# Optimal Sourcing From Alternative Capacitated Suppliers and General Cost Structures

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**Abstract:** Most manufacturers or retailers must procure items or services necessary for their businesses, in an environment that typically includes a number of competing suppliers with varying cost structures, price schemes, and capacities. In this paper, we consider the sourcing problem in which the buyer determines the sources that should be utilized and to what extent, in turn, dictating the total quantity available for the buyer to sell/utilize, subject to stochastic demand/requirement. Our approach advocates not to determine the quantity to be sourced a priori. We allow for capacitated sources and any cost structure in which fixed costs and quantity discounts are special cases. Some simpler versions of this problem are shown to be NP-hard in literature. By proving that the order of the sources is irrelevant for the optimal solution, we devise a dynamic programming model with pseudo-polynomial complexity to solve the multiple supplier sourcing problem to optimality. We propose two extensions: one limits the number of suppliers, and the other allows multi-period sourcing.

**Keywords:** Operations Management; Inventory Control; Value Chain; Sourcing; Procurement

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# 1. Introduction and Related Literature

Consider a manufacturer or retailer who procures (or, 'sources') a certain product or service, to use directly or indirectly in meeting the stochastic demand that she faces. Considering the manufacturing environment as an example, the product that is to be procured (or, the 'item') can be supplied by a finite number of capacitated external suppliers, and the manufacturer must decide which of the sources to utilize and to what extent. One could prefix the procurement quantity based on inventory- and production-related costs, and then find the least costly solution from the available pool of suppliers with corresponding price structures and capacities. However, the optimal sourcing (procurement) decision under stochastic demand requires an integrated approach, using all of the cost parameters and capacity and price information of alternative suppliers simultaneously.

Supplier price and capacity information could be collected by making use of e-business infrastructure or organized industrial associations, or by contacting qualified suppliers, using a request-for-quotations (RFQ). These sources may have different capacities and price structures, but we consider them to be identical in terms of their function, i.e. the item's characteristics do not depend on the supplier. We do not restrict our analysis to a particular cost function for procurement, and we allow, for example, for a separate fixed cost for initiating the use of each source, for logistics costs that might depend on the geographical location of the suppliers, and for non-linear unit variable costs. Progressive or all-units quantity discounts are special cases. Moreover, the "cost-of-doing business" with each supplier might incur non-linear cost factors (Kostamis et al. 2009). The suppliers' capacity utilization might result in re-evaluating the remaining available capacities, inducing quantity-dependent price quotations.

Purchasing is a common operation for all types of businesses. Kaplan and Sawhney (2000) analyze business-to-business e-commerce marketplaces and classify the purchasing market as manufacturing inputs and operating inputs, in terms of what businesses buy and as systematic sourcing and spot sourcing, in terms of how they buy. Our approach applies to any type of manufacturing or operating inputs that face stochastic demand and that are purchased from the spot market: the 'exchanges' and 'yield managers', respectively (Kaplan and Sawhney, 2000). There are numerous web-based platforms on the market that can materialize the sourcing methodologies prescribed in this study. There are general purpose

B2B e-commerce platforms such as Ariba (2014), Fiatech (2014), and 1 Point Commerce (2014) and specific platforms operated by companies for their operations such as the ones by Ford (2014), Foster Wheeler (2014), and Hilton (2014).

We note that our problem environment is extremely general and is not necessarily confined to procurement of goods and a supply chain context. To name some other environments, consider transportation logistics, manufacturing options, carbon offsetting, and the make-or-buy problem. As for transportation logistics, suppose that the materials ordered by a manufacturer or a retailer are shipped by vehicles with certain capacities. For each vehicle utilized, there may exist a fixed cost as well as a unit variable cost and possibly quantity discounts. The total order may be satisfied with a number of vehicles with varying characteristics. As for the manufacturing options, consider a heating process using industrial ovens. Each oven may have a different capacity and a particular cost of operation, including fixed costs. Similarly, consider a production environment with flexible and dedicated machines, in which each machine incurs different set-up and production costs. As for carbon offsetting, consider a socially responsible company that wants to offset its carbon emissions by investing in carbon abatement projects. The company must choose the 'best' (cost minimizing or utility maximizing) way of offsetting, from a number of certified offsetting options with different cost parameters (or utilities) and carbon abatement capacities. Finally, our methodology can be used to find the optimal in-house production versus outsourcing decision (considering the cost aspect of the problem in isolation), as in-house production can be considered one of the available sourcing options. In such situations, it is likely that the total cost of allocating some or all in-house capacity for producing the item would have a non-linear nature, stemming from cost components such as fixed costs, incremental capacity usage costs, and concave or convex capacity allocation (opportunity) costs. The complexity of in-house capacity costs is also illustrated by a Darden School of Business case on Emerson Electric Company (Davis and Page, 1991). The flexibility of our proposed methodology in its ability to handle all kinds of cost functions is one of our major contributions to literature.

Procurement decisions should consider the cost of materials procured, delivery punctuality, the quality of items procured, creation of effective strategic partnerships, possibly the carbon footprint, and the like. Therefore, one of the key processes of effective supply chain management is the supplier selection process, which consists of determining a supplier base (a set of potential suppliers to operate with), the supplier(s) to procure from, and the procurement quantities. We refer the reader to Elmaghraby (2002) for an overview of research on single- and multiple-sourcing strategies. Aissaouia et al. (2007) present a comprehensive review of literature related to several aspects of the procurement function, including the supplier selection process and in-house versus outsourcing decisions. Firms sometimes employ multiple criteria in selecting their suppliers (Ustun and Demirtas, 2008). A recent survey of multi-criteria approaches for supplier evaluation and selection processes is presented by Ho et al. (2010). More recently, Kumar et al. (2014) introduce a supplier selection approach taking carbon footprint of the suppliers into account. In our work, we do not include the multi-criteria supplier evaluation phase. We assume that the supplier base has already been determined and that the immediate supplier selection decisions are based on the cost criterion.

