Simultaneous analysis and design based optimization for paper path and timing design of a high-volume printer

Simultaneous Analysis and Design Based Optimization for Paper Path and Timing Design of a High-Volume Printer

L. Swartjes[a,*], L.F.P. Etman[a], J.M. van de Mortel-Fronczak[a], J.E. Rooda[a], L.J.A.M. Somers[b]

[a] *Control Systems Technology Group*
Department of Mechanical Engineering
Eindhoven University of Technology
Eindhoven, PO Box 513, 5600 MB
The Netherlands
[b] *Océ-Technologies B.V.*
Venlo, PO Box 101, 5900 MA
The Netherlands

Abstract

The design of a high-volume printer for professional use is rather complex. The design of the paper path and the timing of sheets is frequently reengineered as the design of the printer components progresses. This paper presents an optimization model for the combined paper path and timing design problem. The paper path is an optimal physical dimensioning problem, while the timing is an open-loop optimal control problem. The coupled optimization problem is formulated as a simultaneous analysis and design (SAND) problem using a direct transcription of the optimal control problem. Benefits of the chosen formulation for industrial application are the ease of setting up the optimization model for arbitrary printer configurations, and the short computation times. Results of an industrial case are presented.

Keywords: printer, paper path, SAND, optimization, model

1. Introduction

A printer is an electromechanical device capable of transferring an image to a sheet of paper. Various types of printers are on the market, ranging from printers for home use to high-volume printers for professional applications. This paper considers the design of a high-volume printer. In particular, an optimization model is developed to support the design of the, so-called, paper path and timing for such a printer.

Inside the printer, sheets are transported through a metal track. Along their route through the track, sheets visit several modules, which typically includes a paper input module, an image transfer station, a turn track for duplex (two-sided) printing, and a finisher. The paper path represents the physical layout and dimensioning of the metal track with the various printer modules. The timing is the envisioned position of sheets along the paper path with respect to time.

Design of the paper path and timing is a coupled problem. A change in the design of the paper path affects the timing. The other way around, a change in timing typically requires the paper path to be re-designed. This leads to a lot of rework due to the constant changing design in the early phases. Moreover, many changes are induced due to the multidisciplinarity of the design of a printer: each discipline introduces their own specific requirements. All in all, this leads to a highly iterative design process. Creating a paper-path layout with accompanied timing typically takes a lot of time.

A model-based engineering approach may be taken to address this challenge. For instance, Cloet et al. [1] developed a computational model to calculate collision free transportation of sheets through the paper path, assuming simplex (one-sided) printing and homogeneity of the print job, i.e. every paper sheet excites the same dynamics shifted in time. Kruciński [2] and Bukkems et al. [3] addressed media path design and related control challenges in the context of high-speed printers, including single and multiple print engine systems. Additionally, Beckers et al. [4] and Heemels and Muller [5] presented a model that enables the visualization of the flow of papers along the paper path. Given a paper path and timing, the paper flow can be simulated and inspected for possible collisions. Stamps et al. [6] studied the timing design for duplex printing jobs, assuming homogeneity of the print job. They developed a numerical optimization model that includes the duplex printing cycle and the merging of the stream of paper sheets from the duplex loop with the input stream.

Also in other engineering domains, literature may be found that considers related problems. For instance, Cao

*Corresponding author
Email addresses: l.swartjes@tue.nl (L. Swartjes), l.f.p.etman@tue.nl (L.F.P. Etman), j.m.v.d.mortel@tue.nl (J.M. van de Mortel-Fronczak), j.e.rooda@tue.nl (J.E. Rooda), lou.somers@oce.com (L.J.A.M. Somers)

et al. [7] investigated model-predictive control for the path generation of merging automated vehicles. The vehicles on the ramp merge with the vehicles on the main road. The method is intended for online vehicle collision avoidance. The focus of timing design for printers is different: one seeks the timing of each sheet such that system performance (e.g. paper throughput) is maximized while guaranteeing that sheets do not collide.

The timing design problem in the printer relates to trajectory optimization in the field of optimal control (open-loop control). For instance, see Betts [8] for an overview of algorithms for trajectory optimization problems. Balesdent et al. [9] present a survey of multidisciplinary optimization (MDO) methods in launch vehicle design, where trajectory optimization is one of the disciplines. They compare various single-level and multi-level MDO methods for application in the launch vehicle design. Herein, single-level refers to centralized optimization, and multi-level to distributed decision making. For classifications of single-level and multi-level MDO methods, one is referred to Cramer et al. [10], Balling and Sobieszczanski-Sobieski [11], Alexandrov and Lewis [12], Tosserams et al. [13], De Wit and Van Keulen [14], and Martins and Lambe [15]. Allison and Herber [16] review MDO methods specifically for dynamic systems design. They advocate the development of dedicated MDO methods for dynamic systems with balanced consideration of physical and control system design. Examples of papers developing methods for the optimal design and control co-design problem in the context of mechatronic systems design include Ravichandran et al. [17], Affi et al. [18], and Peters et al. [19].

We aim to develop an optimization model for the combined design of the paper path and the timing. Given the application in industry, it is required that the optimization model can be solved using off-the-shelf optimization software. Preferably in an environment such as MATLAB. Solving times of the optimization runs are preferably small to allow rapid prototyping. What is more, the model setup should be such that the paper path configuration can be easily adjusted. This paper describes the model that we have developed.

The timing design is viewed as an open-loop optimal control problem, while the paper path design is a physical system optimal design problem. The key concept in our model development is to treat the coupled system in an all-at-once fashion by incorporating a direct transcription of the optimal control problem Hargraves and Paris [20], Betts [8], Biegler [21] into a simultaneous analysis and design (SAND) formulation of the combined paper path and timing design problem Haftka [22], Arora and Wang [23], Allison and Herber [16]. That is, the equations of motion are discretized and included as algebraic equality constraints, while the decision variables include the physical lengths of the segments of the paper path, the state variables due to the discretized equations of motion, and the control input variables (segment accelerations). This is a different approach compared to the commonly employed nested analysis formulation (NAND) which defines the paper path dimensions and the segment accelerations as decision variables. For each evaluation of the objective function the state variables follow from a nested solution of the equations of motion.

