Modeling the cost of trade finance during the financial crisis of 2008-2009

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MODELING THE COSTS OF TRADE FINANCE DURING THE FINANCIAL CRISIS OF 2008-2009: AN APPLICATION OF DYNAMIC HIERARCHICAL LINEAR MODEL

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Modeling the Costs of Trade Finance during the Financial Crisis of 2008-2009:
An Application of Dynamic Hierarchical Linear Model

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Modeling the Costs of Trade Finance during the Financial Crisis of 2008-2009: An Application of Dynamic Hierarchical Linear Model

Abstract

The authors propose a dynamic hierarchical linear model (DHLM) to study the variations in the costs of trade finance across multiple countries during the global financial crisis of 2008-2009. Specifically, they examine how the impact of a set of four macroeconomic indicators on trade finance costs varied in and around the financial crisis. They find that countries with higher GDP growth faced lower costs of trade finance and countries with higher trade intensity (Trade/GDP) experienced higher trade finance costs in 2009 and 2010. Somewhat surprisingly, the countries with more stock market capitalization compared to GDP also faced higher costs of trade finance during and post crisis. Finally, inflation had a weak statistically significant impact on trade finance costs in 2009. The authors propose extensions to the model and discuss its alternative uses in different contexts.

JEL Classification: C11, F30, F65, G01
Modeling the Costs of Trade Finance during the Financial Crisis of 2008-2009:
An Application of Dynamic Hierarchical Linear Model

Trade finance consists of borrowing using trade credit as collateral and/or the purchase of insurance against the possibility of trade credit defaults (Ahn 2011; Amiti and Weinstein 2011). According to some estimates more than 90% of trade transactions involve some form of credit, insurance, or guarantee (Auboin 2007), making trade finance extremely critical for smooth trades. After the global financial crisis of 2008-2009, the limited availability of international trade finance has emerged as a potential cause for the sharp decline in global trade\(^1\) (e.g., Amiti and Weinstein 2011; Chor and Manova 2012; Haddad, Harrison, and Hausman 2010). As a result, understanding how trade finance costs varied over the period in and around the financial crisis and across countries has become critical for policymakers\(^2\) to ensure adequate availability of trade finance during the crisis period and mitigate the severity of the crisis.

The extant literature documents a large heterogeneity in the way trade finance costs affected nations and firms during the 2008-2009 financial crisis. The World Bank survey evidence suggests that exporters and importers in developing countries faced severe constraints due to limited availability of trade finance (Malouche 2009). But the impact of trade finance costs varied depending on the firm size, sectoral activity, and countries’ integration into the global economy. Chor and Manova (2012) find that trade finance costs adversely affected exports the most in the firms in financially vulnerable sectors. Although there is apparent heterogeneity in the way crisis affected trade finance costs for different nations and firms, there are

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\(^1\) See Levchenko, Lewis, and Tesar (2010) for counter evidence.

\(^2\) Such as the World Trade Organization (WTO), the World Bank (WB), and the International Monetary Fund (IMF)
few studies that attempt to explain this variation systematically across the cross-section of countries and over time.

A systematic investigation of this issue faces a few critical hindrances due to methodologies adopted in the literature and also the relatively short duration of the financial crisis. The studies that used survey methods for understanding the impact of financial crisis on trade finance costs (e.g., Malouche 2009) suffer from various biases introduced by survey method. First, survey responses are likely to have subjective components. To the extent that this subjectivity is common across the survey respondents, a strong bias will be present in their responses. For example, managers belonging to the same country are likely to exhibit common bias in their responses (Baumgartner and Steenkamp 2001). Second, survey responses are also difficult to verify. Managers may over- or under-estimate their trade finance costs systematically, again depending on the countries where their firms operate. Finally, survey research is often done in one cross-section of time. This makes it impossible to capture the variation of trade finance costs over time unless the survey is carried out each year.

To a large extent, studies that use observational data on trade finance overcome the above-mentioned drawbacks related to survey method. However, due to the short duration of the financial crisis of 2008-2009, which lasted only over a few months, it’s difficult to capture time variation in the effects of various trade finance determinants. For example, although it is possible to explain how inflation affected trade finance costs across different countries during the financial crisis, it is very difficult to study the variation in the impact of inflation across multiple countries over time as the number of parameters exceeds the number of observations.

To that end, the objective of our paper is to propose a dynamic hierarchical linear model, which explains the variations in trade finance costs across multiple countries over several years. In the current form the model consists of three types of equations. At the higher level, Observation Equation specifies the relationship
between trade finance costs for each country in each year and a set of macroeconomic variables (e.g., inflation in the country). The coefficients of the predictors in the Observation Equation are allowed to vary across the cross-section and time-series. Next, in the Pooling Equation we specify the relationship between each coefficient from the Observation Equation and a common, cross-country parameter belonging to the same time period. Pooling Equation enables us to capture the average impact of the macroeconomic variable on the trade finance cost in a given time period. Finally, this average impact is likely to depend on its level from the previous period. In order to incorporate the continuity in the effects of macroeconomic variables on trade finance costs, in the final Evolution Equation, the cross-country parameter is modeled to follow a random walk.

Modeling costs of trade finance using dynamic hierarchical model is critical for at least two reasons. First, considering that financial markets in different countries are heterogeneous and governed by different regulations specific to those countries, impacts of the determinants of financial intermediation costs must vary in the global cross-section (Levine, Loayza, and Beck 2000). Depending on the factors such as inflation, capital markets development, etc., trade finance costs will spread out across nations. Therefore, incorporating into an economic model the heterogeneity across countries is critical for estimating the impact of macro and microeconomic drivers of trade finance costs.