In our analysis, we consider a single-item, make-to-stock setting. We address a single period problem and extend it to a multi-period case in Section 4.2. The procurement problem has received much attention, mostly under the deterministic demand assumption (which results in a preset total procurement quantity). When the demand is deterministic, the problem becomes either (i) to determine the set of suppliers to purchase a given quantity, or (ii) to determine the suppliers and the purchasing frequency for a given demand rate. Chauhan and Proth (2003) consider a version of the problem, in which there is a lower and an upper bound for the capacity of each supplier, and the supply costs are concave. They propose heuristic algorithms. Chauhan et al. (2005) show that the problem considered by Chauhan and Proth (2003) is NP-hard. Burke et al. (2008) consider this problem under different quantity discount schemes and capacitated suppliers. They propose heuristic algorithms to solve the problem. Burke et al. (2008) discuss that this particular problem is a version of the 'continuous knapsack problem', in which the objective is to minimize the sum of separable concave functions, and show that this problem is NP-hard. Romeijn et al. (2007) analyze the continuous knapsack problem with nonseparable concave functions and propose a polynomial time algorithm. We note that the supplier selection problem with stochastic demand results in a nonseparable cost function; it is actually not a knapsack problem, because the size of the knapsack (the amount allocated to the suppliers) is itself a decision variable. We provide an exact pseudo-polynomial algorithm to solve the stochastic version of this problem, while not imposing restrictions on the supply cost. We refer the interested reader to Burke et al. (2008) for a further review of the related literature and to

Qi (2007), Kawtummachai and Hop (2005), and Mansini et al. (2012) for different aspects of the problem under deterministic demand. In this study, we contribute to the literature by considering stochastic demand and by including general cost structures.

The stochastic demand version of the procurement problem under capacitated suppliers has also received attention to a certain extent in the literature. Alp and Tan (2008) and Tan and Alp (2009) analyze the problem with two supply options, in a multi-period setting under fixed costs of procurement. Alp et al. (2014) consider an infinite horizon version of this problem with identical suppliers and a linear cost function with a fixed component, which is a special case of ours. Awasthi et al. (2009) consider multiple suppliers that have minimum order quantity requirements and/or a maximum supply capacity, but no fixed cost is associated with procurement. They show that this problem is NP-hard, even when the suppliers quote the same unit price to the manufacturer and propose a heuristic algorithm for the general version. Hazra and Mahadevan (2009) analyze an environment in which the buyer reserves capacity from a set of suppliers through a contracting mechanism. The capacity is reserved before the random demand is observed and allocated uniformly to the selected suppliers. If the capacity is short upon demand realization, the shortage is fulfilled from a spot market at a higher unit price. Our work differs from these articles, because we consider multiple suppliers and general cost functions, and we do not impose a particular structure on the allocation of purchased quantity to the suppliers.

Zhang and Zhang (2011) consider a similar environment to ours. A single item that faces stochastic demand is procured from potential suppliers that have minimum and maximum order sizes, and a fixed procurement cost is considered. They propose a nonlinear mixed integer programming formulation and a branch-and-bound algorithm. Our problem is more general than this, as we do not impose restrictions on the supply cost structures, a situation that cannot be handled by the methodology proposed by the aforementioned authors. Finally, we note that Zhang and Ma (2009) also consider a similar problem for multiple items. They assume that suppliers are capacitated and offer quantity discounts. A mixed integer nonlinear programming formulation that determines the optimal production quantities of each product, purchasing quantities of the raw materials, and the corresponding suppliers to make the purchases is proposed.

In this paper, we build a dynamic programming model to find the optimal solution to the NP-hard procurement problem, under a fairly general setting consisting of stochastic demand, general cost structures, and capacitated suppliers. The computational complexity of the solution that we propose is pseudo-polynomial. We also evaluate the performance of decoupling procurement and production decisions and build managerial insights.

# 2. Modeling Approach

In this section, we analyze the procurement problem in a single-period setting, under a given set of alternative capacitated suppliers, with corresponding general procurement cost functions. The procured quantity also dictates the stock quantity, subject to stochastic demand. There are two decisions in such an environment: Which sources should be utilized and in what quantities? The relevant parameters in determining those quantities are not only procurement costs and supplier capacities, but also the inventory-related cost parameters in the system. Nevertheless, one could either prefix the total order quantity and then decide on the allocation of this to the supplier base in a sequential manner, or make those decisions in an integrated fashion. The former could be a result of factors such as i) the perception that procurement-related (external) parameters and production/inventory-related (internal) parameters need to be treated separately; ii) the time lag between those decisions, e.g., the production department determines required quantities and relays this information to the purchasing department, who makes the purchase with the least cost; iii) lack of sufficient coordination between separate departments within the organization, e.g., making their uncoordinated decisions based on sales targets and forecasts of the company or their separate performance incentives; iv) the conventional market and/or company practice of tendering for bids based on a prefixed quantity; v) lack of sufficient information on the supplier base; and vi) managerial overlook on the potential savings of integration. In the absence of such factors, solving the problem by considering all problem parameters in an integrated way constitutes the basic research question that we address.

