Spectral element model for 2-D electrostatic fields in a linear synchronous motor

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Abstract—This paper presents a fast and accurate 2-D spectral element model for analyzing electric field distributions in linear synchronous motors. The electric field distribution is derived using the electric scalar potential for static cases. The spatial potential and electric field distributions obtained by the spectral element method are validated by finite element analysis. Protrusions, voids and rounded edges in the motor geometry are taken into account without loss of accuracy. Furthermore, the convergence and computational load of the spectral element model is investigated.

I. INTRODUCTION

Increasing demands of the throughput of high-precision positioning systems require more powerful actuators. Higher accelerations and velocities result in higher operating voltages of the actuators. As a result, high electric field strengths are present in the insulation system of the motor. Cavities and protrusions in the motor geometry caused by mounting holes, surface irregularities or corners lead to local electric field enhancement and, subsequently, to partial discharge or dielectric breakdown at a relative low operating voltage [1], [2].

Modeling of the electric field distribution in electrical machines is often performed by finite element or boundary element methods [3]–[7]. These methods, however, are computationally intensive compared to (semi-)analytical techniques such as harmonic modeling [8], conformal or Schwarz-Christoffel mappings [9], [10] or a combination of these models [11]. On the other hand, these analytical methods are not suitable for the modeling of electric field distributions accurately in complex geometries. Harmonic models only apply to either Cartesian or cylindrical coordinate systems. Therefore, curvatures or circular regions cannot be taken into account in rectangular domains. Conformal mappings allow curvatures to be taken into account but only for simple geometries without loss of accuracy [12]. In other engineering fields, the spectral element method has shown, for relative simple geometries, its lower computational cost compared to the finite element method while maintaining high accuracy [13]–[16]. Furthermore, this method allows the extension to nonsquared elements and, therefore, to the modeling of complex geometries [17], [18].

In this paper, the potential and electric field distributions in a linear synchronous motor are obtained by the Spectral Element Method (SEM). The electrostatic field solution is derived using the Galerkin formulation where Legendre-Gauss-Lobatto (LGL) polynomials are applied as basis functions. Curvatures and nonsquared regions, such as rounded corners and individual coil strands, are included in a Cartesian coordinate system by the transfinite interpolation method. This allows electric-failure related causes such as cavities in the insulation system to be investigated by the spectral element model.

II. GEOMETRY

The benchmark topology comprises a periodic section of a three-phase, iron-core, synchronous linear motor with concentrated phase windings and surface mounted permanent magnets, as shown in Fig. 1. The iron core is often, for safety reasons, connected to protective earth as is the back plate of the permanent magnet array. High electric field strengths are expected in the area with wire strands close to the iron core. Furthermore, protrusions such as cavities and corners lead to large electric field enhancement and are often strongly localized in space. Therefore, the area near a tooth of the iron core is investigated as shown in Fig. 2. This section consists of 4 coil strands with individual wire insulation, embedded in an insulating material. A rounding radius, $r_{cor}$, is applied to the sharp-angled corner of the tooth since right-angled corners physically do not exist and otherwise would lead to infinitely high electric field strengths. Furthermore, path $\beta$ is located in between coil strand $C_{s_0}$ and $C_{s_1}$, starting at the wall of the tooth and continues through the center of the voids. This path is used in the verification of the spectral element model. Material properties and dimensions of the investigated linear motor are shown in Table I.

III. SPECTRAL ELEMENT METHOD

In geometries without discontinuities, harmonic modeling techniques, which are derived using the separation of variables,
have an extremely fast convergence rate towards the solution, this property is known as a spectral accuracy [4]. However, harmonic modeling has some strict limitations in terms of applying different types of boundary conditions and modeling non-periodic geometries, in these cases the method loses its spectral properties [17]. The principle of separation of variables is extended in spectral element method, so that a sum of basis functions are used to approximate the solution of a partial differential equation. In order to have the flexibility of applying the Dirichlet or Neumann boundary conditions on a spectral element (or domain), polynomial basis are selected. Therefore, multiple spectral elements can be coupled together to solve a subdomain problem.

