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Citation for published version (APA):
https://doi.org/10.1103/PhysRevE.96.062611

DOI:
10.1103/PhysRevE.96.062611

Document status and date:
Published: 18/12/2017

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

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Diffusing-wave spectroscopy in a standard dynamic light scattering setup

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(Received 11 September 2015; revised manuscript received 15 June 2017; published 18 December 2017)

Diffusing-wave spectroscopy (DWS) extends dynamic light scattering measurements to samples with strong multiple scattering. DWS treats the transport of photons through turbid samples as a diffusion process, thereby making it possible to extract the dynamics of scatterers from measured correlation functions. The analysis of DWS data requires knowledge of the path length distribution of photons traveling through the sample. While for flat sample cells this path length distribution can be readily calculated and expressed in analytical form; no such expression is available for cylindrical sample cells. DWS measurements have therefore typically relied on dedicated setups that use flat sample cells. Here we show how DWS measurements, in particular DWS-based microrheology measurements, can be performed in standard dynamic light scattering setups that use cylindrical sample cells. To do so we perform simple random-walk simulations that yield numerical predictions of the path length distribution as a function of both the transport mean free path and the detection angle. This information is used in experiments to extract the mean-square displacement of tracer particles in the material, as well as the corresponding frequency-dependent viscoelastic response. An important advantage of our approach is that by performing measurements at different detection angles, the average path length through the sample can be varied. For measurements performed on a single sample cell, this gives access to a wider range of length and time scales than obtained in a conventional DWS setup. Such angle-dependent measurements also offer an important consistency check, as for all detection angles the DWS analysis should yield the same tracer dynamics, even though the respective path length distributions are very different. We validate our approach by performing measurements both on aqueous suspensions of tracer particles and on solidlike gelatin samples, for which we find our DWS-based microrheology data to be in good agreement with rheological measurements performed on the same samples.

DOI: 10.1103/PhysRevE.96.062611

I. INTRODUCTION

Since its development, diffusing-wave spectroscopy (DWS) [1–3] has proven to be an important and versatile tool for studying the dynamics, mechanics, and structure of a wide range of soft materials [4–11]. By taking advantage of the fact that the transport of photons through an optically turbid sample can be described as a diffusion process, DWS extends dynamic light scattering (DLS) measurements to the highly multiple scattering regime. It thus enables access to the dynamics of a material at very short time and length scales. The method is particularly useful when combined with the concept of microrheology, where information on the dynamics of tracer particles added to a material are used to extract information on the material’s viscoelastic properties [12–16]. However, the proper analysis of any DWS measurement requires detailed knowledge of the path length distribution $P(s)$ for photons traveling through the sample to the detector. For a number of sample geometries and experimental situations, the calculation or determination of $P(s)$ has been described in previous studies, including for the situation of backscattering from a flat sample cell of infinite thickness, or for transmission through cone-plate cells or flat circular cells of finite diameter and thickness [9,17,18]. Importantly, for sample cells in the shape of a flat slab of thickness $L$, infinitely extended in height and width, $P(s)$ can be expressed in analytical form, and the analysis of DWS data is therefore straightforward [3,17].

For the cylindrical sample cells used in conventional dynamic light scattering setups, however, an analytical expression for $P(s)$ is not available. DWS measurements are therefore usually performed in dedicated instruments that use flat sample cells.

In this paper, we show how DWS measurements can be performed in a standard dynamic light scattering setup, using cylindrical sample cells. We perform simple numerical random-walk simulations to account for the propagation of photons through a cylindrical cell and describe how this information is used to obtain the mean-square displacement of tracer particles from the temporal autocorrelation functions determined in experiments. We further show that by performing measurements at different detection angles, the range of accessible time and length scales can be extended; this is in analogy to standard DWS measurements employing a range of different cell thicknesses. Importantly, our approach also provides a valuable consistency check, especially in the context of microrheology, since measurements taken at different detection angles should yield the same viscoelastic response, even though the corresponding correlation functions must be very different due to the variation in geometry and average path length.

In analogy to a conventional DWS measurement, the transport mean free path $l^* l^*$ is determined by a calibration measurement on tracer particles of well-known uniform size,

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In analogy to a conventional DWS measurement, the transport mean free path $l^*$ is determined by a calibration measurement on tracer particles of well-known uniform size,
suspended in a Newtonian liquid of known viscosity; the expected single-particle dynamics is thus known a priori. Alternatively, the transport mean free path can also be directly determined from the measured scattering intensity as a function of angle, $I(\theta)$ if an initial experimental calibration is combined with results from our simple numerical calculations. In the highly multiple scattering limit, and in the absence of absorption in the sample, $I(\theta)$ is well approximated by a function that depends only on $I^*$, on the corresponding calculated path length distribution $P(s)$, and on a constant $\beta_{\exp}$ that is determined by the experimental setup.

We thus demonstrate that standard goniometer light scattering setups can be successfully used for DWS measurements. Compared to dedicated DWS setups, our method has the advantage of being able to reliably determine the transport mean free path $I^*$ as well as to extend the range of accessible length and time scales, using only a single cylindrical sample cell.