In what follows, we first highlight a major drawback of the sequential approach. Then, we present a dynamic programming model to formulate the problem under consideration and show how the optimal solution can be found in an integrated manner. Finally, we present the results of the numerical study we conducted to investigate (i) the effect of problem parameters on the optimal solution, and (ii) the performance of the sequential approach.

The relevant costs in our environment are the costs of procuring from suppliers and underage and overage costs, all of which are exogenously determined and non-negative. We do not impose any conditions on the costs of procuring from suppliers, and, hence, these costs might assume any form, including fixed costs for procurement, stepwise costs for shipments, costs that imply minimum order quantities, and different forms of quantity discounts. Our approach allows for the underage and overage costs of the remaining inventory level after demand materialization to also assume any form, via the corresponding loss function. We consider capacitated suppliers with fixed and known capacities. We assume full availability of the ordered quantities, and we also assume that the differences between procurement lead times from alternative suppliers can be neglected. In case the latter assumption is significantly violated, different lead times can be approximately incorporated into the model, by considering appropriate costs associated with purchasing from each supplier, reflecting the cost effect of corresponding procurement lead times. Similarly, other non-biddable price factors, such as delivery punctuality, the quality of items procured, and strategic partnership concerns, are also valuated by the manufacturer and reflected in the procurement costs. Naturally, the more differences in non-biddable price factors, the less accurate the cost-based methods (like ours). For a discussion on the valuation of non-biddable price factors, see Kostamis et al. (2009). We summarize our major notation in Table 1.

Table 1: Summary of notation.

| | | |
|---|---|---|
| $N$ | : | Number of alternative suppliers |
| $Q$ | : | Total procurement quantity |
| $U_n$ | : | Capacity of supplier $n$, $n = 1, 2, ..., N$ |
| $q_n$ | : | Quantity procured from supplier $n$ |
| $C_n(q_n)$ | : | Cost of procuring $q_n$ units from supplier $n$, $n = 1, 2, ..., N$ |
| $h$ | : | Overage cost per unit unsold |
| $b$ | : | Underage cost per unit of unmet demand |
| $W$ | : | Random variable denoting the demand |
| $G(w)$ | : | Distribution function of $W$ |

If $q_n$ units are procured from supplier $n$, $n = 1, 2, ..., N$, with a corresponding cost of $C_n(q_n)$, then the total cost of procuring $Q = \sum_n q_n$ units is $PC(Q) = \sum_n C_n(q_n)$, and the resulting average unit procurement cost is $c = PC(Q)/Q$. The problem is to minimize expected total costs $ETC(Q) = PC(Q) + \mathcal{L}(Q)$, where $\mathcal{L}(Q)$ denotes the total expected overage and underage costs, the standard loss function $\mathcal{L}(Q) = h \int_0^Q (Q-w)dG(w) + b \int_Q^\infty (w-Q)dG(w)$ being a special case. In the sequential approach, the total order quantity $Q^o$ is decided without knowing the total cost of procurement. This is because it is unknown, a priori, what the exact allocation of the total order quantity to the supplier base is, or whether the supplier base has the total capacity to meet this order. Once the total order quantity is determined, the allocation is optimized by solving the following problem (P), based on the sales prices and capacities quoted by various suppliers:

$$\begin{aligned} Min. \quad & \sum_n C_n(q_n) \\ st \quad & \sum_n q_n = \min\{Q^o, \sum_n U_n\} \\ & q_n \leq U_n \text{ for all } n. \end{aligned}$$

Note that $Q^o$ is not necessarily equal to the optimal procurement quantity, $\hat{Q} = \sum_n q_n$. As to the determination of the total order quantity $Q^o$, if only the inventory-related costs are considered, then the optimal order quantity is $\hat{Q}^o = \arg\min_Q\{\mathcal{L}(Q)\}$. But this approach results in over-estimation of the required quantity, as it neglects procurement costs. If one prefers to incorporate a linear unit procurement cost of $c$, the resulting optimal order quantity would be $\hat{Q}^o(c) = \arg\min_Q\{cQ + \mathcal{L}(Q)\}$ (in case of standard loss function $\mathcal{L}(Q)$, the solution would then be $\hat{Q}^o(c) = G^{-1}(\frac{b-c}{b+h})$). However, in general, there is no way of knowing what the actual procurement cost will be, until the required quantity is known. One could prepare a list of all possible quantities, but each entry in the list requires solving problem P, which is a knapsack problem with a general objective function. A special case is the fixed-charge continuous knapsack problem (see Haberl, 1999), which is NP-hard with some known pseudo-polynomial algorithms.

A simple approach is to incorporate an estimate of the purchasing cost, $\tilde{c} = \frac{\sum_n C_n(U_n)}{\sum_n U_n}$, and decide on $\hat{Q}^o(\tilde{c})$ accordingly, after which $\hat{Q} = \min\{\hat{Q}^o(\tilde{c}), \sum_n U_n\}$ units are procured by solving problem P. Nevertheless, this approach can be improved: Once the optimal cost of procuring $\hat{Q}$ and the corresponding average unit procurement cost $c = PC(\hat{Q})/\hat{Q}$ are known, $\hat{Q}^o$ can be updated by making use of this information, and so forth. Exploiting this idea, one can come up with the following algorithm (where Step 0 makes use of the computations stated above as the simple approach):

**Step 0.** Set $i = 1$, $\hat{Q}_i = \min\{\hat{Q}^o(\tilde{c}), \sum_n U_n\}$, $c_{i+1} = PC(\hat{Q}_i)/\hat{Q}_i$.

**Step 1.** Set $i = i + 1$. Find $\hat{Q}_i^o(c_i) = \arg\min_Q\{c_i Q + \mathcal{L}(Q)\}$.

**Step 2.** Solve problem P with $Q^o = \hat{Q}_i^o(c_i)$ to decide on the optimal allocation of $\hat{Q}_i = \min\{\hat{Q}_i^o(c_i), \sum_n U_n\}$ to the supplier base.

