Modeling bounded rationality in choice behavior: Relative utility vs. random regret models

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Modeling Bounded Rationality in Choice Behavior
Relative Utility vs. Random Regret Models

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Abstract: To better understand individuals’ decision behavior, bounded rational decision models are getting increasingly more attention. This paper reviews relative utility and random regret models, which both assert that the performance of the considered alternative is dependent on the attributes of one or more foregone alternatives in the choice set. This property of the models is called context dependency. Differences and similarities between the models are outlined, new developments and applications are presented and finally future improvements are discussed.

1. INTRODUCTION

Last decades have witnessed a vast growth in applications of discrete choice models (DCMs) across different domains, including urban planning and transportation planning (e.g., McFadden, 1978; Ben-Akiva and Lerman, 1985; McFadden, 2000; Train, 2009). Traditionally, DCMs have been developed based on the principle of random utility maximization (RUM), which can be seen as an example of a theory of rational decision-making. In that sense, RUM models usually assume that individuals have specific goals, infinite knowledge and consistent preferences all the time (Simon, 1955). However, an increased number of studies pointed out the existence of bounded rationality and proposed alternative theories and models (e.g., Kahnmena and Tversky, 1979; Tversky and Kahneman, 1992; Loomes and Sugden, 1982, 1986; Bell, 1982, 1985).

Rasouli and Timmermans (2015a) reviewed the development and application of bounded rational decision models under condition of certainty. They defined bounded rationality as a “decision-making process and choice behavior in which individuals do not seek the optional choice and/or consider only a subset of the potentially influential attributes, and/or in comparing
choice alternatives do not differentiate between asymptotically small differences in attribute/alternative values, and/or do not consider all alternatives in the choice set”. Among the bounded rational models reviewed, (random) relative utility maximization (RRUM) models and random regret minimization (RRM) models have attracted much interest. Considering their similarity in some assumptions and model specifications, this paper aims to compare the differences and similarities between RRUM and RRM models, discuss improvements of these models to further elaborate the review and provide additional insights into these models that to date have largely been discussed in isolation.

RRUM models (Zhang et al., 2004) assume that judgments about choice alternatives are made relative to one or more reference points, while RRM models (Chorus et al., 2008; Chorus, 2010) assume that choice behavior is driven by avoiding negative emotions rather than the maximization of some form of payoff. Regret (i.e. negative emotion) occurs when one or more non-chosen alternatives outperform the chosen alternative in terms of one or more attributes. Therefore, it is straightforward that the bounded rationality underlying these models lies in the context-dependency of the decisions that individuals make, which tend to be compromise decisions.

After some seminal work in the middle 1980s and early 1990s, little effort has been spent on models, which attempt to specify context effects in terms of the utility of the choice model. Rather, the vast majority of studies have concentrated on the error terms, making assumptions about variables that are unknown. RRUM models and RRM models believe that in the context of applications of choice models in urban and transportation planning, modeling error terms might be not very effective. Rather, it might be more productive trying to capture the causes of the context effects, trying to find specifications in which the error terms represent pure error.

Nevertheless, there is still no generally accepted definition of context, and many scholars proposed their own definitions (e.g., Oppewal and Timmermans, 1991; Simonson and Tversky, 1992). As to RRUM and RRM models, they only consider alternative-specific context (choice set composition), which takes one or more non-chosen alternatives and their attributes in a choice set into account.

2. MODEL SPECIFICATIONS

2.1 RRUM models

The RRUM models were put forward by Zhang et al. (2004), who argued that utility was meaningful only relative to some reference points and that individuals had different interests in each alternative. The model specification can be described as follows:

\[
RU_i = r_i \sum_{j \neq i} (u_i - u_j) = RV_i + \varepsilon_i = r_i \sum_{j \neq i} \sum_m [\beta_m (x_{im} - x_{jm})] + \varepsilon_i
\]  

(1)

where \( RU_i \) indicates the random relative utility of alternative \( i \); \( RV_i = r_i \sum_{j \neq i} \sum_m [\beta_m (x_{im} - x_{jm})] \) indicates the observed term of \( RU_i \); \( \beta_m \) is the parameter with respect to \( m \)th attribute; \( \varepsilon_i \) indicates the unobserved term of \( RU_i \); \( u_i \) and \( u_j \) indicate the utilities of alternative \( i \) and \( j \).
respectively; \( r_i \) is relative interest parameter, which indicates the weight of alternative \( i \); \( x_{im} \) and \( x_{jm} \) indicate the level of \( m \)th attribute of alternative \( i \) and \( j \), respectively. Assuming different distributions of \( \varepsilon_i \), a new family of choice models can be developed.

