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Citation for published version (APA):

DOI:
10.1109/JSTQE.2017.2720966

Document status and date:
Published: 01/01/2018

Document Version:
Accepted manuscript including changes made at the peer-review stage

Please check the document version of this publication:
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Download date: 20. Sep. 2020
Theoretical analysis of a feedback insensitive semiconductor ring laser using weak intracavity isolation

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Abstract—External optical feedback can severely deteriorate the performance of semiconductor lasers. This work proposes an integrated laser design that can withstand tens of percent of off-chip feedback, without requiring the integration of magneto-optic materials. The proposed laser consists of a ring cavity with a weak intracavity optical isolator. Sufficient gain difference between clockwise and counter-clockwise modes leads to unidirectional laser oscillation. Any reflected light is returned to a mode that is below threshold. This significantly reduces interactions between the feedback and the lasing mode. A rate-equation analysis is presented to show that the relative intensity noise changes less than a factor 2 when less than −0.1 dB of the light is fed back into the laser and when intracavity isolation is 10 dB. Linewidth and optical output power change approximately 1% for these values.

Index Terms—Ring lasers, Semiconductor lasers, Optical feedback, Photonic integrated circuits, Rate equation modelling

I. INTRODUCTION

SEMICONDUCTOR lasers can be particularly sensitive to external optical feedback (EOF), as was recognized long ago [1]–[4]. Five feedback regimes have been identified in a distributed feedback laser based on the feedback power ratio and the distance to the reflection [1]. The distance to the reflection only plays a role for ratios below −50 dB. Going from low to high reflection, feedback can cause external cavity modes, line narrowing, extreme line broadening and finally stabilization when the laser field is dominated by the feedback. In many circumstances the strength of the reflection is unpredictable and can fluctuate over time, resulting in an undesirable change in the output laser light. To solve this problem, one or often two Faraday isolators are employed to prevent EOF from returning to the laser cavity altogether [5]–[8]. From [1] it is found that feedback power ratios below −60 dB have no observable effect on the lasing mode of a distributed feedback laser for typical feedback distances.

Attempts to obtain Faraday isolation on chip have been made [9]–[19]. So far the insertion loss and integration complexity have proven a barrier to implementation of non-reciprocal elements however. Recently a class of isolators has been devised which avoids the use of magneto-optic materials altogether. It was found that isolation can be achieved using non-linearities [20], but this type of isolator can only be used as when light is either propagating in the forward or the reverse direction. These devices are therefore not suitable for preventing the detrimental effects of EOF on continuous wave lasers, but can be used for pulsed lasers as was proposed in [21]. Isolation can also be obtained using a time varying refractive index [22]–[25], but none of these have achieved the 60 dB required to fully suppress the effects of EOF.

This work takes a different approach to suppressing the effects of EOF on a continuous wave, single frequency, tunable laser. The principle is to allow light to get back into the laser cavity, but into a different, sub-threshold mode. This is achieved by placing a weak isolator inside the cavity of a ring laser such that one direction is favoured over the other. E.g. the clockwise (cw) mode reaches threshold and is lasing, while EOF returns to the counterclockwise (ccw) mode. With a gain difference of only 0.5 dB the ccw mode is suppressed by approximately 40 dB relative to the cw mode [26]. For large external reflection, EOF contributes significantly to the power in the ccw mode and larger isolation is required, at most 10 dB as will be shown later. It will be shown that by using a weak intracavity isolator the effects of EOF on the lasing mode can be greatly reduced. This is in stark contrast to a ring laser that is forced to oscillate unidirectionally using an external mirror, which was studied using a rate equation analysis in [27]. As was shown in [25], it is possible to achieve the required isolation using InP based waveguides and phase modulators but the principle holds for any integration platform that is able to create 10 dB of isolation. A less detailed study of the steady state behaviour of this laser was already performed by us in [28], while [29] studied the dynamic behaviour of the laser in some detail.

In this paper we will analyze an integrated ring laser with intracavity isolator in detail to obtain a good insight into the performance of the laser when it is subjected to EOF and to determine the requirements on the isolator. Section II first gives an intuitive explanation why a unidirectional ring laser with weak intracavity isolator will be less susceptible to EOF. Section III then goes on to describe the model that is used to analyze the laser providing the assumptions which have been made. Section IV details the analysis of the laser for the steady state, resulting in the average values of the optical intensity and number of charge carriers, and section V introduces the effects of spontaneous emission noise. As a result of this analysis...
the laser, including the response induced by EOF. Because of the intracavity filter the laser is single mode in both directions and the rate equations that need to be considered can be simplified significantly. Since the cw and ccw modes have equal wavelength, they both interact with charge carriers of the same energy level in the SOA. Thus, the laser can be described using a set of three rate equations: two for the complex optical fields in both the cw and ccw modes and one for the number of charge carriers.