Secondly, the impact of the drivers of trade finance costs is also likely to change over time (Beck, Demirgüç-Kunt, and Levine 2009). Such a shift is expected due to the time-varying technological advances, regulatory changes, and evolution of the banking sector competitive environment, etc. As we are studying 2008-2009 global financial crisis, many drivers of the costs may have different impacts during the crisis compared to the pre-crisis period. For example, during the crisis many lenders may prefer borrowers with the top most quality, thus exhibiting a “flight to
quality” (Caballero and Krishnamurthy 2001). Therefore, it is crucial to consider the time-series variation in the effects of the determinants of trade finance costs.

Our model estimates provide several interesting insights into the role of some of the macroeconomic variables in affecting the cost of trade finance. Although the objective of our paper is to introduce a model to provide reliable estimates when limited data are available, the findings from our empirical analysis may show way for future research. Therefore, we do not claim that our empirical findings are theoretically easily explainable. First, we find that for firms from countries with high GDP growth, the cost of trade finance reduces as the financial crisis approaches. Second, we find that firms from countries that have higher inflation faced higher cost of trade finance. Both these findings are consistent with the “flight to quality” theory advanced in the finance literature. Third, we have the counterintuitive finding that firms from countries with higher market capitalisation (relative to GDP) face increasing trade finance costs during the crisis. Finally, we also find that countries with a higher reliance on trade face higher costs of trade finance.

This research aims to make two concrete contributions. First, we introduce a new model for the evolution in trade finance costs across countries through time even when limited time series data are available. Our model can be adopted to study evolution of various other variables such as financial services costs and global trade. Our model is also flexible for inclusion of firm-level heterogeneity. Finally, our research has policy implications. We provide policymakers such as the World Trade Organization (WTO), the World Bank (WB), and the International Monetary Fund (IMF) a tool using which they can predict which countries are likely to be affected more in a crisis. Further, our model paves way to identify the characteristics of the companies which may need more assistance\textsuperscript{3}. Thus, our research removes subjectivity

\textsuperscript{3} This can be achieved by adding more levels in the hierarchical model.
in extending benefits to the affected exporters and importers. It’s likely that even large scale surveys are unable to provide such a granular implication for the policy.

The paper proceeds as follows. In the first section we describe the dynamic hierarchical Bayesian model. We provide the theoretical underpinnings necessary to derive the model. Next we describe the data and variables used in the empirical analysis. In the third section we provide detailed discussion of the results. We conclude the paper with the discussion of the findings.

MODEL DEVELOPMENT

We model how trade finance cost is influenced by the following country-level variables: GDP growth, inflation, stock market capitalization and trade using a dynamic hierarchical linear model (DHLM). A similar modeling approach has been used by previous studies in marketing and statistics (e.g., Gopinath, Thomas, and Krishnamurthi 2014; Landim and Gamerman 2000; Neelamegham and Chintagunta 2004) to capture time-varying relationships. Dynamic linear models (DLM) also use a similar framework, and have been used more extensively (e.g., Ataman, Van Heerde, and Mela 2010; Ataman, Mela, and van Heerde 2008). The key difference between the DLM and the DHLM is the hierarchical structure in the DHLM, which permits us to pool information across different countries to arrive at overall aggregate-level inferences. Shrinking of the country-level parameters to an “average effect” of the key variables across country has been used by other researchers in different contexts (Montgomery 1997; Montgomery and Rossi 1999; Neelamegham and Chintagunta 2004).

A standard DHLM specification consists of three equations – observation equation, pooling equation and evolution equation. The observation equation models the dependent measure of interest as a function of explanatory variables and time-varying parameters. The time-varying parameters in the observation equation are country specific. The pooling equation specifies the relationship among the time-
varying country-level parameters and a new set of parameters that vary only in time. This hierarchical structure pools information across countries for each point in time. This structure permits us to estimate the effect of the variables for each country over time as well as the average effect over time.

We specify country-specific trade finance costs as a function of country-level indicators.

**Observation Equation:** \( y_t = F1_t \theta 1_t + v1_t \); where \( v1_t \sim N(0, \sigma^2 V1) \)  

An observation \( y_t \) at time \( t \) is defined as a vector that consists of the trade finance cost of countries at time \( t \), whereas \( F1_t \) is a matrix that contains the macro-economic variables associated with cost of trade finance for all countries. The vector of parameters \( \theta 1_t \) contains, for all countries, the time-varying parameters, as well as time-varying intercepts. The error term \( v1_t \) is multivariate normal. We specify \( y_t \), \( F1_t \), and \( \theta 1_t \) similar to Neelamegham and Chintagunta (2004) and Gopinath, Thomas and Krishnamurthi (2014).

**Pooling Equation:** \( \theta 1_t = F2_t \theta 2_t + v2_t \); where \( v2_t \sim N(0, \sigma^2 V2) \)  

\( F2_t \) is the matrix of 0’s and 1’s which allows us to adjust the size of \( \theta 2_t \) to that of \( \theta 1_t \). The error distribution \( v2_t \) is multivariate normal.

We specify how the average effect of the country-level time-varying parameters evolves over time. We follow the dynamic linear models (DLM) literature (West and Harrison 1997, p. 34) and model the evolution of these parameters over time as a random walk.

**Evolution Equation:** \( \theta 2_t = G \theta 2_{t-1} + w_t \); where \( w_t \sim N(0, \sigma^2 W) \)  

The random walk specification requires \( G \) to be an identity matrix and \( w_t \) is a multivariate normal error.
ESTIMATION

We need to compute the full joint posterior of the set of parameters \((\theta_1, \theta_2, \text{and the variance parameters } \sigma^2, V1, V2, \text{and } W)\) conditional on observed data. We employ Gibbs sampling (Gelfand and Smith 1990) to generate the posteriors of the parameters. The Gibbs sampler can be applied to obtain the posterior distribution of parameters if we can sample from the complete conditional distributions of each of the model parameters (Carlin, Polson, and Stoffer 1992; Ferreira, Gamerman, and Migon 1997). We provide the specifications of the full conditional distributions in the Appendix A. We adapt the Gibbs sampler from Landim and Gamerman (2000). Figure 1 describes the sampler.