We illustrate and test the use of our approach by performing DWS-based microrheology measurements on a typical solid-like soft material, gelatin, and find the resulting frequency-dependent viscoelastic moduli to be in very good agreement with separately performed rheological measurements.

II. EXPERIMENTAL: MATERIALS AND METHODS

A. Background on DLS, DWS, and microrheology

Standard static and dynamic light scattering experiments are limited to samples that exhibit very little multiple scattering, with the overwhelming majority of detected photons having been scattered only a single time within the sample. A typical setup for such single scattering experiments uses a cylindrical sample cell that is illuminated by a laser, as shown schematically in Fig. 1(a). The detector, typically comprising an optical fiber that is coupled to a photomultiplier tube, can be positioned at a range of detection angles $\theta$, corresponding to scattering wave vectors $q(\theta) = \frac{2\pi}{\lambda} \sin(\theta/2)$, where $n$ is the index of the material and $\lambda$ is the wavelength of the laser in vacuum. For single scattering, the fluctuations in the detected intensity, which reflect the dynamics of the scatterers, are then quantified by the temporal intensity autocorrelation function,

$$g_2(t) = \frac{\langle I(\tilde{t} + t) I(\tilde{t}) \rangle}{\langle I(\tilde{t})^2 \rangle}, \quad (1)$$

where $t$ is the lag time and the brackets $\langle \cdots \rangle$ indicate a time-average over all times $\tilde{t}$. The field autocorrelation function $g_1(t)$, measured at a wave vector $q$, reflects the temporal fluctuations of the electric field. It can be related, via the so-called Siegert relation, to the intensity correlation function, as $g_1(t) \approx \sqrt{g_2(t) - 1}/\beta$, where $\beta$ is the coherence factor [19], a constant that depends on the experimental setup. For a Gaussian distribution of displacements $\Delta x$, the field correlation function $g_1(t)$ is directly linked to the dynamics of scatterers in the sample, as

$$g_1(t) = e^{-\frac{1}{2} \langle \Delta x^2(t) \rangle}, \quad (2)$$

where $\langle \Delta x^2(t) \rangle$ is the time-dependent mean-square displacement of scatterers in the material. For the simplest example, where the scatterers are uniformly sized particles suspended in a Newtonian liquid, the particles undergo ideal Brownian motion, and thus $\langle \Delta x^2(t) \rangle \approx 6Dt$, where $D$ is the particle diffusion coefficient in three dimensions. For this case, the field correlation function has a single exponential form, $g_1(t) = e^{-t/\Gamma}$, where the $q$-dependent decay rate $\Gamma = Dq^2$ is set by $D$.

**Diffusing-wave spectroscopy** (DWS) is an extension of dynamic light scattering measurements to the highly multiple scattering regime. A typical experimental setup for DWS is shown in Fig. 1(b). In contrast to conventional DLS measurements, this technique requires that photons are scattered

![FIG. 1](image-url). Schematic experimental setup for typical dynamic light scattering measurements. (a) Standard dynamic light scattering (DLS) setup employing a cylindrical sample cell and a goniometer, which enables accessing different scattering angles $\theta$. (b) Diffusing-wave spectroscopy (DWS) setup in transmission geometry. The pathways of photons are well-described by a random walk of step size $L^*$. (c) Schematic of random-walk simulations in a cylindrical geometry. The detection angle $\theta$ is defined as for conventional DLS; here it determines the distance $L(\theta)$ between the points of entry and exit of detected photons. (d) Expected distribution $p(z)$ of the $z$ coordinate where a photon exits the cylinder; a fraction $\alpha_z$ (marked area) reaches the detector.
many times, before they reach the detector. For this highly multiple scattering regime, the propagation of photons through the sample can be adequately described as a simple diffusion process, where the details of each single scattering event are no longer relevant. This photon diffusion processes can be accounted for by a single parameter, the so-called mean free path \( l^* \). This characteristic length scale is defined as the average distance a photon travels in the sample before its direction of propagation is randomized. The path of photons through the sample can thus be approximated as an ideal random walk with step size \( l^* \). For such a random walk the path length of photons, and the number of randomizing scattering events, is no longer uniform, as is the case for single scattering. Instead, the correlation function measured in an experiment is determined by contributions from all path lengths \( s \) weighted by the path length distribution \( P(s) \), as

\[
[g_1(t)] = \int_0^\infty P(s)e^{-\frac{k_0^2}{4\sigma^2}(\Delta r^2(t))}r^ds,
\]

where \( k_0 = 2\pi n/\lambda \) is the magnitude of the photon wave vector in the sample and \( s/l^* \) reflects the number of randomizing scattering events experienced by a photon with path length \( s \) [3]. The basis for this simple form of Eq. (3) is that each of the approximately \( s/l^* \) randomizing scattering events contributes to a change of this particular photon path by a squared distance of \( \langle \Delta r^2(t) \rangle \), leading to a partial decorrelation of \( g_1(t) \). The cumulative decorrelations from all these randomizing scattering events thus lead to the functional form in Eq. (3). Knowledge on the path length distribution \( P(s) \) is therefore essential in the analysis of DWS measurements; without such knowledge the measured correlation functions cannot be related to the dynamics of the scatterers. The path length distribution depends sensitively on the geometry of the sample cell used in the experiment. For sample cells in the shape of a flat slab, infinitely extended in both height and width, \( P(s) \) can be expressed in analytical form as a function of \( l^* \) and the thickness \( L \) of the sample cell [3,17].

