Hygro-mechanical structure–property relations for paper sheets

HYGRO-MECHANICAL STRUCTURE–PROPERTY RELATIONS FOR PAPER SHEETS

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Summary Our objective is to establish models which allow one to predict the hygro-mechanical properties of paper sheets based on the properties of the underlying fibrous network. Asymptotic homogenisation is used to determine rigorous effective properties of random networks numerically. The trend predicted by these random networks is however captured quite reasonably by a much simpler analytical model which assumes a regular network of perpendicular fibres. The latter model is fully analytical and is thus a much more powerful tool for developing understanding and designing materials. Predictions made by it compare well with measured hygro-expansivities reported in the literature.

INTRODUCTION AND OBJECTIVES

Significant dimensional variations may occur in paper sheets when they are subjected to changes in moisture content. Gradients of the moisture content in the plane of the sheet, in particular, may result in instabilities and out-of plane deformation of the sheet, which are problematic in e.g. digital printing devices.

Moisture induced deformations are ultimately governed by the swelling of individual fibres, which is transferred through inter-fibre bonds within the fibrous network. The fibres' swelling is highly anisotropic and as a result complex interactions between mechanical and hygro-expansive properties take place in the bonding areas, affecting the overall material response. Matters are further complicated by the fact that the fibre orientation distribution is anisotropic as well and internal stresses may be 'dried in' during paper production [1].

Our objectives are (i) to understand how the hygro-mechanical properties of individual fibres, via the bonds and network, determine the sheet-level hygro-expansion of paper sheets and (ii) based on this understanding, to formulate structure–property relations which allow one to predict the sheet-level hygro-expansivity as a function of microstructural parameters such as single-fibre properties and fibre orientation distribution.

METHODOLOGY

We address the issue by developing microstructural, i.e. network-level, models which incorporate the relevant effects. We then study the overall response of these models and its dependence on the characteristics of the network and fibres. The models are so far of a two-dimensional nature and assume a perfect bonding between fibres where they overlap. In this contribution we focus on the reversible hygro-mechanical response – although progress has also been made on the issue of dried-in stress and strain.

Within this scope, we follow two approaches:

1. Unit cells comprising periodic, but otherwise random fibre networks are generated according to a certain orientation distribution, see Figure 1 (left) for an example. Their overall mechanical and hygroscopic properties are computed by rigorous, asymptotic homogenisation. This involves discretising the networks by finite elements and solving a set

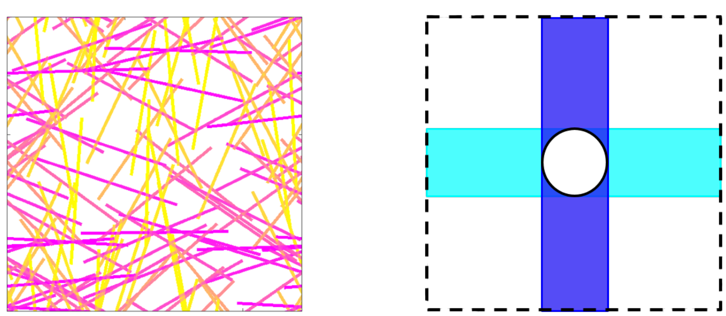

Figure 1: Sketches of the two types of microstructural models used: (left) unit cell with random fibre distribution and (right) idealised unit cell with two fibre families.

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of boundary value problems which deliver the network-level displacement fluctuations due to mechanical as well as hygroscopic loading. Equivalent, homogenised elastic moduli and hygro-expansivities can finally be computed from these fluctuation functions.

2. An extremely idealized model is considered, in which the fibrous network is assumed to consist of infinitely long fibres which are organised in a square, periodic pattern – see the sketch in Figure 1 (right) for a unit cell of this periodic pattern. Crucially, the model does distinguish between free-standing fibre parts and bonded areas. The anisotropy of the network is reflected in the fact that the 'fibre bundles' in the two directions have a different thickness. The ratio of these thicknesses is established by lumping the fibre orientation distribution into two perpendicular contributions. The model can be solved analytically, allowing one to determine its anisotropic hygro-mechanical response as an explicit function of its parameters [1, 2].

RESULTS AND CONCLUSIONS

Comparison of results obtained by the two models (not shown here) demonstrates that the analytical, highly idealised model qualitatively captures the trends predicted by the more detailed numerical model. In fact, a good quantitative agreement is also observed in the range of realistic parameters values.

Figure 2 shows a comparison of predictions made by the simple, analytical model with experimental data reported by Uesaka [3]. Shown are the hygro-expansive coefficients in machine direction and cross direction as a function of the degree of anisotropy of the fibre orientation distribution. The latter is characterised by the ratio of the elastic moduli in machine direction and cross direction – which were measured in the experiments and are obtained as a by-product of the model. The solid and dashed curves represent the model prediction for the two different paper grades considered in the experiments and the square and circular markers indicate the respective corresponding experimental data. The trends exhibited by the experimental data are captured quite accurately by the model – despite the fact that no effort was made to fit the model specifically to this data set. In the model, the dependence on the fibre network's anisotropy can be traced back to the fibre bonds, which are thus shown to play a crucial role.

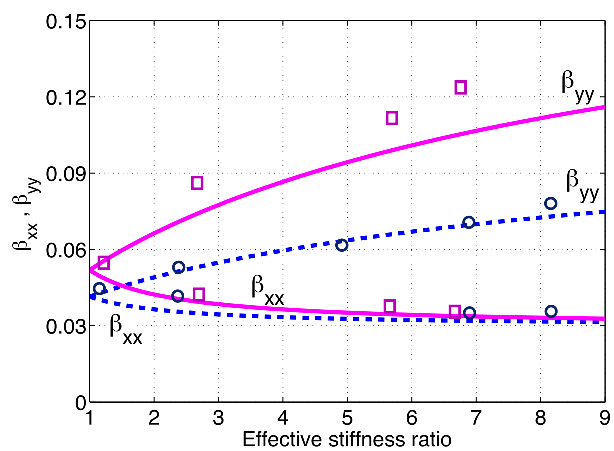

Figure 2: Hygro-expansive coefficients in machine direction, β_{xx}, and cross direction, β_{yy}, as a function of the degree of (mechanical) anisotropy. The curves represent the prediction of the simplified model for two paper grades; the markers are the experimental data for these grades as reported in [3].

References

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