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Large amplitude dynamic behavior of thrust air bearings: modeling and experiments

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Abstract

Large dynamic loading of air thrust bearings may result in undesired contact between bearing surfaces. The counteracting nonlinear aerodynamic forces delivered by the thin air film in the bearing, which determine if contact will occur, may be computed either by solving the nonlinear Reynolds equation or by using a grid with position dependent dynamic bearing coefficients. It is shown by a numerical case study that both numerical approaches give similar response results. Moreover, it is demonstrated that the model based on the nonlinear Reynolds equation accurately predicts the experimental large amplitude dynamic behavior of an axial-symmetric air thrust bearing. Hence, both numerical approaches may be used to predict if bearing surfaces will make contact due to large impulsive external forces.

Keywords: air thrust bearing, large dynamic loading; modeling and experiments; prediction of contact

1. Introduction

One of the key components in high precision systems are thrust air bearings. Their function is to enable a relative motion with low friction and no wear. In order to fulfill this function, it is vital that the two bearing surfaces do not touch each other. Even if a sudden impulsive force or an inertial force due to the acceleration of a component is exerted on the air bearing, the surfaces should not collide. Therefore, it is important to design air bearings such that the two bearing surfaces will never touch each other, even if the air bearing is subjected to specified classes of applied external forces.

Dynamic behavior of air bearings is often studied with a simulation model based on the linearization of the well known Reynolds equation [1]. These models are used to predict the load carrying capacity and the dynamic coefficients (stiffness and damping) of air bearings [2, 3, 4].

If the large displacement behavior of air bearings needs to be studied the linearized Reynolds equation is not applicable anymore and the nonlinear Reynolds equation needs to be solved. The instationary Reynolds equation is solved by a Finite Element Method in [5] to study the stability of thrust and journal air bearings. A similar study is carried out in [6] where a Finite Difference Method is used to study the whirl instability of journal bearings with porous surfaces.

In [7], the bifurcation and the nonlinear dynamic behavior of the rotor in a united gas-lubricated bearing are analyzed for several operating conditions. There the Reynolds equation is solved both by the Finite Difference Method in combination with successive over relaxation (SOR) and by a Differential Transformation Method in combination with a Finite Difference Method. A simplified calculation method to analyze the performance of an aerostatic thrust bearing with multiple pocketed orifice-type restrictors is proposed in [8]. A hybrid approach to analyze the airflow in an aerostatic thrust bearing is proposed in [9]. There the air flow in the region near the restrictor is calculated by solving the full Navier-Stokes equations and that in the outer region is calculated by solving the Reynolds equation.

The large displacement behavior of journal bearings is studied in [10] by using multiple sets of dynamic coefficients. The gap height displacement is determined using the dynamic coefficients obtained by a linearization around a reference gap height. If the gap height displacement becomes relatively large, the dynamic coefficients at the reference gap height are not valid anymore for the current gap height. Then, the dynamic coefficients belonging to a linearization around a gap height closer to the current gap height are used to proceed with the determination of the gap height displacement.

In [11], it was shown that a good agreement exists between the static load carrying capacity measured in experiments and those calculated with several simulation models for a porous axisymmetric thrust air bearing. Considering the arguments mentioned in the first paragraph, it is clear that if large changes in the air film gap occur due to high dynamic loading, experimental validation of the dynamic model predictions becomes even more important.

This work compares the predicted large displacement behavior of a flat single hole thrust air bearing obtained
Nomenclature