It is assumed that the number of carriers and the intensities and frequencies will reach a stable average value after some time—the steady state—after which only minor excursions from these values occur due to spontaneous emission noise. Since only small excursions are assumed, all effects are linearized around the steady state to achieve considerable simplification in describing the effect of noise. It is assumed that all the EOF originates from a single point of reflection outside of the laser cavity, indicated by \( R \) in Fig. 1. For a photonic integrated circuit (PIC) this can for instance be a reflection of a fiber-connector. Because only a single point of reflection is assumed, use can be made of the equations derived in the pioneering paper by Lang and Kobayashi [30].

This study builds further on the ideas of [30], and extends these by including the loss difference induced by the isolator and obtaining a model for a ring laser. It is assumed that the only coupling between the cw and ccw modes is through the EOF and the changes in carrier density. Effects such as backscattering in the waveguides are assumed to be negligible. The isolator is modeled as a loss difference between the cw and ccw modes.

IV. RATE EQUATIONS

To obtain the steady state intensities and frequencies of both modes as well as the number of charge carriers at steady state, this section first formulates a fully deterministic model that does not include EOF. Only after this model is completed, feedback effects are added to the model. Finally stochastic terms which model the influence of spontaneous emission are added. This treatment greatly simplifies the analysis and was done for linear lasers in [26].

The complex electric field strength of the cw and ccw modes and the number of carriers shall be indicated with \( \mathcal{E}_{cw}, \mathcal{E}_{ccw} \) and \( n \) respectively. \( \mathcal{E}_{cw} \) is normalized such that \( |\mathcal{E}_{cw}|^2 \) is equal to the number of photons in the cw mode in the cavity, or more correctly the energy in the cw mode divided by the energy per field quantum, \( \hbar \omega \), where \( \omega \) is the optical angular frequency of the mode. A similar scaling is performed for \( \mathcal{E}_{ccw} \) to obtain the number of photons in the ccw mode.

A. Rate equations for the electric field

First a set of rate equations for the electric field inside the laser cavity is derived. As a starting point we take a traveling wave propagating in the cw direction, which can generally be described by

\[
\mathcal{E}_{cw}(z,t) = \mathcal{E}_{cw,0} \exp \left( i(kz - \omega t) + \int_0^z \frac{\dot{g}(z') - \dot{\gamma}(z')}{2} dz' \right). \tag{1}
\]
Here $\mathcal{E}_{cw,0}$ is the amplitude of the field vector at $z = 0$, $\omega$ is the optical frequency, $t$ is time, $k$ is the wave number, $z$ indicates the position along the cavity, $\hat{g}$ is the power gain per unit length and $\gamma$ denotes the power loss per unit length. Since the points $z = 0$ and $z = L$ represent the same point in the ring and the field in the cavity is assumed to be slowly varying compared to the round trip time (the slowly varying envelope approximation), it should hold that $\mathcal{E}_{cw}(0, t) = \mathcal{E}_{cw}(L, t)$, from which it is found that

$$\mathcal{E}_{cw}(z, t) = \exp(ikL + \frac{1}{2}(g - \gamma)L)\mathcal{E}_{cw}(z, t),$$

where $gL = \int_{0}^{L} \hat{g}(z)dz$ is the round trip gain and $\gamma L = \int_{0}^{L} \hat{\gamma}(z)dz$ is the round trip loss.

$k$ is dependent on both the number of carriers, $n$, and optical frequency $\omega$. It is assumed that both $n$ and $\omega$ will be close to their threshold values when the laser is lasing, which justifies an expansion of $k$ around threshold. This yields

$$k \approx k_{th} + \frac{\partial k}{\partial n}|_{th} (n - n_{th}) + \frac{\partial k}{\partial \omega}|_{th} (\omega - \omega_{th}),$$

where the subscript “th” denotes the value at threshold in the solitary laser. The effective refractive index is defined as $\mu_{e} \equiv kr/\omega$ such that $k = \mu_{e}\omega/c$. $\mu_{e}$ is dependent on both $n$ and $\omega$. This definition is substituted into (3) and the effective group index is defined as $\mu_{g} \equiv \mu_{e} + \omega\partial\mu_{e}/\partial\omega$ to find

$$k \approx \frac{\omega_{th}}{c} \left( \mu_{e, th} + \frac{\partial \mu_{e}}{\partial n}|_{th} (n - n_{th}) + \frac{\mu_{e}}{\omega_{th}} (\omega - \omega_{th}) \right),$$

where $n_{th}$ is the number of carriers at threshold.