We use sequential inference to obtain the posterior distribution of \(\theta_1\) (West and Harrison 1997). Sequential inference implies that for every point in time \(t\), we need to obtain the prior, predictive and updated distribution of the parameters of a DLM. Thus for \(\theta_1\), the prior distribution at a point in time \(t\), is conditional on observations till time \(t-1\) and can be denoted by \(p(\theta_1|y_{t-1})\). The predictive distribution would imply our estimate for \(y_t\) give the information we have till \(t-1\), and is denoted by \(p(y_t|y_{t-1})\). The updated distribution is obtained by using Bayes theorem and it refers to the distribution of \(\theta_1\) after observing \(y_t\), and is denoted by \(p(\theta_1|y_t)\). Subsequently this updated distribution would serve as the prior in the next time period for \(\theta_1\), and is denoted by \(p(\theta_{1,t+1}|y_t)\). Thus, we simulate the posterior values of \(\theta_1\) in a sequential manner till we have reached the last time period in our data.

The posterior distribution of \(\theta_2\) is also obtained by employing the sequential inference method within the forward-filtering backward-sampling algorithm proposed by Fruhwirth-Schnatter (1994) and Carter and Kohn (1994). We discuss this algorithm further in the Appendix C. Once we have obtained the posterior estimates of \(\theta_1, \theta_2, \text{and the variance parameters, our model permits us to generate out of sample forecasts of the dependent variables. Furthermore, we need to make}
assumptions for the prior distributions of $\theta_2$, V1, V2, W and $\sigma^2$ at time t=0. We specify fairly standard, conditionally conjugate forms that have been adopted in prior DLM analysis (Gopinath, Thomas, and Krishnamurthi 2014; Landim and Gamerman 2000; Neelameggham and Chintagunta 2004).

For the variance parameter $\sigma^2$ we specify an inverse gamma prior, for the remaining variance parameters we employ inverted Wishart priors and for $\theta_2$ we use a normal prior. We use a prior of +1 for all the average effects of the country level parameters.

**Figure 1**

Directed Acyclic Graph

**DATA**

For the empirical tests, the data are derived from two sources. The information on trade finance costs is obtained from Loan Pricing Corporation’s
Dealscan database. The information on macroeconomic variables for the countries is obtained from the World Bank. We briefly describe the data sources.

**Dealscan**

Dealscan provides detailed information on loan contract terms including the spread above LIBOR, maturity, and covenants since 1986. The primary sources of data for Dealscan are attachments on SEC filings, reports from loan originators, and the financial press (Sufi 2007). As it is one of the most comprehensive sources of syndicated loan data, prior literature has relied on it to a large extent (e.g., Acharya, Almeida, and Campello 2013; Almeida, Campello, and Hack Barth 2011; Dahiya et al. 2003; Haselmann and Wachtel 2011; Sufi 2007).

Each year a borrower company may make several loan deals and each deal may have several loans or facilities. For trade finance we limit the sample to only those loans where the purpose was identified by Dealscan as one of the following: Trade Finance, CP Backup, Pre-Export, and Ship Finance. Our trade finance costs are measured as the loan price for each loan facility, which equals the loan facility’s at-issue yield spread over LIBOR (in basis points). We aggregate the loans in each year at the level of borrowing company’s home country. Due to the limited number of observations, we don’t differentiate between different types of loans. Instead, the trade finance costs are averaged across different types of loans such as revolver loans, term loans, and fixed-rate bonds.

**The World Bank Data**

We use the World Bank data to get information on the economic and regulatory climate, and extent of development of the banking sector of the countries where the borrowing firms are headquartered. The economic and regulatory climate of a country is captured by the following: GDP growth, Inflation, Stock market capitalization, and Trade.
Countries with high GDP growth are likely to have higher cost of trade finance, particularly during the financial crisis. Typically emerging nations exhibit high GDP growth rate but they also get affected more during the financial crisis because lenders are likely to move their assets to developed nations. Countries with higher inflation will likely have higher cost of trade finance as the rate of returns on the loans will incorporate the rate of inflation. We include stock market capitalization scaled by GDP as a proxy for the capital market development in the country. Countries with higher stock market capitalization are likely to have more developed financial markets. Therefore, the cost of trade finance in such markets is likely to be lower. Finally, we include total trade for the country scaled by the country’s GDP as a measure of trade intensity. We don’t have any specific expectation about the sign of this variable.

We could include many other macroeconomic indicators in the model. However, we are limited by the limited availability of data due to which the parameter to observation ratio in the model is already quite high. If include more variables in the model, we are likely to get estimates of parameter variance that are overly large.

