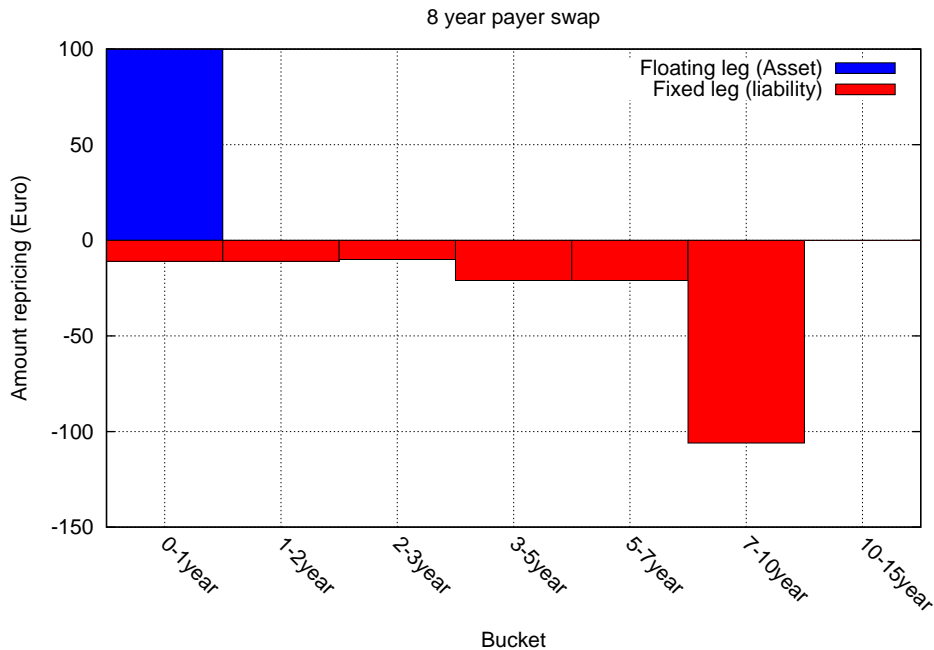


Figure 5: 8 year payer swap

²This is unrealistically high, but it was chosen to enhance the figure

3.4 Swap Value Determination

A swap is an agreement between two counter-parties to exchange cash flows. In a plain vanilla interest rate swap, the swaps which will be at the center of attention during this research; one party pays a fixed interest rate, better known as the fixed leg. The other counter party pays a variable rate, which is called the floating leg. For valuation of swaps one incorporates the notional amount. This is done as one is exposed to interest rate risk over the notional as well, the notional is required when doing swap valuation. When doing the valuation; the model will only look at time $t = 0$. Thus, the model will determine the value as if the swaps were purchased and instantly after purchasing an interest rate shock occurs.

The floating leg is at par at a reset and at initiation. Assume the floating leg to reprice every τ time periods. The floating leg pays a variable rate $E_{t-\tau}(\tau)$ at time t , which is the Euribor rate at $t - \tau$ for a maturity of τ . Therefore only the first payment of the floating leg is known at initiation of the swap. The discount factor which will be used can be gotten from the swap rate for that respective time. The characteristic of the fixed leg to be at par at reset is a very important one which is crucial in swap value determination. This characteristic will be shown in figure 6. Assume a swap to have a notional of 1 which is exchanged at the end of its lifetime and its maturity 4 time periods. Where the tenor of the floating leg is 1 period; τ . The floating leg looks as follows

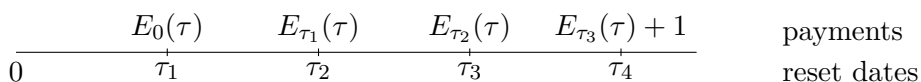


Figure 6: Floating leg payments

The value of the floating leg at τ_3 can be computed as:

$$\begin{aligned} PV_{\text{Float}_{\tau_3}} &= \frac{E_{\tau_3}(\tau) + 1}{1 + E_{\tau_3}(\tau)} \\ &= 1 \end{aligned} \tag{3}$$

If one looks back another time period; at τ_2 the value of the floating leg becomes:

$$\begin{aligned} PV_{\text{Float}_{\tau_2}} &= \frac{PV_{\text{Float}_{\tau_3}} + E_{\tau_2}(\tau)}{1 + E_{\tau_2}(\tau)} \\ &= \frac{1 + E_{\tau_2}(\tau)}{1 + E_{\tau_2}(\tau)} \\ &= 1 \end{aligned} \tag{4}$$

The floating leg which will be used in the remainder of the thesis will be 6 month Euribor, a rate which is very common in swaps. The value of the floating leg can be computed using the formula, assuming that the notional is exchanged at maturity. The value of the floating leg at $t = 0$, can be computed using the following formula. One has to bear in mind that the first payment is known and doesn't change anymore. This payment is $E_0(6M) = \sqrt{1 + sr_{0.5, \text{FIX}}}$ and is fixed; the discount factor however $DF_{0.5}$, used to discount the first payment can change. The discount factor becomes $DF_{0.5} = \frac{1}{\sqrt{sr_{0.5, \text{VAR}} + 1}}$.

$$PV_{\text{Float}} = N \cdot (E_0(6M)) \cdot DF_{0.5} \quad (5)$$

$$= N \cdot \sqrt{1 + sr_{0.5, \text{FIX}}} \cdot \frac{1}{\sqrt{sr_{0.5, \text{VAR}} + 1}} \quad (6)$$

The value of the fixed leg can be computed using the following formula, by simply discounting all the coupon payments and the notional payment:

$$PV_{\text{Fixed}} = \sum_{t=0}^{T-1} N \cdot r \cdot DF_t + (1 + r)N \cdot DF_T \quad (7)$$

where t is the time of payment in years, T is the final payment. N is the notional and r is the rate of the fixed leg. DF_t is the discount factor used at time t . The N will be determined using the model. The discount factors can be derived from the interpolated swap curve. At initiation of a swap agreement, no counter-party is better off in engaging in the swap agreement. This characteristic will be used in order to determine the fixed rate leg r . r will be determined using the notion that at initiation the fixed leg and floating leg have the same value. Which can be done using the following formula:

$$\begin{aligned} \text{Floating leg} &= \text{Fixed leg} \\ N_i &= \sum_{t=1}^{T-1} r_i \cdot N_i \cdot DF_t + (r_i + 1) \cdot N_i \cdot DF_T \\ N_i(1 - DF_T) &= \sum_{t=1}^{T-1} r_i \cdot N_i \cdot DF_t + r_i \cdot N_i \cdot DF_T \\ N_i(1 - DF_T) &= \sum_{t=1}^T r_i \cdot N_i \cdot DF_t \\ r_i &= \frac{N_i(1 - DF_T)}{\sum_{t=1}^T N_i \cdot DF_t} \end{aligned} \quad (8)$$

3.5 Swap curve determination

The swap curve is required as it is used in determining the fixed leg for interest rate swaps. It is the case that the swap curve is only given for certain maturities. The swap curve will have to be determined for every possible maturity. The method which will be used for it is the Nelson Siegel Svensson method. The Nelson Siegel Svensson method is a popular parametrization and is used by central banks and other market participants a lot, to model the term structure of interest rates (Gilli and Große, 2010). The swap curve will be extracted from Bloomberg. The swap curve which will be used is the Euribor Swap Curve. The floating leg will reprice every τ time periods, and will equal the curve at point τ . The Nelson Siegel Svensson method uses six parameters which are used to determine the interpolated swap curve.

$$\begin{aligned} y(t) = & \beta_1 + \beta_2 \left[\frac{1 - e(\frac{-t}{\lambda_1})}{\frac{t}{\lambda_1}} \right] + \beta_3 \left[\frac{1 - e(\frac{-t}{\lambda_1})}{\frac{t}{\lambda_1}} - e(\frac{-t}{\lambda_1}) \right] \\ & + \beta_4 \left[\frac{1 - e(\frac{-t}{\lambda_2})}{\frac{t}{\lambda_2}} - e(\frac{-t}{\lambda_2}) \right] \end{aligned} \quad (9)$$

Where $y(t)$ is the curve in percentage at time t . This method works with a residual sum of squares method, implying that this method will fit the curve by ensuring that the sum of the squared residuals will be as small as possible. The swap curve at $t = 0$ and the interpolated curve look as:

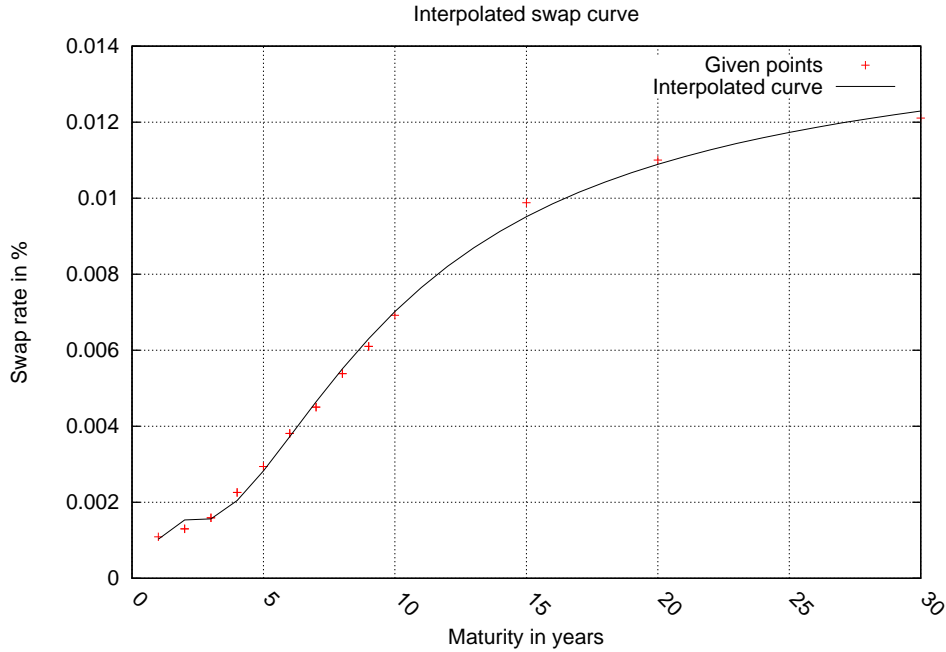


Figure 7: Interpolated swap curve

Swaps can have different tenors; for the floating leg a very common tenor is 6-months and thus the floating rate leg is 6 month Euribor. For the fixed leg the tenor is usually 1 year. Therefore the curve will be interpolated at halve a year points. The interpolated curve can be found in figure 7.