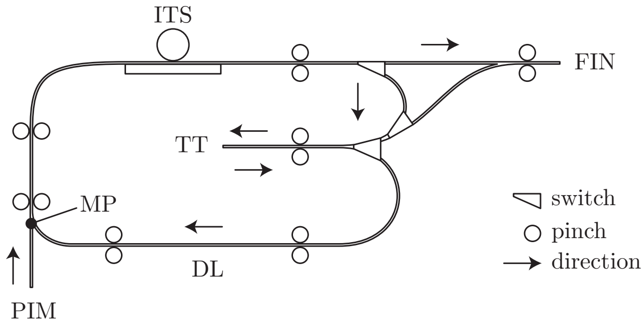

Figure 1: Schematic layout of a paper path

The SAND formulation allows the modular setup of the optimization model for arbitrary paper path configurations; the optimization model can be assembled from the components given a certain paper path configuration. This presents a significant advantage compared to Stamps et al. [6], where a NAND approach was used. With this specific method, a new configuration required a new model to be derived.

The paper is organized as follows. First, the design problem is described. Subsequently, the optimization problem is formulated, which includes the mathematical representation of the various paper path components and the accompanying derivation of design goals, design constraints, and state equations. Then, the industrial case is presented to demonstrate the model-based optimization framework. Finally, some concluding remarks are offered.

2. Design Problem

The paper path and timing are leading in the design of the various components that make up the printer. The design of the paper path and the timing is frequently revisited. In this section, the paper path and timing design problem is explained in further detail.

2.1. Paper Path

The paper path of a printer is defined as the path sheets can follow in the printer. The paper path comprises the various components that perform actions on the sheets. Generally, a paper path includes a track, an *image transfer station* (ITS), a *turn track* (TT), multiple switches, and numerous pinches. In some printer configurations also a cooling station is included, after the ITS.

In Fig. 1, a typical paper path is depicted. Sheets are fed to the paper path by means of a *paper input module* (PIM), via a standardized interface. The sheets move inside the metal track, where pinches actuate the sheets by means of frictional forces. Pinches can either be controlled

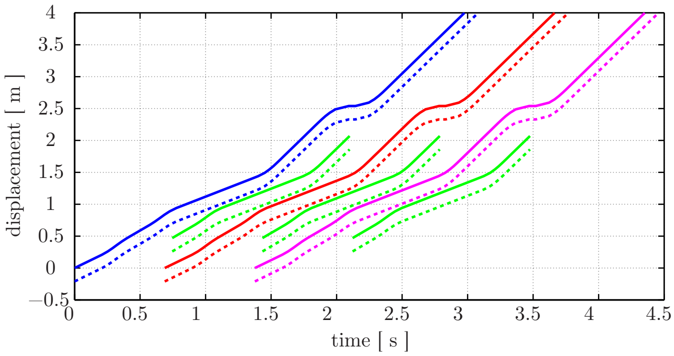

Figure 2: Exemplary partial timing for three sheets

as a group or individually. If pinches are coupled, the same velocity must be imposed by each pinch in the same group. At intersections, switches are used to guide sheets into a specific direction.

The (digital) image is transferred to one side of the sheet at the ITS. This is typically done at a pre-defined constant velocity, to achieve high quality prints and prevent smudging. In the case that both sides are to be printed, in a two-sided (duplex) job, a sheet passes the ITS two times. To this end, sheets are reversed (turned) in the the turn track and guided to the *duplex loop* (DL). The stream of sheets from the duplex loop is merged with the stream of sheets from the paper input module at the *merge point* (MP).

After the printing process for a sheet is finished, the sheet leaves the paper path via another standardized interface to the *finisher* (FIN). The finisher can be any external module, which post-processes sheets or stores sheets.

2.2. Timing

The timing is the envisioned displacement of the leading edge of a sheet with respect to time. Each sheet has, in general, its own characteristic timing, as the properties of sheets may differ. However, it is assumed that every sheet has the same size and the same timing, shifted in time, i.e. a homogenous job.

The determination of the timing can be viewed as the off-line calculation of the optimal time-displacement profiles of subsequent sheets to achieve maximum printer performance (open-loop optimal control problem).

In Fig. 2, an exemplary timing is (partially) given for three sheets. The solid lines represent the position of the leading edge of a sheet. The dashed lines represent the position of the trailing edge of a sheet. The combination of a solid and dashed line of one particular color, represents the timing of one sheet. The green, shorter, lines represent sheets passing for the second time through the image transfer subsection of the paper path. These green lines are of different sheets than the ones depicted with blue, red, and violet in Fig. 2.

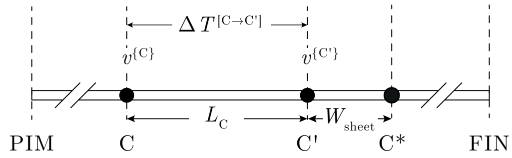

Figure 3: Visual representation of a printer component

3. Optimization Problem Definition

In this section, the optimization model for the combined paper path and timing design problem is derived. To this end, first the representation and dynamics of the printer components is introduced. The chosen description of the printer models the dynamics close to reality. Based on this representation, the design limitations, i.e. the constraints, are derived. Finally, the different design goals are introduced and the complete optimization problem is summarized.

