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The expected outdating versus the fill rate for perishable products in a periodic review inventory system

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Abstract

In this paper we show how the amount of waste in an inventory system with perishable products, periodic review, positive lead time and fixed case pack sizes is affected by the setting of safety stocks. Demand is discrete and stochastic. Demand which is not met from inventory on hand will be lost. We derive approximations for the expected amount of waste as a function of the safety stock and the product and demand characteristics. We first derive approximations in an analytical way and then use regression to improve these approximations. The final approximation is very fast and performs very well: the mean and standard deviation of the approximation error are equal to -0.06% and 0.78%. Moreover, the approximation is applicable in situations with parameter settings which are frequently encountered in practice, and for which no other approximations have been derived in the literature before.

1. Introduction

Retailers are very keen on reducing the amount of outdated perishable products. Perishable products constitute a large part of the total turnover and profit margin of grocery retailers. Waste is seen by them as a loss which not only includes the raw material, but also the costs which have been added in the supply chain such as transportation and labor costs. Furthermore, there is an increasing awareness in society that waste implies a loss of natural resources and should be avoided as much as possible. Finally, retailers may gain market share if they are able to offer fresher products, higher product availability and larger assortments. For example, [1] reports that for some fresh product categories in retail the number of products offered to customers has increased 100% in five years. To avoid large amounts of waste due to increased assortments, advanced inventory management is needed. A key issue in this inventory management is to find the optimal balance between the amount of outdating and the product availability. To this end, we study two relationships between key performance indicators and system parameters for an inventory system with a perishable product. The first relationship shows how the safety stock depends on the system parameters as well as the target product availability. This relationship is studied in [2]. The second relationship shows how the amount of outdating depends on the product availability and the system parameters, such as product and store characteristics. This relationship is studied in this paper.

The first relationship can be used on a tactical/operational level. It facilitates the implementation of an efficient inventory replenishment policy for perishable products, by supporting the setting of the safety stock as a function of the system parameters and the target service level. It can also be used for retailers who aim to reduce average inventory levels, e.g. to free up capital. The relationship informs them how a redesign of the system, leading to changes in the system parameters, influences the amount of safety stock needed.

The second relationship can be used on a strategic and tactical level. Retailers typically impose a budget for the maximum amount of outdating on store managers. The relationship developed in this paper may be used to support the determination of realistic budgets, while taking into account the characteristics of the environment as well as the implications for the maximum product availability which can be obtained given a maximum amount of outdating. The relationship can also be used to support discussions between retail headquarters and suppliers or store managers on how to change the characteristics of the environment in order to reduce outdating while maintaining the current product availability or to increase the sales while maintaining the current outdating. The relationship supports these discussions by quantifying the consequences of the proposed changes. Examples of these proposed changes are increasing the shelf life through innovation in packaging, increasing the delivery frequency, reducing the lead times, or reducing the demand uncertainty. Finally, the relationship can also be used to evaluate the effectiveness of the store manager's inventory management, by comparing among a set of similar stores the ratio between the actual outdating and the outdating based on the approximation developed in this paper.

Very few independent reports are available which quantify the amount of outdating in retail. The USDA [4] estimates that annual food losses in supermarkets in 2005 and 2006 averaged 11.4 percent for fresh fruit, 9.7 percent for fresh vegetables and 4.5 percent for fresh meat, poultry, and sea food. Since the annual sales of perishables in US supermarkets in 2008 were $222 billion [5], this implies annual food losses between $10 and $20 billion in US supermarkets only. To assist retailers in the reduction of outdating, we develop in this paper approximations for outdating, which show how the amount of outdating depends on the system parameters.

The structure of this paper is as follows. In section 2 the literature is reviewed, while in section 3 the model assumptions and notation are introduced. The first approximations are derived analytically in section 4 and tested by means of simulation in section 5. Based on the first approximations and the simulation results a final approximation is derived using linear regression in section 6. Section 7 discusses the managerial implications and section 8 gives the conclusions of this paper.

2. Literature review

Perishable inventory systems with stochastic demand and a fixed lifetime are first studied by Van Zyl [11]. For a perishable item with fixed lifetime equal to m periods, incurring outdating costs for units of inventory available in the system at the end of their lifetime, the optimal replenishment policy in general depends on a $(m-1)$-dimensional vector, which describes the age distribution of the inventory in the system (see [12] and [13]). This makes the computation of optimal policies very complex for large values of m [6]. To deal with this complexity, authors typically have chosen either to simplify the system, e.g. by assuming $m=2$ periods (see

[11], [14] and [15]) or to consider simple replenishment policies like the single critical number policy (see [16], [17] and [18]). All these papers assume the withdrawal policy is first in first out (FIFO), the lead time is zero, there is no lot sizing and the demand is stationary. The single critical number policy does not use any information about the age distribution of the inventory. More recently, several papers have been published on replenishment policies, which take into account partial or full information about the age distribution of the inventory [19], [20], [21], [10], [7] and [22]. These replenishment policies need more detailed information, but outperform the single critical number policies.