This is one of the main reasons why DWS measurements have typically relied on measurements performed in dedicated instruments, employing flat sample cells.

Such dedicated DWS instruments can also offer other important advantages, in particular for measurements on solidlike, nonergodic samples, where the measured, time-averaged correlation functions are not representative of the ensemble-averaged dynamics of the sample [20].

Methods for acquiring ensemble-averaged correlation functions in DWS measurements include the use of double-cell techniques, where either an ergodic sample with slow dynamics [21] or a slowly rotating opaque disk [22] is placed in front of the sample cell. Both these techniques create a slow randomization of the incoming photon paths, resulting in an ensemble-averaging of the collected temporal correlation functions. Either translations of the sample cell or rotations of an opaque disk can also be employed for ensemble-averaging using echo techniques [23], yielding ensemble-averaged correlation functions at long time scales and with excellent statistics [22,24]. While these ensemble-averaging techniques could in principle also be incorporated into a standard goniometer setup, we choose an alternative method, so-called Pusey-averaging. This method uses the measured time-averaged correlation function and the measured ensemble-averaged scattered intensity together with a simple theoretical treatment to provide the ensemble-averaged correlation function [25,26].

The ensemble averaged scattering intensity \( \langle I \rangle_e \) can be readily acquired in separate intensity measurements during which the sample is rotated; the measured dynamics is perturbed by the motion of the sample, but the average scattering intensity is still properly ensemble-averaged. Using the ratio of the ensemble-averaged to the time-averaged scattering intensities \( Y = \frac{\langle I \rangle_e}{\langle I \rangle} \), the ensemble-averaged field autocorrelation function \( g_1(t) \) can then be estimated as a function of the time-averaged correlation function as

\[
g_1(t) = \frac{Y - 1}{Y} + \frac{1}{Y}(\tilde{g}_2(t) - \sigma^2)\text{,}
\]

where \( \tilde{g}_2(t) = 1 + \frac{1}{\beta \sigma^2} + \frac{2}{\beta \sigma^2}t \) is the time-averaged intensity autocorrelation function normalized by the coherence factor \( \beta \), which is obtained from the separate ensemble-averaged measurements, and \( \sigma^2 = \tilde{g}_2(0) - 1 \) characterizes the short-time intercept of \( \tilde{g}_2(t) \).

We can now use the resulting ensemble-averaged field autocorrelation function \( g_1(t) \) to extract viscoelastic properties of the sample, using the micro rheology concept [7,12]. To do so, we employ the local power-law approximation developed by Mason et al. [15,16]. In brief, the method is based on the assumption that the Stokes-Einstein relation, which links the thermal motion of particles in a Newtonian liquid to the viscosity of the surrounding liquid, can be generalized to viscoelastic materials with frequency-dependent linear viscoelastic moduli. The approximation also neglects inertial effects on the motion of the probe particles, which is justified for most soft materials at frequencies below \( \approx 1 \) MHz.

By describing the time-dependent mean-square displacement as a local power law around each data point, the magnitude of the frequency-dependent complex modulus can be expressed in analytical form as

\[
|G^*(\omega)| \approx \frac{k_BT}{\pi a(\Delta r^2(1/\omega))\Gamma(1 + \alpha(1/\omega))},
\]

where \( a \) is the particle radius, \( k_BT \) the thermal energy, \( \alpha(t) = \frac{\partial \ln(\Delta r^2(\gamma))}{\partial \ln(\gamma)} \) is the logarithmic slope of the mean-square displacement as a function of lag time, and \( \Gamma \) denotes the \( \Gamma \) function.

B. Sample preparation

Polystyrene particles (micromod Partikeltechnologie GmbH, Germany) coated with a grafted layer \( M_w = 300 \) g/mol of poly(ethylene glycol) were used as tracer particles in the DWS measurements. The diameter of the particles is \( 1 \) \( \mu \)m and they are provided suspended in water at a concentration of 5 wt%. The test samples with tracers in water are prepared by mixing the stock particle suspension with deionized water (Milli-Q water, \( \sigma > 18 \) MΩ cm at 25°C), to obtain the desired tracer particle concentrations \( C_{\text{tracer}} \). To study the effect of the transport mean free path \( l^* \), which is expected to scale as \( l^* \propto 1/C_{\text{tracer}} \), we prepare a series of samples with particle concentrations ranging from \( C_{\text{tracer}} \approx 0.3 \) to 5 wt%.
power-law fit to the data (dashed line), yielding an exponent
the distance
s/l⋆\approx l⋆/R
room temperature. in the experiment, and subsequently allowed to cool down to
magnetic stirrer, transferred to the cylindrical sample cell used
variation of the detection angle from
3, ALV GmbH, Germany), equipped with a 50 mW solid-
in a static and dynamic light scattering setup (ALV CGS–
USA) and 1.25 wt% of tracer particles at elevated temperatures
5 wt% gelatin powder (type A, from porcine skin, Sigma,
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the detector. polarized and a horizontal polarizing filter placed in front of
in vertical-horizontal mode, with the incoming light vertically
from surfaces in the setup, our measurements are performed
FIG. 1(c) i st h u s
DWS requires a
DWS measurements. In particular, as a result of the long path
standard goniometer setups may be suitable for performing
in this paper could be performed with any standard goniometer
setup; however, not all laser sources that are included in
standard goniometer setups may be suitable for performing
DWS measurements. In particular, as a result of the long path
lengths of the photons through the sample, DWS requires a
laser source with a sufficiently long coherence length. Our
simulations indicate that for values of l*/R \approx 0.01, with
R = 4.33 mm, the largest relevant path length are around 1–3 m, as seen in Fig. 2(a). For the samples used in our experiments, the smallest value of l* (for the 2.5 wt% sample) was around 150 \mu m [see Fig. 3(g)], corresponding to l*/R \approx 0.35. For this most concentrated sample, the relevant path lengths are below \approx 1 meter, as seen in Fig. 2(a) for an experiment at detection angle \theta = 90°. For the laser used in our setup (WhisperIT 532 nm, Pavillion Integration Corporation, San Jose, CA), we have obtained specifications from the manufacturer, stating a spectral bandwidth of less than 100 MHz, corresponding to a coherence length of at least \approx 1 m. While a direct measurement