3.1. Component Definition

The paper path is modeled as a chain of one-dimensional components. A component is represented as given in Fig. 3. The component spans between point C and C$'$, and has a length of L_C. Upon arrival of the leading edge of a sheet at point C, the sheet enters the component. W_{sheet} is the width of a sheet of paper. Therefore, when the leading edge reaches point C*, the trailing edge reaches C$'$, i.e. the respective sheet is fully deflected. A one-dimensional description of the paper path suffices, as it is assumed no tearing or skewing of sheets occurs.

In Fig. 3, $v^{\{C\}}$ is used to denote the velocity of the leading edge, upon reaching point C. Similarly, $s^{\{C\}}$ denotes the displacement upon reaching point C, that is, the distance travelled from the paper input module up to point C. The time it takes for the leading edge to move from point C to C$'$ is denoted by $\Delta T^{[C\to C']}$.

The physical start and end points of the component, C and C$'$, are referred to as control points. Between two control points, a constant acceleration is assumed. To allow a non-constant acceleration profile within a component (between C and C$'$), the component can be subdivided into smaller segments. For each segment a constant acceleration is assumed. If a component is subdivided into segments, an equidistant spacing of the inserted control points is assumed. As an example, the component of Fig. 3 is subdivided into three segments, which is depicted in Fig. 4. Herein, $L^{\{k\}} = L^{\{k+1\}} = L^{\{k+2\}}$, and $L^{\{k\}} + L^{\{k+1\}} + L^{\{k+2\}} = L_C$.

So basically, the paper path consists of single-segment and multi-segment components. We denote the set $\mathcal{K} = \{1, 2, \ldots, K\}$ to hold the identifiers of all segments in the paper path. The segment acceleration, duration and length, are denoted by, respectively, $a^{\{k\}}$, $\Delta T^{\{k\}}$ and $L^{\{k\}}$, with $k \in \mathcal{K}$. The displacement and velocity of the leading edge of a sheet are defined on the edges of the segments, i.e. at the control points. These are denoted by $s^{\{k\}}$, and $v^{\{k\}}$,

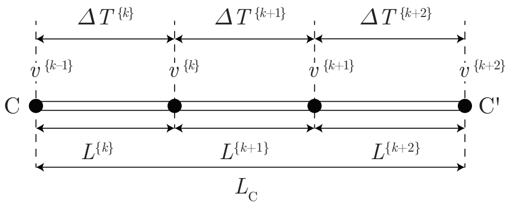

Figure 4: Segmentation of the component

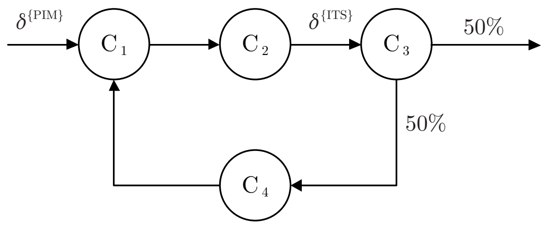

Figure 5: Paper flow as a manufacturing line

respectively, with $k \in \mathcal{K}^0$. $\mathcal{K}^0$ is the set $\mathcal{K}$ extended with the identifier zero: $\mathcal{K}^0 = \{0\} \cup \mathcal{K}$.

3.2. Mathematical Representation

The following simultaneous analysis and design optimization model is defined for the combined paper path and timing design problem:

$$\begin{array}{rl} \text{find} & \vec{z} = \left[\vec{\Delta T}, \vec{L}, \vec{v}, \mathrm{IDT}\right] \\ \text{minimize} & f(\vec{z}) \\ \text{subject to} & \vec{h}(\vec{z}) = 0 \\ & \vec{g}(\vec{z}) \leq 0 \end{array} \tag{1}$$

The decision variables are $\vec{\Delta T} = \{k \in \mathcal{K} : \Delta T^{\{k\}}\}$, $\vec{L} = \{k \in \mathcal{K} : L^{\{k\}}\}$, $\vec{v} = \{k \in \mathcal{K}^0 : v^{\{k\}}\}$, and IDT. Here, IDT refers to the *inter-departure* time of sheets, i.e. the time between two consecutive releases of sheets from the PIM.

The equations of motion are included as equality constraints by means of direct transcription. Since for each segment a constant acceleration is assumed, the state equations for a segment become:

$$v^{\{k\}} = v^{\{k-1\}} + a^{\{k\}}\Delta T^{\{k\}} \quad \forall k \in \mathcal{K} \tag{2}$$

$$s^{\{k\}} = s^{\{k-1\}} + v^{\{k-1\}}\Delta T^{\{k\}} + \frac{1}{2}a^{\{k\}}\left\{\Delta T^{\{k\}}\right\}^2 \tag{3}$$

Using $L^{\{k\}} = s^{\{k\}} - s^{\{k-1\}}$, and substituting Eqn. (2) into Eqn. (3) to eliminate acceleration $a^{\{k\}}$ yields:

$$\left(v^{\{k-1\}} + v^{\{k\}}\right)\Delta T^{\{k\}} = 2L^{\{k\}} \quad \forall k \in \mathcal{K} \tag{4}$$

which is bi-linear in terms of the decision variables. The segment acceleration $a^{\{k\}}$ can be easily obtained using Eqn. (2) once $\vec{v}$, $\vec{L}$, $\vec{\Delta T}$, and IDT have been solved from Eqn. (1).

3.3. Throughput Consideration

One of the decision variables in the optimization problem is the IDT. The IDT determines the *inter-arrival time* (IAT) of sheets at the different components. This is illustrated in Fig. 5, a process-flow representation of the paper path of Fig. 1. The same reasoning is used as in Stamps et al. [6].