The frequency independent round trip gain $G_1$ and the frequency dependent round trip gain $G_2$ are now defined as

$$G_1 \equiv \exp \left( \frac{\omega_{th}L}{c} \left( \mu_{e, th} + \frac{\partial \mu_{e}}{\partial n}|_{th} (n - n_{th}) + \frac{1}{2}(g - \gamma)L \right) \right),$$

$$G_2 \equiv \exp \left( i(\omega - \omega_{th})\tau_{r} \right),$$

such that $\mathcal{E}_{cw}(z, t) = G_1 G_2 \mathcal{E}_{cw}(z, t)$. When $\tau_{r} \equiv \mu_{e} L/c$. Here it is assumed that the free spectral range of the filter is large compared to the linewidth of the laser, such that it can be considered spectrally flat. Since multiplication by $\exp(i\omega_{th}t)$ is equivalent to a time delay of $\tau_{r}$, the field at time $t$ can be expressed in terms of the electric field at time $t - \tau_{r}$ as

$$\mathcal{E}_{cw}(t) = G_1 \exp(-i\omega_{th}\tau_{r})\mathcal{E}_{cw}(t - \tau_{r}).$$

It is now possible to define the slowly varying envelope $E_{cw}(t)$ such that $\mathcal{E}_{cw}(t) = \exp(-i\omega_{th}t)E_{cw}(t)$. By substitution into (6) the envelope at time $t$ can be expressed in terms of the envelope at time $t - \tau_{r}$, as

$$E_{cw}(t) = G_1 E_{cw}(t - \tau_{r}).$$

The round trip time is assumed to be short compared to the time scales with which the envelope $E_{cw}(t)$ varies and the time derivative of $E_{cw}(t)$ is approximated by a difference quotient such that the time derivative of the envelope becomes

$$\dot{E}_{cw}(t) \approx \frac{E_{cw}(t) - E_{cw}(t - \tau_{r})}{\tau_{r}},$$

$$\approx \frac{G_1 - 1}{\tau_{r}} E_{cw}(t),$$

where the dot in $\dot{E}_{cw}(t)$ denotes a time derivative.

Subsequently the fraction in (8b) is linearized around threshold. To this end it is realized that $i\omega_{th}\mu_{e, th} L/c$ is an integer multiple of $2\pi$ and does not contribute to $G_1$. When the laser is lasing, the gain compensates for almost all of the losses of the cavity. Therefore, the net gain, $g - \gamma \approx \partial g/\partial n(n - n_{th})$, is small while the laser is lasing such that the argument of the exponent in (5a) is small. Carrying out a linearization of $G_1$ around threshold and substituting the result into (8b) yields

$$\dot{E}_{cw}(t) = \frac{1}{2} \xi_{cw}(1 + i\alpha)N(t)E_{cw}(t),$$

where $N(t) \equiv n(t) - n_{th}$ and

$$\xi_{cw} \equiv \frac{L}{\tau_{r}} \frac{\partial g}{\partial n}, \quad \alpha \equiv \frac{2\omega_{th}}{c} \frac{\partial \mu_{e}}{\partial \omega}.\tag{10}$$

A similar argument can be made for the electric field propagating in the ccw direction where the effect of the isolator is modeled by a gain difference by replacing $\gamma$ with $\gamma + \Delta\gamma$ in (5a). This yields

$$\dot{E}_{ccw}(t) = -\frac{1}{2} \Delta\Gamma \xi_{ccw}(t) + \frac{1}{2} \xi_{ccw}(1 + i\alpha)N(t)\dot{E}_{ccw}(t),\tag{11}$$

where

$$\xi_{ccw} \equiv \exp \left( -\frac{1}{2} \Delta\gamma L \right) \xi_{cw},\tag{12a}$$

$$\Delta\Gamma \equiv \frac{2 \alpha}{\tau_{r}} \exp \left( -\frac{1}{2} \Delta\gamma L \right).\tag{12b}$$

$\Delta\Gamma$ models the gain difference that would be expected from the isolator. $\xi_{ccw}$ is different from $\xi_{cw}$, which is explained by the difference in threshold for both modes caused by the gain difference.

### B. Rate equations for the number of charge carriers

To complete the deterministic model without EOF, a rate equation for the number of charge carriers is subsequently obtained. This equation is found by considering the SOA as a big reservoir of carriers where each carrier represents an electron in the excited state. Again a similar procedure was performed in [26] for a linear laser. The current analysis modifies this analysis slightly to include the effects of the two modes inherently present in a ring laser.

Several processes can be identified that add or remove carriers from this reservoir. First of all, the SOA is supplied with an injection current. This current adds carriers to the reservoir and is modeled by a rate $j(t)$.