Considering that the relative utility is actually defined at the attribute level, Zhang et al. (2013) generalized the RRUM model as follows:

\[
RU_i = f(g(x_{im} - x_{jm}), r_i, w_{ij} \mid j \neq i)
\]  

(2)

where \( RU_i \) indicates the random relative utility of alternative \( i \); \( r_i \) is relative interest parameter; \( w_{ij} \) is a weight parameter reflecting the influence of alternative \( j \) on the choice of alternative \( i \); \( x_{im} \) and \( x_{jm} \) denote the level of the \( m \)th attribute of alternative \( i \) and \( j \), respectively. Using different function forms of \( f(\cdot) \) and \( g(\cdot) \), different types of RRUM models can be developed and context dependency can be flexibly represented.

### 2.2 RRM models

Similar to the concept of relative utility, Chorus et al. (2008) proposed the concept of random regret minimization, which is rooted in regret theory (Bell, 1982; Fishburn, 1982; Loomes and Sugden, 1982). Contrary to RUM models, RRM models claim that choice behavior is driven by minimizing regret. In the original specification, the context effect is reflected in the comparison of the considered alternative and the best foregone alternative in a choice set. The systematic regret of alternative \( i \) can be written as follows:

\[
R_i = \max_{j \neq i} \left\{ \sum_m \max(0, \beta_m(x_{jm} - x_{im})) \right\}
\]  

(3)

where \( R_i \) denotes the systematic regret of alternative \( i \), which acts as the observed term in random regret; \( \beta_m \) is the parameter with respect to the \( m \)th attribute; \( x_{im} \) and \( x_{jm} \) denote the level of the \( m \)th attribute of alternative \( i \) and \( j \), respectively.

Later, Chorus (2010) proposed another version of the RRM models based on the following considerations: first, the original RRM model postulates that regret is only experienced with respect to the best foregone alternative rather than all other alternatives in the offered choice set; second, the specification’s likelihood function in the original RRM model is non-smooth, which creates difficulties in model estimation. To distinguish these two versions of the RRM models, in the remainder the authors refer to the one proposed in Chorus et al. (2008) as the original RRM model, and the one proposed in Chorus (2010) as the new RRM model. The specification of the new RRM model can be written as follows:

\[
R_i = \sum_{j \neq i} \sum_m \ln \left[ 1 + \exp(\beta_m(x_{jm} - x_{im})) \right]
\]  

(4)

where \( R_i \) denotes the observed term in random regret of alternative \( i \); \( \beta_m \) is the parameter with respect to the \( m \)th attribute; \( x_{im} \) and \( x_{jm} \) denote the level of the \( m \)th attribute of alternative \( i \) and \( j \), respectively. Replacing the inner max operator in Eq. (3) with a logsum, its likelihood function
becomes smooth, which facilitates model estimation. Replacing the outer max operator in Eq. (3) with a summation, all other foregone alternatives in the offered choice set are taken into consideration equally.

3. THEORETICAL COMPARISONS

In this part, we will discuss the differences and similarities between RUM, RRUM and RRM models. Note that for the RUM model we assume the utility function has a linear-additive form. Model specifications are presented in Eq. (1), (3) and (4).

Conclusion 1: the RRUM model is mathematically equivalent to the RUM model if all the relative interest parameters are equal.

This conclusion is also known as the Relative Utility Theorem, whose proof can be found in Appendix A of Zhang et al. (2004). This conclusion emphasizes the importance of the relative interest parameter in RRUM model. Without the relative interest parameters, the RRUM model is nothing but a standard RUM model.

Conclusion 2: RRUM model reduces to the RUM model if the choice set is binary.

Consider a binary choice set \{1, 2\}. Assume relative utility of alternative 1 and alternative 2 are defined as \(RV_1 = r_1 \sum_{m=1}^{M} \beta_m(x_{1m} - x_{2m})\) and \(RV_2 = r_2 \sum_{m=1}^{M} \beta_m(x_{2m} - x_{1m})\), respectively. Therefore, the difference in the relative utilities of the two alternatives equals:

\[
RV_1 - RV_2 = r_1 \sum_{m=1}^{M} \beta_m(x_{1m} - x_{2m}) - r_2 \sum_{m=1}^{M} \beta_m(x_{2m} - x_{1m})
\]

\[
= r_1 \sum_{m=1}^{M} \beta_m(x_{1m} - x_{2m}) + r_2 \sum_{m=1}^{M} \beta_m(x_{1m} - x_{2m})
\]

\[
= (r_1 + r_2) \sum_{m=1}^{M} \beta_m(x_{1m} - x_{2m})
\]

\[
= (r_1 + r_2)(V_1 - V_2)
\]

where \(V_1\) and \(V_2\) indicate the standard utility of alternative 1 and alternative 2, respectively. According to Eq. (5), it is intuitive that the RRUM model reduces to the RUM model if the choice set is binary.