In Fig. 5, $\delta^{\{\mathrm{PIM}\}}$ and $\delta^{\{\mathrm{ITS}\}}$ represent, respectively, the throughput at the PIM and the ITS. It is assumed an interleaving policy is used in case of double sided printing, which means that at the merge process one sheet from the duplex loop is alternated with one sheet from the PIM. This results in a situation where 50% of the sheets leaves the paper path after the ITS, while 50% continues for their second pass. The thoughput of sheets at the ITS is two times the throughput at the PIM:

$$\delta^{\{\mathrm{ITS}\}} = 2\delta^{\{\mathrm{PIM}\}} \tag{5}$$

The reciprocal of this expression, in terms of the interarrival times, equals

$$2\mathrm{IAT}^{\{\mathrm{ITS}\}} = \mathrm{IAT}^{\{\mathrm{PIM}\}} = \mathrm{IDT} \tag{6}$$

Therefore, given the IDT, the interarrival times of the individual components are known.

3.4. Design Limitations

The design limitations for the paper path and timing design problem relate to the physical limitations and the inter-sheet behavior. For the inter-sheet behavior, we distinguish collision prevention, mutual exclusion, and merging of sheets. All of these limitations are expressed as inequality and equality constraint functions.

Physical limitations. The physical limitations are defined by the pinches used in the printer and the space the printer may occupy. To this end, a minimal and maximal velocity and paper path length is defined. The minimal and maximal velocity of pinches is defined by, respectively, $v_{\min}$ and $v_{\max}$. For the length, the segment length between two control points is considered. $L_{\min}$ and $L_{\max}$ denote respectively the minimal and maximal length of such a segment.

Collision Prevention. Consider control point k with velocity $v^{\{k\}}$. To prevent collision, a sheet must have travelled at least its own width, before the next sheet arrives at k. The leading edge of the successor sheet arrives $\mathrm{IAT}^{\{\mathrm{k}\}}$ seconds later. Taking into account an additional clearance distance, $d_{\mathrm{clearance}}$, collision avoidance at point k requires $v^{\{k\}} \cdot \mathrm{IAT}^{\{k\}} \geq W_{\mathrm{sheet}} + d^{\{k\}}_{\mathrm{clearance}}$. Since between successive control points a constant acceleration applies, the velocity change between two successive control points is linear. To guarantee collision avoidance along the entire paper path, it is therefore sufficient to impose the collision avoidance constraints at the control points only:

$$v^{\{k\}} \cdot \mathrm{IAT}^{\{k\}} \geq W_{\mathrm{sheet}} + d^{\{k\}}_{\mathrm{clearance}} \quad \forall k \in \mathcal{K}^0 \tag{7}$$

Mutual Exclusion. Some components can hold only a single sheet at a time. Before a new sheet may enter, the previous sheet should have left the component. This is a more strict condition compared to the collision avoidance constraints presented above. For components that require mutual exclusion of sheets we introduce an additional constraint to enforce full deflection of a sheet before a new one may enter. To this end, point C* in Fig. 3 is used: if the leading edge of a sheet reaches this point, its trailing edge reaches C$'$, which means that a sheet is fully deflected. The mutual exclusion constraint becomes:

$$\sum_{m \in M_C} \Delta T^{\{m\}} \le \mathrm{IAT}^{\{C\}} \quad \forall C \in \mathcal{M} \tag{8}$$

where M_C is the set of identifiers of the segments corresponding to component C, $\mathcal{M}$ the set of mutual exclusive components, and $\mathrm{IAT}^{\{C\}}$ the IAT of component C.

Merging. At the merge point, we need a constraint to prevent collisions during merging. During a duplex print job, every $\Delta T^{[\mathrm{PIM}\to\mathrm{MP}]} + \lambda \cdot \mathrm{IAT}^{\{\mathrm{MP}\}}$ with $\lambda \in 2\mathbb{N}^+$, the leading edge of a sheet from the PIM reaches the merge point, see Fig. 5. Note that, $\mathbb{N}^+$ are the integer numbers, without zero. For the odd λ values, sheets from the duplex loop reach the merge point. In order to prevent collisions, the timing of sheets from the duplex loop should be such that every $\lambda \in 2\mathbb{N}^+ + 1$, the leading edge of a sheet from the duplex loop reaches the merge point. Such a timing is obtained if the following constraint is satisfied:

$$\Delta T^{[\mathrm{MP}\to\mathrm{MP}']} = \kappa \cdot \mathrm{IAT}^{\{\mathrm{MP}\}} \quad \kappa \in 2\mathbb{N}^+ + 1 \tag{9}$$

where $\Delta T^{[\mathrm{MP}\to\mathrm{MP}']}$ represents the time needed for a leading edge to travel from the merge point (first pass) to the merge point (second pass). Note that, κ is a discrete variable. It represents the number of sheets on their way along the paper path between points MP and MP$'$. This variable is treated as a parameter and not included as discrete design variable in the optimization problem.

3.5. Design Goals

In this paper, four design goals are considered for the design: energy, duration, cost, and productivity. These design goals are defined in terms of decision variables, namely $\vec{z} = \left[\Delta\vec{T}, \vec{L}, \vec{v}, \mathrm{IDT}\right]$, and are given as follows.

Minimal time Minimize the time spent by sheets inside the paper path, i.e. $f_1(\vec{z}) = \sum_{k\in\mathcal{K}} \Delta T^{\{k\}}$.

Minimal cost Minimize the material cost, due to the size of the paper path, i.e. $f_2(\vec{z}) = \sum_{k\in\mathcal{K}} L^{\{k\}}$.

Minimal energy Minimize the transportation energy of sheets in the paper path, i.e. $f_3(\vec{z}) = \sum_{k\in\mathcal{K}} \left(v^{\{k\}} - v^{\{k-1\}}\right)^2$.

Maximal productivity Maximize the utilisation of the ITS, i.e. $f_4(\vec{z}) = \mathrm{IDT}$.

The objective function is defined as the weighted sum of these four goals:

$$f(\Delta\vec{T}, \vec{L}, \vec{v}, \mathrm{IDT}) = \sum_{i=1}^{4} w_i f_i(\vec{z}) \tag{10}$$

The weighting values determine the importance of the respective goals. Objective function f is quadratic.