The aqueous gelatin gel is prepared by mixing water with 5 wt% gelatin powder (type A, from porcine skin, Sigma, USA) and 1.25 wt% of tracer particles at elevated temperatures of \approx 60 °C. The mixture is homogenized for 30 min using a magnetic stirrer, transferred to the cylindrical sample cell used in the experiment, and subsequently allowed to cool down to room temperature.

C. Light scattering experiments
All dynamic light scattering experiments are performed in a static and dynamic light scattering setup (ALV CGS–3, ALV GmbH, Germany), equipped with a 50 mW solid-state laser (\lambda = 532 nm) and a goniometer that allows for variation of the detection angle from \theta \approx 20° to \theta \approx 160°. Measurements are performed in cylindrical cells with outer diameter 10 mm and inner diameter 8.65 mm; the cell radius relevant to the propagation of photons in the sample cell [see Fig. 1(c)] is thus \bar{R} \approx 4.33 mm. Measurements of 30 s duration are performed at detection angles between 30° and 150° in steps of 10°. To minimize the detection of stray light, reflected from surfaces in the setup, our measurements are performed in vertical-horizontal mode, with the incoming light vertically polarized and a horizontal polarizing filter placed in front of the detector.

For the gelatin samples we use separate experiments on the same sample to determine the ensemble-averaged scattering intensities (I) as well as the coherence factor \beta needed for the Pusey averaging method. In these separate experiments the sample cells are slowly rotated during data acquisition; we perform three such measurements at each scattering angle, each lasting 10 s.

In principle, the light scattering measurements we describe in this paper could be performed with any standard goniometer setup; however, not all laser sources that are included in standard goniometer setups may be suitable for performing DWS measurements. In particular, as a result of the long path lengths of the photons through the sample, DWS requires a laser source with a sufficiently long coherence length. Our simulations indicate that for values of l*/R \approx 0.01, with R = 4.33 mm, the largest relevant path length are around 1–3 m, as seen in Fig. 2(a). For the samples used in our experiments, the smallest value of l* (for the 2.5 wt% sample) was around 150 \mu m [see Fig. 3(g)], corresponding to l*/R \approx 0.35. For this most concentrated sample, the relevant path lengths are below \approx 1 meter, as seen in Fig. 2(a) for an experiment at detection angle \theta = 90°. For the laser used in our setup (WhisperIT 532 nm, Pavillion Integration Corporation, San Jose, CA), we have obtained specifications from the manufacturer, stating a spectral bandwidth of less than 100 MHz, corresponding to a coherence length of at least \approx 1 m. While a direct measurement

FIG. 2. Simulation results. (a) Path length distribution P(s) for different values of l*/R, calculated at a fixed detection angle \theta = 90° and for a cell radius \bar{R} = 4.33 mm. (b) Master curve of scaled path length distributions, showing that the shapes of P(s) calculated at different l*/R are similar. (c) Corresponding average path lengths \bar{s} as a function of l*/R. As expected, we find a scaling \bar{s} \propto 1/l*\approx l⋆/R, as indicated by a power-law fit to the data (dashed line), yielding an exponent m = −0.975 ± 0.05. (d) Average number of scattering events \bar{\delta}/l*≈ L(\theta)^2 that would be expected for an unrestricted random walk.
of the coherence length of a laser requires setups that are typically only available in specialized optics laboratories, test measurements on samples with known tracer dynamics and path length distributions, as performed here, can serve as a test to probe the limitations of the coherence length of the laser. In practice, the coherence length of the laser used will thus set a limit on the sample concentration or scattering mean free path $l^*$ that can be characterized.