\begin{itemize}
  \item $A_{in}$ Area at inlet of air bearing gap (= $2\pi R_1 h$), m$^2$
  \item $A_{or}$ Cross-sectional area of orifice (= $\pi R_1^2$), m$^2$
  \item $C_{d,in}$ Discharge coefficient at inlet of area bearing gap, (dimensionless)
  \item $C_{d,or}$ Discharge coefficient of orifice, (dimensionless)
  \item $D$ Damping of air bearing, (Ns)/m
  \item $D_0$ Damping at frequency $\omega = 0$, (Ns)/m
  \item $D_\infty$ Damping at frequency $\omega = \infty$, (Ns)/m
  \item $D_Z$ Damping in standard linear solid model, (Ns)/m
  \item $F$ Force, N
  \item $\bar{F}$ Amplitude of impulsive excitation force, N
  \item $F_0$ Static air bearing force at bearing gap $h_0$, N
  \item $F_{ab}$ Force delivered by the air bearing, N
  \item $F_e$ Dynamic part of air bearing force, N
  \item $F_{ex}$ External force applied to air bearing, N
  \item $F_{ex, st}$ Static external force, N
  \item $F_s$ Static part of air bearing force, N
  \item $\bar{F}_e, \bar{F}_s$ Columns with discrete (non-) linear equations
  \item $g$ Gravitational acceleration, m/s$^2$
  \item $h$ Gap height in the air bearing, m
  \item $h_0$ Static or mean air bearing gap, m
  \item $K$ Pressure recovery factor, (dimensionless)
  \item $M$ Air bearing mass, kg
  \item $m_{gap}$ Mass flow into the air bearing gap, kg/s
  \item $m_{in}$ Mass flow from orifice to air bearing gap, kg/s
  \item $m_{or}$ Mass flow through the orifice, kg/s
  \item $p$ Pressure of the air in the bearing gap, Pa
  \item $p_a$ Ambient pressure, Pa
  \item $p_{in}$ Pressure at inlet air bearing gap, Pa
  \item $p_{or}$ Pressure of the air underneath the orifice, Pa
  \item $p_s$ Supply pressure, Pa
  \item $p_{th}$ Theoretical pressure at inlet air bearing gap, Pa
  \item $r$ Cylindrical radial coordinate, m
  \item $R_1$ Radius of orifice = inner radius air bearing, m
  \item $R_2$ Outer radius of the air bearing, m
  \item $Re$ Reynolds number (= $n_{in}/(\pi \mu R_1)$), (dimensionless)
  \item $\Re$ Real part operator, (dimensionless)
  \item $R_s$ Specific gas constant, J/(kgK)
  \item $S$ Stiffness of the air bearing, N/m
  \item $S_0$ Stiffness of air bearing at frequency $\omega = 0$, N/m
  \item $S_\infty$ Stiffness of air bearing at frequency $\omega = \infty$, N/m
  \item $S_Z$ Stiffness in standard linear solid model, N/m
  \item $T$ Temperature, K
  \item $t$ Time, s
  \item $t_{im}$ Time duration of applied impulse force, s
  \item $v$ Squeeze velocity in air bearing (= $dh/dt$), m/s
  \item $\bar{v}$ Column of unknowns
  \item $Z$ Impedance of the air bearing, N/m
  \item $\kappa$ Adiabatic expansion coefficient, (dimensionless)
  \item $\mu$ Dynamic viscosity of air, kg/(ms)
  \item $\omega$ Excitation frequency, rad/s
  \item $\nabla$ Gradient operator, 1/m
  \item $\nabla\cdot$ Divergence operator, 1/m
\end{itemize}

from the nonlinear Reynolds equation with the behavior obtained using multiple sets of dynamic coefficients that follow from the linearized Reynolds equation. Moreover, the air bearing behavior predicted by the nonlinear Reynolds equation is compared with the measured behavior in experiments for both a static and an impulsive external load. The orifice model proposed by Holster and Jacobs [12] has been used to determine the airflow through the orifice to the region in between the two bearing surfaces of the aerostatic thrust bearing.

This paper is organized as follows. In section 2, the mathematical models used in this study are elaborated. Section 3 compares the impulsive load response of a linear model, a nonlinear model, and a model that uses multiple interpolated coefficients. Section 4 describes the experimental setup used for the experimental validation and Section 5 compares the experimental and predicted response results for both static loads and impulsive loads. Finally, in section 6, the conclusions are drawn.

2. Mathematical models

In this section, the nonlinear lubrication model and the interpolated coefficients model are presented. First the governing nonlinear equations to model the transient behavior of a flat single hole thrust air bearing are discussed. Then the numerical method to solve these equations is given. Thereafter, the interpolated coefficients model is discussed.