3.6 Hedge Performance

One wants to be able to make a judgement about swap performance. There are a few ways in which the hedge performance will be evaluated. The first key performance indicator is duration of equity. As mentioned earlier, duration is a measure of interest rate sensitivity, duration of equity is a measure of sensitivity of equity to interest rates. The peer bank analysis concluded that this metric was always used to target a specific risk profile. The duration of equity can be computed using the formula 10. Where A are the total amount of different asset types and L are the total amount of different liabilities.

$$DUR_{\tilde{E}} = \frac{\sum_a^A PV_a \cdot DUR_a - \sum_l^L PV_l \cdot DUR_l}{PV_{\tilde{E}}} \quad (10)$$

The second KPI is EVAR. This KPI is chosen as it is often used in

the decision making process with regard to swaps³; and secondly as DNB requires it supervisee to report this risk metric. Equity will also be plot versus shocks in interest rates, to get an visual representation of equity under certain interest rate shocks. EVAR is in two scenarios; a shock up and a shock down. It can be computed using the following formula:

$$EVAR^+ = PV_{\tilde{E},+200bps} - PV_{\tilde{E},0bps} \quad (11)$$

$$EVAR^- = PV_{\tilde{E},-200bps} - PV_{\tilde{E},0bps} \quad (12)$$

$$(13)$$

Finally the performance swaps will also be evaluated under non-parallel shifts in the yield curve. Two different scenarios will be tested; one in which the short end will be increased more than the long end, whereas in the other scenario the long end will be increased more than the short end. Under a normal yield curve⁴: increasing the short end will result in a flattened yield curve; increasing the long end would result in a steepened yield curve. As resulted from the peer bank analysis, none of the banks incorporate non-parallel shifts in the yield curve in their hedging decisions. In the steepening yield curve scenario, the yield curve remain unchanged at $t = 0$, but it shift up 200 basis points at end of the horizon, usually 30 years. For each t in between the yield curve will shift up linearly. Therefore, for a yield curve with an x year horizon the steepened yield curve will look as follows:

$$y^u(t) = y^i(t) + \frac{2}{x} \cdot t \quad (14)$$

Where $y^i(t)$ is the initial yield of time t , and $y^u(t)$ is the yield of the steepened curve at time t . For the flattened curve, the curve is shifted the other way around, the 200 basis points are subtracted instead of added.

$$y^f(t) = y^i(t) - \frac{2}{x} \cdot t \quad (15)$$

The performance of the swaps will be evaluated by computing the equity in the different scenarios, with and without swaps. It can be computed using formulas 11 and 12, but then using different curves; $EVAR^f$ for the flattened yield curves and $EVAR^u$ steepened yield curves. Figure 8 shows an initial, a flattened and a steepened yield curve using the above methodology.

³It is closely linked to duration of equity

⁴One which is upward sloping

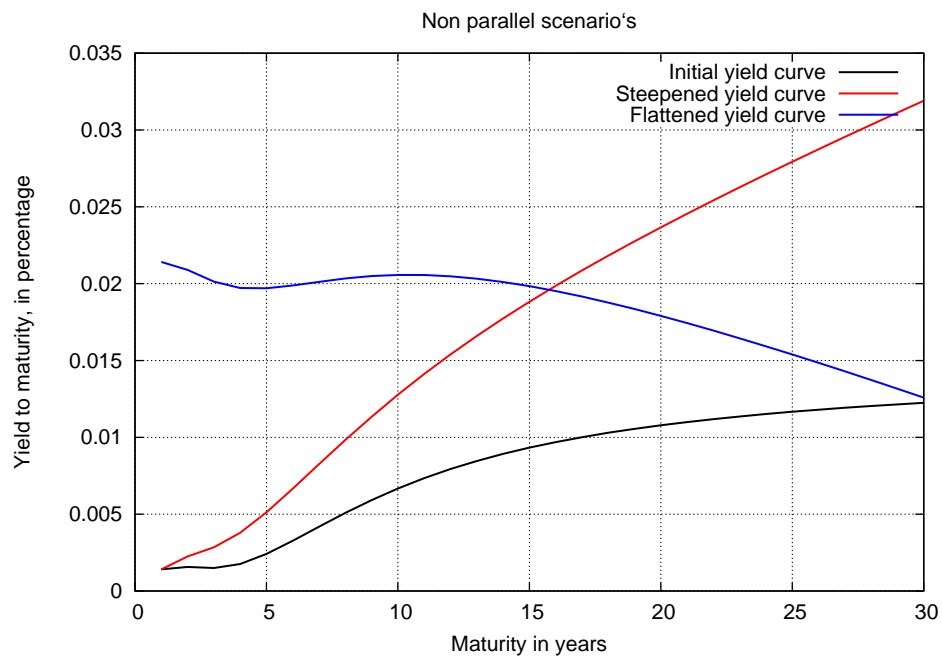


Figure 8: Non parallel shifts in yield curve

As in the current economic environment the yield curve is very flat, the “flattening” procedure has actually caused the yield curve to become inverted, downward sloping.

4 Model

In this section, the mathematical model which determines the optimal portfolio of swaps will be specified. The model determines what swaps offset the bucket's sensitivity to interest rates, it will furthermore determine how the swaps have to be adjusted to meet the desired duration of equity. The first input for the model will be gap analysis; which is based on the cash flow pattern for the assets and liabilities, accompanied by their respective curves used for discounting. Secondly the current swap curve has to be extracted from Bloomberg. The third parameter is the target duration of equity. Finally, the maturity which one wants to use for each bucket has to be selected. For each bucket; only a few possible maturities are possible; only the maturities that mature in that particular time interval. The swap portfolio and several KPIs will be the output of the model.

\tilde{A}	Assets	
\tilde{L}	Liabilities	
\tilde{E}	Equity	
\tilde{S}	Swaps	
A	amount of asset types	
L	amount of liability types	
I	amount of different buckets	
S	Amount of different swaps	
q	index for assets, liabilities and swaps	
M	total amount of months	
J_q	total amount of individual q	
a	index for asset type, running from	1 to A
l	index for liability type, running from	1 to L
i	index for bucket, running from	1 to I
j_q	index for individual component, running from	1 to J
w	parallel shift in yield curve, in basis points, running from	0 to 1
m	index for the month, running from	1 to M
$B_{i,q,w}$	Size of bucket i for category q in scenario w	
$G_{i,w}$	Gap for bucket i in scenario w	
$PV_{q,w}$	Present Value for category q under scenario w	
Dur_q	Duration for category q	
$EVAR_V^{\text{scen}}$	Equity at risk in a scenario with or without swaps	
$CF_{m,q,j,w}$	Cash flow in month m for category q for item j in scenario w	
$DV01_{i,q}$	DV01 of type q for bucket i	
$DF_{m,q,w}$	Discount factor for month m for category q in scenario w	
N_s	Notional for swap s	
D_s	Maturity for swap s	
r_s	Fixed rate for swap s	
τ	Tenor of the floating leg	
$C_{q,m}$	curve of category q at time m	
κ	Tenor of the fixed leg	
T_s	Type of swap s	
F_s	Indicator for type of leg for swap s	
sr_m	Swap rate in month m at time 0	
x	Target duration equity (years)	
Z	Amount of iterations	

The different types of categories will be subdivided below

$$q = \begin{cases} \tilde{A} & \text{if category type is asset} \\ \tilde{L} & \text{if category type is liability} \\ \tilde{S} & \text{if category type is swap} \end{cases} \quad (16)$$

$$T_s = \begin{cases} 1 & \text{if swap type is } \textit{payer} \\ 2 & \text{if swap type is } \textit{receiver} \end{cases} \quad (17)$$

$$F_s = \begin{cases} 1 & \text{if leg type is } \textit{fixed} \\ 2 & \text{if leg type is } \textit{floating} \end{cases} \quad (18)$$

As a swap consists of two legs, one which is an asset whereas the other is a liability, for every swap two present values and also two durations have to be computed. One for the floating leg and one for the fixed leg. As the present value of each leg will respond differently to changes in interest rates. The duration of each leg is also different as the floating leg is less sensitive to changes in interest rates.