Conclusion 3: the original RRM model is mathematically equivalent to the RUM model if the choice set is binary.

Consider a binary choice set \{1, 2\}. Assume that \(\beta_m(x_{2m} - x_{1m}) \geq 0, \ m \in \{1, 2, ..., m_1\}\), while \(\beta_m(x_{2m} - x_{1m}) < 0, \ m \in \{m_1 + 1, m_1 + 2, ..., M\}\). Therefore, the systematic regret of the two alternatives equals:

\[
R_1 = \sum_{m=1}^{M} max\{0, \beta_m(x_{2m} - x_{1m})\} = \sum_{m=1}^{m_1} \beta_m x_{2m} - \sum_{m=1}^{m_1} \beta_m x_{1m}
\]

\[
R_2 = \sum_{m=1}^{M} max\{0, \beta_m(x_{1m} - x_{2m})\} = \sum_{m=m_1+1}^{M} \beta_m x_{1m} - \sum_{m=m_1+1}^{M} \beta_m x_{2m}
\]

Therefore, the difference in systematic regret of the two alternatives is equal to:

\[
R_1 - R_2 = (\sum_{m=1}^{m_1} \beta_m x_{2m} - \sum_{m=1}^{m_1} \beta_m x_{1m}) - (\sum_{m=m_1+1}^{M} \beta_m x_{1m} - \sum_{m=m_1+1}^{M} \beta_m x_{2m})
\]

\[
= (\sum_{m=1}^{m_1} \beta_m x_{2m} + \sum_{m=m_1+1}^{M} \beta_m x_{2m}) - (\sum_{m=m_1+1}^{M} \beta_m x_{1m} + \sum_{m=1}^{m_1} \beta_m x_{1m})
\]
\[
\sum_{m=1}^{M} \beta_m x_{2m} - \sum_{m=1}^{M} \beta_m x_{1m} = -(V_1 - V_2)
\] (6)

Since regret uses the rule of minimization while utility uses the rule of maximization, Eq. (6) shows that the original RRM model reduces to the RUM model when the choice set is binary.

**Conclusion 4:** The new RRM model reduces to the RUM model if the choice set is binary.

The proof of this conclusion can be found in Appendix 2 of Chorus (2010). It is interesting to see that all models reduce to RUM models in the case of binary choice set. This is because that in DCMs, utility (or regret) itself is a relative concept, and this relativity lies in the pairwise comparison. So when the choice set is binary, RRUM models and RRM models are identical to RUM models.

**Conclusion 5:** Both RRM models are special cases of the RRUM model.

This conclusion can be drawn easily from Eq. (2). The only difference is their specific function forms of \( f() \) and \( g() \): Eq. (1) takes a linear form with a relative interest parameter, and considers all foregone alternatives in a choice set; Eq. (3) takes a non-smooth polyline form, and only considers the best foregone alternatives in a choice set; Eq. (4) takes a smooth exponential form, and considers all foregone alternatives in a choice set.

### 4. RECENT DEVELOPMENTS

After Zhang et al. (2004), some further work was done using the concept of relative utility, especially the attempt to link it to prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992). Given the property of loss aversion, a model that integrates the concepts of relative utility and prospect was proposed (Zhang et al., 2010; Zhang et al., 2013). This model takes the form of the value function in cumulative prospect theory and evaluates the relative utility in gains and losses separately. The specification of this model can be expressed as follows:

\[
RV_i = r_i \left\{ \sum_k \left[ \gamma_k^+ \left( d_{ijk}^+ \Delta x_{ijk} \right)^\alpha - \gamma_k^- \lambda \left( -d_{ijk}^- \Delta x_{ijk} \right)^\beta \right] \right\}
\] (7)

where \( RV_i \) indicates the observed term in the random relative utility of alternative \( i \); \( r_i \) is a relative interest parameter; \( \gamma_k^+ \) and \( \gamma_k^- \) are coefficients that need to be estimated; \( \lambda, \alpha \) and \( \beta \) are parameters in the value function of CPT, which are fixed to simplify model estimation; \( d_{ijk}^+ \) and \( d_{ijk}^- \) are dummy variables, \( d_{ijk}^+ \) is set to 1 if \( \Delta x_{ijk} \) is non-negative, otherwise 0, \( d_{ijk}^- \) is set to 1 if \( \Delta x_{ijk} \) is negative, otherwise 0; \( \Delta x_{ijk} = x_{ik} - x_{jk} \), which defines the attributes difference between two alternatives.