A. Elliptic equation for the electrostatic field

The potential, \( \varphi \), and the electric field strength, \( \vec{E} \), distributions can be derived from Maxwell’s equations, where the time derivatives are assumed to be zero. In general, the electrostatic equation can be written as

\[
\nabla \cdot (\epsilon \nabla \varphi) = -\rho, \tag{1}
\]

consequently, the electrical field is given by

\[
\vec{E} = -\nabla \varphi, \tag{2}
\]

where \( \rho \) is the electric charge density and \( \epsilon \) the permittivity of the material. As it can be seen, (1) is an elliptic partial differential equation, or in case when the permittivity of the material is isotropic and homogeneous the Poisson equation is obtained.

B. Galerkin formulation on a spectral element

In order to couple elements together in a complex geometry, but also to impose boundary conditions on the edges of the elements. The Legendre-Gauss-Lobatto (LGL) basis functions are used and given by,

\[
L_{GL_i}(x) = L_{i+1}(x) - L_{i-1}(x), \tag{3}
\]

where \( L_i(x) \) is the Legendre polynomials along the \( x \)-axis and defined as,

\[
L_{i+1}(x) = \frac{2i + 1}{i + 1} x L_i(x) - \frac{i}{i + 1} x L_{i-1}(x), \tag{4}
\]

where the first and second basis are \( L_0 = 1 \) and \( L_1 = x \), resulting in a three-term recursion. The solution from (1) can be approximated with the sum of basis functions from (4). The polynomial basis functions are evaluated at their roots, as indicated in Fig. 3a, in a \([-1, 1]\) range using Lagrange interpolation basis functions [19]

\[
\varphi(x) = \sum_{j=0}^{N} \varphi_j l_j(x), \tag{5}
\]

where \( \varphi_j \) is the solution on the roots and \( l_j \) is the Lagrangian polynomial basis

\[
l_j(x) = \prod_{i=0 \atop i \neq j}^{N} \frac{x - x_i}{x_j - x_i}, \tag{6}
\]

where \( x_i \) and \( x_j \) are the roots of the LGL basis functions, defined in (3). On the numerator of (6) a factored polynomial is obtained containing the roots of LGL basis. The coefficients in the denominator of (6) are also known as the barycentric weights, \( \omega_j \), and are given by

\[
\omega_j = \prod_{i \neq j}^{N} \frac{1}{x_j - x_i}. \tag{7}
\]

These barycentric weights are independent of the solution and, therefore, are computed in advance to reduce the computational load of the spectral element model [20]. In the same manner, using the barycentric formulation, the derivative and the quadratures on the Lagrange interpolation polynomial roots are obtained.

The formulation of the spectral element method in nodal form is more convenient, because the similar Galerkin method applied are widely employed in finite element models. Therefore, the numerical tools for it are already implemented and verified for their effectiveness.
a smooth function defined in (1). Equation (1) is multiplied by the electrostatic equation. To obtain the weak form of the electrostatic formulation is used which results in the weak form of the electrostatic equation for mapped interior elements becomes, 

\[
\left[ M \right] \left[ \phi \right] = \left[ F \right]
\]

The weak form of the electrostatic equation is given by,

\[
\int_{-1}^{1} \int_{-1}^{1} \left( \rho - \nabla \cdot (\epsilon \nabla \phi) \right) \phi \, d\xi \, d\eta = 0.
\]

For a more generic representation, (8) should be multiplied with a weight function appropriate to the used approximation. Since in this paper LGL nodes are used, this weight function is unity [21]. Applying the product rule to (8) and using the first Green identity, it is rewritten as,

\[
\int_{\Gamma} \epsilon \nabla \phi \cdot \hat{n} \, d\Gamma - \int_{-1}^{1} \left( \int_{-1}^{1} \nabla \gamma \cdot (\epsilon \nabla \phi) \, d\xi \, d\eta \right) = \int_{-1}^{1} \int_{-1}^{1} \rho \gamma \, d\xi \, d\eta.
\]

Equation (9) holds for a square domain \((\xi, \eta)\) from Fig. 3b and is flexible in terms of using it for applying the boundary conditions but also to couple the different elements. The main reason for this flexibility is that the circular integral on the left hand side of (9) gives direct access to the potential on the boundary. The circular integral in (9) is the Gauss’s law in integral form. Therefore, the inward and outward flux of a single element can be forced to any value.