III. RESULTS AND DISCUSSION

A. Simulation of photon paths through the sample

To properly interpret experimental data in a setup with a cylindrical cell, the path length distribution $P(s)$ of photons traveling through the sample is required both as a function of the detection angle $\theta$ and the transport mean free path $l^*$. To achieve this, we perform numerical simulations of photons traveling through a cylindrical cell, assuming that they undergo an ideal random walk with step size $l^*$. In the two-dimensional coordinate system given in Fig. 1(c), photons are released at point $(x/R = -1 + l^*/R, y/R = 0)$, where $l^*$ is the transport mean free path and $R$ is the radius of the cylindrical cell. Subsequently, each photon is propagated in steps of $l^*/R$, where each step proceeds in a random (3D) direction. At the point where the photon exits the cell ($x^2 + y^2 > R^2$), we evaluate the number $M$ of scattering events and record the detection event with respect to the observed detection angle $\theta$.

We do this by dividing the surface of the cylindrical cell into $n_{\text{bins}}$ angular bins, spanning from 0 to 180° (taking into account the symmetry around the $x$ axis). In addition, to take into account the three-dimensional nature of photon transport in the real geometry, we consider the fact that each realization of a two-dimensional photon path represents a whole range of possible three-dimensional paths with an identical number of scattering events $M$ and identical $(x, y)$ paths. Since in the $z$ direction the photon also performs a (one-dimensional) random walk, we can readily express the probability distribution $p(z)$ for the photon to end up at a position $z$ after propagating $M$ random steps. What is relevant here is the fraction of those paths that will reach the detector, as illustrated in Fig. 1(d). We assume that all photons with $|z| < \Delta z$ are detected; in accord with the resolution of the angular bins, we set $\Delta z = \frac{\pi}{2n_{\text{bins}}}$. Then, the fraction $\alpha_c$ of

FIG. 3. Simulation results versus experiments for colloidal particles in water ($\phi = 0.625\%$, $d = 1\,\mu m$): Measured (blue circles) and calculated (red line) field correlation function $g_2(t)$ for a detection angle of $\theta = 50^\circ$ (a), $\theta = 90^\circ$ (b), and $\theta = 130^\circ$ (c). In (d) $\ln(g_2(t))$ as a function of $t$ for the same values of $\theta = 50^\circ$ (d), $\theta = 90^\circ$ (e), and $\theta = 130^\circ$ (f). (g) $l^*$ obtained from fitting simulation data to experiments, as a function of $\theta$ for different $\phi$. (h) The same $l^*$ values plotted as a function of $\phi$ for different values of the detection angle $\theta$; we find approximately $l^* \propto 1/\phi$, as illustrated by the dotted line. The data is also in fair agreement with predictions from Mie scattering theory [28–30], shown as a dashed line.
contributing three-dimensional paths is given as

\[ \alpha_z = \text{erf}\left(\frac{3\Delta z}{l^*\sqrt{M}}\right), \tag{6} \]

where erf is the error function, and \(\sqrt{l^*/3}\) is the effective one-dimensional step size in the \(z\) direction. We thus account for diffusion in the \(z\) direction in our statistics of path length distributions by, instead of adding 1, adding a contribution \(\alpha_z\) to the angular bin corresponding to each simulated photon path: \(f(n_{\text{bin}}) \to f(n_{\text{bin}}) + \alpha_z\).

Each bin thus represents a detection area of surface area \(A_{\text{bin}} \approx l^* R^2/\pi\). The cumulative value of each angular bin, after propagating \(N\) photons and normalizing by \(N\), thus defines a (dimensionless) scattering intensity as \(I_{\text{bin}} := f(n_{\text{bin}})/N\), representing the probability for a photon to reach the detection area corresponding to bin number \(n_{\text{bin}}\).

In addition to recording the angle where the photons end up, we also record, for each angle \(\theta(n_{\text{bin}})\), a distribution of the number of scattering events, by adding a contribution \(\alpha_z\) to a bin accounting for the number of scattering events at each angle \(\theta(n_{\text{bin}})\). The bins are linearly spaced, with bin number 100 representing a number of approximately \((L(\theta)/l^*)^2\) scattering events, which corresponds to an expected average number of scattering events for a distance \(L(\theta)\) between the entry point and the detection point of the photons.\(^1\) We use 300 bins per angle \(\theta(n_{\text{bin}})\), thus accounting for up to three times the expected typical number of scattering events; higher numbers, while not counted, in practice are extremely rare in our simulations and do not significantly affect the resulting path length distributions.

To achieve good statistics in the calculated path length distributions, the paths for a large number of photons have to be simulated. In the actual experiments, obtaining good statistics is usually not a problem, due to the enormous number of photons that are propagated.

In our experiments we use a laser with 50 mW of power at a wave length of 532 nm; this corresponds to \(\approx 10^{17}\) photons entering the sample cell, every second. Such numbers are beyond the capability of computer simulations; in comparison, for our calculations we typically simulate \(10^6\) photons propagating through the sample, which is enough to yield reasonable statistics, and relatively smooth calculated path length distributions. For details of our code for the random-walk simulations and for analyzing the experimental light scattering data, see Appendix B 1 or the online repository [27].

### B. Scaling properties of \(P(s)\)

Typical obtained simulation results for \(P(s)\) are shown in Fig. 2(a); these curves are calculated at a fixed angle \(\theta = 90^\circ\) for different values of \(l^*/R\). Interestingly, while the average path length decreases with increasing \(l^*\), the shapes of these path length distributions appear surprisingly similar.