2.1. Governing equations

Figure 1 shows schematically the air bearing under consideration. The air bearing is axial symmetric with a radius of $R_2$. The orifice has a radius of $R_1$. The gap height $h$ is assumed to be uniform along the bearing surface. Air is supplied through the supply air inlet with a known pressure $p_s$. The air is pressed through the orifice after which the air pressure drops to a yet unknown pressure $p_{or}$. Subsequently, the air flows through the air bearing gap to the ambient air at pressure $p_a$.

The pressure distribution in the fluid film of the air bearing is determined by the instationary Reynolds equa-
A Bernoulli equation is negligibly small. This latter mass flow follows from the orifice, because the volume of the bearing chamber is

\[ \text{mass flow into the air bearing gap at the orifice} \]

This mass flow is assumed to be the same as that through the orifice, because the volume of the bearing chamber is negligibly small. This latter mass flow follows from the Bernoulli equation

\[ \dot{m}_{\text{gap}} = \frac{\pi h^2 R_1 p}{6 \mu R_e T} \frac{\partial p}{\partial r} \bigg|_{R_1} . \]  

This mass flow is assumed to be the same as that through the orifice, because the volume of the bearing chamber is negligibly small. This latter mass flow follows from the Bernoulli equation

\[ \dot{m}_{\text{or}} = C_{d,or} \frac{A_{or}}{R_e T} \phi(p_s, p_{or}) , \]

with

\[ \phi(x, y) = \begin{cases} \frac{2 \kappa}{\kappa - 1} \left( \frac{2}{\kappa} \right)^{\frac{x}{\kappa} - \frac{\kappa + 1}{\kappa}} & \text{if } \frac{y}{x} \geq \left( \frac{2}{\kappa} \right)^{\frac{x}{\kappa} - \frac{\kappa + 1}{\kappa}} , \\ \frac{2 \kappa}{\kappa + 1} \left( \frac{2}{\kappa} \right)^{\frac{x}{\kappa} - \frac{\kappa + 1}{\kappa}} & \text{if } \frac{y}{x} < \left( \frac{2}{\kappa} \right)^{\frac{x}{\kappa} - \frac{\kappa + 1}{\kappa}} , \end{cases} \]

where \( C_{d,or} \) is the discharge coefficient of the orifice, \( A_{or} = \pi R_1^2 \) is the area of the orifice, \( R_e \) is the specific gas constant of air, \( T \) is the air temperature and \( \kappa \) is the adiabatic expansion coefficient [13]. Furthermore, it must be equal to the mass flow to underneath the orifice to the inlet of the air bearing gap

\[ \dot{m}_{\text{in}} = C_{d,\text{in}} \frac{A_{in}}{R_e T} \phi(p_{or}, p_{th}) , \]

where \( C_{d,\text{in}} \) is the discharge coefficient at the inlet of the air bearing gap, \( A_{in} = 2\pi R_1 h \) is the area of the inlet of the air bearing gap and \( p_{th} \) is the theoretical pressure value just after the inlet of the air bearing gap.

When the air has entered the air bearing gap a pressure recovery occurs where the pressure increases from \( p_{th} \) to \( p_{in} \). This pressure recovery is determined by the \( K \) factor

\[ K = \frac{p_{or} - p_{in}}{p_{or} - p_{th}} . \]  

The empirical relation between the Reynolds number Re and the \( K \) factor has been determined in [14]. This relation has also been used in [12] and reads

\[ K = 0.2 + 0.5(1 - \exp(-\text{Re}/1200))^2 , \]

where the Reynolds number is given by

\[ \text{Re} = \dot{m}_{\text{in}} / (\pi R_1 \mu) . \]