$$PV_{q,w} = PV_{s,F_s,w} \quad (19)$$

$$DUR_q = DUR_{s,F_s} \quad \text{if } q = \tilde{S} \quad (20)$$

4.1 Gap definition

With regard to the gap analysis; these are the underlying relationships:

$$m = 1, \dots, M \quad (21)$$

$$i = 1, \dots, I \quad (22)$$

$$i = \begin{cases} 1 & \text{for } m = 1, \dots, p \\ 2 & \text{for } m = p + 1, \dots, u, \\ \vdots & \\ I - 1 & \text{for } m = k + 1, \dots, g \\ I & \text{for } m = g + 1, \dots, M \end{cases} \quad (23)$$

Given a cash flow pattern for the assets and liabilities, the gap analysis can be computed using the following formulas:

$$B_{i,q,w} = \begin{cases} \sum_j \sum_{m \in i} \sum_a^A CF_{m,a,j,w} \cdot DF_{m,a,w} & \text{if } q = \tilde{A} \\ - \sum_j \sum_{m \in i} \sum_l^L CF_{m,l,j,w} \cdot DF_{m,l,w} & \text{if } q = \tilde{L} \end{cases} \quad (24)$$

The swaps will be treated in the gap analysis as mentioned in table 1. For the counter party agrees to a payer swap, the floating leg can be considered an asset, as in a payer swap one pays fixed and receives float. The fixed leg can be considered a liability in payer swap. In a receiver swap it is the other way around. For a swap, its fixed rate has to be determined according to the formula discussed in the methodology. Bear in mind that this equation holds at swap initiation, also remember at initiation $w = 0$;

$$\begin{aligned}
PV_{s,F_s=2,w} &= PV_{s,F_s=1,w} \\
N_s &= \sum_{m=\kappa}^{D_s-\kappa} r_s \cdot N_s \cdot DF_{m,\tilde{S},w} + (r_s + 1) \cdot N_s \cdot DF_{D_s,\tilde{S},w} \\
N_s(1 - DF_{D_s,\tilde{S},w}) &= \sum_{m=\kappa}^{D_s-\kappa} r_s \cdot N_s \cdot DF_{m,\tilde{S},w} + r_s \cdot N_s \cdot DF_{D_s,\tilde{S},w} \\
N_s(1 - DF_{D_s,\tilde{S},w}) &= \sum_{m=\kappa}^{D_s} r_s \cdot N_s \cdot DF_{m,\tilde{S},w} \\
r_s &= \frac{N_s(1 - DF_{D_s,\tilde{S},w})}{\sum_{m=\kappa}^{D_s} N_s \cdot DF_{m,\tilde{S},w}} \\
r_s &= \frac{(1 - DF_{D_s,\tilde{S},w})}{\sum_{m=\kappa}^{D_s} DF_{m,\tilde{S},w}} \tag{25}
\end{aligned}$$

The cash flow pattern for swaps can be found hence:

$$CF_{m,\tilde{S},s,w} = \begin{cases} N_s \cdot \sqrt{1 + sr_\tau} DF_{\tau,\tilde{S},w} & \text{if } m = \tau, F_s = 2 \\ 0 & \text{if } m \neq \tau, F_s = 2 \\ 0 & \text{if } m \bmod \kappa \neq 0, F_s = 1 \\ r_s \cdot N_s \cdot DF_{m,\tilde{S},w} & \text{if } m \bmod \kappa = 0, m \neq D_s, F_s = 1 \\ (1 + r_s) \cdot N_s \cdot DF_{m,\tilde{S},w} & \text{if } m = D_s, F_s = 1 \end{cases} \tag{26}$$

$$B_{i,\tilde{S},w} = \begin{cases} \sum_s^S \sum_{m \in i} CF_{m,\tilde{S},s,w} & \text{if } F_s = 2, T_s = 1 \\ \sum_s^S \sum_{m \in i} CF_{m,\tilde{S},s,w} & \text{if } F_s = 1, T_s = 2 \\ - \sum_s^S \sum_{m \in i} CF_{m,\tilde{S},s,w} & \text{if } F_s = 2, T_s = 2 \\ - \sum_s^S \sum_{m \in i} CF_{m,\tilde{S},s,w} & \text{if } F_s = 1, T_s = 1 \end{cases} \tag{27}$$

Where $DF_{m,q,w}$ can be determined using the respective curve $C_{m,q}$.

$$DF_{m,q,w} = \begin{cases} \min \left(1; \frac{1}{(1+C_{m,\tilde{A}}+0.0001*w)^{m/12}} \right) & \text{if } q = \tilde{A} \\ \min \left(1; \frac{1}{(1+C_{m,\tilde{L}}+0.0001*w)^{m/12}} \right) & \text{if } q = \tilde{L} \\ \min \left(1; \frac{1}{(1+C_{m,\tilde{S}}+0.0001*w)^{m/12}} \right) & \text{if } q = \tilde{S} \end{cases} \quad (28)$$

The $\min = 1$ is there because DNB requires their supervisee to floor at 0.0%.

The resulting gap profile will be by adding the different buckets together:

$$\begin{aligned} G_{i,w} &= \sum_q B_{i,q,w} && \forall i \\ &= B_{i,\tilde{A},w} + B_{i,\tilde{L},w} + B_{i,\tilde{S},w} && \forall i \end{aligned} \quad (29)$$

4.2 Interest Rate sensitivity

With regard to interest rate sensitivity, the $DV01_{q,i}$ has to be determined for all q and i . It can be derived by subtracting the total size of a bucket in a +1 basis point scenario by the total size of a bucket in the base scenario of 0 basis points. This has to be done for both the assets and the liabilities. And when add together, one gets the $DV01$ of a specific bucket.

$$DV01_{i,q} = \begin{cases} (B_{i,\tilde{A},1} - B_{i,\tilde{A},0}) \cdot 10^4 & \text{if } q = \tilde{A} \\ (B_{i,\tilde{L},1} - B_{i,\tilde{L},0}) \cdot 10^4 & \text{if } q = \tilde{L} \end{cases} \quad (30)$$

The total $DV01$ of bucket i can be determined using:

$$DV01_i = \sum_q DV01_{i,q} \quad \text{for } i = (2, \dots, I), q = \{\tilde{A}, \tilde{L}\} \quad (31)$$

For the swaps the $DV01$ is the change in value of the swap given a 1 basis points shock. Determined by considering both legs, the ‘‘asset’’ leg and the ‘‘liability’’ leg.

$$DV01_{s,\tilde{S}} = \begin{cases} (PV_{s,F_s=2,1} - PV_{s,F_s=1,1}) \cdot 10^4 & \text{if } T_s = 1 \\ (PV_{s,F_s=1,1} - PV_{s,F_s=2,1}) \cdot 10^4 & \text{if } T_s = 2 \end{cases} \quad (32)$$

The value of each leg of the swap can be found using the following formula:

$$\begin{aligned}
PV_{s,F_s,w} &= \sum_{m=1}^{D_s} CF_{m,\tilde{S},s} \cdot DF_{m,\tilde{S},w} \\
&= \sum_{m=1}^{D_s} CF_{m,\tilde{S},s} \cdot \frac{1}{(1 + C_{m,\tilde{S}} + 0.0001 * w)^{m/12}} \quad \text{if } F_s = 1 \quad (33)
\end{aligned}$$

$$\begin{aligned}
PV_{s,F_s,w} &= N_s \sqrt{1 + C_{\tau,\tilde{S}}} \cdot DF_{\tau,\tilde{S},w} \\
&= N_s \sqrt{1 + C_{\tau,\tilde{S}}} \cdot \frac{1}{(1 + C_{\tau,\tilde{S}} + w \cdot 10^{-4})^{m/12}} \quad \text{if } F_s = 2 \quad (34)
\end{aligned}$$

The duration of the assets and liabilities can be computed using the following formulas:

$$\begin{aligned}
DUR_q &= \begin{cases} \left(- \left(\frac{\sum_{a=1}^A \sum_{m=1}^M (DF_{m,q,1} \cdot \sum_j^J CF_{m,q,j})}{\sum_{a=1}^A \sum_{m=1}^M (DF_{m,q,0} \cdot \sum_j^J CF_{m,q,j})} \right) + 1 \right) * 10^4 & \text{for } q = \tilde{A} \\ \left(- \left(\frac{\sum_{l=1}^L \sum_{m=1}^M (DF_{m,q,1} \cdot \sum_j^J CF_{m,q,j})}{\sum_{l=1}^L \sum_{m=1}^M (DF_{m,q,0} \cdot \sum_j^J CF_{m,q,j})} \right) + 1 \right) * 10^4 & \text{for } q = \tilde{L} \\ \left(- \left(\frac{PV_{q,1}}{PV_{q,0}} \right) + 1 \right) * 10^4 & \text{for } q = \{\tilde{A}, \tilde{L}\} \end{cases} \quad (35)
\end{aligned}$$

The above methodology can be used to determine the duration for the individual legs of the swaps, by using formulas 33 and 34:

$$DUR_{s,F_s} = \left(- \left(\frac{PV_{s,F_s,1}}{PV_{s,F_s,0}} \right) + 1 \right) * 10^4 \quad \text{for } F_s = 1, 2 \quad (36)$$