**Step 3.** Compute the average unit cost associated with purchasing $\hat{Q}_i$ units, $c_{i+1} = PC(\hat{Q}_i)/\hat{Q}_i$.

**Step 4.** If the solution converges (i.e. if $|\hat{Q}_i - \hat{Q}_{i-1}| < \epsilon$, where $\epsilon$ is a small enough constant) or the algorithm is run for a sufficiently long time, quit with $Q = \hat{Q}_i$. Otherwise, go to Step 1.

Naturally, the sequential approach described above does not necessarily find the optimal solution. Any approach (such as dynamic programming, DP) that considers the allocation of an additional unit will not guarantee optimality either, as the solution may change drastically by this additional unit. Furthermore, the problem cannot be seen as a special case of a knapsack problem with a non-separable objective function, because the 'knapsack size' (i.e., the total amount to be purchased and allocated to the suppliers) is also a decision variable. Consequently, the problem requires a different solution approach.

Nevertheless, the following DP formulation can be used to solve the integrated problem of finding optimal procurement decisions, including the procurement quantity, with $f_n(x)$ defined as the minimum total cost of

(i) procuring from the partial supplier base $\{n, n+1, \ldots, N\}$ and

(ii) the expected overage and underage of the total quantity purchased from the full supplier base $\{1, ..., N\}$,

when $x$ units are already procured from the partial supplier base $\{1, 2, \ldots, n-1\}$.

The Procurement Problem (PP):

$$
\begin{aligned}
\text{for } 0 \le x \le \sum_{i=1}^{N} U_i: \quad f_{N+1}(x) &= \mathcal{L}(x), \\
\text{for } 0 \le x \le \sum_{i=1}^{n-1} U_i: \quad f_n(x) &= \min_{y:\, x \le y \le x+U_n} \{C_n(y-x) + f_{n+1}(y)\} \quad \text{for } 2 \le n \le N, \\
f_1 &= \min_{y:\, 0 \le y \le U_1} \{C_1(y) + f_2(y)\}.
\end{aligned}
$$

**Theorem 1.** *The minimum cost attained by the optimal solution of PP is given by $f_1$ for any arbitrary order of suppliers numbered from 1 to $N$.*

**Proof :** Let us number the suppliers from 1 to $N$. Any order can be used. The Procurement Problem is to find the optimal procurement quantities $q_n^*$ for $n \in \{1, ..., N\}$ that minimize the total cost of procuring from the supplier base $\{1, 2, ..., N\}$ and the expected overage and underage cost, i.e.,

$$\begin{aligned}
& C_1(q_1^*) + C_2(q_2^*) + \cdots + C_N(q_N^*) + \mathcal{L}(q_1^* + q_2^* + \cdots + q_N^*) \\
= & \min_{\substack{0 \le q_1 \le U_1, \\ ..., \\ 0 \le q_N \le U_N}} \{C_1(q_1) + C_2(q_2) + \cdots + C_N(q_N) + \mathcal{L}(q_1 + q_2 + \cdots + q_N)\} \\
= & \min_{0\le q_1\le U_1,...,0\le q_N\le U_N} \{C_1(q_1) + C_2(q_2) + \cdots + C_N(q_N) + f_{N+1}(q_1 + q_2 + \cdots + q_N)\} \\
= & \min_{0\le q_1\le U_1,...,0\le q_{N-1}\le U_{N-1}} \Big\{C_1(q_1) + \cdots + C_{N-1}(q_{N-1}) \\
& \qquad + \min_{0\le q_N\le U_N} \{C_N(q_N) + f_{N+1}(q_1 + q_2 + \cdots + q_N)\}\Big\} \\
= & \min_{0\le q_1\le U_1,...,0\le q_{N-1}\le U_{N-1}} \{C_1(q_1) + \cdots + C_{N-1}(q_{N-1}) + f_N(q_1 + \cdots + q_{N-1})\} \\
= & \min_{0\le q_1\le U_1,...,0\le q_{N-2}\le U_{N-2}} \Big\{C_1(q_1) + \cdots + C_{N-2}(q_{N-2}) \\
& \qquad + \min_{0\le q_{N-1}\le U_{N-1}} C_{N-1}(q_{N-1}) + f_N(q_1 + q_2 + \cdots + q_{N-1})\Big\} \\
= & \min_{0\le q_1\le U_1,...,0\le q_{N-2}\le U_{N-2}} \{C_1(q_1) + \cdots + C_{N-2}(q_{N-2}) + f_{N-1}(q_1 + \cdots + q_{N-2})\} \\
\cdots & \\
= & \min_{0\le q_1\le U_1} \{C_1(q_1) + f_2(q_1)\} = f_1.
\end{aligned}$$

Note that the above result does not depend on the ordering of the suppliers due to the commutative property of the addition operator, hence, it does not depend on the initial choice of ordering, and, therefore, the theorem holds for any arbitrary order of suppliers. □

Let $q_n^*(x)$ be such that $x \le q_n^*(x) \le x + u_n$ and

$$C_n(q_n^*(x)) + f_{n+1}(x + q_n^*(x)) \quad \le \quad C_n(y - x) + f_{n+1}(y) \;\; \forall y : x \le y \le x + u_n,$$

for any given value of $x$. Then, the optimal quantity procured from supplier $n$, $\tau_n$, is given by

$$\tau_1 = q_1^*(0), \tau_n = q_n^*\left(\sum_{i=1}^{n} \tau_i\right) \text{ for } 2 \le n \le N.$$

The total optimal procurement quantity is given by $Q^* = \sum_{i=1}^{N} \tau_i$. The computational complexity of this DP is $O\left(N\left(\sum_n U_n\right) \max_n(U_n)\right)$.