3.6. Summary

Combining the equations derived in the previous subsection, the optimization problem becomes as follows:

$$\begin{array}{lll}
\text{find} & \Delta\vec{T}, \vec{L}, \vec{v}, \mathrm{IDT} & \\
\text{min} & f(\Delta\vec{T}, \vec{L}, \vec{v}, \mathrm{IDT}) & \\
\text{s.t.} & 2L^{\{k\}} - \left(v^{\{k-1\}} + v^{\{k\}}\right)\Delta T^{\{k\}} = 0 & \forall k \in \mathcal{K} \\
 & W_{\text{sheet}} + d^{\{k\}}_{\text{clearance}} - v^{\{k\}} \cdot \mathrm{IAT}^{\{k\}} \le 0 & \forall k \in \mathcal{K}^0 \\
 & \sum_{m\in M_C} \Delta T^{\{m\}} \le \mathrm{IAT}^{\{C\}} & \forall C \in \mathcal{M} \\
 & \Delta T^{[\mathrm{MP}\to\mathrm{MP}']} - \kappa \cdot \mathrm{IAT}^{\{\mathrm{MP}\}} = 0 & \kappa \in 2\mathbb{N}+1 \\
 & v^{\{k\}} - v_{\max} \le 0 & \forall k \in \mathcal{K}^0 \\
 & v_{\min} - v^{\{k\}} \le 0 & \forall k \in \mathcal{K}^0 \\
 & \Delta T^{\{k\}} - \Delta T_{\max} \le 0 & \forall k \in \mathcal{K} \\
 & \Delta T_{\min} - \Delta T^{\{k\}} \le 0 & \forall k \in \mathcal{K} \\
 & L^{\{k\}} - L_{\max} \le 0 & \forall k \in \mathcal{K} \\
 & L_{\min} - L^{\{k\}} \le 0 & \forall k \in \mathcal{K} \\
 & \mathrm{IDT} - \mathrm{IDT}_{\max} \le 0 & \\
 & \mathrm{IDT}_{\min} - \mathrm{IDT} \le 0 & \\
\text{given} & \mathcal{K}, \mathcal{M}, \kappa & \\
 & \mathrm{IAT}^{\{k\}} & \forall k \in \mathcal{K}^0 \\
 & v_{\min}, v_{\max}, \Delta T_{\min}, \Delta T_{\max} & \\
 & L_{\min}, L_{\max}, \mathrm{IDT}_{\min}, \mathrm{IDT}_{\max} &
\end{array}$$

Note that the actual value of IAT depends on the component C: the IAT variable is to be substituted by IDT or $\frac{1}{2}$IDT based on the considered component, as explained in the throughput consideration subsection. Secondly, it is noted that additional design requirements, such as a specific minimal length of a component, may be easily included. Finally, it is assumed that configuration choices defined by the sets $\mathcal{K}$, $\mathcal{M}$ and variable κ are input to the optimization model. Therefore, the optimization problem is continuous for a given configuration. The equality constraints representing the discretized equations of motion (first constraint), are quadratic. Hence, the optimization problem is non-convex.

4. Industrial Case

In the industrial case, the paper path of Fig. 6 is considered. This is almost the same paper path as in Fig. 1, though leaving out the functionality to feed a sheet directly to the finisher after turning and neglecting the pinches. Sheets will be printed duplex. After the enter at the PIM, the image on one side is printed in the ITS. Afterwards, the sheets are turned in the TT. Finally, the other side is

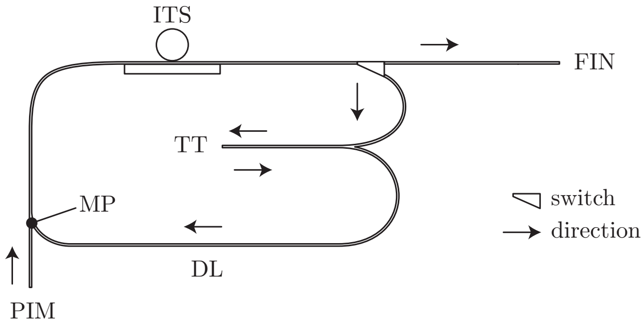

Figure 6: Schematic layout of the paper path

printed by passing the ITS the second time before leaving the printer by means of the FIN.

For the industrial case, the following assumptions are made.

1. All pinches are individually controlled.
2. The sheets have the A4 paper size, i.e. $W_{\text{sheet}} = 0.21$ meter.
3. The overall clearance distance is 5 millimeters, i.e. $d^{\{k\}}_{\text{clearance}} = 0.005$ meter $\forall k \in K$.
4. The minimal stop time in the turn track is 5 milliseconds.
5. The PIM processes sheets at 1 meter per second, the MP at 1.2 meters per second, and the ITS at 0.8 meters per second.
6. The acceleration is bounded between minus and positive 10 meters per second squared.
7. The velocity can vary between 0 and 2 meters per second.
8. The duration has a maximum of 3 seconds.
9. The IDT has a maximum of 2 seconds, i.e. a minimum of 60 pages per minute at the ITS.
10. The length of a component cannot be smaller than W_{sheet} and cannot exceed 15 meters.
11. The total length of the paper path cannot exceed 15 meters.
12. The length of the ITS must be larger than 0.5 meters, and the switch must be larger than 0.4 meters.
13. For feasibility, the duplex loop must be at least 6 meters.
14. κ is set to 15 sheets.
15. All parts of the paper path, where no constant velocity is required, are subdivided into 6 segments.

The total number of elements considered, i.e., the cardinality of the set $\mathcal{K}$, is 135. Additionally, the number of variables is 287 and the number of constraints is 296.