Duration for equity, which is one of the most used indicators in practice can be determined by dividing the product of all the assets and their respective duration, minus the product of all the liabilities and their respective durations by equity. As any swap has two legs of which one is an asset whereas the other is a liability. The formula for equity duration will be as follows:

$$\begin{aligned}
DUR_{\tilde{E}} = & \frac{\sum_{a=1}^A PV_a \cdot DUR_a + \sum_{s=1|F_s=1,T_s=2 \text{ OR } F_s=2,T_s=1}^S PV_s \cdot DUR_{s,F_s}}{PV_{\tilde{E}}} \\
- & \frac{\sum_{l=1}^L PV_l \cdot DUR_l + \sum_{s=1|F_s=1,T_s=1 \text{ OR } F_s=2,T_s=2}^S PV_s \cdot DUR_{s,F_s}}{PV_{\tilde{E}}} \quad (37)
\end{aligned}$$

4.3 Solution Method

The decision variable will be the different notionals for the individual swaps of the swap portfolio; the portfolio aims for a target duration of equity. As for each swap its rate and maturity are known, the decision variable is the notional of each swap. The type of each swap will be determined as well. Equity duration is a metric which is used in practice a lot. Practitioners have to decide for themselves what duration of the equity is desirable, given the current flat yield curve a duration of the equity of around 2 years is common, as aiming for a higher duration is not yielding a lot currently, while the risk does increase. Thus the optimization problem will become as follows:

Choose:

$$N_i \quad \forall i \in I \setminus 1 \quad (38)$$

Such That:

$$Dur_{\tilde{E}} \quad \approx x \quad (39)$$

In hedging one tries to find a portfolio of hedging instruments which fluctuate with same magnitude but in the opposite direction of the underlying one is trying to hedge. How the individual gaps respond to interest rates has to be determined. The sensitivity of assets and liabilities in a specific gap, will need to be hedged by a swap which reacts with same magnitude but in the opposite direction. $DV01_i$, which is the DV01 of bucket i , is computed using the following formula 31. As one swap is responsible for that particular bucket, offsetting that sensitivity will have to be done using a correct type of swap⁵. A payer swap is needed if there is a positive gap and one wants to hedge “downwards”. Whereas a receiver swap will be required when one wants to hedge “upwards”.

⁵Either a payer or a receiver swap

$$T_{s=i} = \begin{cases} 1 & \text{for } DV01_i \leq 0 \\ 2 & \text{for } DV01_i > 0 \end{cases} \quad (40)$$

The notional required to offset that swap will be:

$$N_{i=s} = \frac{DV01_i}{DV01_{s,\bar{s}}} \quad (41)$$

The first swap which will be determined is the swap with the largest maturity. This is done because when one starts with the swap off the smallest duration, the cash flow patterns and sensitivity are affected by every consecutive swap with larger maturity, therefore this model starts with the swap of the largest maturity. Given the above methodology the swap will be chosen and its cash flow pattern will be determined. Based on this cash flow pattern the gap analysis will be changed and the swap in an earlier bucket is the next swap to be determined. As the DV01 of a swap affects multiple buckets this procedure is repeated Z times, in order to be as accurate as possible. After each iteration one will be left with two swaps of each maturity; this will be evened out by the sort swaps algorithm. The entire procedure will run for less than a few seconds and will equalize all bucket DV01 mismatches which existed in the gap analysis for all buckets.

This procedure will be repeated for each bucket in the gap analysis. Providing the swaps which adjust the gap analysis such that the desired risk profile is met. The model will close all the positions based on DV01. The initiation algorithm looks as follows.

Data: Initiation Algorithm

Result: Swap portfolio which reduces the duration of equity close to 0 within Z iterations.

```

begin
   $y = 0$ 
  while  $y < Z$  do
    for  $\forall s$  do
       $p = 0$ 
      if  $DV01_{S-p, \tilde{B}} > 0$  then
         $T_{S-p} = 2$ 
      else
         $T_{S-p} = 1$ 
      end
       $N_{S-p} = \frac{DV01_{S-p, \tilde{B}}}{DV01_{S-p, \tilde{S}}}$ 
       $p = s$ 
    end
     $y = y + 1$ 
    Loop ;
  end
  Swap Portfolio with all parameters for each swap
   $DUR_{\tilde{E}} = U$ 
end

```

Algorithm 1: Initiation Algorithm

U being the duration of equity once the initiation algorithm has been run. After this algorithm has been run, the interest rate sensitivity on basis of $DV01$ will have been reduced for all the buckets except the first one, the initial values for each of the notionals is known: N_s^\bullet , the optimization algorithm can be run. This algorithm will take into account the gap profile as determined by the initiation algorithm. And will add⁶ an equal amount N_s^\diamond to each swap in order to acquire the final notionals N_s^* and thus, the desired risk profile. This will ensure the target duration of equity is achieved.

⁶or subtract

Data: Optimization Algorithm

Result: Swap portfolio which targets a specific duration of equity of x .

```
begin
   $DU R_{\tilde{E}}^{\text{target}} = x$ 
   $N_s^\diamond = \frac{x-U}{\left( \frac{-\sum_s DV01_{s,\tilde{S}}}{PV_{\tilde{E}}} \right)}$ 
  for  $\forall s$  do
    if  $T_s = 1$  then
       $N_s^\star = N_s^\bullet + N_s^\diamond$ 
    else
       $N_s^\star = N_s^\bullet - N_s^\diamond$ 
    end
  end
  Optimal swap portfolio
end
```

Algorithm 2: Optimization Algorithm

It is also possible for a user which wants to take on some additional risk to use the model and synthetically create mismatches anywhere. It is important to get an indication of hedge performance, how the swap portfolio is affecting the position of a firm given certain interest rate scenarios. In this way, one can model the balance sheet's sensitivity to interest rates. For most banks; an increase in interest rates will decrease their equity; they will therefore choose hedging instruments which gain value as interest rates go up, in order to hedge the losses in other parts of their balance sheet. A traditional bank, one which uses short-term funding to fund long term assets would require payer swaps in order to hedge for the risk of rising increasing rates⁷.

4.4 Swap performance

The main purpose of hedging is to reduce the probability and magnitude of adverse movements in equity as a result of interest rates. Before one can determine an effective hedging strategy one needs to get grips with balance sheet dynamics; how is the balance sheet affected by interest rates. Also whilst keeping flooring in mind. Flooring being the fact that interest rates should not go below a certain rate. Currently DNB requires banks to floor at 0%. Perhaps in the near future, if rates drop even further DNB might require banks to floor at lower rates.

⁷If interest rates would decrease the bank would gain value

The first key performance indicator (KPI), will be duration of equity. Which can be calculated using formula 42. This KPI is chosen as it used in practice a lot gives an accurate indication to interest rate sensitivity⁸. The second KPI will be EVAR. This is the value of equity under an interest rate shock of 200 basis points up and down. This KPI is chosen as DNB requires banks to report EVAR. Furthermore, using 10 basis points increments, equity in 40 interest rate scenarios will be investigated.

$$DUR_{\tilde{E}} = \frac{\sum_{a=1}^A PV_a \cdot DUR_a + \sum_{s=1|F_s=1, T_s=2 \text{ OR } F_s=2, T_s=1}^S PV_s \cdot DUR_{s, F_s}}{PV_{\tilde{E}}} - \frac{\sum_{l=1}^L PV_l \cdot DUR_l + \sum_{s=1|F_s=1, T_s=1 \text{ OR } F_s=2, T_s=2}^S PV_s \cdot DUR_{s, F_s}}{PV_{\tilde{E}}} \quad (42)$$

As described in the methodology section, the EVAR will be evaluated in multiple scenarios including non parallel shifts in the yield curve, as it also provides additional insight in how well the swaps perform under non parallel shifts in the yield curve. The first two EVAR calculations, which also have to be reported to DNB, are the ones in which equity at risk will be determined in a 200 basis points parallel shock up and down. Furthermore, EVAR will be computed under two different scenarios. The first scenario is one in which the yield curves are steepened, whereas the second scenario will be one in which the yield curves are flattened. $EVAR_V^{\text{scen}}$, will be denoted as follows. There are 4 different scenarios. V indicates whether swaps are present or if they are absent, 1 is for swaps being present, 0 is for swaps being absent.

$$EVAR_V^{\text{scen}} \begin{cases} EVAR_V^+ & \text{for equity at risk in a +200 bps scenario} \\ EVAR_V^- & \text{for equity at risk in a -200 bps scenario} \\ EVAR_V^f & \text{for equity at risk in flattened yield curve scenario} \\ EVAR_V^u & \text{for equity at risk in steepened yield curve scenario} \end{cases} \quad (43)$$

Where $EVAR_1^+$, the equity at risk in the +200 basis point scenario with

⁸for smaller shocks in interest rates.

swaps, can be computed as follows.

$$\begin{aligned}
EVAR_1^+ &= \left(\sum_{a=1}^A PV_{(a,200)} + \sum_{s=1|F_s=1,T_s=2 \text{ OR } F_s=2,T_s=1}^S PV_{(s,200)} \right. \\
&\quad \left. - \sum_{l=1}^L PV_{(l,200)} - \sum_{s=1|F_s=1,T_s=1 \text{ OR } F_s=2,T_s=2}^S PV_{(s,200)} \right) \\
&\quad - \left(\sum_{a=1}^A PV_{(a)} + \sum_{s=1|F_s=1,T_s=2 \text{ OR } F_s=2,T_s=1}^S PV_{(s)} \right. \\
&\quad \left. - \sum_{l=1}^L PV_{(l)} - \sum_{s=1|F_s=1,T_s=1 \text{ OR } F_s=2,T_s=2}^S PV_{(s)} \right) \quad (44)
\end{aligned}$$

For equity at risk without swaps the calculation will be as follows:

$$\begin{aligned}
EVAR_0^+ &= \left(\sum_{a=1}^A PV_{(a,200)} - \sum_{l=1}^L PV_{(l,200)} \right) \\
&\quad - \left(\sum_{a=1}^A PV_{(a)} - \sum_{l=1}^L PV_{(l)} \right) \quad (45)
\end{aligned}$$

$$\begin{aligned}
EVAR_0^- &= \left(\sum_{a=1}^A PV_{(a,-200)} - \sum_{l=1}^L PV_{(l,-200)} \right) \\
&\quad - \left(\sum_{a=1}^A PV_{(a)} - \sum_{l=1}^L PV_{(l)} \right) \quad (46)
\end{aligned}$$

For the scenarios with steepened and flattened yield curve the same methodology applies, expect different curves have been used to determine the present values.