The balances of mass flows and the pressure recovery result in the following set of equations

\[ F_1 = \begin{bmatrix} \dot{m}_{\text{gap}}(p_{in}) - \dot{m}_{\text{in}}(p_{or}, p_{th}) \\ \dot{m}_{\text{gap}}(p_{in}) - \dot{m}_{\text{or}}(p_{or}) - K(\text{Re}) \end{bmatrix} = 0 \]

for the unknown pressures \( p_{or}, p_{th}, \) and \( p_{in} \). At the outer edge \( (r = R_2) \) the pressure in the air bearing equals the ambient pressure

\[ p = p_a . \]

The motion of the air bearing pad follows from Newton’s second law

\[ M \ddot{h} = F_{ab}(h, \dot{h}) - M g - F_{ex}(t) , \]

where \( M \) is the air bearing mass and \( g \) is the gravitational acceleration. Furthermore, \( \dot{\text{and}} \) \( \ddot{\text{denote the first and second time derivative respectively. The}} \)

\[ F_{ab} = 2\pi \int_{R_1}^{R_2} \left( r p(h, \dot{h}) \right) dr + \pi R_1^2 p_{or} - \pi R_2^2 p_a . \]

Note that the force exerted by the ambient air is taken into account. Equations [1], [9], [10], [11], and [12] determine the pressure in the air bearing.

2.2. Numerical solution method

An approximate solution of the governing equations in subsection 2.1 is obtained using the Finite Difference Method for the spatial derivative and a Crank-Nicolson method for the time integration. This results in the following set of discrete nonlinear equations that is solved with a Newton method. The set of nonlinear equations is written in the form

\[ F(x_{j+1}; x_j) = 0 , \]
with

\[
\mathbf{E}_j = \begin{bmatrix}
    h_j \\
    v_j \\
    p_{or,j} \\
    p_{n,j} \\
    p_{0,j} = p_{in,j} \\
    p_{1,j} \\
    \vdots \\
    p_{l,j}
\end{bmatrix},
\]  \quad (14)

where the subscript \( i \) denotes quantities at the discrete location \( r_i \) (\( i = 0, \ldots, I \)), the subscript \( j \) denotes quantities at the discrete time \( t_j \) (\( j = 0, \ldots, J \)), and \( v = \frac{dh}{dt} \). \( \mathbf{E}_4 \) contains the orifice model described in (9), \( \mathbf{E}_2 \) contains the equation of motion (11) in discretized form where (11) is rewritten into an equivalent set of first order equations

\[
\mathbf{E}_2 = \left[ \begin{array}{c}
    h \\
    v \\
    \end{array} \right]_{j+1} - \left[ \begin{array}{c}
    h \\
    v \\
    \end{array} \right]_j - \Delta t \left[ \begin{array}{c}
    \left( \frac{F_{ex}(p)-F_{ex}-Mg}{M} \right) \\
    \end{array} \right]_{j+\frac{1}{2}} = 0 , \quad (15)
\]

where the subscript \( j+\frac{1}{2} \) indicates that the corresponding quantity has to be evaluated in between the neighboring time steps, e.g. \( y_{j+\frac{1}{2}} = (y_{j+1} + y_j)/2 \). The discretized Reynolds equation (11) is described in \( \mathbf{E}_3 \) as

\[
\mathbf{E}_3 = p_{i,j+1}h_{j+1} - p_{i,j}h_{j} - \frac{\Delta t}{24\mu} \left( c_{1,i}p_{i-1}^2 - c_{2,i}p_{i}^2 + c_{3,i}p_{i+1}^2 \right)_{j+\frac{1}{2}} = 0 , \quad \quad (16)
\]

\[i = 1, \ldots, I - 1 .\]

where

\[
c_{1,i,j} = \frac{r_{i-1}h_{i+j}^2 + r_ih_{i+j}^2}{r_{i}(r_{i+1} - r_{i-1})(r_{i+1} - r_{i})} , \quad (17)
\]

\[
c_{3,i,j} = \frac{r_{i}(r_{i+1} - r_{i-1})(r_{i+1} - r_{i})}{r_i(r_{i+1} - r_{i-1})h_{i+j}^2 + r_{i+1}h_{i+j}^2} , \quad (18)
\]

\[
c_{2,i,j} = -(c_{1,i,j} + c_{3,i,j}) . \quad (19)
\]

Note that \( h \) is in this case uniform and therefore not depending on \( r_i \). The boundary condition on the outer edge of the air bearing is given by

\[
\mathbf{E}_4 = p_l - p_o = 0 . \quad (20)
\]

The transient gap height is obtained by solving the set of equations (15) using Newton’s method subsequently at the time steps \( t_j, j = 1, \ldots, J \).