In order to perform the calculation in a nonsquared geometry, a computational domain like the one from Fig. 3c with curved boundaries is used. For curved boundaries, the mapping, \(\tilde{M}\), of the domain edges is performed using the transfinite interpolation

\[
\tilde{M}(\xi, \eta) = (1 - \eta)\tilde{M}_1(\xi) + \eta\tilde{M}_3(\xi) + (1 - \xi)\tilde{M}_4(\eta) + \xi\tilde{M}_2(\eta) - \xi\eta\tilde{M}_3(1) + \xi(1 - \eta)\tilde{M}_1(1) + \eta(1 - \xi)\tilde{M}_3(0) + (1 - \xi)(1 - \eta)\tilde{M}_1(0)
\]

where \(\tilde{M}_1-4\) are the boundary curves composed by \(\Gamma_x\) and \(\Gamma_y\) component in a physical \((x, y)\) coordinate system. To solve the elliptic problem under mapping, the contravariant base vectors are introduced, they are perpendicular to \(\xi\) and \(\eta\) axes respectively [22]. Subsequently, the weak form of the electrostatic equation for mapped interior elements becomes,

\[
- \int_{-1}^{1} \int_{-1}^{1} \left( F^1 \frac{\partial \gamma}{\partial \xi} + F^2 \frac{\partial \gamma}{\partial \eta} \right) \, d\xi \, d\eta = \int_{-1}^{1} \int_{-1}^{1} \rho \gamma J \, d\xi \, d\eta,
\]

where \(J\) is the Jacobian of the transformation and \(F^1\) and \(F^2\) are the contravariant fluxes that correspond to \(\xi\) and \(\eta\) axis, respectively, they perform the differential operations under mapping. For a homogeneous permittivity of a region \(\epsilon_r\), the contravariant fluxes result in a Laplacian [18].

C. The performance of the Spectral Element Method

In order to model the geometry shown in Fig. 1 with SEM, the geometry is divided into smaller spectral elements. For convenience, each element contains the same material property or smooth source distribution. In Fig. 4 the division of the geometry into elements and nodes is shown. Any discontinuities in the spatial quantities, such as materials and sources, will lower the convergence of the solution. To build the global
Fig. 5. Nonzero elements in the global matrix for the degree of the polynomial basis, $N$, equal to 6.

matrices for all elements from the geometry, the matrix elements for each spectral element are placed in an appropriate order so that shared lines and points from the geometry are also coupled in the global matrix. In the Fig. 5 the nonzero elements of SEM linear system is shown. On the diagonal, the blocks matrices corresponding to the interior LGL nodes are placed followed by the shared lines and points. For a small number of spectral elements and with a higher order polynomial, the matrix will become dense with less nonzero matrix elements. Therefore, in order to keep the desired sparsity of the system the number of basis functions and the number of elements is balanced.

In the Fig. 6 the convergence of the SEM solution compared with an accurate solution estimated by finite element analysis (FEA) is shown. The accurate solution is obtained from a converged FEA mesh simulation, i.e., doubling the mesh density does not result in a change of the electric potential more than 0.1% . The computation points marked with "o" correspond to the maximum absolute error of the solution computed on path $\beta$, depicted in Fig. 2. With each computation point the degree of polynomial basis, $N$, is increased so the results are shown for $N$ is equal to 2, 4, 6, 8 and 10, respectively. From Fig. 6 it can be seen that with the increase of the unknowns number the solution on SEM nodes is converging exponentially. However, the exponential convergence for SEM does not always hold. For instance, when an element with sharp corner is surrounded by elements with different material properties the singularities will occur in the corners. The singularities will cause the derivatives of the solution to jump to infinity limiting the convergence in this case to linear [19]. For the geometry in this paper, $N = 6$ is chosen to limit the absolute error to 3 Volts.