5 Numerical Results

A numerical example will be given below, which will illustrate how the model works. Assume a firm to have 1 type of asset and 1 type of liability. The swaps it wants to use, have a tenor of 6-Months for the floating leg and 1 year for the fixed leg. There are three curves which will be used to discount the different categories; the assets, liabilities and swaps. The different curves used can be found in the appendix. For the asset, $J_a = 50$ individual cash flows patterns have been generated; with their maturity following a normal distribution with an average of $\mu_a = 6$ years and a standard deviation of $\sigma_a = 2$ years, where 1 is the minimum. The notional equals 100 and the coupon payment is set at 3%. For the liability, $J_l = 50$ individual cash flows have been generated; with their maturity following a normal distribution with an average of $\mu_l = 4$ years and a standard deviation of $\sigma_l = 3$ years, with a minimum of 1 year. Also for the liability cash flows the notional equals 100 but the rate paid is 1.5%. The model will do 10 iterations; $Z = 9$. For the swap curve interpolation, figure 7 shows the plot of the interpolated swap curve, the data can be found in the appendix. Thus:

$$\begin{aligned} A &= 1 \\ L &= 1 \\ \tau &= 0.5 \\ \kappa &= 1 \\ Z &= 9 \end{aligned}$$

For the gap analysis, the following buckets have been defined.

$$m = 1, \dots, 300 \quad (47)$$

$$i = 1, \dots, 7 \quad (48)$$

$$i = \begin{cases} 1 & \text{for } m = 1, \dots, 12 & 0-1 \text{ year} \\ 2 & \text{for } m = 13, \dots, 24 & 1-2 \text{ year} \\ 3 & \text{for } m = 25, \dots, 36 & 2-3 \text{ year} \\ 4 & \text{for } m = 37, \dots, 60 & 3-5 \text{ year} \\ 5 & \text{for } m = 61, \dots, 84 & 5-7 \text{ year} \\ 6 & \text{for } m = 85, \dots, 120 & 7-10 \text{ year} \\ 7 & \text{for } m = 121, \dots, 300 & 10-15 \text{ year} \end{cases} \quad (49)$$

The maturities for each of the swaps has to be within the interval of the that particular bucket. Since the fixed leg tenor, κ is 1 year, the maturity of the swaps has to be in multiplicatives of that tenor; 1 year. The following swap maturities have been defined. The smallest bucket, the first one, will

not be defined as this is the bucket to which everything is swapped to⁹.

$$D_s = \begin{cases} 2 \text{ year} & \text{for } s = 2 \\ 3 \text{ year} & \text{for } s = 3 \\ 5 \text{ year} & \text{for } s = 4 \\ 6 \text{ year} & \text{for } s = 5 \\ 9 \text{ year} & \text{for } s = 6 \\ 14 \text{ year} & \text{for } s = 7 \end{cases} \quad (50)$$

The initial gap analysis based on the generated cash flows is visualized in figure 9.

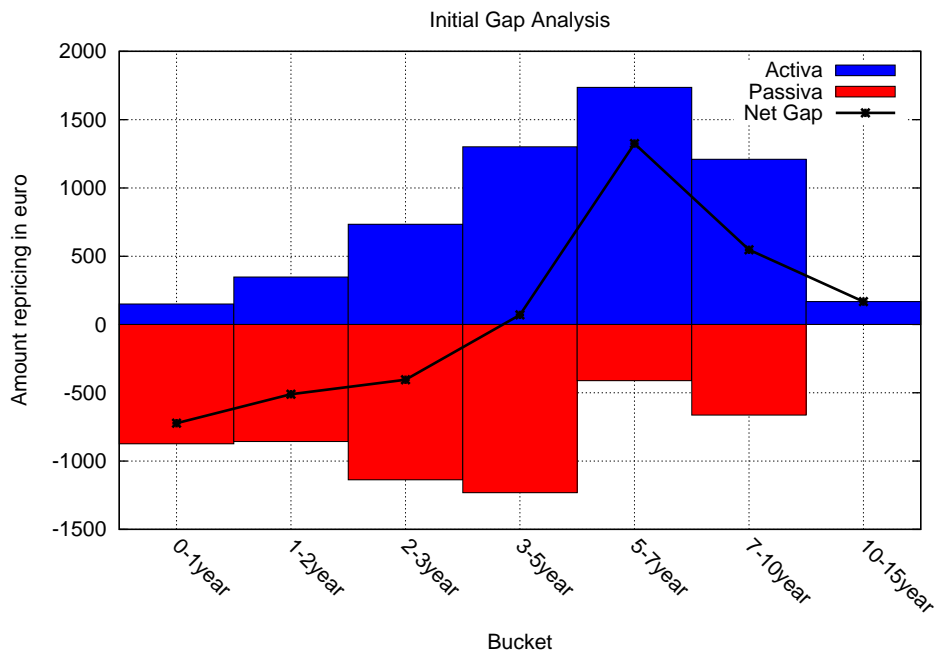


Figure 9: Initial Gap Analysis

These simulated assets and liabilities, yield equity worth €473.1. The duration of the equity is 27.1 year. Which can be considered large exposure to interest rate risk; as a 100 basis point increase in interest rates will reduce equity with approximately 27.1%. The balance sheet looks as follows.

⁹If there is a mismatch in bucket 5 for example, it will be closed by creating a mismatch in the first bucket, as depicted in table 1

Assets	Market value (€)	Duration (year)
<i>Asset</i>	€ 5.645,77	5,77
<i>Swaps</i>	€ -	0,00
Total	€ 5.645,8	5,77
Liabilities	Market value (€)	Duration (year)
<i>Liability</i>	€ 5.172,71	3,82
<i>Swaps</i>	€-	0,00
<i>Total liabilities</i>	€ 5.172,7	3,82
<i>Equity</i>	€ 473,1	27,122707

Table 2: Starting balance sheet

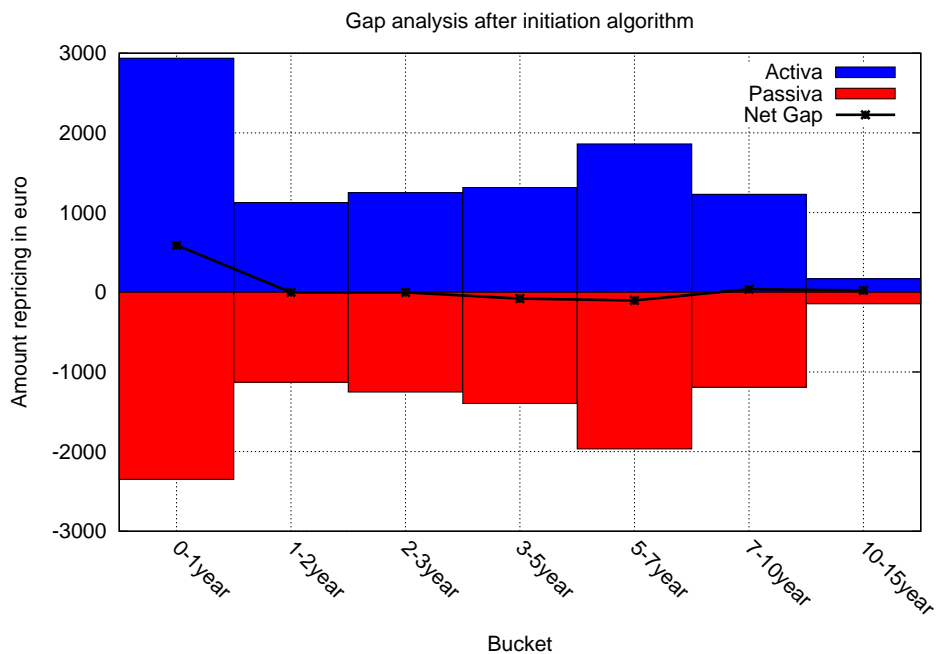


Figure 10: Initial Gap Analysis

After the initiation algorithm, this is the new balance sheet. The value of the equity remain unchanged, as a swap is worth zero at initiation. The duration of the equity however, has been changed: the new duration of the equity is -0.1 year.

Assets	Market value (€)	Duration (year)
<i>Asset</i>	€ 5.645,77	5,77
<i>Swaps</i>	€ 3.191,88	1,07
Total	€ 8.837,7	4,07
Liabilities	Market value (€)	Duration (year)
<i>Liability</i>	€ 5.172,71	3,82
<i>Swaps</i>	€ 3.191,9	5,11
<i>Total</i>	€ 8.364,6	4,31
<i>Equity</i>	€ 473,1	-0,145383

Table 3: Balance sheet after initiation step

The swap portfolio which achieves this gap analysis and balance sheet can be found in table 4.