Dealscan contains information on loans made to companies at the level of each facility within deal packages. A company may make several deals each year and each deal may have several facilities. As our objective is to study the phenomenon at the national level, we aggregate the information about trade finance costs at the country level and merge the data with macroeconomic variables. Our interest is in modelling trade finance costs around the financial crisis of 2008-2009. Therefore, we use a 5-year time series starting in 2006 and ending in 2010. This gives us a reasonable window that contains pre-crisis, during the crisis, and post-crisis periods. A longer window would help in obtaining more reliable estimates for our model parameters. However, we are constrained by the number of years for which the data are available to us from Dealscan. After merging the two databases, our final sample consists of
eight countries for which we have information on trade finance costs as well as macroeconomic indicators for all the five years. The eight countries are: Brazil, Ghana, Greece, Russia, Turkey, Ukraine, United Kingdom (UK), and the United States (USA).

| Table 1 |
| Descriptive Statistics |

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
<th>Mean</th>
<th>Q3</th>
<th>Maximum</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade Finance Cost above LIBOR (basis points)</td>
<td>40</td>
<td>20.00</td>
<td>71.21</td>
<td>150.56</td>
<td>189.43</td>
<td>263.12</td>
<td>700.00</td>
<td>155.50</td>
</tr>
<tr>
<td>GDP Growth %</td>
<td>40</td>
<td>-14.80</td>
<td>-0.25</td>
<td>3.49</td>
<td>2.57</td>
<td>6.41</td>
<td>9.16</td>
<td>5.16</td>
</tr>
<tr>
<td>Inflation %</td>
<td>40</td>
<td>0.76</td>
<td>2.88</td>
<td>6.55</td>
<td>10.53</td>
<td>14.36</td>
<td>80.75</td>
<td>13.32</td>
</tr>
<tr>
<td>Stock Market Cap/GDP</td>
<td>40</td>
<td>9.61</td>
<td>24.16</td>
<td>65.03</td>
<td>62.96</td>
<td>99.77</td>
<td>146.91</td>
<td>43.65</td>
</tr>
<tr>
<td>Trade/GDP</td>
<td>40</td>
<td>22.14</td>
<td>43.29</td>
<td>52.85</td>
<td>54.11</td>
<td>61.15</td>
<td>104.31</td>
<td>22.28</td>
</tr>
</tbody>
</table>

We report the descriptive statistics for the sample in Table 1. Average trade finance costs are approximately 190 basis points above LIBOR. Mean GDP growth is just 2.57%, reflecting the lower growth during the financial crisis. Although average inflation is at 10.53%, the median inflation is at a moderate 6.55%. On average stock market capitalization/GDP ratio is around 63% while trade/GDP ratio is around 54%. More detailed summary statistics for the trade finance costs are depicted in Figure 2.

Figure 2 effectively captures the variation in the trade finance cost over time and across 8 countries. However, there are two important observations. First, with the exception of Greece, the rest of the countries experienced a large increase in trade finance costs going from 2008 to 2009. Both Brazil and UK show a large jump of close to 300 basis points above LIBOR. Second, except for Brazil and Greece, the
trade finance costs for the rest six countries came down in 2010 from their peak in 2009.

This suggests that the financial crisis impacted trade finance costs quite uniformly across the sample.
Figure 2

Trade Finance Costs
However, there are marked differences in the trade finance costs patterns as well. For example, the costs from 2006 to 2008 vary distinctly for the sample. For Greece, Turkey, and Russia the trade finance costs first came down in 2007 and then went up in 2008. For Ghana and the UK, the trade finance costs smoothly went up from 2006 to 2008. Brazil and Ukraine experienced decrease in the trade finance costs pre-crisis. Thus, our sample shows a wide variation in the trade finance costs.

To complete the discussion around the descriptive analysis, in Figure 3 we present the scatter plots and correlations. The diagonal of the chart represents the distribution of each variable (GDP Growth, Inflation, Market Capitalization/GDP, Trade/GDP, and Trade Finance Rate) broken out by year. Each year is given a common color code, which are visible in the panels on the upper side of the diagonal. For example, the series in 2006 is orange, 2007 is light green, 2008 is dark green, 2009 is blue, and 2010 is magenta. The upper panels report overall correlations in grey color and year-specific correlations in their respective colors. Finally, the bottom panels show scatter plots using separate colors for the years.

In the first panel of the diagonal of Figure 3, we see GDP Growth distribution changing from one year to other. In particular, we see that the whole distribution shifts to left for 2009 (blue series), indicating the negative impact of financial crisis. A similar pattern emerges for Inflation. Market Capitalization and Trade distributions don’t show such apparent shifts. This is to be expected because both the variables are scaled by the GDP. However, Trade/GDP distribution shows hardly any variation across time. In the last panel, we see the distribution of the trade finance costs. Not surprisingly in 2009 and 2010 the distributions are shifted to right.

For our research, the most critical set of panels is on the extreme right, which reports the correlations between our dependent variable, trade finance cost, and the four independent variables. Overall, GDP Growth, Inflation, and Market Capitalization are negatively while Trade is positively related to trade finance cost.
More interestingly, we see a wide variation in the correlations over the five-year period. For example, GDP Growth has high degree of positive correlation with trade finance rate in 2006 (0.606) and 2007 (0.632) but it turns negative in the next three years (-0.248 in 2008, -0.686 in 2009, and -0.0835 in 2010). Even when the correlation doesn’t flip signs from year to year, the magnitudes of the correlations vary a lot. For instance, the correlation between Trade and Trade Finance Costs changes from 0.611 in 2006 to 0.184 in 2010. This underscores the need for a time-varying parameter model for modelling trade finance costs. We don’t discuss the scatter plots separately in detail as their summaries are already reported in the upper panels.
Figure 3
Scatter Plots and Correlations
RESULTS

Our main results are summarized in Figure 4, which reports the estimates for Pooling Equation (θ2). There are four panels, each depicting the changes in θ2 for one of the four independent variables over 2006-2010. For each set of estimates we show the 90% confidence interval (CI). We find that at the overall sample level, GDP Growth has a positive impact on Trade Finance Costs and this impact shows a declining trend from 2006 to 2009. There is no change in the impact from 2009 to 2010. In other words, companies from high GDP Growth countries faced higher cost of trade finance before the financial crisis. However, as we move towards the financial crisis, this impact reduces monotonically supporting the “flight to quality” story.