Secondly, stimulated emission affects the number of carriers in addition to its effect on the number of photons in the cavity. Since each stimulated emission event consumes a carrier and produces a photon, the rate at which carriers are consumed is
equal to the rate at which photons are generated. The latter can be found from (9) and (11). At steady state, stimulated emission compensates for the photons lost during a round trip, yielding a term $-\Gamma I$, where $J \equiv I_{cw} + J_{ccw}$ and $\Gamma \equiv (\exp(\gamma L) - 1)/\tau_r$ represents all the optical losses in the entire cavity except the SOA, i.e. including the output coupling losses. Small deviations from steady state result in a slightly modified loss rate for the carriers. Combined, both effects result in a carrier loss rate of $-(\Gamma + \xi_{cw} N(t)) I_{cw}(t) - (\Gamma + \xi_{ccw} N(t)) I_{ccw}(t)$.

All other processes that have an effect on the number of carriers in the SOA are grouped together. Their effect on the number of carriers is linearized around threshold with respect to the number of carriers. This yields an average carrier lifetime, $T$, and a resulting loss of carriers $n(t)/T$. Together, these terms yield

$$\dot{n}(t) = j(t) - n(t)/T - (\Gamma + \xi_{cw} N(t)) I_{cw}(t) - (\Gamma + \xi_{ccw} N(t)) I_{ccw}(t).$$  \hspace{1cm} (13)

By substituting $n(t) = n_{th} + N(t)$ and $J(t) = j(t) - n_{th}/T$, one obtains

$$\dot{N}(t) = J(t) - N(t)/T - (\Gamma + \xi_{cw} N(t)) I_{cw}(t) - (\Gamma + \xi_{ccw} N(t)) I_{ccw}(t).$$  \hspace{1cm} (14)

C. External optical feedback and noise

To the deterministic model, effects of EOF are added according to [30], where it is realized that light only couples from the cw into the ccw mode and not the other way around. This is only true if the effect of backscattering can be neglected. For weakly guiding waveguides, such as those used in modern InP platforms, this is generally true as the backscattering is approximately $-50 \text{dB mm}^{-1}$ [31]. The intensity of the slowly varying envelope, $I_{x}(t)$, and the phase, $\phi_{x}(t)$, are defined such that $E_{x}(t) = \sqrt{I_{x}(t)} \exp(\imath \phi_{x}(t))$ where $x$ represents the cw or ccw mode. By separating the real and imaginary parts of the rate equations a new set of equations is found for the intensity and phase of the light in both modes. Together with the rate equation for the carriers, this results in a set of five equations. Noise terms are added to these equations as was done in [32]. On average, intensity noise will contribute photons at a rate $R_s$ to each mode. The average contribution to the phase and the number of carriers equals $0$. We define $F_y$ to be the zero-average noise, where $y$ indicates the quantity to which the noise is added. Finally, this results in the equations

$$\dot{I}_{cw}(t) = \xi_{cw} N(t) I_{cw}(t) + R_s + F_{I,cw}(t)$$  \hspace{1cm} (15a)

$$\dot{I}_{ccw}(t) = (\xi_{ccw} N(t) - \Delta \Gamma) I_{ccw}(t) + R_s + F_{I,ccw}(t)$$  \hspace{1cm} (15b)

$$\dot{\phi}_{cw}(t) = \frac{1}{2} \xi_{cw} \alpha N(t) + F_{\phi,cw}(t)$$  \hspace{1cm} (15c)

$$\dot{\phi}_{ccw}(t) = \frac{1}{2} \xi_{ccw} \alpha N(t) + F_{\phi,ccw}(t)$$  \hspace{1cm} (15d)

Here $R_s$ is the rate of spontaneous emission into a single mode, $|\kappa| \equiv \sqrt{\Gamma_{iso} I_{cw} R/\tau_r}$ is the feedback rate, $\Gamma_{iso}$ is defined as the power transmission of the isolator for the cw mode relative to the transmission for the ccw mode and represents the isolation provided by the isolator, $T_{out}$ is the power transmission coupled out of the cavity by the output coupler, $R$ is the power reflection of the external reflector, $\arg \kappa$ is the change in phase the light accumulates while propagating outside the laser cavity. The feedback delay time, $\Delta \phi_{r}(t) \equiv \phi_{cw}(t - \tau) - \phi_{ccw}(t)$ is the phase difference between the field in the cw and ccw modes and $\psi_0 \equiv \arg \kappa - \omega_{th} \tau$ is the phase delay due to propagation outside of the laser cavity.