For the configuration given in Fig. 6 and given the assumptions stated above, the optimization problem is generated and solved using the MATLAB Optimization Toolbox. As the optimization is a non-linear optimization problem, the `fmincon` algorithm [24] from MATLAB 2013a is used; `fmincon` is the function that can solve non-linear optimization problems. `fmincon` returns a numerical solution, subject to a user-defined tolerance. As the problem is non-convex, the found optimum can be a local optimum. `fmincon` is set to use the SQP algorithm and the tolerances are set to `1e-6`. The gradients of the problem are explicitly supplied to the optimization function.

For the case study, first the influences of the weights in the objective function are investigated, whereafter the ease of extending the optimization problem definition is investigated by adding an optional component.

Four different settings are considered for the weighting factors. Each setting emphasizes one of the goals; all weights are set to 1 except for the one with emphasis, which has weight 10. Each run of the optimization problem typically needs just three or four seconds on a standard PC to complete. In Fig. 7 and Fig. 8, the results are depicted with the used weight in the legend. Figure 7 depicts the velocity versus the control points, while Fig. 8 depicts the acceleration versus the control points. The difference in the returned timings is explained in the next paragraphs.

Emphasis on minimizing the sheet travel time, $w_1 = 10$, yields the highest accelerations imposed on the sheets such that the highest possible sheet velocities are obtained. This can be observed, as the velocity, in general, is higher than with the other solutions. Going as quick as possible, will result in the lowest travel time.

Emphasis on minimizing the physical length of the paper path, $w_2 = 10$, results in a timing with reduced accelerations and velocities compared to the first case. This can be explained by the fact that if a sheets travels at lower speeds, less time (and therefore paper path) is needed to make the sheets come to a complete stop.

If the focus is on minimizing energy, $w_3 = 10$, accelerations are set to the smallest possible values. As known from the laws of kinematics, energy is directly related to the acceleration. Hence, a lower acceleration leads to less energy consumption.

Finally, if the focus on maximizing productivity, $w_4 = 10$, the solution is close again to the solution obtained for the first case, but with lower and more gradually changing accelerations. As the IDT is dependent on sheets size and the value of κ, there is not a direct physical explanation for this result.

Next, the optional cooling station is considered located between the ITS and the switch. The cooling station operates at a speed of 0.5 meters per second, and the velocity of sheets should stay constant during the cooling process. Inclusion of the new component required three additional lines of code in the MATLAB specification of the optimization problem. This ease of change in the configuration of the paper path is due to the SAND-based formulation of the paper path and timing design optimization problem. Note that the objective function with focus on maximizing productivity is used.

Figures 9 and 11 present the resulting displacement of sheets, as well as the displacements of the original situ-

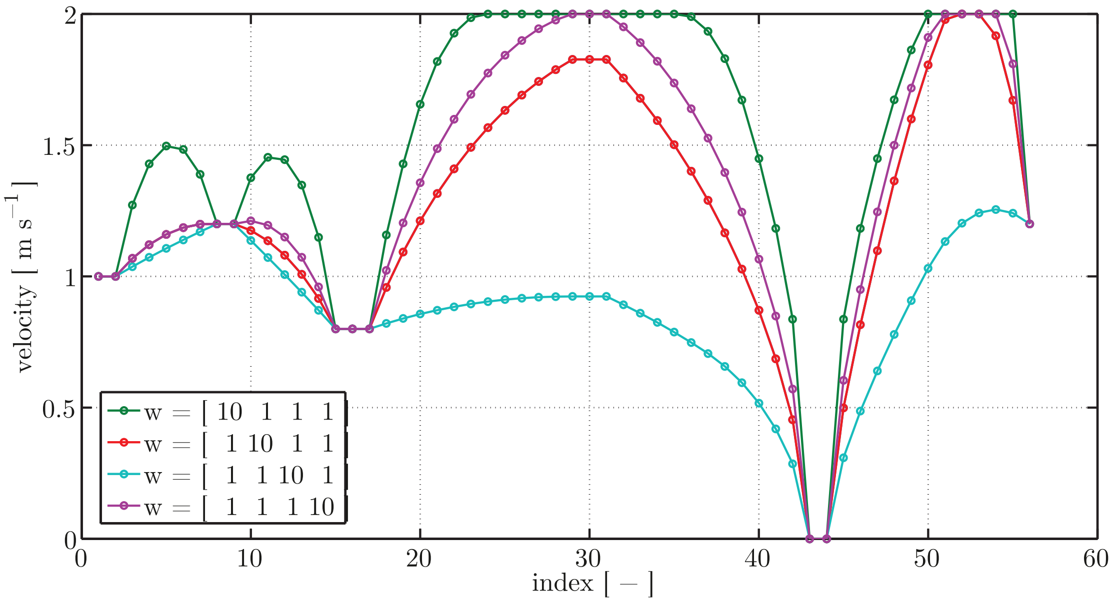

Figure 7: Influence of the weights on the velocity versus the control points

ation without cooling station. Figures 10 and 12 present the corresponding velocities for both cases, with and without the cooling station. We observe that the length of the paper path stays around 10 meters, whereas the duration increases with approximately two seconds. Also, the instance at which the turn track is reached occurs later. The overall velocity behavior stays the same. The consequence of including the cooling station is visible between time instances $t \approx 1.7$ and $t \approx 2.6$, i.e. the flat part with a constant velocity of 0.5 meters per second. Between these time instances, the sheets move slowly trough the cooling station in order to cool down.

5. Concluding Remarks

The SAND-based optimization problem formulation of the combined paper path and timing design problem allows for easy specification and adjustment of the paper path configuration. Solution times for our industrial printer case stay within a couple of seconds. The ease to reconfigure and the short computational times appeared to be necessary ingredients for the use by design engineers in the industrial environment. Compared to an earlier developed framework based on NAND by Stamps et al. [6], the newly developed optimization environment provided a significant improvement. Desirable extensions for future work are to allow for inhomogeneous print jobs (e.g. a mix of A3 and A4), to account for uncertainties in system parameters, and to enable run-time recalculation of the timing. This presents new challenges to the formulation of the optimization problem.

