

Description of the data generating system utilized in “prediction-error identification of LPV systems: a nonparametric gaussian regression approach”

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Description of the data generating system utilized in “Prediction-Error Identification of LPV Systems: A Nonparametric Gaussian Regression Approach”

By

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I. INTRODUCTION

In this report, we give the exact coefficient function matrices a_i, b_j of the *Linear Parameter-Varying* (LPV) process model utilized in [1], together with the coefficient function matrices c_i, d_j of the considered noise dynamics, i.e., the corresponding full *Box Jenkins* (BJ) model.

II. LPV-BJ MODEL DESCRIPTION

Consider a *multi-input multi-output* (MIMO) data generating LPV system described in discrete-time by the following difference equations:

$$A_0(p, k, q^{-1})\check{y}(k) = B_0(p, k, q^{-1})u(k), \quad (1a)$$

$$D_0(p, k, q^{-1})v(k) = C_0(p, k, q^{-1})e(k), \quad (1b)$$

$$y(k) = \check{y}(k) + v(k), \quad (1c)$$

where $k \in \mathbb{Z}$ is the discrete time, q is the forward time-shift operator, i.e., $qx(k) = x(k+1)$, $u : \mathbb{Z} \rightarrow \mathbb{U} = \mathbb{R}^{n_u}$ is the input, $\check{y}, y : \mathbb{Z} \rightarrow \mathbb{Y} = \mathbb{R}^{n_y}$ are the noiseless and noisy outputs respectively, $p : \mathbb{Z} \rightarrow \mathbb{P}$ is the so-called scheduling variable with compact range $\mathbb{P} \subseteq \mathbb{R}^{n_p}$, $v : \mathbb{Z} \rightarrow \mathbb{Y}$ is a coloured noise process, and $e : \mathbb{Z} \rightarrow \mathbb{Y}$ is a white noise process with normal (Gaussian) distribution, i.e., $e(k) \sim \mathcal{N}(0, \Sigma_e)$ with covariance $\Sigma_e \in \mathbb{R}^{n_y \times n_y}$. The p -dependent operators $A_0(p, k, q^{-1})$ and $B_0(p, k, q^{-1})$ that describe the process model (1a) are matrix polynomials in q^{-1} of degree n_a and n_b respectively:

$$A_0(p, k, q^{-1}) = I_{n_y} + \sum_{i=1}^{n_a} a_i(p, k, i)q^{-i}, \quad (2a)$$

$$B_0(p, k, q^{-1}) = \sum_{j=0}^{n_b} b_j(p, k, j)q^{-j}, \quad (2b)$$

where I_{n_y} is the n_y -dimensional identity matrix and the matrix functions $a_i(p, k, i) : \mathbb{P} \times \dots \times \mathbb{P} \rightarrow \mathbb{R}^{n_y \times n_y}$ and $b_j(p, k, j) : \mathbb{P} \times \dots \times \mathbb{P} \rightarrow \mathbb{R}^{n_y \times n_u}$ are shorthand notations for $a_i(p, k, i) = a_i(p(k), \dots, p(k-i))$ and $b_j(p, k, j) = b_j(p(k), \dots, p(k-j))$. These functions are assumed to be smooth and bounded functions on \mathbb{P} . In a similar fashion, for the noise model (1b), the relations $D_0(p, k, q^{-1})$ and $C_0(p, k, q^{-1})$ are defined as

$$C_0(p, k, q^{-1}) = I_{n_y} + \sum_{i=1}^{n_c} c_i(p, k, i)q^{-i}, \quad (3a)$$

$$D_0(p, k, q^{-1}) = I_{n_y} + \sum_{j=1}^{n_d} d_j(p, k, j)q^{-j}, \quad (3b)$$

where $d_j(p, k, j) : \mathbb{P} \times \dots \times \mathbb{P} \rightarrow \mathbb{R}^{n_y \times n_y}$ and $c_i(p, k, i) : \mathbb{P} \times \dots \times \mathbb{P} \rightarrow \mathbb{R}^{n_y \times n_y}$ are the coefficient functions matrices of the monic polynomials matrices in q^{-1} of degree n_c and n_d , respectively.

In [1, Section 5], a MIMO LPV-BJ model in the form of (1) is considered with $n_y = 2, n_u = 2, n_p = 2$. The full description of this model is given below.

III. COEFFICIENT FUNCTIONS OF THE PROCESS DYNAMICS

$$b_0(p, k, 0) = \begin{bmatrix} 1 - \exp(-0.6p_1(k)) & 0.64 - 0.72 \exp(0.7p_1(k)) \\ 0.3 - 0.4p_1^2(k) + 0.5p_2(k) & 0.2 + 0.98 \tan^{-1}(0.66p_2(k)) \end{bmatrix} \quad (4a)$$

$$b_1(p, k, 1) = \begin{bmatrix} 0.24 - 0.32p_1^2(k) + 0.4p_2(k-1) & 0.22 \exp(0.4p_1(k-1)) \\ 0.16 + 0.9 \tan^{-1}(0.63p_2(k)) & 0.22 - 0.5p_1^2(k) + 0.45p_2(k-1) \end{bmatrix} \quad (4b)$$

$$b_2(p, k, 2) = \begin{bmatrix} 0.16 + 0.64 \tan^{-1}(0.8p_2(k-2)) & 0.14 + 0.7 \tan^{-1}(0.6p_2(k-2)) \\ 0.64 - 0.64 \exp(-0.6p_1(k-1)) & 0.17 - 0.32p_1^2(k) + 0.32p_2(k-1) \end{bmatrix} \quad (4c)$$

$$a_1(p, k, 1) = \begin{bmatrix} 0.2 + 0.12p_2^2(k-1) & 0 \\ 0 & 0.2 + 0.35 \tan^{-1}(p_1(k)) \cos(p_1(k-1)) \end{bmatrix} \quad (4d)$$

$$a_2(p, k, 2) = \begin{bmatrix} 0.19 + 0.15 \tan^{-1}(p_1(k-1)) \cos(p_2(k-2)) & 0 \\ 0 & 0.17 + 0.11p_2^2(k-1) \end{bmatrix}. \quad (4e)$$

IV. COEFFICIENT FUNCTIONS OF THE NOISE DYNAMICS

$$d_1(p, k, 1) = \begin{bmatrix} 0.3 + 0.3\sqrt{|(p_1(k))|} & 0 \\ 0 & 0.45 + 0.45 \sin(p_2(k)) \end{bmatrix} \quad (5a)$$

$$d_2(p, k, 2) = \begin{bmatrix} 0.34 + 0.34 \sin(p_2(k-1)) & 0 \\ 0 & 0.23 + 0.23\sqrt{|p_1(k-2)|} \end{bmatrix} \quad (5b)$$

$$c_1(p, k, 1) = \begin{bmatrix} 0.3 + 0.45p_1^3(k) + 0.3p_1^2(k-1) & 0 \\ 0 & 0.3 + 0.45p_2^2(k-1) \end{bmatrix} \quad (5c)$$

$$c_2(p, k, 2) = \begin{bmatrix} 0.24 + 0.36p_1^2(k-1) & 0 \\ 0 & 0.24 + 0.36p_2^3(k-2) + 0.24p_2^2(k-1) \end{bmatrix}. \quad (5d)$$

REFERENCES

- [1] M. A. H. Darwish, P. B. Cox, I. Proimadis, G. Pillonetto, and R. Tóth, "Prediction-error identification of LPV systems: A nonparametric Gaussian regression approach." *To be submitted to Automatica*.