Inflation has an overall positive impact on the cost of trade finance. In contrast to the impact of GDP Growth, the impact of Inflation slowly rises over 2006-2010. However, the 90% posterior probability band includes 0 in 4 out of 5 years indicating less reliability of the estimates. Nonetheless, the overall pattern suggests that the companies belonging to the countries with more inflation faced higher trade finance costs in the financial crisis. This is also consistent with the “flight to quality” interpretation.

The impacts of Stock Market Capitalization and Trade are depicted in the lower panels in Figure 4. We obtain roughly similar patterns for these two variables. The impact of Stock Market Capitalization increases from approximately 0.4 in 2006 to 1.8 in 2010. This suggests that the companies from countries with higher stock market capitalization relative to the GDP faced increasing costs of trade finance during the financial crisis. This finding is somewhat counterintuitive because we used Stock Market
Figure 4

Estimates of Pooling Equation ($\theta_2$)
Capitalization as a proxy for development of financial markets. During the financial crisis one would expect lower costs of trade finance for the countries where financial markets are well developed. Instead we find a result in the opposite direction.

Similarly, the impact of Trade/GDP ratio on the cost of trade finance also increases during the financial crisis. We introduce this variable in the model to measure the trade intensity of a country. Our results indicate that during the financial crisis the countries with more reliance on trade faced higher costs of trade finance. We believe that to some extent this is expected because higher reliance on trade might make these countries riskier in a financial crisis.

Overall, our model is able to capture the time-varying impact of the four macroeconomic variables on the cost of trade finance. We find that the impact of Inflation was somewhat weak because in four out of five years the CI included 0. Nonetheless, considering that there are only 40 observations in our sample, our model did an excellent job of estimating 25 estimates of $\theta_2$ (5 each for four macroeconomic indicators and the intercept). In addition, our model also estimated another 200 estimates for $\theta_1$ that we describe next.

Figures 5A-5D report the country-specific estimates ($\theta_1$) for each macroeconomic indicator over 2006-2010. Note that the importance of these estimates is lower than the estimates of $\theta_2$. This is because, due to a large number of parameters, we are unlikely to get many statistically significant estimates. Further, the information content in $\theta_2$ is more valuable because these estimates capture the dynamic nature of the impacts of macroeconomic variables on trade finance costs at an aggregate level. Therefore, they are more useful for generalizing the results to countries that are not contained in the sample.
Figure 5A
Estimates for Observation Equation ($\theta_i$) for GDP Growth
Figure 5B
Estimates for Observation Equation ($\theta_i$) for Inflation

- Brazil
- Ghana
- Greece
- Russia
- Turkey
- Ukraine
- United Kingdom
- USA

Year

Inflation Coefficient
Figure 5C
Estimates for Observation Equation ($\theta_t$) for Stock Market Capitalization/GDP

![Diagram showing estimates for different countries including Brazil, Ghana, Greece, Russia, Turkey, Ukraine, United Kingdom, and USA over the years 2006 to 2010. The X-axis represents the years, and the Y-axis represents the Market Capitalization/GDP Coefficient.]
Figure 5D
Estimates for Observation Equation ($\theta_i$) for Trade/GDP
At the individual country-level, we find that the standard errors of the estimates are too large for GDP Growth and Inflation. As a result, all the 16 panels shown in Figures 5A and 5B have estimates with CIs that contain 0. We believe that this is indicative of the complexity of the task of modeling so many parameters from a small set of observations. Nonetheless, in Figures 5C and 5D we find several instances where the CIs don’t contain a 0. We show the impact of Stock Market Capitalization on trade finance costs for each country in Figure 5C. For Brazil, we find that although in the first 3 years this impact is flat, in 2009 and 2010, the estimates increase significantly in 2009 and 2010. Similarly, in Greece, Russia, Ukraine, the UK, and the USA we find significant increase in over the same two years.

Figure 5D reports the estimates for the impact of Trade intensity on trade finance costs for each country over 2006-2010. Once again we find that several of these estimates are statistically significant. In particular, the estimates are large over 2009 and 2010 across the board. This suggests that our model is able to capture the dramatic effect of the finance crisis on trade finance costs.

DISCUSSION

In this research we attempt to shed light on the following questions: How can we develop a model that captures the evolution of trade finance for countries that face changing environments with very short series of data? How can we account for the changing effects of macro-level variables on the cost of trade finance?

We addressed these questions by proposing a Bayesian model that is both hierarchical and dynamic. The hierarchical Bayesian formulation permits us to pool data across different countries while providing country-level parameter estimates. Thus, although we have only a few observations for each country, we are able to combine
information from other countries to obtain reliable estimates for the impact of at least some of the macroeconomic indicators. Next, to account for the evolution of trade finance costs of countries, we specify the parameters in the hierarchical model to be dynamic, that is, time-varying.

The dynamic hierarchical Bayesian model enjoys a critical advantage: it can easily be scaled up. First, we can add another level in the model hierarchy. This would permit us to study the problem at a more granular level. For example, we can analyse the time-varying effect of firm-level drivers on trade finance costs. Further, we can add more macroeconomic variables that are likely to impact the trade finance costs of a country (at present, we study the effect of four macro-economic indicators).

Our model can also be applied to other syndicated loan costs and not just trade finance. As a demonstration, in Appendix B we show the results of applying this model to syndicated loan rates. As the data is more widely available, we can present more extensive graphs for this sample. Overall, we have 56 countries in this sample. Figures B-1 shows the scatterplots and correlations. Figure B-2 reports the estimates of $\theta_2$ for the same four macroeconomic variables that we used in this article. Figures B-3a to B-3d report the estimates for each macroeconomic variable at the country level.