# 3. Numerical Study

We conducted a numerical study to investigate (i) the effect of problem parameters on the optimal solution (Sections 3.1 and 3.2) and (ii) the performance of the sequential approach (Section 3.3). We considered the following setting: The demand has a Gamma distribution with coefficient of variation ($CV$) values of 0.5, 1, 1.5, and with expected values, $E[W]$, of 20, 40, 50, and 60. Demand is assumed to be discrete in this section for ease of exposition. The cost parameters are $h = 1$, $b = 2, 5, 10, 50$, and 200. We consider three sets of suppliers. In the first set (Supplier Base 1), there are $N = 5$ alternative suppliers ($n = 1, 2, \ldots, 5$) with capacities $U_n$ = 40, 20, 20, 10, and 10, respectively. There exists a fixed-cost component of ordering from supplier $n$, with $K_n = 40, 20, 20, 10$, and 10, respectively, and a linear unit variable cost component of $c_n$, in which $c_1 \in \{1.5, 2, 2.5\}$, $c_2$ and $c_3 \in \{2, 2.5, 3\}$, $c_4$ and $c_5 \in \{2.5, 3, 3.5\}$. This set resembles a situation in which the supplier base consists of a variety of suppliers, in terms of cost and capacity. In the second set (Supplier Base 2), there are also $N = 5$ alternative suppliers, but their capacities are $U_n$ = 60, 10, 10, 10, and 10, respectively. We set the fixed cost of ordering from supplier $n$ as $K_n = 60, 10, 10, 10$, and 10, respectively, and we set a linear unit variable cost component of $c_n$, as $c_1 \in \{1.0, 1.5, 2.0\}$, and $c_2$ to $c_5 \in \{2.5, 3, 3.5\}$. This set resembles a situation in which there is one dominant supplier in the supply base, and the rest are relatively smaller suppliers. The third set (Supplier Base 3) also consists of $N = 5$ alternative suppliers, but their capacities are $U_n$ = 24, 22, 20, 18, and 16, respectively. We set the fixed cost of ordering from supplier $n$ as $K_n = 24, 22, 20, 18$, and 16, respectively, and a linear unit variable cost component of $c_n$, as $c_1 \in \{1.8, 2.3, 2.8\}, c_2 \in \{1.9, 2.6, 2.9\}, c_3 \in \{2.0, 2.5, 3.0\}, c_4 \in \{2.1, 2.6, 3.1\}$, and $c_5 \in \{2.2, 2.7, 3.2\}$. This set resembles a situation in which there is no dominant supplier, and all suppliers are comparable in capacity.

## 3.1 Effects of Demand Variability and Cost Parameters

The following insight that simple inventory/production models generate holds for the procurement problem to some extent: As the unit underage cost increases (while keeping all other problem parameters constant), the total quantity procured from the suppliers and the total expected costs of the operation increase. As any cost component of a supplier increases, the supplier is preferred less by the buyer, and the total procurement quantity, if any, from that supplier decreases. The optimal total procurement quantity does not necessarily increase as the variability of demand increases (see Table 2), because the risk of being left with unsold goods (as in obsolescence) outweighs the risk of goodwill loss, due to relatively high procurement and overage costs.

Table 2: The optimal procurement decision at different coefficients of demand variation and underage costs. Supplier Base 1; $E[W] = 40$; and $c_n = 1.5, 2, 2, 3$, and 3, for $n = 1, \ldots, 5$.

| | b=2 | b= 5 | b=10 | b=50 | b=200 |
|---|---|---|---|---|---|
| CV = 0.5 | $(0,0,0,0,0)$ | $(40,0,0,0,0)$ | $(40,0,0,0,0)$ | $(40,20,17,0,0)$ | $(40,20,20,10,0)$ |
| CV = 1.0 | $(0,0,0,0,0)$ | $(0,20,0,0,0)$ | $(40,0,0,0,0)$ | $(40,20,20,10,0)$ | $(40,20,20,10,10)$ |
| CV = 1.5 | $(0,0,0,0,0)$ | $(0,0,0,0,0)$ | $(40,0,0,0,0)$ | $(40,20,20,10,10)$ | $(40,20,20,10,10)$ |

We also observe that the optimal solution might be extremely sensitive to cost parameters. For example, when $CV = 1.0$, $b = 5$, $N = 3$, $U_n = 40, 20, 10$, $K_n = 40, 20, 10$, and $c_n = 1.5, 2.5, 2.5$, for $n = 1, 2, 3$, respectively, the optimal solution is $(37, 0, 0)$. When we keep all parameters the same, except for $c_1 = 2$, instead of 1.5, the optimal solution becomes $(0, 0, 10)$, which represents not only a 73% decrease in total procurement quantity, but also a completely different supplier selection. This example shows that the optimal solution of a particular situation could significantly change, even when a single parameter changes, indicating a lack of robustness, which emphasizes the importance of having a methodology appropriate for finding the optimal solution.