D. Steady state values

Using the model, the steady state values are obtained. It is assumed that the intensity and frequency of the light eventually reach a steady state, represented by a fixed point in the three dimensional state space spanned by $I_{cw}, I_{ccw}$ and $N$. Because of the noise terms, $F_y$, the laser will make minor excursions around this point. Assuming these excursions are small compared to any non-linearities in the laser, it is possible to find the steady state values from (15) by neglecting these small excursions, i.e. by taking $F_y(t) = 0$. It follows from (15a) that

$$\bar{N} = \frac{-R_s}{\xi_{cw} I_{cw}},$$  \hspace{1cm} (16)

where the bar denotes that the value is only valid in the steady state.

From (15c) and (15d) it follows that $\Delta \phi_{r}(t)$ becomes time independent in the steady state. Assuming $|\kappa| \sqrt{I_{cw}/I_{ccw}} \gg (\xi_{cw} - \xi_{ccw}) \alpha \bar{N}$, the time-independent solution of the resulting differential equation yields $\sin(\psi_0 + \Delta \phi_{r}) \approx 0$ and $\cos(\psi_0 + \Delta \phi_{r}) \approx 1$. The latter result can be used to simplify (15b), removing its dependency on the feedback phase. This can be explained because the EOF is the dominant effect coupling the two modes directly. This allows the phase of the ccw mode to lock to the phase of the cw mode. In other words, a change in feedback phase is accompanied by a change in the phase difference between the two modes. The resulting simplified, quadratic equation can then be solved for $I_{ccw}$ yielding

$$I_{ccw} = \left( \frac{\kappa \sqrt{I_{cw}} + \sqrt{\kappa^2 I_{cw} + 4 \left( \frac{\xi_{cw} R_s}{\xi_{cw} R_s + \Delta \Gamma} \right) R_s}}{2 \left( \frac{\xi_{cw} R_s}{\xi_{cw} R_s + \Delta \Gamma} \right)} \right)^2. \hspace{1cm} (17)$$

where it is worthwhile to notice that $I_{cw} = I_{cw} \kappa^2 / \Delta \Gamma^2$ for $R_s \perp 0$ as it provides an approximation for the ratio of optical power in both modes.

It is possible to express (15e) in terms of $I_{cw}$ using these results, and it is found that

$$0 = J - \Gamma \left( I_{cw} + I_{ccw}(I_{cw}) \right) - N(I_{cw}) \left( \frac{1}{\tau_r} + \xi_{cw} I_{cw} + \xi_{ccw} I_{ccw} \right)$$  \hspace{1cm} (18)
Table I
VALUES USED FOR PHYSICAL QUANTITIES

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differential gain</td>
<td>( \xi )</td>
<td>( 10^{3} )</td>
<td>s(^{-1} )</td>
</tr>
<tr>
<td>Injection current over threshold</td>
<td>( J )</td>
<td>( 10^{17} )</td>
<td>s(^{-1} )</td>
</tr>
<tr>
<td>Loss rate</td>
<td>( \Gamma )</td>
<td>( 10^{13} )</td>
<td>s(^{-1} )</td>
</tr>
<tr>
<td>Carrier life time</td>
<td>( T )</td>
<td>( 10^{-9} )</td>
<td>s(^{-1} )</td>
</tr>
<tr>
<td>Spontaneous emission rate</td>
<td>( R_s )</td>
<td>( 10^{13} )</td>
<td>s(^{-1} )</td>
</tr>
<tr>
<td>Feedback delay time</td>
<td>( \tau )</td>
<td>( 10^{-7} )</td>
<td>s</td>
</tr>
<tr>
<td>Cavity length</td>
<td>( L )</td>
<td>2 cm</td>
<td></td>
</tr>
<tr>
<td>Relative transmission of isolator</td>
<td>( T_{iso} )</td>
<td>3 dB</td>
<td></td>
</tr>
<tr>
<td>Outcoupler transmission</td>
<td>( T_{out} )</td>
<td>3 dB</td>
<td></td>
</tr>
<tr>
<td>External reflectivity</td>
<td>( R )</td>
<td>(-20)</td>
<td>dB</td>
</tr>
<tr>
<td>Group index</td>
<td>( \mu_e )</td>
<td>3.7</td>
<td></td>
</tr>
<tr>
<td>Linewidth enhancement parameter</td>
<td>( \alpha )</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Numerically solving this equation yields a value for \( \bar{I}_{cw} \) which can subsequently be used to find \( I_{cw} \) and \( N \).

The output power is studied numerically, using the parameter values indicated in Table I. Most of these values are typical for semiconductor lasers. To give a feeling for the magnitude of these numbers, the following equalities might prove insightful. The injection rate, \( J \), is equivalent to an injection current of approximately 16 mA over the threshold current. The photon lifetime, \( 1/\Gamma \), is equivalent to a loss of 6.5 dB cm\(^{-1} \). The feedback delay time, \( \tau \), is equivalent to a reflection at approximately 10 m from the laser. To improve process tolerances, coupling of the ring cavity to the output waveguide is done using multimode interferometers. These components usually act as 3 dB-splitters, which explains the number for \( T_{out} \).