The dynamic hierarchical Bayesian model has a few limitations as well. It is a computationally intensive method. An increase in the longitudinal aspect of the data (for example, the number of years), or an increase in the cross-sectional aspect of the data (for example, the number of countries) leads to an exponential increase in the computational time. The cross-sectional aspect of the data also places special demands on the memory requirements of the computer.
Our model estimates provide several interesting insights into the role of some of the macroeconomic variables in affecting the cost of trade finance. First, we find that for firms from countries with high GDP growth, the cost of trade finance reduces as the financial crisis approaches. Second, we find that firms from countries that have higher inflation faced higher cost of trade finance. Both these findings are consistent with the “flight to quality” theory advanced in the finance literature. Third, we have the counterintuitive finding that firms from countries with higher market capitalisation (relative to GDP) face increasing trade finance costs during the crisis. Finally, we also find that countries with a higher reliance on trade face higher costs of trade finance. Although this was not the main focus of our research, we believe that a more detailed scrutiny of these findings will likely benefit the future research in this area.
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APPENDIX A: POSTERIOR COMPUTATIONS

We compute the joint posterior using MCMC and we give below the full conditional distributions that we used.

A.1. Sampling from the full conditional distributions of the variance parameters:

Let \( n \) denote the number of time periods for which we estimate the model, and \( K \) denote the total number of countries. The dimensions of the variance parameters are as follows: \( \sigma^2 \) is scalar, \( V_1 \) is \( K \times K \), \( V_2 \) is \( 5K \times 5K \), and \( W \) is \( 5 \times 5 \).

The prior distribution for \( \sigma^2 \) is independent inverse gamma while it is independent inverted-Wishart distributions for the rest of the variance parameters. Hence: \( \sigma^2 | D \sim IG (n_0, S_0) \), \( V_1 | D \sim IW (n_{1_0}, S_{1_0}) \), \( V_2 | D \sim IW (n_{2_0}, S_{2_0}) \) and \( W | D \sim IW (n_{W_0}, S_{W_0}) \). We follow Ferreira et al. (1997) and specify an Inverse Gamma distribution with an expected value of 1.5. In line with Landim and Gamerman (2000) and Neelamegham and Chintagunta (2004) we specify the degrees of freedom for the independent inverted Wishart distribution to be \( 2d + 1 \), where \( d \) is the size of the variance matrix. The prior mean of the distribution is assumed to be proportional to identity matrix. We estimate our model on a sample of 8 countries (\( K=8 \)) which lead to the following prior distributions: \( \sigma^2 \sim IG (3, 3) \), \( V_1 \sim IW (17, 17I_8) \), \( V_2 \sim IW (91, 91I_40) \) and \( W \sim IW (11, 11I_5) \).

When the prior is drawn from an independent inverted Gamma distribution, Ferreira et al. (1997) derive its inverted Gamma posterior. We follow the same for our study and thus \( \sigma^2 \) has shape and scale parameters, respectively specified as:

1. \( 0.5\{3 + n(5k + k + 5)\} \) and \( 0.5 \{3 + \sum_{t=1}^{n}(y_t - F_1 \theta_1)^'V_1^{-1}(y_t - F_1 \theta_1) + \sum_{t=1}^{n}(\theta_1^t - F_2 \theta_2^t)^'V_2^{-1}(\theta_1^t - F_2 \theta_2^t) + \sum_{t=1}^{n}(\theta_2^t - G \theta_2^t)^'W^{-1}(\theta_1^t - G \theta_2^t)\} \)

Given an independent inverted Wishart prior, Landim and Gamerman (2000) derive the inverted Wishart posterior. We adopt the same for our study. Thus \( V_1 \), \( V_2 \) and \( W \) have degrees of freedom and scale parameters, respectively specified as:

2. \( 17 + n \) and \( 17 \times 0.5I_8 + \sum_{t=1}^{n}(y_t - F_1 \theta_1)^'\sigma^{-2}(y_t - F_1 \theta_1) \)
3. \( 91 + n \) and \( 91l_{40} + \sum_{t=1}^{n}(\theta_1 t - F2_t \theta_2 t)')\sigma^{-2}(\theta_1 t - F2_t \theta_2 t) \)

4. \( 11 + n \) and \( 11 * 0.1l_5 + \sum_{t=1}^{n}(\theta_2 t - G \theta_2 t)')\sigma^{-2}(\theta_1 t - G \theta_2 t) \)

**A.2. Sampling from the full conditional distribution of the process parameters**

**Theta 1: \( \theta_1 \)**

Sequential inference is used to sample from \( \theta_1 \). Thus, as discussed before, this implies that for each time \( t \), we obtain a prior, predictive and updated distribution of \( \theta_1 \). The prior and the updated (posterior) distribution of \( \theta_1 \) have been derived in Landim and Gamerman (2000). We follow their approach. Thus the updated posterior of \( \theta_1 \) is a multivariate normal distribution with the following moments:

Mean: \( F2, \theta_2, + \sigma^2 V2 F1, (F1, \sigma^2 V2 F1,') + \sigma^2 V1)^{-1}(y_t - F1, F2, \theta_2, ) \)

Variance: \( (\sigma^2 V2 - \sigma^2 V2 F1, (F1, \sigma^2 V2 F1,') + \sigma^2 V1)^{-1}F1, \sigma^2 V2) \)