## 3.2 Effects of Flexibility

In this section, we analyze the impact of flexibility on optimal procurement decisions. We call one problem environment 'more flexible' than another when there is at least one more procurement option to choose from. In our numerical tests, we frequently observe that the total procurement quantity does not decrease as the problem environment becomes more flexible. Nevertheless, our numerical experiments reveal that a more flexible environment may also lead to lower procurement quantities. Such a situation is observed when a more appealing (e.g., cheaper per unit, when the order size is sufficiently high) procurement alternative is introduced to the supplier base, and it is not necessary to place a high order size, to benefit from economies of scale in the former situation, by utilizing this new supplier. An example of this situation can be illustrated by the following instance: Supplier Base 1, $E[W] = 40, CV = 0.5, b = 5, c_n = 2.5, 3, 3, 2.5$, and 2.5, for $n = 1, \ldots, 5$, respectively. Let us first set $U_3 = U_5 = 0$, i.e., only suppliers 1, 2, and 4 are available with $U_1 = 40, U_2 = 20$, and $U_4 = 10$. In this case, the optimal solution is $(34, 0, 0, 0, 0)$. When we make this system more flexible by letting $U_3 = 20$, and $U_5 = 10$, the optimal solution becomes $(0, 0, 0, 10, 10)$, decreasing the total procurement by 41%. In the former situation, the buyer does not prefer to procure 20 units (as in the latter case), because Supplier 1 is short in capacity, and Supplier 2 is a more expensive option. The fixed cost of Supplier 1 leads to the procurement of a larger quantity in the optimal solution. In the latter situation, the introduction of Supplier 5, a cheaper option, makes it unnecessary to utilize Supplier 1 with its high fixed cost; 20 units turn out to be optimal when the trade-off between the underage and fixed costs are resolved. This phenomenon is observed in several more problem instances with similar conditions, also for Supplier Base 2 and 3. On the other hand, this is attributed to the existence of suppliers with diverse cost and capacity structures, e.g., when there is a dominant supplier. When we decreased this diversity in our numerical tests by trimming the cost differences among the suppliers in any particular supplier base, we consistently observed a decrease in the number of cases in which this phenomenon is observed. Obviously, in the limit when all suppliers are identical, increasing flexibility does not lead to a decrease in total procurement quantity.

## 3.3 Value of the Integrated Approach

Finally, we compare the optimal solution of the integrated approach with the solution found by the sequential approach as presented in Section 2. The average cost deviation percentages relative to the optimal solution over all the problems in our test bed are presented in Figure 1.

The sequential approach performs well, when the underage cost is either extremely low or is the dominating cost factor: the sequential approach yields the optimal solution when $b = 2$ and the average cost deviation is 0.34% for $b = 200$, in our test bed. This is because when $b$ is as low as 2 in our test bed, the optimal policy is trivially to always backorder; when $b$ is high, procurement takes place in large quantities, sometimes consuming the full available capacity, which is the other extreme trivial solution. Nevertheless, when the underage cost is neither dominating nor insignificant, the value of the integrated approach over the sequential approach appears to be significant. The average cost deviation over all cases considered is 12.18% and 4.67% when $b = 5$ and 10, respectively; the maximum is 85.81%, which also demonstrates the importance and non-triviality of finding the optimal solution.

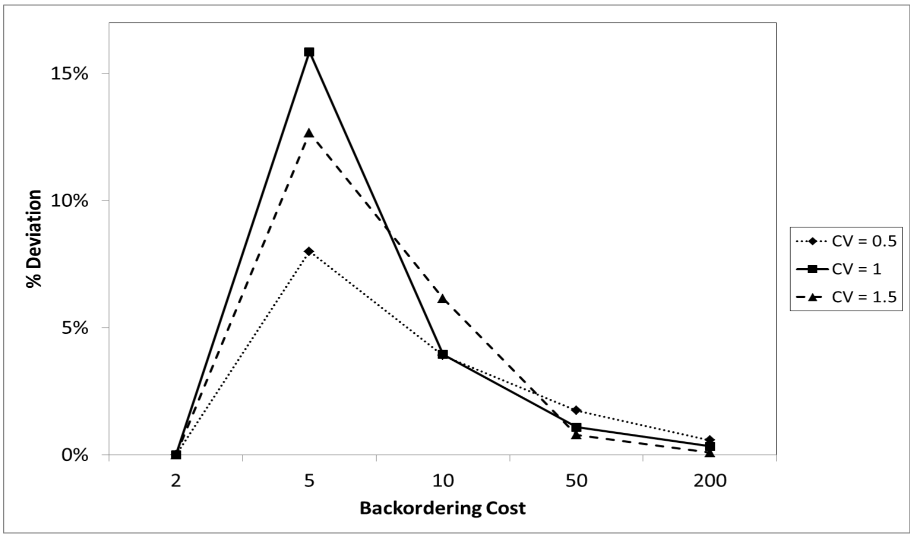


Figure 1: Percent of cost deviation due to sequential approach.

# 4. Extensions

In what follows, we model two important extensions of the basic model. We note that the properties that hold in Theorem 1 also hold in these extensions, which we do not show for brevity.

## 4.1 Limited Number of Suppliers

The practice of working with multiple suppliers at the same time is a means of mitigating, to some extent, the risk of supply and yield uncertainties. Nevertheless, working with too many suppliers might also bring extra operational burden on the buyer; for example, this might loosen the control over the quality and punctuality of the delivered items, or the buyer might be willing to establish long-term business relations with a selected number of suppliers. A plausible and commonly adopted strategy in practice is 'dual sourcing'. In this section, we extend our model to include a limit on the number of suppliers with which the buyer does business.

We first introduce a new state variable, $k = 0, 1, ..., K$, which is defined as the number of suppliers selected for business. We define the recursive cost function $f_n(x, k)$ as before, with the addition that a positive procurement is made from $k$ suppliers within the supplier base $\{1, 2, ..., n-1\}$.