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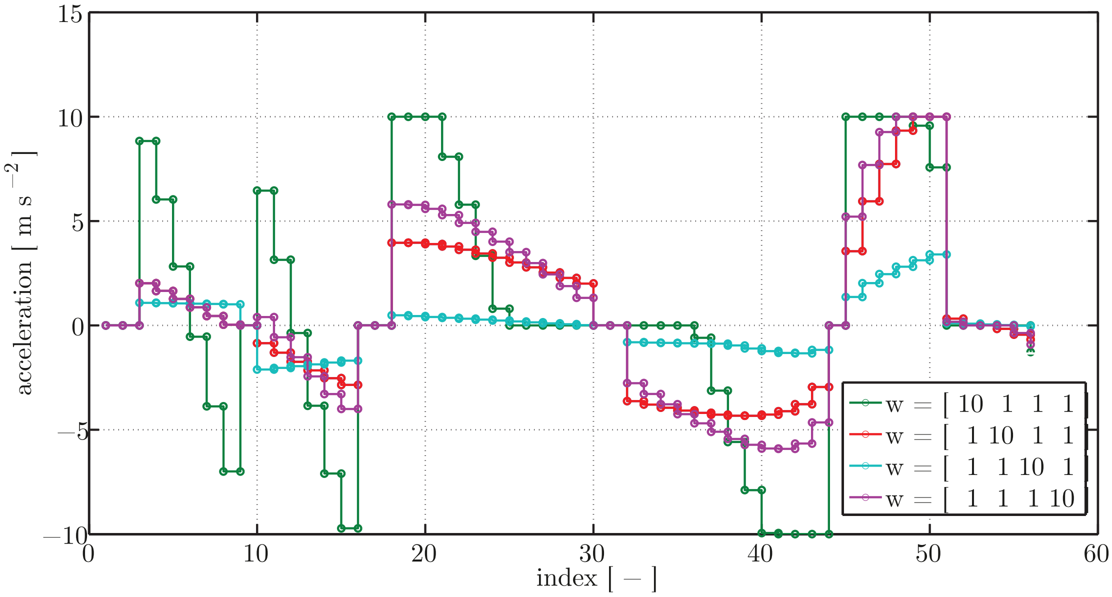

Figure 8: Influence of the weights on the acceleration versus the control points

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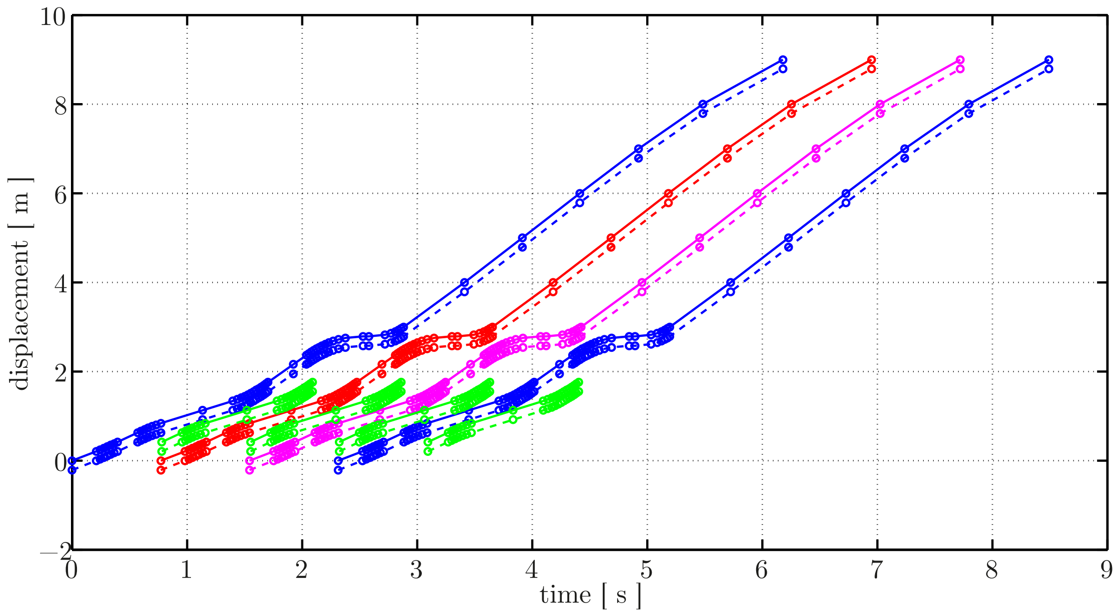