The cavity length, \( L \), is taken rather long, and is usually on the order of 100 \( \mu \)m. Inclusion of an isolator such as presented in [25] does however require the addition of several components to the laser cavity. These are two phase modulators and a spectral filter. The phase modulators each need to sinusoidally modulate the optical phase with an amplitude of approximately 1.2 rad at a frequency of several GHz requiring a length of several mm each. The spectral filter requires a free spectral range of several GHz and can be implemented using serial Mach-Zehnder filters. This requires about 1 cm of waveguides. Finally the SOA needs to supply more gain to compensate for the extra losses that are introduced by the phase modulators and the spectral filter increasing its length. 2 cm is therefore taken as a more appropriate cavity length.

The simulation results for \( I_{cw} \) are found in Fig. 2. This figure shows trends that are expected. \( I_{cw} \) decreases for increasing amounts of EOF, while an increase in isolation counteracts this change. As EOF increases, \( I_{cw} \) will start to grow, consuming a number of carriers while doing so. Since both the cw and ccw modes interact with the same carrier population, this results in a reduction of \( I_{cw} \). When the laser receives strong EOF compared to the gain difference inside the cavity, the ccw mode will become the lasing mode. In this regime the cw mode provides very little output power and it is therefore not interesting to operate the laser in this regime. If the amount of isolation is adequate, this effect becomes negligible however.

V. SMALL SIGNAL ANALYSIS

To predict the dynamic behavior of the laser, the changes in field and carriers are linearized around steady state. Assuming the fluctuations in field strength and carrier number are small compared to their steady state values, the laser is best modeled by a system of linear equations that is driven by the noise forces. The problem is subsequently converted to the frequency domain by making use of a Fourier transform, \( 2\pi x(t) = \int \exp(i\omega t)x(\omega)\mathrm{d}\omega \). Finally it is realized that the deterministic phases of both modes lock to each other owing to (15d), meaning that \( \bar{I}_{cw}, \bar{I}_{ccw} \) and \( \bar{N} \) are all independent of \( \dot{\phi}_{cw} \) and \( \dot{\phi}_{ccw} \). This allows for a study of the behavior of the intensities and carrier number independent of the phases and results in

\[
\begin{pmatrix}
F_{1,cw}(\omega) \\
F_{1,ccw}(\omega) \\
F_N(\omega)
\end{pmatrix} = Z
\begin{pmatrix}
d\bar{I}_{cw}(\omega) \\
d\bar{I}_{ccw}(\omega) \\
dN(\omega)
\end{pmatrix},
\]

where

\[
Z =
\begin{pmatrix}
(i\omega - \xi_{cw}\bar{N}) & 0 & -\xi_{cw}\bar{I}_{cw} \\
\zeta_{ccw,cw} & \zeta_{ccw,ccw} & -\zeta_{ccw}\bar{I}_{ccw} \\
\zeta_{cw,cw} & \zeta_{cw,ccw} & \zeta_{cw}\bar{N}
\end{pmatrix},
\]

\[
\zeta_{ccw,cw} = -\frac{1}{2}\frac{\mu}{\pi}\left|\frac{\omega}{\omega_c}\right|^2 \exp(-i\omega t),
\zeta_{ccw,ccw} = i\omega - \xi_{ccw}\bar{N} + \frac{1}{2}\Delta\Gamma
\]

and \( \zeta_{cw,cw} = i\omega + \frac{1}{2} + \frac{\mu}{\pi}\frac{\omega}{\omega_c}\bar{I}_{cw} + \xi_{cw}\bar{I}_{cw} \). Let the first vector be called \( \mathbf{F} \), the matrix \( Z \) and the last vector \( d\mathbf{X} \). \( Z \) is the linearized coupling between the various deviations from steady state and is found from (15). It is now possible to express \( d\mathbf{X} \) in terms of \( \mathbf{F} \) and \( Z \) by matrix inversion, e.g. through Cramer's rule, which yields \( d\mathbf{X} = \mathbf{YF} \). The components of \( \mathbf{Y} \) that are used in this paper can be found in the Appendix.