The steady-state gap height follows from the steady-state form of the balance of forces (11), hence

\[
F_{ax}(h) = F_{ex,st} + Mg , \quad (21)
\]

where \( F_{ex,st} \) is a static external force. The steady-state pressure distribution is obtained from the set of equations \( \mathbf{E}_1 = 0 \), the modified equations \( \mathbf{E}_3 = 0 \) and \( \mathbf{E}_4 = 0 \). The modified equations \( \mathbf{E}_3 = 0 \) contain the discretization of the steady-state Reynolds equation

\[
F_{3,i} = c_{1,i}p_{i}^2 - c_{2,i}p_{i}^2 + c_{3,i}p_{i+1}^2 = 0 , \quad (22)
\]

\[i = 1, \ldots, I - 1 .\]

The force delivered by the air bearing \( F_{ab} \) is computed using (21). The unknown steady-state gap height \( h \) is solved from (21) using a secant method.

2.3. Interpolated coefficient model

Dynamic bearing coefficients of the air bearing are determined by a harmonic variation of the bearing gap \( h(t) = h_0 + \Re \{ h \exp(j\omega t) \} \) around a mean bearing gap \( h_0 \). The obtained stiffness \( S(\omega) \) and damping \( D(\omega) \) coefficients of the bearing depend on the excitation frequency \( \omega \) due to the compressibility of the air. At small excitation frequencies the stiffness and damping coefficients approach values \( S(\omega \rightarrow 0) = S_0 \) and \( D(\omega \rightarrow 0) = D_0 \), respectively. At high excitation frequencies they approach the asymptotic values \( S(\omega \rightarrow \infty) = S_\infty \) and \( D(\omega \rightarrow \infty) = 0 \), see e.g. (10).

A relatively simple model that describes the behavior of the air bearing under a specified transient load is the standard linear solid (SLS) model. This model consists of two springs and a damper, as schematically shown in Figure 2. It inhibits the stiffness \( S_\infty \) at high rates \( \frac{dh}{dt} \) of the change in bearing gap and a stiffness \( S_0 \) and damping \( D_0 \) at very small rates of changes in the bearing gap. From Figure 2 it follows that the relation between the force \( F(t) \) and the gap height \( h(t) \) is

\[
F + \frac{D_0}{S_\infty - S_0} \frac{dF}{dt} = S_0h + S_\infty \frac{D_0}{S_\infty - S_0} \frac{dh}{dt} . \quad (23)
\]

If \( F(t) \) is varied harmonically with a small amplitude at frequency \( \omega \) the bearing gap will also vary harmonically with frequency \( \omega \) as (23) is a linear differential equation in \( h \). Then the impedance \( Z = \frac{dF}{dh} = S + jD \) of the system is

\[
Z(\omega) = \frac{S_0 + j\omega S_\infty \frac{D_0}{S_\infty - S_0}}{1 + j\omega \frac{D_0}{S_\infty - S_0}} . \quad (24)
\]
Hence, with this proposed model the impedance [24] of the air bearing is approximated by a rational polynomial of first order [10].