**Theta 2: \( \theta_2 \)**

We introduce the following additional notations \( y^n = \{y_1,...,y_n \}, \theta_1^n = \{\theta_1, ..., \theta_1 \} \) and \( \theta_2^n = \{\theta_2, ..., \theta_2 \} \). The set of variance parameters is denoted by \( \psi \) and thus \( \psi = \{ \sigma^2, V1, V2, W \} \). We can think of equations (2) and (3) as a multivariate dynamic model with observations \( \theta_1 \) and state parameters \( \theta_2 \), conditional on \( \theta_1^n \) and \( \psi \). This implies that given \( \theta_1^n \), \( \theta_2^n \) is independent of \( y^n \). This structure would permit us to implement the forward-filtering backward-sampling algorithm proposed by Fruhwirth-Schnatter (1994) and Carter and Kohn (1994). We adopt their approach and this enables us to write the posterior distribution of \( \theta_2^n \) as follows:

\[
p(\theta_2^n | \theta_1^n, \psi) = p(\theta_2 | \theta_1, \psi) \Pi_{t=1}^{n-1} p(\theta_2 | \theta_2_{t+1}, ..., \theta_2, \theta_1^n, \psi)
\]
The second term in the right hand side of the above equation reduces to 
\( p(\theta_{2t}|\theta_{2t+1}, \theta_{1n}, \psi) \) due to the conditional structure of the dynamic linear model. We generate an observation from the full conditional distribution of \( \theta_2 \) as follows:

1. We apply the standard sequential updating results for normal multivariate dynamic models to compute for \( t = 1...n \), the moments \( m_t \) and \( C_t \) of the posterior \( E(\psi | \psi) \) as shown below:

\[
p(\theta_{2t}|\theta_{1t}, \psi) \sim N(m_t, C_t),
\]

where \( m_t = G m_{t-1} + R_t F_{2t} Q_t^{-1}(\theta_1 - F_2 G m_{t-1}) \), \( C_t = R_t - R_t F_2 Q_t^{-1} F_2 R_t \), \( Q_t = \sigma^2 V + F_2 R_t F_2' \) and \( R_t = \sigma^2 W + G C_{t-1} G' \). We define \( m_0 \) to be equal to a vector whose means vary around \((2,-2)\) with initial variance \( C_0 \) equal to an identity matrix.

2. The final state vector \( \theta_{2n} \) for \( t=n \), can be sampled from the marginal distribution \( p(\theta_{2n}|\theta_{1n}, \psi) = N(m_n, C_n) \), where \( m_n \) and \( C_n \) can be obtained by following the definitions in step 1.

3. Subsequently, for \( t=n-1,...0 \), we sample from \( p(\theta_{2t}|\theta_{2t+1}, \theta_{1n}, \psi) \) at every time period conditional on the latest values of \( \theta_{2t+1} \) just sampled. We specify the multivariate normal distribution for this stage as:

\[
p(\theta_{2t}|\theta_{2t+1}, \theta_{1n}, \psi) \sim N(h_t, H_t),
\]

where \( H_t = (C_t^{-1} + G' (\sigma^2 W)^{-1} G)^{-1} \) and \( h_t = H_t (C_t^{-1} m_t + G' (\sigma^2 W)^{-1} \theta_2_{t+1}) \).

The three steps described above would lead to a draw \( \theta_{2n}, \theta_{2n-1}, ... \theta_2 \) from the full conditional posterior.
Appendix B, Figure B-1: Scatterplots and Correlations for Syndicated Loans

<table>
<thead>
<tr>
<th>Year</th>
<th>Loan Rate</th>
<th>GDP Growth</th>
<th>Inflation</th>
<th>Market Cap</th>
<th>Trade</th>
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<tr>
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<td>0.486</td>
<td>0.545</td>
<td>0.514</td>
<td>0.514</td>
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<tr>
<td>2007</td>
<td>-0.236</td>
<td>-0.227</td>
<td>-0.204</td>
<td>0.572</td>
<td>0.572</td>
</tr>
<tr>
<td>2008</td>
<td>-0.204</td>
<td>-0.204</td>
<td>-0.113</td>
<td>0.555</td>
<td>0.555</td>
</tr>
<tr>
<td>2009</td>
<td>-0.113</td>
<td>-0.113</td>
<td>-0.153</td>
<td>0.538</td>
<td>0.538</td>
</tr>
<tr>
<td>2010</td>
<td>-0.199</td>
<td>-0.199</td>
<td>-0.217</td>
<td>0.573</td>
<td>0.573</td>
</tr>
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</table>
Overall Sample Level Effects

![Graph showing trends in GDP Growth, Inflation, Market Capitalization, and Trade over the years 2006 to 2010.](image)

Figure B-2
Figure B-3a

<table>
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<tr>
<th>Country</th>
<th>Argentina</th>
<th>Australia</th>
<th>Austria</th>
<th>Bahrain</th>
<th>Belgium</th>
<th>Brazil</th>
<th>Canada</th>
<th>Chile</th>
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<th>Croatia</th>
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<th>Germany</th>
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<th>Greece</th>
<th>Hong Kong</th>
<th>Hungary</th>
<th>India</th>
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<th>Japan</th>
<th>Kazakhstan</th>
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<th>Luxembourg</th>
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<th>Norway</th>
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<td><img src="chart72.png" alt="" /></td>
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</tbody>
</table>
Figure B-3b

Country Level Effects

Argentina | Australia | Austria | Bahrain | Belgium | Brazil | Canada | Chile
---|---|---|---|---|---|---|---

China | Colombia | Croatia | Cyprus | Czech Republic | Denmark | Egypt | Finland

France | Germany | Ghana | Greece | Hong Kong | Hungary | India | Indonesia

Ireland | Italy | Japan | Kazakhstan | Korea (South) | Luxembourg | Malaysia | Mexico

Netherlands | New Zealand | Norway | Panama | Peru | Philippines | Poland | Portugal

Qatar | Russia | Saudi Arabia | Singapore | Slovenia | South Africa | Spain | Sweden

Switzerland | Thailand | Turkey | Ukraine | United Arab Emirates | United Kingdom | USA | Vietnam
Figure B-3c
Figure B-3d