The papers which derive explicit expressions for the expected outdating as a function of the system parameters often make severe simplifying assumptions, since the exact analysis of perishable systems is extremely hard. Most papers assume either a case pack size equal to one, zero lead time, Poisson demand, continuous review and/or a shelf life equal to two periods. These papers include [23], [24], [25], [26], [27], [28], [29], [30] and [31]. In this paper we derive an approximation for the expected outdating for the more general case with a shelf life varying from 2 days to 10 days, fixed case pack sizes, a lead time and review period equal to one or two days, and a wide set of demand distributions. After deriving two analytical expressions to approximate the expected outdating, we improve the quality of these approximations using regression. This approach is based on the methodology introduced by Ehrhardt in [32] and [33], who used regression to approximate the parameters in a non-perishable inventory system using cost parameters. The same methodology has also been applied in [34] for an inventory system with non-perishable products using a service level criterion.

3. Model assumptions and notation

We study a single echelon perishable inventory system having a positive lead time L, a review period R, a fixed case pack size Q and stochastic discrete demand per day with mean μ and variance σ^2. Demand which is not met from stock is lost. The perishable product has a fixed lifetime equal to m days, defined as the remaining shelf life when the item arrives in the store. The shelves are assumed to have ample capacity.

To limit the amount of outdating, retailers aim to offer the products in such a way, that the oldest products are sold first; this is the so called first in first out (FIFO) withdrawal policy. Inventory replenishment is done periodically and in small batches ([3], [6]). In the model, the sequence of events during a day is as follows: first demand is subtracted from the inventory during the day, at the end of the day outdated inventory is removed, remaining inventory is counted, and performance measures such as the service level are calculated, goods arrive and are stacked on the shelves, and finally the orders are placed.

The EWA-replenishment policy, introduced in [7], is used to determine the timing and quantity of orders since this policy performs well for perishable products. It is a relatively simple policy and applicable in situations with positive lead times and fixed case pack sizes. The EWA-replenishment policy is a variation of a simple base stock policy: instead of ordering when the inventory position drops below the reorder level, it orders when the inventory position minus the estimated outdating drops below the reorder level. The outdating is estimated based on the age distribution of the actual inventory and by assuming that demand in the next $(L+R-1)$ days will be equal to the expected demand. If the inventory position minus this estimated outdating is less than the reorder level s, a minimal integer number of case packs, each with size Q, will be ordered such that the inventory position after ordering will at or above s. The reorder level s is equal to $s=(L+R)\mu+ss$, with ss denoting the safety stock. The two main performance measures of the perishable inventory system are the relative outdating z, defined as the ratio between the expected daily outdating and the expected daily demand, and the customer service level P_2, also known as the fill rate. The fill rate is defined as the percentage of demand which can be satisfied from the shelf immediately.

4. Approximations

In this Section we derive two approximations for the relative outdating z. The first approximation for the relative outdating is based on the assumption that there is ample inventory on hand in the system due to a very large safety stock. Let us assume without loss of generality that at period zero, a large order has been placed to replenish the safety stock. This safety stock will arrive L days later in the store and will be outdated m days later, i.e. at day $L+m$. The EWA-policy always looks $L+R-1$ days in the future to see whether any inventory will be outdated in this period. So the earliest day, EWA can recognize this outdating is at the first review moment at (or immediately following) day $L+m-(L+R-1)=m-R+1$. This is at day

$\left\lceil \frac{m-R+1}{R} \right\rceil \cdot R$, which is equal to $\rho = \left\lfloor \frac{m}{R} \right\rfloor \cdot R$. The variables $\lceil x \rceil$ resp. $\lfloor x \rfloor$ denote the nearest integer higher than (resp. lower than) or equal to x. At that moment the remaining age of the inventory which was ordered on day zero is equal to $L+m-\rho$. Therefore, EWA will estimate that all inventory except the average demand during $L+m-\rho$ days will be outdated within the next $L+R-1$ days. Since EWA also aims to order as much as is needed to bring the inventory position minus the estimated outdating back to or above the reorder level s, the quantity ordered at day ρ is equal to $\left\lceil \frac{s-[L+m-\rho]\cdot\mu}{Q} \right\rceil \cdot Q$. Since this order is ordered every ρ days, the expected relative outdating can be approximated as follows:

$$z^A = \frac{1}{\rho\mu} \cdot E\left[\left(\left\lceil \frac{s-[L+m-\rho]\cdot\mu}{Q} \right\rceil \cdot Q - D_\rho\right)^+\right] \tag{1}$$

where x^+ is simply equal to $\max\{0,x\}$.