Type	Maturity (year)	Notional	Fixed rate leg	Floating rate leg
1	14	€ 157.13	0,90%	6M - Euribor
1	9	€ 525.54	0,62%	6M - Euribor
1	6	€ 1445.15	0,37%	6M - Euribor
1	5	€ 129.87	0,28%	6M - Euribor
2	3	€ 414.72	0,16%	6M - Euribor
2	2	€ 519.47	0,15%	6M - Euribor

Table 4: Swaps to purchase after initiation algorithm

The swaps have a notional amount of approximately 3000, which given a balance sheet total of over 5000, seems to be a fair result. A traditional bank will require more payer swaps than receiver swaps, which is also the case in this example. Swap performance will be measured by exposing the position of the firm to different interest rate scenarios. The firm with the swaps should experience less problems due to these interest rate scenarios. Furthermore, one can see that the notional with the 6 year maturity is pretty high;1445. In reality, a practitioner might split this swap into two or even three smaller ones; for risk mitigation purposes, in order to reduce counter party credit risk.

5.1 Optimization Algorithm

Assume the management of the firm aims at a duration of equity of 3 years. The swaps found in the initiation algorithm will need to be altered in order to reach this target duration. The optimization algorithm can be used to achieve this. The new gap analysis can be found in figure 11:

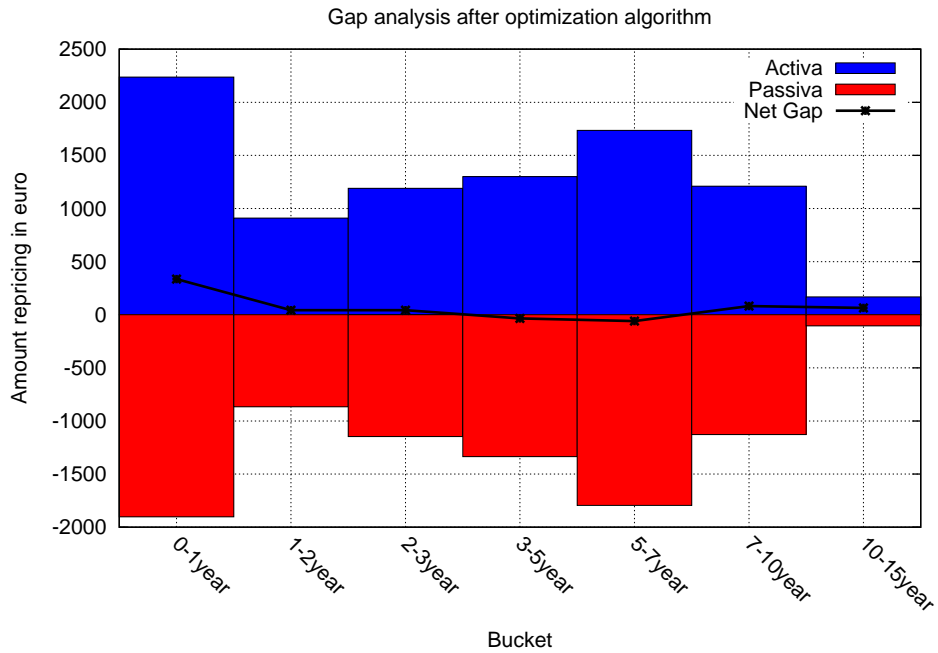


Figure 11: Gap Analysis after optimization

The balance sheet after optimization looks as follows; one can see that the target duration of equity has been achieved.

Assets	Market value (€)	Duration (year)
<i>Asset</i>	€ 5.645,77	5,77
<i>Swaps</i>	€ 3.106,02	1,14
Total	€ 8.751,8	4,13
Liabilities	Market value (€)	Duration (year)
<i>Liability</i>	€ 5.172,71	3,82
<i>Swaps</i>	€ 3.106,0	4,81
<i>Total</i>	€ 8.278,7	4,19
<i>Equity</i>	€ 473,1	3,000235

Table 5: Balance sheet after optimization algorithm

And these are the swaps which are required.

Type	Maturity (year)	Notional	Fixed rate leg	Floating rate leg
1	14	€ 114,2	0,90%	6M-Euribor
1	9	€ 482,61	0,62%	6M-Euribor
1	6	€ 1402,22	0,37%	6M-Euribor
1	5	€ 86,94	0,28%	6M-Euribor
2	3	€ 457,65	0,16%	6M-Euribor
2	2	€ 562,4	0,15%	6M-Euribor

Table 6: Swaps to purchase after optimization

5.2 Swap Performance

Especially when looking at the gap analysis, one has no indication of swap performance. It does give an indication of where repricing mismatches but it doesn't give an idea with regard to swap performance. The first indicator for swap performance is duration of equity. Duration of equity has been reduced from 27.1 years to approximately -0.1 year. Using the above methodology, this could be reduced further, by using the optimization algorithm to target a duration of equity of 0.

For the equity in different interest rate scenarios let's consider two different settings. The first setting being the firm with the swap portfolio after the initiation algorithm. The second setting being the firm with the swap portfolio after the optimization algorithm. These two settings will be exposed to parallel shocks in interest rates, first of all just by ranging from -200 basis points, to 200 basis points, and secondly a visual representation with 10 basis points increments.

5.2.1 Performance initiation algorithm

The EVAR in a 200 basis point shock up and down can be found in the table below. One can see that the variation in equity in extreme scenarios has been reduced immensely.

	Swaps	No Swaps
$EVAR^+$	€ 1,16	€ -232,85
$EVAR^-$	€ -8,85	€ 69,88
Difference	€ 10,02	€ 302,72

Table 7: Equity with and without swaps, initiation algorithm

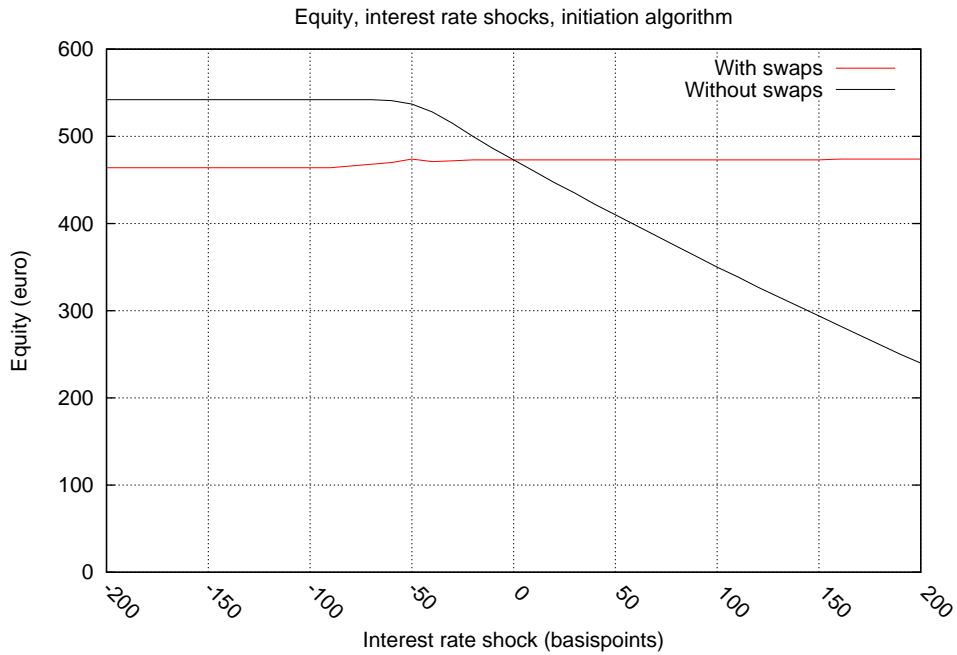


Figure 12: Equity in different interest rate shocks, initiation algorithm

The curve in figure 12 shows the performance of the swap portfolio given parallel shocks in the yield curve. As one can see, the curve is way less volatile than it used to be, indicating a good hedge against parallel shocks in the yield curve.

5.2.2 Performance optimization algorithm

For the second setting, in which the swaps have been altered to reach a target duration of 3, this is what equity look like given a 200 basis point shock up and down.