Country Level Effects

Argentina
Australia
Austria
Bahrain
Belgium
Brazil
Canada
Chile
China
Colombia
Croatia
Cyprus
Czech Republic
Denmark
Egypt
Finland
France
Germany
Ghana
Greece
Hong Kong
Hungary
India
Indonesia
Ireland
Italy
Japan
Kazakhstan
Korea South
Luxembourg
Malaysia
Mexico
Netherlands
New Zealand
Norway
Panama
Peru
Philippines
Poland
Portugal
Qatar
Russia
Saudi Arabia
Singapore
Slovenia
South Africa
Spain
Sweden
Switzerland
Thailand
Turkey
Ukraine
United Arab Emirates
United Kingdom
USA
Vietnam

Year
APPENDIX C: SIMULATION STUDY

In line with Landim and Gamerman (2000) we use a matrix variate Dynamic Linear Model for our simulation study. We specify the model below. Our notations are consistent with Landim and Gamerman (2000).

\[(A2.1) \quad Y_t = F_1 \theta_1 + v_1 \quad \text{where } v_1 \sim N(0, V_1, \Sigma)\]

Here \(Y_t\) is a 8 x 2 matrix \(F_1\) is a 8 x 16 matrix, \(\theta_1\) is a 16 x 2 matrix and, with \(t\) varies from 1 to 50. The error term \(v_1\) is a matrix variate normal. An \(n \times p\) matrix \(Z\) that follows a matrix-variate normal distribution can be denoted by \(Z \sim N(M, C, \Sigma)\) which would mean that \(\text{vec}(Z) \sim N_{np}(\text{vec}(M), \Sigma \otimes C)\), where \(\text{vec}\) represents the column vectorization of \(Z\) (Dawid, 1981), \(N_{np}(, ,)\) represents the \(np\)-variate normal distribution, \(C\) is a \(n \times n\) and \(\Sigma\) is a \(p \times p\) matrix.

For the parameters at the observation level we define the following equations:

\[(A2.2) \quad \theta_1 = F_2 \theta_2 + v_2 \quad \text{where } v_2 \sim N(0, V_2, \Sigma).\]

Here \(\theta_2\) is a 4 x 2 matrix and \(F_2\) is a 16 x 4 matrix.

For the system level, we have

\[(A2.3) \quad \theta_2 = G \theta_2_{t-1} + v_2 \quad \text{where } w_t \sim N(0, W, \Sigma).\]

Here \(G_t\) is a 4x4 matrix, and we choose \(G_t = I_4\).

We specify below the parameter values and the regressor matrices we use for simulating the data:

\[F_1 = \begin{bmatrix} 1 & X_{1t} & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 1 & X_{2t} & 0 & 0 & \ldots & \ldots & \ldots & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \end{bmatrix} X_{9t} \]

\[\theta_1 = \begin{bmatrix} \alpha_{11t} & \alpha_{12t} \\ \beta_{11t} & \beta_{12t} \\ \vdots & \vdots \\ \alpha_{81t} & \alpha_{82t} \\ \beta_{81t} & \beta_{82t} \end{bmatrix} \]
The regressor variables $X, Z$ and $N$ were all generated according to a standard normal distribution. The variances we used for simulating the data were

$$
\Sigma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix},
$$

V1 = \begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 4 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 4 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 4 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix}

V2 = 10I_{10}$ and $W = I_4$.

The initial value of $\theta_2$ used for simulating the data was taken as

$$
\begin{bmatrix} 10 & 10 \\ 1 & 1 \\ 10 & 10 \\ 1 & 1 \end{bmatrix}
$$

The estimation procedure is similar to that described in Appendix A. In line with Landim and Gamerman (2000) we specify the prior distributions of the variance parameters as: $V1 \sim IW(17,8.5I_4)$, $V2 \sim IW(33,33I_{10})$, $W \sim IW(9,0.9I_4)$, $\Sigma \sim IW(5,5I_2)$ and $\theta_2|\Sigma \sim N(0,100I_2)$. We use the same initial values adopted by Landim and Gamerman (2000), and
thus we take $\theta_1,^0$ to be a full matrix of 10's, for all $t$ and the variance matrices to be all equal to identity. We run the MCMC for 10000 iterations and we use a burn in period of 5000 for the draws.

We reproduce the same figures as in Landim and Gamerman (2000) to demonstrate that the posterior is recovering the simulated parameters nicely. The recovery plot for the 8 parameters of $\theta_2$ is shown in Figure C-1a and Figure C-1b. The points represent the simulated values, the solid line represents the recovered value and the dotted line represents the 95% credibility interval. As all the simulated parameters are within the 95% credibility interval, we can state that the parameter recovery has been good.

INSERT FIGURE C-1a ABOUT HERE

INSERT FIGURE C-1b ABOUT HERE

We also examine the recovery of the variance parameters. We note that the parameterization of the normal matrix-variate distribution do not permit us to identify each of the individual variance terms. Hence we focus on the full variances that are obtained through the Kronecker product of $\Sigma$ with each of the three variance matrices, i.e. $\Sigma \otimes V_1$, $\Sigma \otimes V_2$ and $\Sigma \otimes W$. Figure 6 contains the box plots of the diagonal elements of $\Sigma \otimes V_2$ whose theoretical values are 100. We find that the diagonal elements are estimated to be around 150. Thus the variance parameters are overestimated a little bit. We suspect that an increase in the number of iterations might address this issue.

INSERT FIGURE C-2 ABOUT HERE
Figure C-1a

RECOVERY PLOT OF FIRST 4 PARAMETERS OF \( \theta_2 \)

Figure C-1b

RECOVERY PLOT OF LAST 4 PARAMETERS OF \( \theta_2 \)
Figure C-2

BOX PLOTS OF VARIANCES $\Sigma V_2$