In fact, we can overlay the curves from Fig. 2(a) and create a master curve, as shown in Fig. 2(b). To obtain this master curve, we have rescaled the path length with a factor \(\tilde{s}\) and multiplied the magnitude with the same factor; it turns out that \(\tilde{s}\) is the average path length, defined below in Eq. (7).

Any practical use of the calculated path length distributions requires that \(P(s)\) data are available for any arbitrary value of \(l^*\). To address this problem, we calculate path length distributions for different values of \(l^*/R\) and examine the scaling properties of these path length distributions.

In contrast to an ideal random walk, the path of photons through our cylindrical sample is constrained by the geometry. Nevertheless, the essential scaling properties of a random walk still hold approximately for the path length distributions simulated here. In particular, for an \textit{unrestricted} random walk of step size \(l^*\), we expect the mean-square displacement \(\langle \Delta R^2 \rangle\) to be given as \(\langle \Delta R^2 \rangle = Ml^*\), with \(M\) the number of steps. Considering the average path length,

\[ \tilde{s} := \int_0^\infty s P(s) \, ds, \tag{7} \]

we can estimate the average number of scattering events \(\tilde{M}\) to travel to a point at distance \(L(\theta)\) from the origin to be approximately given as \(\tilde{M} \approx \langle L(\theta) \rangle l^*\), as would be the case for a completely unrestricted random walk.

As the path length is \(s \approx Ml^*\), the average path length should scale with \(L(\theta)\) and \(l^*\) as \(\tilde{s} \approx L(\theta)^2/l^*\). Conversely, at fixed detection angle \(\theta\) and thus fixed distance \(L(\theta)\), we would clearly expect a scaling of \(\tilde{s} \propto 1/l^*\).

To test this scaling, we examine the \(P(s)\) data with respect to both \(l^*\) and the detection angle \(\theta\), where a variation of the latter corresponds to a variation of the distance \(L(\theta)\) between the entry and detection points of the photons. In Fig. 2(c) we plot the average path length \(\tilde{s}\) as a function of \(l^*/R\) for simulation data calculated at a single detection angle \(\theta = 90^\circ\). Indeed, the data is in excellent agreement with a scaling of \(\tilde{s} \propto 1/l^*\); the dashed line in Fig. 2(c) shows a power-law fit to the data, yielding an exponent of \(-0.975 \pm 0.05\). This scaling is a consequence of the self-similarity of random walks, which enables us to approximate each random walk with a “coarse grained” version of larger step size; this scaled random walk essentially follows the same path, but, as a result of the increased step size, exhibits a reduced contour length.

In contrast to this simple scaling as a function of \(l^*\), if we examine the average number of scattering events as a function of \(L(\theta)\), we find significant deviations from the naïvely expected scaling \(\tilde{s}/l^* \propto L(\theta)^2\), as shown in Fig. 2(d). The symbols in this figure show the simulation data for different values of fixed \(l^*/R\), and the solid lines show the simple prediction discussed above, a power law with exponent 2. In hindsight, it is clear that such deviations should be expected, as, in contrast to the \(l^*\)-dependence at fixed detection angle, a variation of \(L(\theta)\) implies a significant modification of the effective sample geometry. It is thus evident that calculations of \(P(s)\) at different detection angles are necessary. However, the simple scaling properties with respect to \(l^*\), highlighted in Fig. 2(b), can be exploited to obtain accurate path length distributions for arbitrary \(l^*\) values, based on simulations performed at a single value of \(l^*/R\).
C. Determination of $l^*$ for samples with known tracer dynamics

Typically, when DWS experiments are used to perform microrheology measurements, uniformly sized tracer particles are added to the soft material of interest. If the transport mean free path $l^*$ is known, the dynamics of these particles can be extracted, yielding direct information on the viscoelastic properties of the surrounding material. Vice versa, if the viscoelastic properties of the surrounding material are a priori known, then $l^*$ can be extracted from a DWS measurement. A requirement is that the particles scatter much more strongly than the material, ensuring that any detected dynamics are related only to the particle dynamics, and not to fluctuations within the surrounding sample. If this criterion is fulfilled, the simplest method for determining $l^*$ is to measure the scattering of uniformly sized, spherical particles, suspended in a Newtonian background liquid of known viscosity.