IV. VERIFICATION WITH FINITE ELEMENT ANALYSIS

For the investigated synchronous linear motor it is assumed that the electric potential at the edges of the core, back plate and permanent magnet is zero. Furthermore, the electric potential at the circumference of coil strand $CS_a$ to $CS_d$ is equal to 1000, 950, 750 and 700 V, respectively. In total, the geometry is divided into 43 regions, 106 lines and 60 corners with each region having $N = 6 \times 6$, resulting in a total of 1665 unknowns. The roots distribution for a LGL polynomial of the 6th degree is shown in Fig. 4.

To validate the results of the spectral element model, the topology shown in Fig. 2 is modeled with 2-D finite element method using Cedrat FLUX2D [23]. The mesh of the finite element analysis near coil strand $CS_a$ and $CS_b$ is shown in Fig. 7. The mesh consists of 17226 triangular second-order elements.

Fig. 7. Mesh of the finite element method near the wall of the tooth and strands $CS_a$ and $CS_b$. 

Fig. 6. Comparison between SEM and FEM of the electric potential along path $\beta$. 
elements and 35559 nodes. Thus, the finite element model requires over 10 times the number of nodes compared to SEM.

A. Geometry without voids

The investigated geometry, presented in Fig. 2, is first assessed without the presence of voids. Therefore, the relative permittivity of the voids are equal to that of the surrounding insulation material, \( \varepsilon_{\text{ins}} \). The potential distribution obtained by the spectral element method is shown in Fig. 8 whereas the potential distribution obtained by the finite element analysis (FEA) is shown in Fig. 9. From these figures it is clear that the results from SEM, globally, correspond with those of the finite element analysis. A more detailed analysis of the discrepancy between SEM and FEA is performed on path \( \beta \), starting at the wall of the tooth and continues through the center of the voids parallel to \( x \)-axis. The modulus of the electric field strength, obtained by SEM, for path \( \beta \) is shown in Fig. 10a. The discrepancy between SEM and FEA on this path is shown, on a logarithmic scale, in Fig. 10b. The average discrepancy between SEM and FEA is 1.5 % whereas a peak discrepancy of 11.1 % is observed. The largest discrepancy occurs near \( x \) is equal to 1.5 mm where low electric field strengths are present. This discrepancy is caused by the relative large width of the segment, resulting in a locally low resolution of the roots distribution and hence lower spatial accuracy.

The electric field strength on path \( \zeta \), near the rounded corner of the tooth, is investigated and an average discrepancy of 3.6 % between SEM and FEM is observed.

B. Geometry with voids

The geometry of Fig. 2 is investigated with the inclusion of air-filled voids. Therefore, the relative permittivity of the voids is equal to that of air, \( \varepsilon_{\text{air}} \). The modulus of the electric field strength, obtained by SEM, for path \( \beta \) is shown in Fig. 11a. The discrepancy between SEM and FEA on this path is shown, on a logarithmic scale, in Fig. 11b. The average discrepancy between SEM and FEA is 1.5 % while a peak discrepancy of 10.1 % is observed. The cause of these discrepancies is equal to those of Section IV-A and applies also on geometries with void inclusions. However, the accuracy of the spectral element model can be improved, without a significant increase in computational cost, by dividing elements with large widths into multiple elements. In this case, the discrepancy occurs in a region of with low electric field strengths and is, therefore, of little interest.

V. Conclusion

In this paper, 2-D potential and electric field distributions in a linear synchronous machine are obtained by a model based on the spectral element method. Circular elements and
curvatures such as individual wire strands and corners are taken into account by the transfinite interpolation method. The potential and electric field strength distribution obtained by the spectral element method is compared to results from the finite element analysis with on average 1.5 % discrepancy. Voids, protrusions and rounded corners in the geometry of the linear synchronous motor are successfully modeled without loss of accuracy. Furthermore, the computational load is significantly lower since the computational matrix for the spectral method is relatively sparse and over 10 times smaller compared to one from the finite element analysis. Therefore, the presented model is suitable in optimization procedures requiring fast calculation of electric field distributions in motor geometries or the rapid evaluation of electric-failure related causes in electric machines.

REFERENCES