Let $G_n(x,k) = \min_{y:x<y\le U_n} \{C_n(y-x) + f_{n+1}(y,k+1)\}$. This function reflects the minimum total costs of procuring from the immediate supplier $n$ by utilizing $k+1$ suppliers within the supplier base $\{1,2,...,n\}$. Then, the following formulation solves the problem:

$$
\begin{aligned}
\text{for } 0 \le x \le \sum_{i=1}^{N} U_i: \quad & f_{N+1}(x,K) && = \mathcal{L}(x), f_{N+1}(x,k) = \infty \quad \forall k = 0,1,...K-1 \\
\text{for } 0 \le x \le \sum_{i=1}^{n-1} U_i: \quad & f_n(x,k) && = \min\{G_n(x,k), f_{n+1}(x,k)\} \quad \text{for } 2 \le n \le N, 0 \le k \le K \\
& f_1(0,0) && = \min\{G_1(0,0), f_2(0,0)\}.
\end{aligned}
$$

## 4.2 Multiple Periods

In this section, we extend our original model to multiple periods. In the beginning of the planning horizon, the buyer collects bids from a number of suppliers for each period of the planning horizon. The bids quoted by the suppliers may be period dependent, including the possibility of no supply. In every period, the buyer observes non-stationary stochastic demand. End-of-period inventory or backorder is carried to the next period, with a certain cost. We assume that the procurement lead time is zero, but our model can be easily extended to accommodate positive lead times.

For this extension, we need to introduce the 'time' stage into the formulation and redefine the state variable $x$. Let $T$ be the total number of periods in the planning horizon and $t$ be the time index corresponding to periods. Similar to the original model, we number the suppliers arbitrarily from 1 to $N_t$ in every period $t$, where $N_t$ is the number of suppliers placing a bid in period $t$. Let $U_{tn}$ be the maximum capacity of supplier $n$ in period $t$. Subscript $n$ is still used to denote suppliers 1, 2, ..., $N_t$. The state variable $x$ and the recursive cost function are defined as follows:

$x$: inventory level (on-hand inventory or backorders) at a given stage $(t,n)$, when procurement decisions have been made for suppliers 1, 2, ..., $n-1$ in period $t$;

$f_{tn}(x)$: minimum total expected cost of procuring from the partial supplier base $\{n, n+1, ..., N\}$ in period $t$ from the full supplier base in periods $t+1, t+2, ..., T$, when the inventory level is $x$.

In this recursive function, when $n \ge 2$ for a given $t$, $x$ is interpreted as the inventory level in period $t$, after procurement decisions from the supplier base $\{1,2,...,n-1\}$ are made (cf. the single period model). When $n = 1$ for a given $t$, $x$ corresponds to the on-hand inventory or backorder at the beginning of period $t$, which is carried over from period $t-1$. When $n = N_t + 1$ for a given period $t$, $x$ corresponds to the total inventory on-hand or backorders at the end of period $t$, just before the demand is observed. The following formulation can be used to find the optimal procurement decisions in all periods:

$$
\begin{aligned}
f_{T+1,1}(\cdot) &= 0 \\
f_{tn}(x) &= \begin{cases} \mathcal{L}_t(x) + E_{D_t}\left[f_{t+1,1}(x - D_t)\right] & \text{if } n = N_t + 1 \\ \min_{y:x\le y\le x+U_{tn}} \{C_{tn}(y-x) + f_{t,n+1}(y)\} & \text{if } n = 1,2,...,N_t \end{cases} \quad \forall t = 1,2,...,T.
\end{aligned}
$$

We note that this problem is equivalent to a "generalized" stochastic lot-sizing problem, in which the procurement cost functions that we use correspond to a generalized cost of production, using single or multiple internal and/or external resources in the lot-sizing problem.

# 5. Conclusions

In this paper, we consider the sourcing decisions of a retailer or a manufacturer for a particular product or service. There are basically two decisions: determining the quantity to be procured and selecting the suppliers to procure from. In this study, we develop a unified approach that combines these two decisions. We allow for stochastic demand and capacitated production facilities. Our modeling approach is capable of handling sourcing problems in a wide range of environments, as we do not impose restrictions on the relevant cost components. The procurement problem and its several variations are proven to be NP-hard in literature, however, we develop a dynamic programming model with a state definition, which makes the solution algorithm pseudo-polynomial. We achieve this by proving that the order of the sources is irrelevant for the optimal solution. As this property is inherent in the dynamic programming logic that we propose, the possible extensions of the basic model also possess the same property and remain efficient. We have modeled two such extensions: one limiting the number of suppliers, the other allowing multi-period sourcing.

We derive the following managerial insights through numerical studies:

- An increase in the availability of sourcing options (a more flexible system) may lead to a decrease in the total quantity procured, when there are suppliers with diverse cost and capacity structures, e.g., when there is a dominant supplier.

- The optimal solution to the sourcing problem is not necessarily robust, as a change in even a mere cost parameter might completely change the optimal course of action. In case robustness is sought (for reasons such as ensuring product uniformity or decreasing administrative costs of procurement), strategic partnership, vertical integration, or making instead of buying are some possible means of eliminating or reducing such parameter dependence.

- As it is also common to newsvendor models, the total quantity procured by the manufacturer does not necessarily increase as variability of demand increases. For relatively low service level requirements, the total quantity procured decreases as the variability of the demand increases; whereas a reverse effect is observed otherwise.

- There is significant value in integrating the decisions as to the supplier selection and the procurement quantity, particularly for moderate service level requirements.