To test our approach, and for calibration of the transport mean free path $l^*$, we here perform a series of experiments using samples with different concentrations of uniformly sized polystyrene particles (coated with poly-ethylene glycol, $M_w \approx 300$ g/mol, 1 μm diameter, purchased from micromod GmbH, Germany) suspended in water. We measure the field autocorrelation function $g_1(t)$ of the scattered light for these samples and test how the correlation functions predicted from our photon path simulations compare to these experimental data. As shown in Figs. 3(a)–3(c), where we show data on a suspension of particles at a volume fraction $\phi = 0.625\%$ and detection angles $\theta = 50^\circ, 90^\circ$, and $130^\circ$, we obtain remarkably good agreement between experiments (shown as blue circles) and simulations (shown as red lines), where $l^*$ is the only adjustable parameter. While the dynamics is expected to be purely Brownian, with $<\Delta r^2(t)>$ increasing linearly with time $t$, due to the broad path length distribution of photons passing through the sample, $g_1(t)$ deviates significantly from the single exponential decay that would be observed in single scattering experiments. This can be more clearly seen in Figs. 3(d)–3(f), where $\ln[g_1(t)]$ is plotted as a function of time; in such a plot, an exponential decay would appear as a straight line, as illustrated by the dashed lines, which show exponential fits to the short-time regime of $g_1(t)$. The nonexponential shape of the data thus becomes evident and is captured very well by the curves predicted from our simulations, shown as red lines. Fits performed for the same sample, but at different angles, should yield the same $l^*$ values. Indeed, we obtain good agreement between the $l^*$ values extracted from the data in Figs. 3(a)–3(c); we obtain $l^* = 540$ μm, $l^* = 533$ μm, and $l^* = 523$ μm at angles of $\theta = 50^\circ, 90^\circ$, and $130^\circ$, respectively. In fact, we obtain good agreement between measurements taken at different angles for all the concentrations studied, with volume fractions ranging from $\phi = 0.313\%$ to 2.5%. As shown in Fig. 3(g), the fitted $l^*$ values as a function of $\theta$ exhibit only small variations. Somewhat larger deviations are observed for the sample with the lowest concentration, at the largest detection angles. We attribute this to the fact that this sample has the longest $l^*$, combined with the shortest distances $L(\theta)$ between entry point and exit point of the photons; $l^* \approx 1$ mm and $L(\theta) \approx 2.2$ mm and thus $L(\theta)/l^* \approx 2.2$. In this case the path length of photons is no longer adequately described as an ideal random walk.

Besides these discrepancies at small values of $L(\theta)$, the fitted $l^*$ values depend only on the volume fraction, irrespective of the detection angle. To examine the $\phi$ dependence of the data, in Fig. 3(h) we plot $l^*$ as a function of $\phi$, observing approximately the expected scaling $l^* \propto 1/\phi$ [31], as indicated by the dotted line. The data is also in fair agreement with Mie scattering calculations plotted as a dashed line in Fig. 3(h) [29,30]. The calculations are performed using the web application available on the website of LS Instruments, Switzerland [28], using as input parameters the particle size, the wavelength of the laser $\lambda = 532$ nm, as well as a refractive index of $n_{PS} = 1.598$ for the particles and $n_{H2O} = 1.33$ for water.

D. Obtaining $l^*$ from intensity measurements

In the absence of absorption, the intensity detected at each angle should be fully determined by the transport mean free path of photons in the sample. While absorption is relatively straightforward to include in the current data analysis, here we choose to neglect its effects since absorption is relatively weak in the aqueous samples studied; the typical absorption length is much longer than the typical path length of photons through the samples. As a result, after calibration using a reference sample of known dynamics, a simple measurement of the scattering intensity at different angles on the sample of interest is sufficient for determining its $l^*$. To validate this, we compare the scattering intensities predicted from the simulations with those measured in experiments performed on our polystyrene suspensions.

In Fig. 4(a) we plot the scattering intensity $I_{\text{sim}}$ as a function of detection angle $\theta$, as predicted from the photon path simulations. The shape of these curves is very different from those typically obtained in single scattering experiments on dilute suspensions, where generally the intensity is highest at small detection angles, corresponding to low $q$ values. In the highly multiple scattering regime, however, the intensity is generally highest for detection points closest to the entry point of photons into the sample, which corresponds to large $\theta$ values.

Importantly, we find that the angular dependence of the recorded scattered intensity for the tracer particle suspensions agrees remarkably well with the behavior predicted from our simulations, as shown in Fig. 4(b). For both simulations and experiments, we observe a ratio of $\approx 40$ between the intensities measured at $\theta = 30^\circ$ and $\theta = 150^\circ$. Moreover, comparing Fig. 4(b) with Fig. 4(a), we observe that the shapes of the simulated intensity curves are very similar to those of the experimental data. In fact, the two data sets can be superposed simply by scaling the simulated curves with single factor $\beta_{\text{exp}}$, the value of which depends on experimental parameters such as the size of the detection area, and the distance between the detector and sample. We find that good agreement between the measured and simulated curves is obtained for a value of $\beta_{\text{exp}} \approx 8 \times 10^{10}$ Hz, as shown in Fig. 4(c).

Thus, given $\beta_{\text{exp}}$, a good estimate of $l^*$ can be determined for a sample of unknown properties solely by measuring the scattered intensity at different angles.

E. Test on a viscoelastic material

Finally, we test the use of our method in the context of microrheology, where the measured correlation functions and
the corresponding tracer particle mean-square displacements $(\Delta r^2(t))$ are used for determining the viscoelastic properties of a sample. As a test material we use a common solid-like soft material, an aqueous gelatin gel at a concentration of 5 wt%. The plateau storage modulus of this material is on the order of 1 kPa, which means that for the case of micron-sized tracer particles we need to be able to access particle displacements at subnanometer length scales. DWS is ideally suited for this, since the displacements of all the tracer particles encountered by a photon on its path through the sample cumulatively contribute to changing the total photon path length.