	Swaps	No Swaps
$EVAR^+$	€ -25,76	€ -232,85
$EVAR^-$	€ 0,65	€ 69,88
Difference	€ 26,41	€ 302,73

Table 8: Equity with and without swaps, optimization algorithm

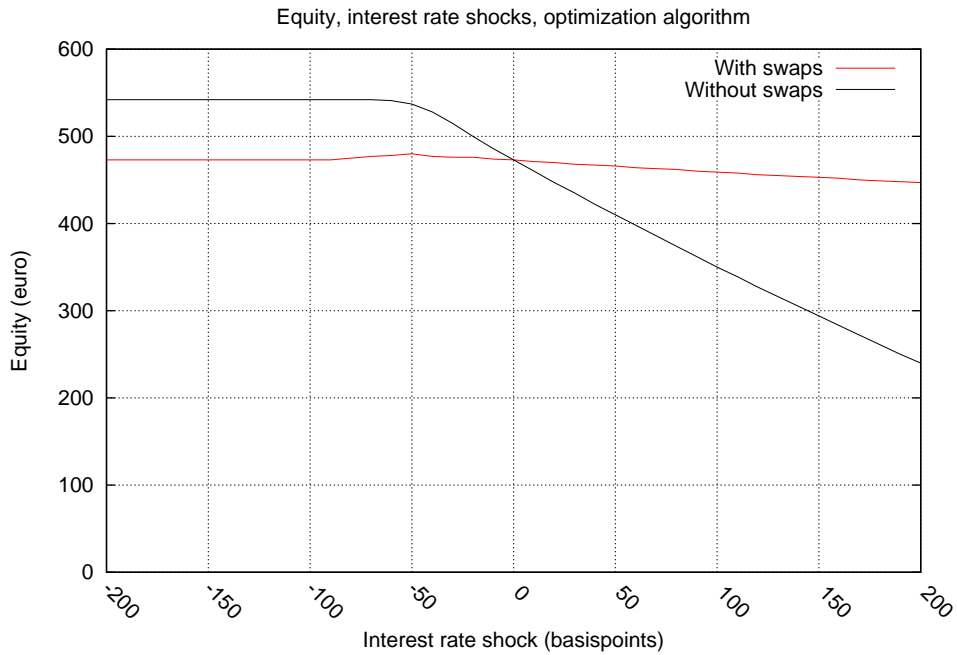


Figure 13: Equity in different interest rate shocks, optimization algorithm

There is an inverse relation between duration and the value of equity given interest rate shocks. This means that for an asset or liability with a positive duration; its value will reduce if rates increase. On the other hand, the value of equity will increase if interest rate go down. Which can also be seen from the results.

5.3 Non parallel shifts in yield curve

As mentioned two different scenarios will be evaluated. The first scenario will be where the respective yield curves have been steepened, shocked up 200 basis points at the long end. Because in the numerical example there are no cash flow after 15 years, 15 years will be the horizon at which the curves will be altered. The equity in the base case was €473.1. For the two different scenarios; the results can be found below:

	Swaps	No swaps
$EVAR^u$	€-4,5	€- 148,1
$EVAR^f$	€-0,43	€- 100,6

Table 9: Non parallel shifts in yield curve

As one can see, the swaps perform well in the steepening scenario the

$EVAR^s$, as sensitivity to equity is in the steepening scenario is reduced almost 30 fold. In the flattening scenario $EVAR^f$ would be reduced 50 times if swaps were used. The swap portfolio of the initiation algorithm was used.

5.4 No Flooring

Currently DNB requires banks to floor at 0.0%. However, as mentioned earlier multiple rates have dropped below this level, it might be thinkable that DNB will no longer require banks to floor at this level. The performance of the model should not be affected as a result of not flooring, as it uses the DV01 which is determined independently of whether or not flooring will be used. In figures 14 and 15 one can see the swap performance for the two different swap portfolios; when there is no flooring at 0.0%. Perhaps this plot shows the effectiveness of the model better; as the previous graphs had the effect of flooring which caused the curve to look a bit odd.

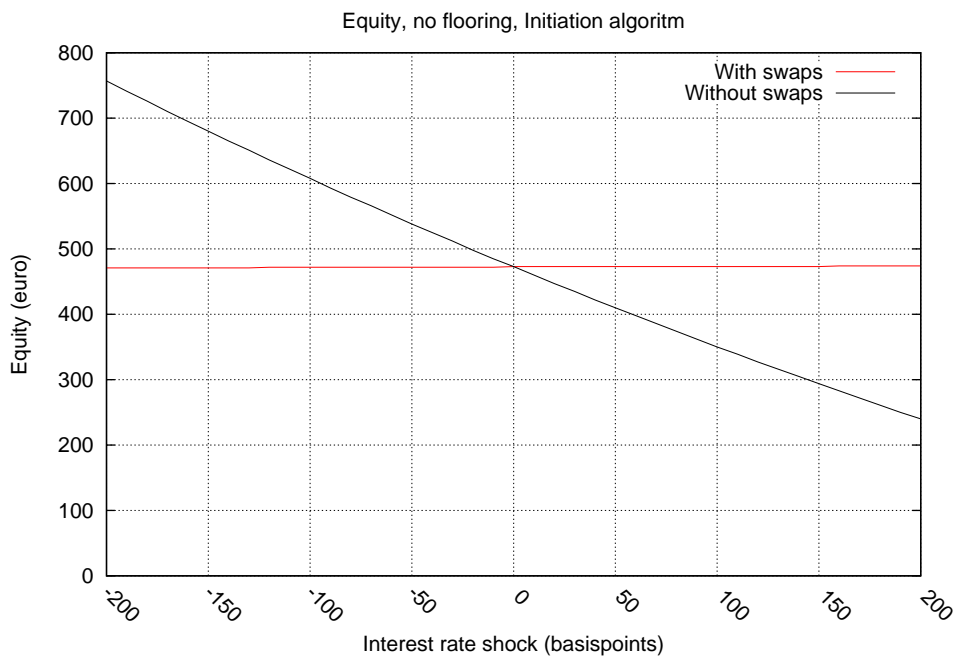


Figure 14: Equity , no flooring, initiation algorithm

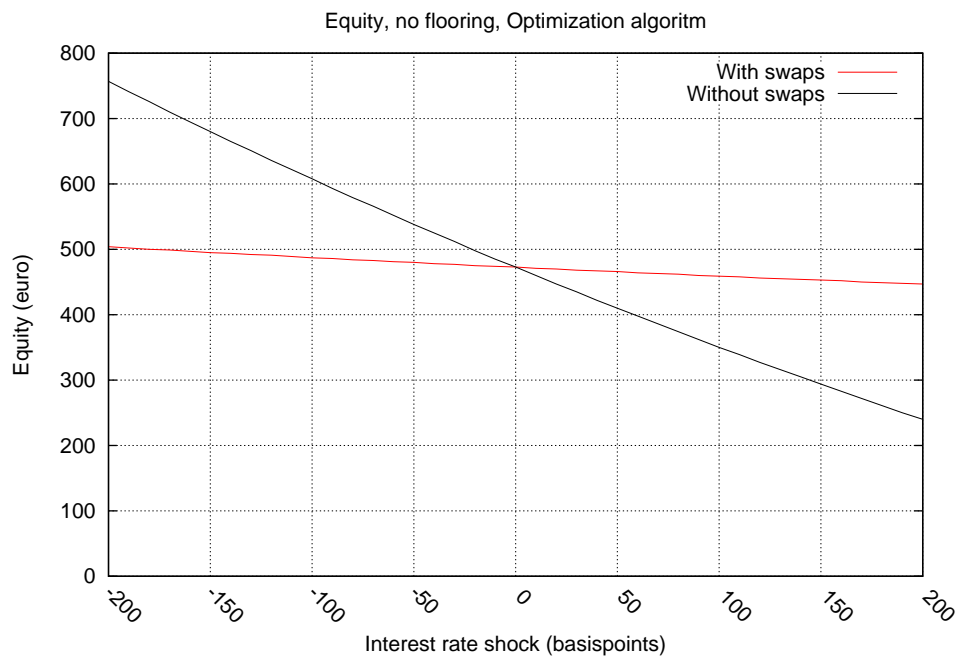


Figure 15: Equity , no flooring, optimizat algorithm

Especially in figure 14, the hedge performance can be seen clearly, yielding almost a straight line for every interest rate shock. For figure 15 the line is also straight but a bit downward sloping. This has to do with the fact that as the target duration was set at 3 years, which implies that value will be lost whenever rates increase. With a duration of equity of 3 years, the firm will gain value when rates drop.

6 Conclusion & Future work

The peer bank analysis concluded that other banks use duration of equity to target a desired risk profile. The hedging instruments which are used are usually plain vanilla interest rate swaps. The tool used in the decision making process is a gap analysis. The interest rate sensitivity of underlying buckets - $DV01_i$ - is used to determine the notionals of the individual swaps. Non-parallel shifts in the yield curve are not taken into account in the decision making process. Banks usually do not use their interest vision into their hedge decisions.

Partially based on the findings of the peer bank analysis, the methodology on which the model is based, was determined. This implies that the model uses a gap analysis and based on the $DV01$ of the underlying buckets the swap portfolio is determined. It tackles repricing risk which, as being considered the main source of interest rate risk, is an achievement. One could use a 20 year swap to close a gap in the three year bucket, but the gap analysis ensures that this is not possible and a gap in the three year bucket will be hedged with a swap of an appropriate maturity. Furthermore the model also allows the user to add a swap manually and to adjust the wanted gap mismatch of each bucket, if one wants to take on some extra risk. And finally the model also allows for meeting a desired risk profile by being able to target a required duration of equity accurately.

The results suggest that the swap portfolio hedges the position of the firm very effectively. Duration of the equity was reduced from 27.1 years to -0.1 years. The volatility in equity was reduced 30 fold, from €301 to only €10. Also when considering multiple scenarios in which yield curves moved in a non-parallel fashion the swaps seemed to perform well. After the correct parameters have been entered the model runs automatically leaving enough flexibility for the user to tailor it to its specific needs.

One does have to bear in mind, that given the current economic climate, the probability of rates going down might be way smaller than the probability of rates going up. Assume a bank to have a negative duration of equity; this implies that the bank would gain money would interest rates go up. One would therefore get swaps now which would protect against rates going down. But as rates are at all time lows one can argue whether one should protect against the scenario of rates dropping even further. Therefore the model should only be used, when the user is very aware of the circumstances and the environment.