A. Relative Intensity Noise

From \( d\mathbf{X} \) it is possible to find the RIN spectrum which is defined as

\[
\text{RIN}(\omega) \equiv \frac{|d\bar{I}_{cw}(\omega)|^2}{\bar{I}_{cw}^2}
\]

and where

\[
d\bar{I}_{cw}(\omega) = \nu_{cw,cw}(\omega)F_{1,cw}(\omega) + \nu_{cw,ccw}(\omega)F_{1,ccw}(\omega) + \nu_{cw,N}(\omega)F_N(\omega)
\]
following from the matrix inversion. It is apparent that terms of the form $F^*_{\text{cw}, \omega}(\omega)F_{\text{iso}, \omega}(\omega)$ will appear when calculating the RIN. These are found using the procedure outlined in [32], and result in

$$\langle |F_{\text{iso}, \omega}|^2 \rangle = R_s(2\bar{I}_s + 1)$$ \hfill (23a)
$$\langle |F_{\text{cw}, \omega}|^2 \rangle = R_s(2\bar{I}_s + 1)$$ \hfill (23b)
$$\langle |F_{\phi, \omega}|^2 \rangle = \frac{R_s}{2\bar{I}_s}$$ \hfill (23c)
$$\langle |F_{N, \omega}|^2 \rangle = 2R_s$$ \hfill (23d)
$$\langle F_{\text{cw}, \omega}^*F_{\omega} \rangle = -R_s$$ \hfill (23e)
$$\langle F_{\text{cw}, \omega}^*F_{\omega} \rangle = -R_s.$$ \hfill (23f)

Note that $R_s$ is the rate of spontaneous emission into one mode, and consequently a factor 2 is found in (23e).

The components of $\Upsilon$ together with (23) now provide a means for calculating the RIN spectrum. The equations for the gain difference $\Delta \Gamma$ and feedback rate $\kappa$ are expressed in terms of the reflectivity of the external reflection causing the feedback, $R$, and the relative transmission of the isolator, $T_{\text{iso}}$. It is realized that this number can be related to the gain difference induced by the isolator as $T_{\text{iso}} = \exp(-\Delta \Gamma L)$ and it is found from the definition of $\Delta \Gamma$ that $\Delta \Gamma = 2(1 - \sqrt{T_{\text{iso}}})/\tau_r$. The feedback rate signifies the number of photons that return to the laser cavity each second, relative to the number of photons present in the cavity. During each round trip of the light, a fraction $T_{\text{out}}$ out will couple out of the cavity. A fraction $R$ of this light will be reflected back towards the laser and finally a fraction $T_{\text{out}}$ of the reflected light will couple back into the cavity. In the cavity the light first passes through the isolator before it enters the SOA. Since most of the interactions between the modes and carriers occur in the SOA, only the fraction of photons that passes the isolator is considered as EOF. It then follows that $\kappa = \sqrt{T_{\text{iso}}T_{\text{out}}R/\tau_r}$.

The previous results are studied numerically to find effects of EOF on the RIN and linewidth of the laser. To this end (22) is substituted into (21) which can then be solved using $\bar{I}_s$, $\bar{I}_{\text{cw}}$ and $\bar{N}$ obtained from the steady state analysis. As with the steady state analysis, the parameter values indicated in Table 1 were used unless indicated otherwise.

The RIN spectrum is found as shown in Fig. 3. From this figure it is apparent that the amount of EOF only significantly influences the RIN at low frequencies. To highlight this dependency, Fig. 4 shows the RIN in the limit for $\omega \downarrow 0$ as a function of $R$ and $T_{\text{iso}}$. This figure shows that the low frequency RIN is fairly constant for small values of $R$. After some maximum feedback strength, the low frequency RIN starts to increase rapidly however. This point is dependent on $T_{\text{iso}}$ and from this we can derive the maximum amount of EOF that is adequately suppressed by the isolator. For an isolation of 10 dB and feedback of $-0.1$ dB, i.e. almost all of the laser light is reflected back into the cavity, the RIN only increases by a factor of 2. When compared to the 60 dB isolation traditionally required to fully suppress the effects of EOF, this is a substantial improvement.

![Figure 3. RIN spectrum for various values of $R$, using the values specified in Table 1 for the intracavity isolation and other parameters. The relaxation oscillation frequency is clearly visible as the peak. Especially for higher amounts of feedback, there is a clear ripple in the spectrum. This ripple is caused by a resonance via the external reflection, the ccw mode and the carriers. The frequencies at which these ripples occur are directly related to the feedback delay time $\tau_F$. For lower and higher frequencies the RIN changes in a similar way as for the frequencies shown. Increasing amounts of EOF result in an increase in the RIN at low frequencies.](image1)

![Figure 4. Low frequency RIN as a function of $R$ and $T_{\text{iso}}$. The red curve corresponds to the values in Fig. 3 for $\omega \downarrow 0$. It is clear that higher amounts of isolation provide a higher immunity to EOF.](image2)

### B. Linewidth

The linewidth of the laser is obtained by considering the deviations from the steady state in (15c). It is found that

$$d\nu(\omega) = \frac{1}{2\pi} \left( \frac{1}{2} \xi_{\text{cw}} d\nu(\omega) + F_{\phi, \omega}(\omega) \right).$$ \hfill (24)

The first term of this equation can be expanded using $d\mathbf{X} = \Upsilon \mathbf{F}$. The power spectral density of the instantaneous frequency deviation is then equal to $|d\nu(\omega)|^2$ and the linewidth is its limit for $\omega \to 0$. The noise correlations in the resulting equation are found according to equations 23. The power spectral density of the frequency noise is then found as in Fig. 5.