Note that in this approximation we only consider the outdating of the large order, which aims to replenish the large safety stock. The outdating due to smaller replenishments in between these large orders is ignored in this approximation and therefore, we expect this approximation to underestimate the relative outdating.

The second approximation is based on the following assumptions: the inventory position at an arbitrary review period just after (potentially) ordering is uniformly distributed between $s-1$ and $s-1+Q$ (a result which holds exact for a periodic system for non-perishables with backordering; see [35]), unmet demand is backlogged and no outdating takes place in the first $L+m-1$ days. Then the expected outdating quantity at the end of day $L+m$ is equal to $E[(s-1+\Delta-D_{L+m})^+]$, with $\Delta \sim u[0,Q]$. As a result, the relative outdating can be approximated by:

$$z^B = \frac{1}{\rho\mu} E[(s-1+\Delta-D_{L+m})^+] = \frac{1}{\rho\mu\cdot Q}\sum_{i=1}^{Q} E[(s-1+i-D_{L+m})^+] \tag{2}$$

Both approximations above, z^A and z^B, contain a factor $E[(c-X)^+]$ with c a constant and X a discrete stochastic variable. This factor can simply be calculated as follows:

$$E[(c-X)^{+}]=\sum_{x=0}^{c}(c-x)P(X=x).$$

5. Simulation experiments

To test these approximations we simulated a wide range of perishable inventory systems. These systems are based on the modeling assumptions mentioned in section 3. For the discrete demand distribution we used the fitting procedure of Adan *et al* [36], which includes the binomial, negative binomial, Poisson and geometric distribution. We varied the input parameters as described in Table 1. The range of values for the lead time, review period, shelf life, case pack size and demand distributions is similar to the experiments in [2], with two exceptions. First, in this paper we focus on perishable products with a large potential to reduce the absolute amount of outdating. Therefore we study products with a shelf life less than or equal to 10 days. Second, we extend the range of the case pack cover (defined as the ratio between the case pack size and the average daily demand) to include the value $\lceil m/2 \rceil$. This is based on the design of experiments in [6]. They varied the ratio between the case pack size and the average demand during the shelf life between 0.33 and 0.625 for slow moving perishable items. Fast moving items have a lower ratio. Therefore we study four lot-sizing policies: the lot-for-lot policy (i.e. $Q=1$), $Q=\mu$, $Q=2\mu$ and $Q=\lceil m/2 \rceil \cdot \mu$.

For each combination of input parameters we simulated the system for all integer values of the safety stock, starting with zero and ending when either the fill rate exceeded 99% or the relative outdating exceeded 30%. Based on the guidelines in [37], per simulation experiment we performed at least 10 replications, each consisting of 50 weeks as the warming-up periods and ending with 1000 weeks in which the statistics are recorded. We replicated until we reached an absolute precision for the fill rate $P_2 \pm 0.002$ with 95% confidence.

PLEASE PUT TABLE 1 HERE.

The 14.274 simulation experiments revealed that the average error of approximations z^A and z^B is 2.28% respectively 0.87%, while the standard deviation of this error is equal to 3.27% and 3.43%. Using the observation that approximation z^A often underestimates the relative outdating, we also tested $\max\{z^A, z^B\}$ as an approximation. The average and standard deviation of the error in this approximation is 0.73% and 3.31%.

6. Approximation based on regression

To improve the approximations we use the simulation results and linear regression. The dependent variable in the regression model is our final approximation for the relative outdating z^*. The independent variables included in the regression are: σ/μ, ss/μ, $(ss+Q-1)/\mu$, $(1-P_2^*)$, $(Q/\mu-R)^+$, z^A and $\max\{z^A, z^B\}$. Regression is used to find the coefficients a_i, $i=0,\ldots,7$, which are used in the final approximation for the relative outdating:

$$\begin{aligned} z^* = (&a_0 + a_1\cdot\sigma/\mu + a_2\cdot ss/\mu + a_3\cdot(ss+Q-1)/\mu + a_4\cdot(Q/\mu-R)^+ + a_5\cdot(1-P_2^*) \\ &+ a_6 z^A + a_7\max\{z^A, z^B\})^+ \end{aligned}$$

