Description of the data generating system utilized in “prediction-error identification of LPV systems: a nonparametric gaussian regression approach”

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By

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I. Introduction

In this report, we give the exact coefficient function matrices a_i, b_j of the *Linear Parameter-Varying* (LPV) process model utilized in [1], together with the coefficient function matrices c_i, d_j of the considered noise dynamics, i.e., the corresponding full *Box Jenkins* (BJ) model.

II. LPV-BJ model description

Consider a *multi-input multi-output* (MIMO) data generating LPV system described in discrete-time by the following difference equations:

$$A_0(p,k,q^{-1})\breve{y}(k) = B_0(p,k,q^{-1})u(k), \tag{1a}$$

$$D_0(p,k,q^{-1})v(k) = C_0(p,k,q^{-1})e(k), \tag{1b}$$

$$y(k) = \breve{y}(k) + v(k), \tag{1c}$$

where $k \in \mathbb{Z}$ is the discrete time, q is the forward time-shift operator, i.e., $qx(k) = x(k+1)$, $u : \mathbb{Z} \to \mathbb{U} = \mathbb{R}^{n_\mathrm{u}}$ is the input, $\breve{y}, y : \mathbb{Z} \to \mathbb{Y} = \mathbb{R}^{n_\mathrm{y}}$ are the noiseless and noisy outputs respectively, $p : \mathbb{Z} \to \mathbb{P}$ is the so-called scheduling variable with compact range $\mathbb{P} \subseteq \mathbb{R}^{n_\mathrm{p}}$, $v : \mathbb{Z} \to \mathbb{Y}$ is a coloured noise process, and $e : \mathbb{Z} \to \mathbb{Y}$ is a white noise process with normal (Gaussian) distribution, i.e., $e(k) \sim \mathcal{N}(0, \Sigma_e)$ with covariance $\Sigma_e \in \mathbb{R}^{n_\mathrm{y} \times n_\mathrm{y}}$. The p-dependent operators $A_0(p,k,q^{-1})$ and $B_0(p,k,q^{-1})$ that describe the process model (1a) are matrix polynomials in q^{-1} of degree n_a and n_b respectively:

$$A_0(p,k,q^{-1}) = I_{n_\mathrm{y}} + \sum_{i=1}^{n_\mathrm{a}} a_i(p,k,i)q^{-i}, \tag{2a}$$

$$B_0(p,k,q^{-1}) = \sum_{j=0}^{n_\mathrm{b}} b_j(p,k,j)q^{-j}, \tag{2b}$$

where I_{n_y} is the $n_\mathrm{y}-$dimensional identity matrix and the matrix functions $a_i(p,k,i) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_\mathrm{y} \times n_\mathrm{y}}$ and $b_j(p,k,j) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_\mathrm{y} \times n_\mathrm{u}}$ are shorthand notations for $a_i(p,k,i) = a_i(p(k), \ldots, p(k-i))$ and $b_j(p,k,j) = b_j(p(k), \ldots, p(k-j))$. These functions are assumed to be smooth and bounded functions on $\mathbb{P}$. In a similar fashion, for the noise model (1b), the relations $D_0(p,k,q^{-1})$ and $C_0(p,k,q^{-1})$ are defined as

$$C_0(p,k,q^{-1}) = I_{n_\mathrm{y}} + \sum_{i=1}^{n_\mathrm{c}} c_i(p,k,i)q^{-i}, \tag{3a}$$

$$D_0(p,k,q^{-1}) = I_{n_\mathrm{y}} + \sum_{j=1}^{n_\mathrm{d}} d_j(p,k,j)q^{-j}, \tag{3b}$$

where $d_j(p,k,j) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_\mathrm{y} \times n_\mathrm{y}}$ and $c_i(p,k,i) : \mathbb{P} \times \ldots \times \mathbb{P} \to \mathbb{R}^{n_\mathrm{y} \times n_\mathrm{y}}$ are the coefficient functions matrices of the monic polynomials matrices in q^{-1} of degree n_c and n_d, respectively.

In [1, Section 5], a MIMO LPV-BJ model in the form of (1) is considered with $n_\mathrm{y} = 2, n_\mathrm{u} = 2, n_\mathrm{p} = 2$. The full descritption of this model is given below.

III. Coefficient functions of the process dynamics

$$b_0(p,k,0) = \begin{bmatrix} 1-\exp(-0.6p_1(k)) & 0.64-0.72\exp(0.7p_1(k)) \\ 0.3-0.4p_1^2(k)+0.5p_2(k) & 0.2+0.98\tan^{-1}(0.66p_2(k)) \end{bmatrix} \tag{4a}$$

$$b_1(p,k,1) = \begin{bmatrix} 0.24-0.32p_1^2(k)+0.4p_2(k-1) & 0.22\exp(0.4p_1(k-1)) \\ 0.16+0.9\tan^{-1}(0.63p_2(k)) & 0.22-0.5p_1^2(k)+0.45p_2(k-1) \end{bmatrix} \tag{4b}$$

$$b_2(p,k,2) = \begin{bmatrix} 0.16+0.64\tan^{-1}(0.8p_2(k-2)) & 0.14+0.7\tan^{-1}(0.6p_2(k-2)) \\ 0.64-0.64\exp(-0.6p_1(k-1)) & 0.17-0.32p_1^2(k)+0.32p_2(k-1) \end{bmatrix} \tag{4c}$$

$$a_1(p,k,1) = \begin{bmatrix} 0.2+0.12p_2^2(k-1) & 0 \\ 0 & 0.2+0.35\tan^{-1}(p_1(k))\cos(p_1(k-1)) \end{bmatrix} \tag{4d}$$

$$a_2(p,k,2) = \begin{bmatrix} 0.19+0.15\tan^{-1}(p_1(k-1))\cos(p_2(k-2)) & 0 \\ 0 & 0.17+0.11p_2^2(k-1) \end{bmatrix}. \tag{4e}$$

IV. Coefficient functions of the noise dynamics

$$d_1(p,k,1) = \begin{bmatrix} 0.3+0.3\sqrt{|(p_1(k))|} & 0 \\ 0 & 0.45+0.45\sin(p_2(k)) \end{bmatrix} \tag{5a}$$

$$d_2(p,k,2) = \begin{bmatrix} 0.34+0.34\sin(p_2(k-1)) & 0 \\ 0 & 0.23+0.23\sqrt{|p_1(k-2)|} \end{bmatrix} \tag{5b}$$

$$c_1(p,k,1) = \begin{bmatrix} 0.3+0.45p_1^3(k)+0.3p_1^2(k-1) & 0 \\ 0 & 0.3+0.45p_2^2(k-1) \end{bmatrix} \tag{5c}$$

$$c_2(p,k,2) = \begin{bmatrix} 0.24+0.36p_1^2(k-1) & 0 \\ 0 & 0.24+0.36p_2^3(k-2)+0.24p_2^2(k-1) \end{bmatrix}. \tag{5d}$$

References

[1] M. A. H. Darwish, P. B. Cox, I. Proimadis, G. Pillonetto, and R. Tóth, "Prediction-error identification of LPV systems: A nonparametric Gaussian regression approach." *To be submitted to Automatica.*