The frequency dependent stiffness and damping coefficients belonging to the standard linear solid model follow from [23] as

\[ Z(\omega) = S_Z(\omega) + j\omega D_Z(\omega), \tag{25} \]

where the stiffness coefficient is

\[ S_Z(\omega) = \frac{S_0 + \omega^2 S_\infty}{1 + \omega^2} \left( \frac{D_0}{S_\infty - S_0} \right)^2 \tag{26} \]

and the damping coefficient is

\[ D_Z(\omega) = \frac{D_0}{1 + \omega^2} \left( \frac{D_0}{S_\infty - S_0} \right)^2 \tag{27} \]

The behavior of the air bearing is approximated by the standard linear solid model with the relation [23] between the load force \( F_{ab} \) and the bearing gap \( h \). The load carrying capacity can be seen as the superposition of a static force \( F_s \) and a dynamic force \( F_d \), hence

\[ F_{ab} = F_s(h) + F_d(v, h). \tag{28} \]

The static air bearing force \( F_s \) is given by

\[ F_s = F_0 + S_0 \cdot (h - h_0), \tag{29} \]

where \( F_0 \) is the static air bearing force at gap height \( h_0 \) that follows directly from the solution of the steady-state Reynolds equation. The relation between the dynamic force and the air bearing gap reads

\[ F_d + \frac{D_0}{S_\infty - S_0} \frac{dF_d}{dt} = S_\infty \frac{D_0}{S_\infty - S_0} \frac{dh}{dt}. \tag{30} \]

The standard linear solid model [28], [29], and [30] is used together with the [11] to predict the change in air bearing gap \( h(t) \) under a prescribed transient applied load force \( F_{\text{app}}(t) \). For small time intervals, the change in gap height will be small. However, for larger time intervals, the change in the air bearing gap is in general not small anymore, and the bearing coefficients calculated at a reference gap \( h_0 \) will not be valid anymore for newly calculated gaps \( h(t) \). Therefore, the reference gap height \( h_0 \) at which the dynamic bearing coefficients are determined to predict the change in gap height needs to be updated every time step.

The interpolated coefficient model in this paper uses the standard linear solid (SLS) model with the dynamic bearing coefficients \( S_0, D_0, \) and \( S_\infty \) calculated at a number of specified gap heights. These gap heights and bearing coefficients are stored in a database. The equation of motion [11] with [28] is solved using MATLAB Simulink. At each time step, a set of dynamic coefficients is retrieved by interpolating the computed values stored in the database.

3. Impulsive load response

In this section, the transient gap height response predicted by the nonlinear model, the interpolated coefficients model, and the linear model are compared. The following parameters are used in the computations: \( R_1 = 2 \cdot 10^{-4} \) m, \( R_2 = 4 \cdot 10^{-2} \) m, \( p_s = 2 \cdot 10^5 \) N/m², \( p_a = 10^5 \) N/m², \( T = 293 \) K, \( \mu = 1.805 \cdot 10^{-5} \) kg/(ms), \( R_s = 287 \) J/(kgK), \( \kappa = 1.405, M = 3.5 \) kg, \( g = 9.81 \) m/s², \( C_{d,\text{in}} = 0.9, C_{d,\text{orr}} = 0.8 \). The impulsive input force used in the three models is

\[ F = \hat{F} \frac{1}{2} \left( 1 + \sin \left( \frac{2\pi t}{t_{\text{im}} - \frac{\pi}{2}} \right) \right), \quad 0 \leq t \leq t_{\text{im}}, \tag{31} \]

where \( \hat{F} = 100 \) N and \( t_{\text{im}} = 0.01 \) s. Figure 3 shows the transient gap height response obtained by the three models.

In the nonlinear model the discretization parameters were \( I = J = 1000 \) with a simulation time range of 0.035 seconds. For the interpolated coefficient model the same time discretization is used. Further, the dynamic coefficients are computed for gap heights 2, 3, 4, ..., 50 µm for frequencies 1 Hz and 5 \cdot 10^5 Hz, which approximate the static (\( \omega \to 0 \) to compute \( S_0 \) and \( D_0 \)) and infinite high frequency (\( \omega \to \infty \) to compute \( S_\infty \)) response. These dynamic bearing coefficients were used in the interpolated coefficient model [23] described in the previous section to predict the dynamic air bearing behavior for large bearing gap changes. Since the gap height response for the interpolated coefficients model varies in the range \( 18 - 46 \) µm, interpolation between coefficients is extensively carried out in the result shown in Figure 3. In the results of the linear model, only the bearing coefficients at the initial steady-state gap height for a static load of \( Mg = 34.3 \) N is used.