The linewidth is equal to the frequency noise for $\omega \downarrow 0$. Fig. 6 shows its value as a function of both $R$ and $T_{\text{iso}}$. It is clear from this picture that even for very small isolation, the changes in linewidth remain small. Furthermore, the change in linewidth can almost completely be attributed to the reduction of the power in the mode, $\bar{I}_{\text{cw}}$, due to cross-gain saturation. It follows approximately a $1/\bar{I}_{\text{cw}}$ trend, similar to what would be expected for a Fabry-Perot laser. Even for $R = 1$, 3 dB of isolation is sufficient to reduce the change in linewidth to approximately 18%. 10 dB of isolation even reduces this number to $1\%$. As with the RIN this is in stark contrast to the 60 dB that is traditionally required.
In this paper a novel type of integrated laser was theoretically analysed. The cavity of this laser includes a weak optical isolator to provide a gain difference between the two propagation directions. This forces the laser to lase unidirectionally. Because the EOF returns to another, sub-threshold mode, external cavity modes are non-existent in this type of laser and the influence of EOF on the laser output is very small.

Since the required isolation is less than 10 dB, the isolator can be implemented on-chip. The output coupler can be implemented using a multi-mode interferometer and the spectral filter can be fabricated using a series of asymmetric Mach-Zehnder interferometers as was done in [33]. As all components of the laser have already been demonstrated, this type of laser is a viable candidate for realization.

The RIN spectrum and linewidth were calculated as indicators of the stability of the laser output. Both of these properties were obtained as a function of both the feedback rate and the intracavity isolation by utilizing a rate equation analysis. From this analysis it follows that only 3 dB of intracavity isolation is sufficient to make the laser immune to ~30 dB of EOF. For 10 dB of isolation it is found that EOF as strong as ~0.1 dB does not affect the laser to a great extend. In that case the changes in RIN are limited to a factor 2 and are calculated to be $6.4 \times 10^{-17}$ rad$^{-1}$. The intensity of the lasing mode and its linewidth are affected by approximately 1%.

The main advantage of integrating the isolator inside the laser cavity as opposed to outside the cavity is the greatly reduced amount of isolation required, making it much easier to realize. Such a weak isolator also shows less insertion loss. This can potentially be leveraged to increase the wall plug efficiency of the laser, while maintaining feedback insensitivity.

### Appendix

The small signal coefficients used in this paper can be found by inverting matrix $Z$ from (19). They yield

$$Z^{-1} = \mathbf{Y}$$

$$= \begin{pmatrix} u_{cw,cw}(\omega) & u_{cw,ccw}(\omega) & u_{cw,N}(\omega) \\ u_{ccw,cw}(\omega) & u_{ccw,ccw}(\omega) & u_{ccw,N}(\omega) \\ u_{N,cw}(\omega) & u_{N,ccw}(\omega) & u_{N,N}(\omega) \end{pmatrix}$$

where

$$u_{cw,cw}(\omega) = \frac{1}{\Delta} \left( i\omega + \frac{1}{T} + \xi_{cw}I_{cw} + \xi_{ccw}I_{ccw} \right)$$

$$u_{cw,ccw}(\omega) = -\frac{1}{\Delta} \left( i\omega - \xi_{ccw}N + \frac{\Delta \Gamma}{2} \right)$$

$$u_{cw,N}(\omega) = \frac{1}{\Delta} \left( i\omega - \xi_{ccw}N + \frac{\Delta \Gamma}{2} \right)$$

$$u_{N,cw}(\omega) = -\frac{1}{\Delta} \left( i\omega - \xi_{ccw}N + \frac{\Delta \Gamma}{2} \right)$$

$$u_{N,ccw}(\omega) = -\frac{1}{\Delta} \left( i\omega - \xi_{ccw}N + \frac{\Delta \Gamma}{2} \right)$$

$$u_{N,N}(\omega) = \frac{1}{2} i\omega \Delta \Gamma - \omega^2 - i\omega N (\xi_{cw} + \xi_{ccw}) +$$

$$\xi_{cw} \xi_{ccw} N^2 - \frac{1}{2} \xi_{ccw} N \Delta \Gamma$$

are the components of this matrix that are used in this paper to calculate the RIN and linewidth of the cw mode. $\Delta$ is the determinant of $Z$.

### References


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