Figure 9: Displacement of sheets without cooling

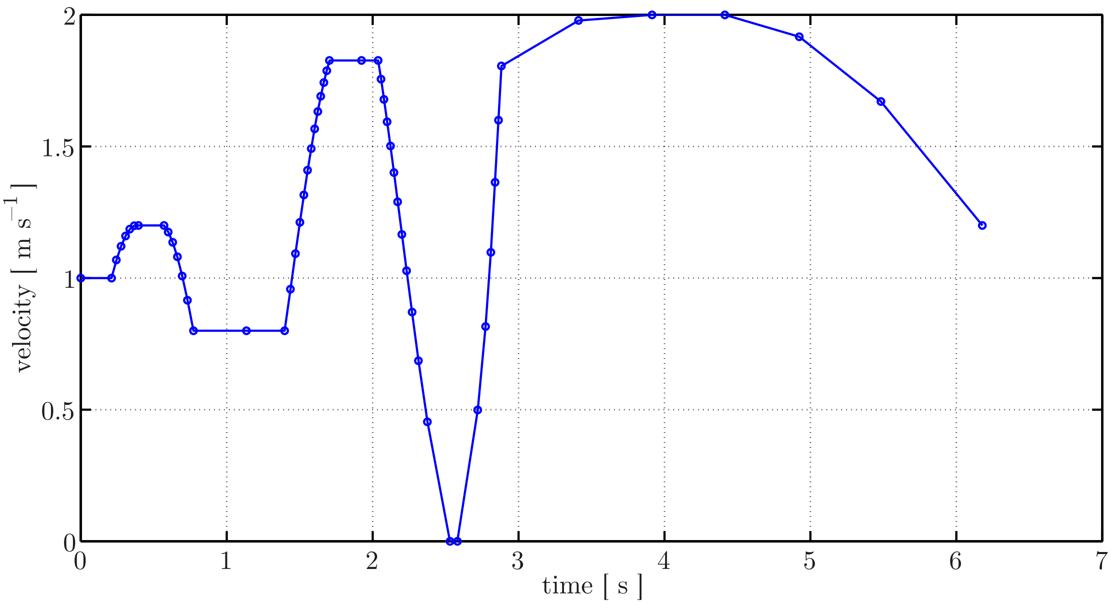

Figure 10: Velocity of sheets without cooling

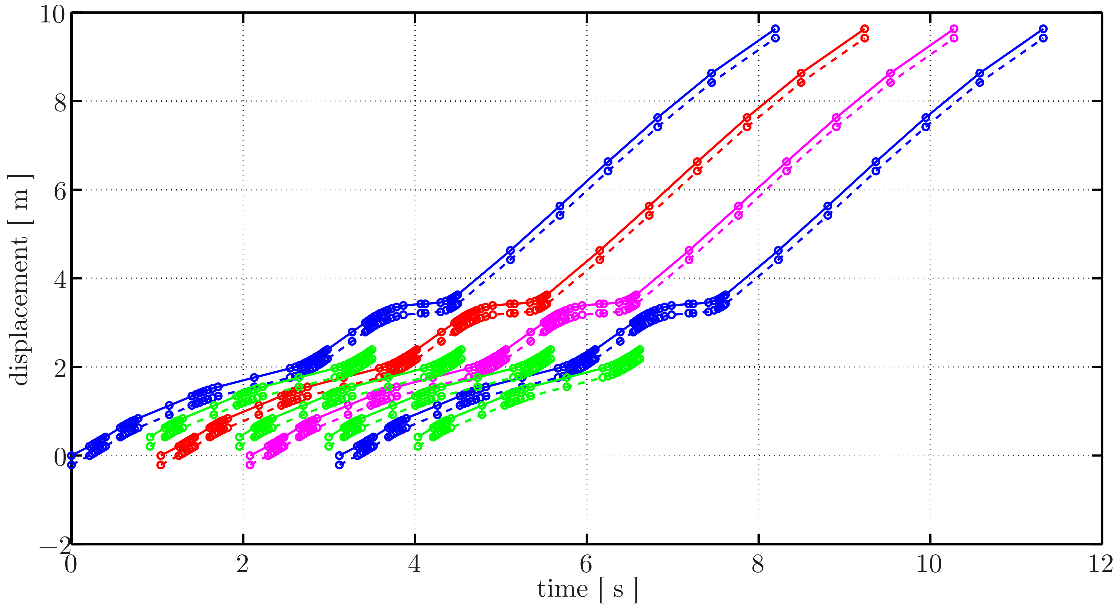

Figure 11: Displacement of sheets with cooling

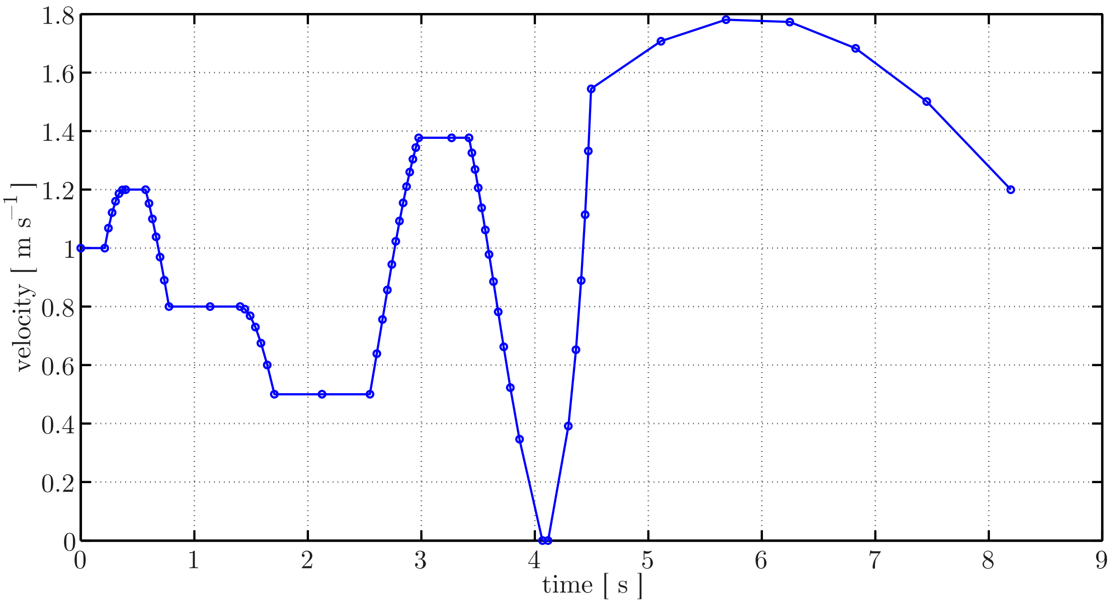

Figure 12: Velocity of sheets with cooling