Note that the multi-period sourcing problem possibly enables exploiting the economies of scale. This makes it more valuable to use the approach that we propose, than to follow the sequential approach in each period.

Our problem also applies to other environments, such as retailers using capacitated vehicles to replenish their inventory, or to production environments with alternative in-house capacitated production facilities. One needs to interpret our problem environment accordingly, such as 'the supplier' being translated into 'alternative in-house capacitated production facilities', or 'in-house manufacturing capability' being translated into 'outsourcing possibility'.

# References

1 Point Commerce. 2014. http://www.1commerce.com

Aissaouia, N., M. Haouaria, E. Hassini. 2007. Supplier selection and order lot sizing modeling: A review. *Comput. & Oper. Res.* 34: 3516–3540.

Alp, O., T. Huh, T. Tan. 2014. Inventory Control with Multiple Set-Up Costs. *Manuf. Serv. Oper. Manag.* 16: 89–103.

Alp, O., T. Tan. 2008. Tactical Capacity Management under Capacity Flexibility. *IIE Trans.* 40: 221–237.

Ariba. 2014. http://www.ariba.com

Awasthi, A., S.K. Chauhan, J-M. Proth. 2009. Supplier Selection Problem for a Single Manufacturing Unit under Stochastic Demand. *Int. J. of Prod. Econ.* 117: 229–233.

Burke, G.J., J. Carillo, A.J. Vakharia. 2008. Heuristics for Sourcing from Multiple Suppliers with Alternative Quantity Discounts. *Eur. J. of Oper. Res.* 186: 317–329.

Burke, G.J., J. Geunes, H.E. Romeijn, A.J. Vakharia. 2008. Allocating Procurement to Capacitated Suppliers with Concave Quantity Discounts. *Oper. Res. Lett.* 36: 103–109.

Chauhan, S.S., A. Eremeev, A. Kolokolov, V. Servakh. 2005. Concave Cost Supply Management for Single Manufacturing Unit, in: P.M. Pardalos, D.W. Hearn (Eds.), Supply Chain Optimisation Product/Process Design, Facility Location and Flow Control. Springer US. 167–174.

Chauhan, S.S., J-M. Proth. 2003. The Concave Cost Supply Problem. *Eur. J. of Oper. Res.* 148: 374–383.

Davis, E.W., K.L. Page. 1991. Emerson Electric Company ACP Division: The Fan Subpack Sourcing Decision. Darden School of Business Case. University of Virgina, Charlottesville, VA, USA.

Elmaghraby, W.J. 2000. Supply Contract Competition and Sourcing Policies. *Manuf. & Serv. Oper. Manag.* 2: 350–371.

Fiatech. 2014. http://fiatech.org

Ford. 2014. https://fsp.portal.covisint.com/web/portal/home

Foster Wheeler. 2014. http://eprocure.fwc.com

Haberl, J. 1999. Fixed-Charge Continuous Knapsack Problems and Pseudogreedy Solutions. *Math. Programming.* 85: 617–642.

Hazra, J., B. Mahadevan. 2009. A Procurement Model Using Capacity Reservation. *Eur. J. of Oper. Res.* 193: 303–316.

Hilton. 2014. http://www.hiltonworldwide.com /development/performance-advantage/supply-management/e-procurement/

Ho, W., X. Xu, P.K. Dey. 2010. Multi-Criteria Decision Making Approaches for Supplier Evaluation and Selection: A literature review. *Eur. J. of Oper. Res.* 202: 16–24.

Kaplan, S., M. Sawhney. 2000. E-Hubs: The New B2B Marketplaces. *Harvard Business Review.* May-June. 97–103.

Kawtummachai, R., N.V. Hop. 2005. Order Allocation in a Multiple-Supplier Environment. *Int. J. of Prod. Econ.* 93-94: 231–238.

Kostamis, D., D.R. Beil, I. Duenyas. 2009. Total-Cost Procurement Auctions: Impact of Suppliers' Cost Adjustments on Auction Format Choice. *Manage. Sci.* 55: 1985–1999.

Kumar, A., V. Jain, S. Kumar. 2014. A Comprehensive Environment Friendly Approach for Supplier Selection. *OMEGA-Int. J. of Manage. Sci.* 42: 109–123.

Mansini, R., M. W.P. Savelsbergh, B. Tocchella. 2012. The Supplier Selection Problem with Quantity Discounts and Truckload Shipping. *OMEGA-Int. J. of Manage. Sci.* 40: 445–455.

Qi, X. 2007. Order Splitting with Multiple Capacitated Suppliers. *Eur. J. of Oper. Res.* 178: 421–432.

Romeijn, H.E., J. Geunes, K. Taaffe. 2007. On a Nonseparable Convex Maximization Problem with Continuous Knapsack Constraints. *Oper. Res. Lett.* 35: 172–180.

Tan, T., O. Alp. 2009. An Integrated Approach to Inventory and Flexible Capacity Management under Non-Stationary Stochastic Demand and Set-up Costs. *OR Spectr.* 31: 337–360.

Ustun, O., E. A. Demirtas. 2008. An Integrated Multi-Objective Decision-Making Process for Multi-Period Lot-Sizing with Supplier Selection. *OMEGA-Int. J. of Manage. Sci.* 36: 509–521.

Zhang, G., L. Ma. 2009. *Optimal Acquisition Policy with Quantity Discounts and Uncertain Demands Eur. J. of Oper. Res.* 47: 2409–2425.

Zhang, J-L., and M-Y. Zhang. 2011. Supplier Selection and Purchase Problem with Fixed Cost and Constrained Order Quantities under Stochastic Demand. *Int. J. of Prod. Econ.* 129: 1-7.