In Figure 3 it is seen that the decrease in gap height predicted by the linear model is much larger than the decrease predicted by both the nonlinear and the interpolated coefficients model. The linear model predicts that the two bearing surfaces will collide (obviously, negative gap heights cannot occur in reality), in contrast to the predictions of both the nonlinear model and the interpolated coefficients model. Furthermore, the latter two models predict that the gap height recovers faster to the initial gap height. The gap heights obtained by the nonlinear and the interpolated coefficients model nearly coincide. The largest difference in predicted gap height occurs between 0.007 and 0.017 seconds, which is probably caused by the approximation of the air bearing performance by the standard linear solid model.

4. Experimental setup

Figure 4 schematically shows the used experimental setup which is depicted in Figure 5. A force transducer is mounted on a large steel block. On top of the force transducer an air bearing pad is mounted with the orifice side
of the bearing facing upward. The air bearing has a radius of $R_2 = 30 \text{ mm}$ and an orifice radius of $R_1 = 0.25 \text{ mm}$. On top of the air bearing pad an air bearing counter surface is placed. This counter surface is mounted to an air lubricated pivot to improve the parallel alignment between the two bearing surfaces of the investigated air bearing. The pivot is connected to an air lubricated linear guide. The linear guide also contains a pressure chamber that makes it possible to apply an adjustable (static) external force on the air bearing.

The average value of two Lion C7-C capacitive displacement sensors is used to measure the air bearing gap. The zero gap height point of the displacement sensors is determined by pressing the two bearing surfaces onto each other using the pressure chamber force without pressurizing the air bearing itself. The supply pressure was measured with a Fluke 700PD7 pressure module and the mass flow through the orifice was measured with an Alicat Scientific M-10SLPM-D mass flow meter. The measurements were conducted in a room at constant temperature of 294 K.

5. Experimental results

In this section, the responses belonging to static and impulsive loads obtained by the nonlinear model described in subsections 2.1 - 2.2 are compared with those measured in the experiments.

5.1. Static external force

Measurements of the static load carrying capacity were performed by applying a static force using the pressure chamber in the linear guide. The applied force was measured by the force transducer and the according gap height was measured with the capacitive displacement sensors.
During the measurements, a small tilt was observed between the two bearing surfaces. This tilt was probably caused by irregularities in the roughness and profile of the bearing surfaces, a small misalignment between the orifice and the counter surface, and a not perfectly perpendicular application of the piston force on the counter surface.

The numerical results were obtained using the same parameter values as given in section 3. However, now with an air bearing that had dimensions $R_1 = 0.25$ mm, $R_2 = 30$ mm, and a supply pressure of $P_s = 5$ bar. As the dimensions of the bearing were similar to those in [12], the values for the discharge coefficients were assumed to be the same as those used in that investigation, hence $C_{d,or} = 0.8$ and $C_{d,in} = 0.9$. The measured gap heights were smaller than those obtained by the nonlinear model. This deviation was probably caused by the roughness of the air bearing surfaces, a not modeled deformation in the experimental setup, and the observed tilt between the two bearing surfaces. Figure 6 shows the roughness of the roughest air bearing surface. This dominating surface roughness has a standard deviation of $\sigma = 0.37 \mu$m. The counter surface of the air bearing was much smoother. In order to compare the trends in gap heights obtained from the experiments and the nonlinear model, the non modeled elastic deformation in the experimental setup is measured. Based on the estimates for the surface roughness and the elastic deformation, the experimental data is shifted $5 \mu$m to the right.

Figure 7 shows the calculated and adjusted measured load capacities for a supply pressure of 5 bar. The calculated and adjusted measured load capacities coincide for large gap heights. For smaller gap heights, up to 16 $\mu$m, the obtained load capacities do not coincide and the maximum predicted static load capacity is not reached in the experiments. This might be an effect due to the observed tilt between the two bearing surfaces. The tilt causes the air to flow more easily from the orifice to ambient through the region with the largest bearing gap. Therefore, the real
pressure distribution in the air between the bearing surfaces will not be axial symmetric as assumed in our simulation model and furthermore, on average it will be smaller than the one calculated with the simulation model. Hence, because of the observed tilt, the measured load carrying capacity will be smaller than the one calculated with our simulation model.