The fill rate P_2 in this equation is approximated by solving the following equation, which is equal to Approximation 2 in [2]:

$$\begin{aligned} \frac{1-P_2^*}{P_2^*} &= \frac{1}{QR\mu}\sum_{i=1}^{Q}\{E[(D_{L+R}-i-s+1)^+] - E[(D_L-i-s+1)^+]\} \\ &= \frac{1}{QR\mu}\left(\sum_{i=1}^{Q}\sum_{d=i+s}^{\infty}(d-i-s+1)P(D_{L+R}=d) - \sum_{i=1}^{Q}\sum_{d=i+s}^{\infty}(d-i-s+1)P(D_L=d)\right) \end{aligned}$$

Although Approximation 2A in [2] performs slightly better than Approximation 2, we use the latter in this paper, since it is easier and faster to compute.

We divided the 14,274 simulation experiments in two sets: a set containing 13,274 experiments to determine the regression coefficients and a holdout set containing 1000 randomly selected experiments to test the quality of the approximations. When we apply regression to the entire set of 13,274 experiments and add the lead time, the review period and eight dummy variables (for the different values of the shelf life) as extra independent variables, the average and standard deviation of the approximation error is equal to -0.21% and 1,67%. To get better regression results, it is essential to divide the entire set of experiments in subsets in which the shelf life, the review period and the lead time are fixed. Thus we get 36 (=9x2x2) subsets for which we derive the regression coefficients and the R^2-adjusted. The results are reported in Table 2a till Table 2d.

PLEASE PUT TABLES 2a, 2b, 2c AND 2d HERE.

This final approximation is very good: the average approximation error in the hold-out sample is equal to -0.06% with a standard deviation equal to 0.78%. This shows a considerable improvement compared to the performance of the first two approximations. At the same time, the standardized coefficients (not reported here) show that approximations z^A and $\max\{z^A, z^B\}$ are the two most important independent variables. In some cases z^A is very close to the actual relative outdating (e.g., if $m = 2$, $L = 2$ and $R = 2$), while in other cases $\max\{z^A, z^B\}$ is a better approximation. Thanks to regression in combination with the subdivision of the dataset, an appropriate weight can be given to each of the approximations and to the other independent variables. Since most perishables will have a relative outdating less than 10%, we also computed the average and standard deviation of the approximation error for the associated subset of experiments. The average approximation error is then equal to -0.14% with standard deviation equal to 0.41%, showing that the performance is also very good in the region where the parameter setting is most in line with empirical settings.

7. Managerial implications

The final approximation for the relative outdating z^*, derived in this paper, in combination with the approximation for the fill rate for a perishable product (formula (3), derived in [2]) enable retailers to make trade-offs between the customer service and the amount of outdating. We illustrate this with an example. Assume a retailer sells a perishable product with the following product, supply and demand characteristics: $L = 1$, $R = 1$, $Q = 1$, $\sigma^2 / \mu = 2$, $m = 5$ and $\mu = 2$. The exchange curve for this product is depicted in Figure 1 (the line with $m = 5$).

If the retailer wants to have a fill rate equal to 98%, the safety stock for this product should be equal to 7 units. This yields 20% outdating. If the retailer does not want more than 10% outdating, he will reduce the safety stock to 4 units and get a fill rate equal to 93% (see Figure 1).

PLEASE PUT FIGURE 1 HERE.

PLEASE PUT FIGURE 2 HERE.

Figures 1 and 2 also show how the exchange curves change if the retailer redesigns the system, e.g. by increasing the shelf life with 1 day (see the lines for $m = 5$ and $m = 6$ in Figure 1) or decrease the demand uncertainty (see the lines with $\sigma^2 / \mu = 2$ and $\sigma^2 / \mu = 1$ in Figure 2). Likewise he can evaluate the impact on the amount of outdating and the service level caused by a reduction in the case pack size, the lead time or the review period or an increase in the average demand, e.g. due to assortment reduction.

8. Conclusions and future research

The final approximation for the relative outdating derived in this paper is very fast, performs very well, and is applicable for a wide range of perishable products. It enables retailers to make trade-offs between the relative outdating and the customer service level, when making strategic or tactical decisions on the (re)design of the perishable inventory system. Options for future research include: deriving approximations for perishable products with shelf life larger than 10 days, with non-stationary demand and/or with Last-In-First-Out (LIFO) customer withdrawal behavior.

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