Hedging is done in order to reduce risk and remain in control of a firm's financial position. Hedging with derivatives does have some serious implications for the accounting part as well. The financial world has been developing new instruments and tools which can be used to hedge risks. The accounting practice is struggling to keep up with these financial innovations, resulting in new and adjusted accounting practices frequently. Hedge accounting is the accounting practice in which fair value adjustments of the derivatives can be used to offset the fair value adjustments of the hedge items, in the financial statements of the firm. Not applying hedge accounting will result in higher volatility in the profit and loss which is unwanted; as hedging is done in order to reduce volatility. Hedge accounting will have big operational impact, perhaps even as big as the derivative trading itself. A model which would aid practitioners with their hedge accounting practice could be very useful.

Work into determining when to get out of a swap might be a valuable contribution as well. As mentioned earlier, hedging with swaps is not a hedge and forget situation. As the situation is changing constantly, as a result of for example fluctuating interest rates; new deposits, withdrawals and changing asset and liability compositions. The hedges will need to be adjusted constantly to have an hedge as effective as possible. It can also be the case that due to certain circumstances it can be cheaper to get out of the swap and get a new one. Therefore a framework which aids in the decision making process of when it is cheaper to exit a swap and get into a new swap at new rates might be a valuable contribution.

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Appendix

Peer Bank Analysis

The peer bank analysis can be considered the first phase and it involved face to face meetings with employees from other banks which are using derivatives. The objective of this phase was to become acquainted with the problem quickly and to get a broad understanding of how other market participants are using interest rate derivatives to reduce their interest rate risk. A questionnaire had been made and the questions revolved around two main themes; *How are derivatives used, using what methodology?* And the second theme was: *When should swaps not have been used?* Three banks have been visited in order to get understanding how other banks are using interest rate derivatives to reduce interest rate risk. The overall idea was the same for each bank. There were a few distinct differences present however. The results of the peer bank analysis and the literature study were determined how the hedging strategy is set up.

Bank 1

Bank X, the measure used for targeting a desired risk profile was duration of equity. This would be accomplished by using a gap analysis. Interest rate sensitivity was expressed in terms of DV01. A swap would be purchased in the total of all the DV01 would exceed certain limits. As the sum of the DV01 of the individual buckets is very closely linked to duration of the equity, they would hedge whenever their duration of equity was not within a certain range. When they had a clear-cut interest vision this would be incorporated in their decision-making process. Non parallel shifts in the yield curve were not taken into consideration in the decision making process, they were monitored, but not incorporated in the decision making process. Advice from this bank was that the person actually having the do the swap transactions has to be very aware of the process behind it, such that the swaps which are being purchased; are actually the swaps which are required. As swap misuse can lead to big losses.

Bank 2

Bank Y, as for bank X, also duration of equity was leading in the swap determination. A gap analysis is used to determine what swaps to use in order to get their overall duration at a target level. Bank Y has recently reduced the target duration of equity; since the yield curve has become very flat, increasing the duration of equity does not yield much more while the risk does increase. The new duration of equity which is targeted is still mildly positive. Non parallel shifts in yield curve are not taken into account when doing the hedges. They are monitored but they do not influence the hedging

decisions. Furthermore, bank Y uses derivatives purely to hedge risks, not create additional value; an interest rate vision is present but it does not influence the hedging decision. Another measure which is also being monitored is earnings at risk. Earnings at risk is an indicator for the earnings perspective of a firm, however, this it is not incorporated in the decision making. As advice was given that if one wants to start using derivatives to hedge interest rate risk, one should keep things simple. Therefore start off with only plain vanilla interest rate swaps as hedging instruments. Furthermore, one can invent numerous complex measures to target, but in the end it all boils down to duration and equity at risk.

Bank 3

Bank Z, the risk measure they use is also duration of equity. In their decision making process they target a certain duration of the equity. The tool used in the decision making process is a gap analysis. DV01 of underlying buckets are brought to within the required bandwidth. The duration they target is slightly different than the duration bank Y targets. Non parallel shifts in the yield curve are not taken into account.

Questionnaire

- Context; what kind of bank is bank X and what kind of assets and liabilities does bank X have?
- How does bank X measure interest rate risk exposure? What measures are used to guide hedging decisions?
- What are for bank X the biggest risks arising from fluctuating interest rates?
- How do you take the fact that the duration of certain financial instruments is affected by interest rates into account? For example deposit behaviour and mortgage prepayments?
- What were the biggest motivators to start derivative trading?
- Which derivatives does bank X use and why?
- How do you determine the maturity and notional of a derivative?
- Can you walk me through the process of deciding which derivative is suited?
- What have been the effects of derivative trading, operationally and financially? Have models and processes been changed significantly? Did compliance to law and regulations cause internal changes?
- How have you set up collateral management? To what extent does your own liquidity influence decision making? How do you monitor the hedges?
- Do you take non-parallel shifts in the yield curve into account?
- To what extent do you take the expected direction of the interest rates into consideration? How do you determine this expected direction?
- How do you measure the effectiveness of your hedges?
- What accounting choice have you made with regard to hedge accounting?
- What is your experience with regard to finding a suitable counter party? Is it easy? How do you judge the creditrisk of the counterparty?
- When did interest rate derivatives not have the desired effect? When should derivatives not have been used.
- If you were a consultant; what are the do's and don'ts with regard to derivative trading; what were unforeseen effects?

Used curves

Three different curves have been used for the three different categories; assets, liabilities and swaps. First of all the given points for the swap curve will be shown, the third column contains the interpolated points. The points which will be used are the points which were given, the interpolated points will be used for points which no value was yet determined. This is the curve which is used to discount swap cash flows.

Time (yrs)	Given	Interpolation	Time (yrs)	Given	Interpolation
0,5	0,14%	0,00%	15,5		0,97%
1	0,11%	0,10%	16		0,99%
1,5		0,16%	16,5		1,00%
2	0,13%	0,15%	17		1,02%
2,5		0,15%	17,5		1,03%
3	0,16%	0,16%	18		1,04%
3,5		0,18%	18,5		1,06%
4	0,23%	0,20%	19		1,07%
4,5		0,24%	19,5		1,08%
5	0,29%	0,28%	20	1,10%	1,09%
5,5		0,33%	20,5		1,10%
6	0,38%	0,37%	21		1,11%
6,5		0,42%	21,5		1,12%
7	0,45%	0,46%	22		1,13%
7,5		0,51%	22,5		1,14%
8	0,54%	0,55%	23		1,14%
8,5		0,59%	23,5		1,15%
9	0,61%	0,63%	24		1,16%
9,5		0,67%	24,5		1,17%
10	0,69%	0,70%	25		1,17%
10,5		0,73%	25,5		1,18%
11		0,76%	26		1,19%
11,5		0,79%	26,5		1,19%
12		0,82%	27		1,20%
12,5		0,85%	27,5		1,20%
13		0,87%	28		1,21%
13,5		0,89%	28,5		1,21%
14		0,91%	29		1,22%
14,5		0,93%	29,5		1,22%
15	0,99%	0,95%	30	1,21%	1,23%

Table 10: Interpolated swap curve data

For the assets and liabilities the following curves are used:

m	$C_{a,m}$	$C_{l,m}$
12	0,20%	0,29%
24	0,35%	0,33%
36	0,46%	0,39%
48	0,60%	0,47%
60	0,78%	0,55%
72	0,97%	0,64%
84	1,16%	0,74%
96	1,35%	0,84%
108	1,53%	0,93%
120	1,70%	1,03%
132	1,76%	1,11%
144	1,82%	1,18%
156	1,87%	1,24%
168	1,91%	1,29%
180	1,95%	1,34%
192	1,99%	1,38%
204	2,02%	1,42%
216	2,04%	1,45%
228	2,07%	1,48%
240	2,09%	1,52%

Table 11: Curves for assets and liabilities

Non parallel scenarios

The following curves were used for the two different scenarios of non-parallel shifts in the yield curves. A horizon of 15 years was taken.

Year	Steep	Flat
1	0,17%	2,03%
2	0,36%	1,95%
3	0,50%	1,81%
4	0,69%	1,72%
5	0,90%	1,66%
6	1,13%	1,61%
7	1,36%	1,57%
8	1,59%	1,52%
9	1,80%	1,46%
10	2,01%	1,39%
11	2,21%	1,32%
12	2,41%	1,23%
13	2,59%	1,15%
14	2,78%	1,05%
15	2,95%	0,95%

Table 12: Swap curve in different scenarios

For the assets the curves look as follows

Year	Steep	Flat
1	0,34%	2,07%
2	0,62%	2,09%
3	0,86%	2,06%
4	1,14%	2,07%
5	1,45%	2,12%
6	1,77%	2,17%
7	2,10%	2,23%
8	2,42%	2,28%
9	2,73%	2,33%
10	3,03%	2,37%
11	3,10%	2,43%
12	3,15%	2,49%
13	3,20%	2,54%
14	3,25%	2,58%
15	3,28%	2,62%

Table 13: Curves used for discounting assets

And for the liabilities:

Year	Steep	Flat
1	0,42%	2,15%
2	0,60%	2,06%
3	0,79%	1,99%
4	1,00%	1,94%
5	1,22%	1,89%
6	1,44%	1,84%
7	1,67%	1,81%
8	1,91%	1,77%
9	2,13%	1,73%
10	2,36%	1,70%
11	2,57%	1,64%
12	2,78%	1,58%
13	2,97%	1,50%
14	3,16%	1,42%
15	3,34%	1,34%

Table 14: Curves used for discounting liabilities