Figure 8 shows the calculated and measured mass flow through the orifice as a function of the (adjusted) gap height. The numerical and experimental results correspond well. The results obtained by the simulation model might be improved by using a measured discharge coefficient instead of the assumed one described in [12]. Furthermore, the effect of the bearing roughness and tilt on the mass flow is unknown and might affect the mass flow.

The results presented here are for a supply pressure of 5 bar. Similar behavior was seen for supply pressures of 4 bar and 6 bar.

5.2. Impulsive external force

The air bearing was brought into an initial steady-state with a supply pressure of \( p_s = 4 \) bar. Then an "impulsive" load, generated with a modal hammer, was applied to the air bearing, see Figure 4, and the resulting gap height behavior was measured using the capacitive displacement sensors. The input force was measured with the force transducer in the tip of the modal hammer. The measured input force was used as applied transient external force in the nonlinear simulation model described in section 2.2. The same parameter values as in section 5.1 were used, except for the supply pressure which was changed to \( p_s = 4 \) bar.

Figure 9 shows the predicted gap height behavior when the bearing is subjected to the impulsive force. Herein, the gap height difference compared to the steady-state gap height is presented. The air bearing mass is equal to \( M = 3.6 \) kg and \( p_s = 4 \) bar.

![Figure 9: Modeled and measured gap height variation with respect to the static gap height for an impulsive input force with \( M = 3.6 \) kg and \( p_s = 4 \) bar.](image)

Figure 10: Modeled and measured gap height variation with respect to the static gap height for an impulsive input force with \( M = 4.45 \) kg and \( p_s = 4 \) bar.

3.5 kg. The predicted initial response, which is important to determine if contact between the bearing surfaces occurs, agrees very well with the measured response. On a larger time scale, the behavior of the predicted and measured gap heights starts to deviate. As can be seen the gap height first reduces by about 5 \( \mu m \) when the impulsive external force is applied. Then due to the dynamics of the system the gap height starts to oscillate in time. As the air bearing has damping, the amplitude of the vibrational motion of the oscillating bearing decreases in time. Investigation of the dominant frequency components of the measured signal in this area, indicates that the measured harmonics for larger times are caused by vibrations of the fixtures holding the displacement sensors. These fixtures are visible in Figure 5. Similar results are seen in Figure 10 where gap height differences obtained from simulations and experiments are shown for an air bearing with mass \( M = 4.45 \) kg.

The frequency of the initial response for the air bearing with \( M = 4.45 \) kg (Figure 10) is slightly larger than the frequency of the initial response of the air bearing with \( M = 3.6 \) kg (Figure 9). Although the mass has increased, the steady-state gap height has decreased and due to the nonlinear behavior of the air bearing the stiffness has increased. This relative increase in stiffness is larger than the relative increase in mass resulting in a higher response frequency for the \( M = 4.45 \) kg case. The frequency of the response for larger times is not affected by the change in mass because it is originating from the fixtures holding the displacement sensors.

6. Conclusions

The gap height response to an impulsive force obtained by a nonlinear model, interpolated coefficient model and linear model have been compared. The linear model over-
estimates the change in gap height. The change in gap height predicted by the nonlinear model and interpolated coefficient model agree very well.

The gap height response predicted by the nonlinear model has been compared with the response measured in experiments for a static and an impulsive load. In both cases, the predicted and measured gap height behaviors agree very well. Hence, both the nonlinear and interpolated coefficients model can be used very well to accurately predict the performance of air bearings. In particular, both models can be used to predict if contact will occur between the bearing surfaces due to large impulsive forces on the air bearing. Finally, the good correspondence between experimental and simulation results also implicitly validates the model of Holster and Jacobs [12] used for the pressure drop across the orifice under dynamic loading conditions.

References