Several applications of RRUM models can be found, such as choices of destinations and stop patterns (Zhang et al., 2004), influence effect of travel information on modal choice (Zhang & Fujiwara, 2004; Fujiwara et al., 2004), choice set generation (Zhang et al., 2005), departure time and route choice (Zhang et al., 2010; Zhang et al., 2013). In addition, a review of RRUM models was presented (Zhang, 2015).

Unlike few developments and applications in RRUM models, the RRM models first introduced by Chorus et al. (2008) have attracted much more attention. After the original RRM
model and the new RRM model, several other versions of RRM models have been developed and some critical problems have been discussed.

The generalized RRM model (Chorus, 2014) has been developed from the new RRM model considering model flexibility, which lies in its ability to capture and distinguish the taste for an attribute and the non-linearity of the attribute’s regret function. This model is created by recasting a fixed constant (i.e. 1) into an attribute-specific regret weight parameter. Its specification can be written as follows:

\[ R_i = \sum_{j \neq i} \sum_m \ln \left[ \gamma_m + \exp \left[ \beta_m (x_{jm} - x_{im}) \right] \right] \]  

where \( \gamma_m \) denotes regret weight parameter. If \( \gamma_m = 1, \forall m \), then it is intuitive that generalized RRM model will reduce to the new RRM model. Typically, \( \gamma_m \in [0, 1], \forall m \). The recent \( \mu \)RRM model (van Cranenburgh et al., 2015) derived from the new RRM model by arguing that the latter is not scale-invariant and allowing the variance of the error term of random regret to be estimated. Its model specification can be described as follows:

\[ RR_i = R_i + \epsilon_i = \sum_{j \neq i} \sum_m \ln \left( 1 + \exp \left[ \frac{\beta_m}{\mu} (x_{jm} - x_{im}) \right] \right) + \epsilon_i, \quad \epsilon_i \sim i.i.d. EV(0, \mu) \]  

where \( \mu \) is scale parameter. This \( \mu \)RRM model with different values of \( \mu \) comes to different models: if \( \mu \) is insignificant different from one, then it becomes the new RRM model; if \( \mu \) is arbitrarily large, then it comes to RUM model; if \( \mu \) is arbitrarily close to zero, then it comes to the pure RRM model, which has the following model specification:

\[ R_i = \sum_m \beta_m x_{im}, \quad \text{where} \quad x_{im} = \begin{cases} \sum_{j \neq i} \max \left( 0, x_{jm} - x_{im} \right), & \text{if } \beta_m > 0 \\ \sum_{j \neq i} \min \left( 0, x_{jm} - x_{im} \right), & \text{if } \beta_m < 0 \end{cases} \]  

Jang et al. (2016a) assumed a perception effect that attribute-level regret is proportional to attribute’s magnitude. Therefore, they proposed new forms of regret function by replacing \( (x_{jm} - x_{im}) \) with \( (x_{jm} - x_{im}) / (x_{im} \theta_m) \) in the original RRM model and the new RRM model. The model specifications are presented as follows:

\[ R_i = \max_{j \neq i} \left\{ \sum_m \max \left( 0, \beta_m \frac{x_{jm} - x_{im}}{(x_{im}) \theta_m} \right) \right\} \]  

\[ R_i = \sum_{j \neq i} \sum_m \ln \left[ 1 + \exp \left[ \beta_m \frac{x_{jm} - x_{im}}{(x_{im}) \theta_m} \right] \right] \]  

\( \theta_m \) is a parameter valued between zero and one to capture the perception effect. If \( \theta_m \) is smaller than one, then the perception effect is diminished in some degree; if \( \theta_m \) is equal to zero, then these models comes to the original RRM model and the new RRM model. Two data sets (one
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Concerned stated preference and the other concerned revealed preference) were applied to test the model fit, and results revealed that these models’ performance were better.

Besides these new forms of RRM models, some hybrid models were also developed, such as the hybrid model of regret and utility (Chorus et al., 2013; Kim et al., 2016), the hybrid model of regret and disappointment (de Carvalho et al., 2016).

Besides these developments in model specification, some studies focus on empirical comparison of different versions of RRM models. As already mentioned above, the difference between the original RRM model and the new RRM model lies in two aspects: first, the original RRM model postulates that regret is only experienced with respect to the best foregone alternative while the new RRM model deals with all other alternatives in the offered choice set; second, the specification’s likelihood function in the original RRM model is non-smooth whereas the new RRM model’s is smooth. According to that, Rasouli and Timmermans (2015b) proposed another form, which defined regret as max attribute-level regret between chosen alternative and all foregone alternatives (denoted as RRsum model):

\[ R_i = \sum_{j \neq i} \sum_m \max\{0, \beta_m (x_{jm} - x_{im})\} \]  

Comparing the model fit of Eq. (3), (4) and (13) in the context of shopping destination choice, they found that the original RRM model performs best, and the new RRM model worst. Similar results were found by Jang et al. (2016a), which was in the context of shopping center choice and travel model choice. It seems that the property of user friendliness of the new RRM model sacrifices its model fit. Recall the curves of attribute-level regrets of the original RRM model and the new RRM model (see Chorus, 2010), their difference might lie in the predictive ability of small difference (around zero). In addition, taking all foregone alternatives equally into consideration is not defendable either.

In addition, Jang et al. (2016b) paid their attention to measurement error of RRM models. Given the different function forms between RUM models and RRM models, their study argued the assumption of i.i.d error term in RRM models should be critically assessed. Although it is acknowledged that biased estimation results from measurement error exist in both types of models, uncertainty tended to accumulate in RRM model since comparisons of alternatives are involved. By comparing the biased estimation results of RUM model and the original RRM model in some different context, this study found that bias caused by the original RRM model specification is higher than in the RUM model. In addition, an approach to deal with this problem was also presented.

5. FURTHER IMPROVEMENTS

Although some studies confirmed the advantages of RRUM models in capturing individuals’ bounded rational choice behavior, this model largely went unnoticed. However, over the last few years, so-called reference-based models have attracted increasing attention. Examples include regret-based models, disappointment-based models, etc. It seems timely to re-address the relative utility models as it can be shown that regret-based models and the other mentioned models are
nothing but special cases of RRUM models. The remainder of this section discusses several potential improvements of RRUM models as well as in RRM models.

First, the recent elaborations of regret models could also be investigated for relative utility models, and vice versa. Second, although Eq. (2) mentioned that \( f(\cdot) \) and \( g(\cdot) \) could have varied function forms, only linear-additive form was tested in RRUM models. Therefore, other different forms could be investigated and compared with linear-additive form. Third, Rasouli and Timmermans (2015b) argued that taking all foregone alternatives equally into account may not be defendable on some choice domains. Therefore, inspired by Eq. (2), one might consider giving each foregone alternative a weight \( w_{ji} \) to measure their influence to the considered alternative. Such model can be expressed as follows:

\[
R_i = \sum_{j \neq i} w_{ji} \sum_{m} \max\{0, \beta_m (x_{jm} - x_{im})\}
\]  

Fourth, to date only the alternative-specific context was discussed in the contexts of RRUM models and RRM models. Alternative-specific context includes the number of alternatives and their attributes, the correlated structure of attributes and the availability of alternatives. However, Zhang (2015) defined another two different types of context: the time-specific and individual-specific context. The time-specific context refers to the individuals’ previous and future choice behavior. The individual-specific context refers to the choice behavior of others’ in individuals’ social network. It is interesting and relevant to examine relative utility and random regret models that include these comparisons or a combination of the three types of contexts.

Fifth, although theoretical comparisons of RRUM and RRM models are presented in Section 3, empirical results of comparisons of these models in different context of different domains are still needed. Sixth, here only RRUM and RRM models under condition of certainty are discussed. However, uncertainty (or risk) is a more common factor in the real world. Therefore, one might be interested in the comparison of these models in terms of theoretical and empirical aspects and their applications under condition of uncertainty.

6. CONCLUSIONS

In this paper, we reviewed the basic concepts underlying relative utility and random regret models, compared their differences and similarities, discussed their new elaborations and applications, and argued their potential improvements. Basically, RRUM models and RRM models take into account context dependency, which reveals individuals’ bounded rationality. Several conclusions may be drawn: 1) RRUM model comes to RUM model if all relative interest parameters are equal; 2) the RRUM model reduces to the RUM model if the choice set is binary; 3) the original RRM model turns into the RUM model if the choice set is binary; 4) the new RRM model reduces to the RUM model if the choice set is binary; 5) both RRM models are special cases of RRUM models; Future research could focus on the combination of RRUM models and RRM models, model developments in time-specific context and individual-specific context and model applications and comparisons under condition of uncertainty.
REFERENCES

de Carvalho, E.S., Rasouli, S. & Timmermans, H.J.P. 2016, “Modeling dynamic route choice behavior under uncertainty using concepts of regret and disappointment”. In Transportation Research Board 95th Annual Meeting (No. 16-5291).
McFadden, D., 1978, Modelling the choice of residential location, Institute of Transportation Studies, University of California.