We perform measurements on the gelatin sample for detection angles ranging from $\theta = 30^\circ$ to $\theta = 150^\circ$. The sample is highly nonergodic, as indicated by the fact that the intercept of the measured intensity correlation functions $g_1(t)$ varies significantly between measurements. We therefore use the Pusey-averaging procedure to obtain a good estimate of the ensemble averaged concentration functions from the measured, time-averaged concentration functions, as outlined in the experimental section. As expected, and shown in Fig. 5(a), the resulting ensemble-averaged field correlation functions $g_1(t)$ vary with the detection angle $\theta$. These correlation functions do not decay significantly; they reach a plateau at values of $g_1(t) > 0.9$ at the longest time scales accessed in the experiments. This reflects the fact that the gelatin sample has a relatively high modulus and the thermal motion of the tracer particles is therefore limited to short length scales. Using the procedure outlined above, we obtain the transport mean free path $l^*$ of the gelatin sample directly from the measured scattered intensities, using the intensity scaling factor $\beta_{\text{exp}}$ as determined from the measurements on our pure tracer suspensions. The mean-square displacements of tracer particles in the gelatin sample are obtained by numerically inverting Eq. (3), using the measured $g_1(t)$, $l^*$, and the calculated $\theta$-dependent path length distribution $P(s)$ as input. The corresponding mean-square displacements are shown in Fig. 5(b). Given the highly nonergodic nature of the sample studied, the data taken at different detection angles are in fair agreement; note that the magnitudes of the accessed particle displacements are in the subnanometer range. We can now convert these data to viscoelastic moduli, using the microrheology concept [7,12] and the local power-law approximation [15,16], developed by Mason et al. The magnitude of the resulting complex modulus $|G^*(\omega)|$ is on the order of 1 kPa and depends only weakly on frequency, as shown in Fig. 5(c). The curves obtained for different detection angles exhibit significant variations, as shown in the inset, where we plot the low frequency plateau values $G^* = |G^*(\omega = 5 \text{ rad/s})|$ as a function of detection angle (the frequency of 5 rad/s is indicated as a dotted line in the main plot). Since $|G^*(\omega)|$ is approximately inversely proportional to the mean-square displacement $\langle \Delta r^2(t = 1/\omega) \rangle$, these variations in the magnitude of the complex modulus directly reflect those observed in the tracer mean-square displacements.

A simple error analysis (see Appendix A) suggests that the main sources of error are on the one hand statistical errors in the intensity correlation function as a result of the finite measurement duration [32] and, on the other hand, errors introduced via the Pusey averaging procedure via the error in the intensity ratio $Y = \frac{I_{\text{sim}}}{I_{\text{exp}}}$ between the time-averaged and the ensemble-averaged scattering intensities.

The total corresponding relative error in the modulus, $\Delta G / G$, can be expressed as a function of the relative errors in the intensity ratio, $\Delta Y / Y$, and the ratio between the probed time scale $t$ and the measurement duration $T$, as

$$\frac{\Delta G}{G} \approx \frac{\Delta (\Delta r^2(t))}{\langle \Delta r^2(t) \rangle} \approx \frac{\Delta Y}{Y} + \frac{3}{g_1^2 \ln(g_1)} \frac{1}{T}. \quad (8)$$

We can estimate the error in determining the intensity ratio $Y$ from the standard deviation of the three ensemble-averaged intensity measurements taken at each angle as $\Delta Y \approx Y \Delta I_e / I_e$, where $\Delta I_e$ is taken as the standard deviation of the three intensity measurements, and $I_e$ is the average ensemble-averaged intensity. The relative error in the mean-square displacement $\epsilon_{\text{MSD}} = \Delta (\Delta r^2(t)) / \langle \Delta r^2(t) \rangle$ that results from the error in $Y$ can be estimated as $\epsilon_{\text{MSD}} \approx \frac{\Delta (\Delta r^2(t))}{\langle \Delta r^2(t) \rangle} / g_1(\Delta r^2(t)) / \Delta Y / Y$. As $g_1(\Delta r^2(t)) / \Delta Y \approx 1 / Y^2$ and $\Delta (\Delta r^2(t)) / \langle \Delta r^2(t) \rangle \approx -\Delta Y / Y / \ln(g_1(\Delta r^2(t)))$, we find $\epsilon_{\text{MSD}} \approx -\ln(g_1(\Delta r^2(t))) / \Delta Y / Y$. Because the modulus $|G^*(\omega)|$ is essentially given as the inverse of the mean-square displacement, it has the same typical relative error $\epsilon_G \approx \epsilon_{\text{MSD}}$. 

FIG. 4. Obtaining $l^*$ from intensity measurements: Calculated and measured intensities as a function of $\theta$ and $l^*$. (a) Intensity as a function of $\theta$ for simulations employing different values of $l^*$. (b) Concentration dependence of the measured scattering intensity (for different angles $\theta$). (c) Comparison between experimental and simulation results; scaled by a single constant $\beta_{\exp}$ good agreement between simulation and theory is obtained at all concentrations studied. Thus, once $\beta_{\exp}$ is determined, the measured transport mean free path $l^*$ can be directly deduced from $I(\theta)$.
For our measurements we find typical values $\epsilon G^* \approx 0.5$, as shown in the inset of Fig. 5(c), where the corresponding error bars are shown for each angle.

While the magnitude of these errors is indeed significant, the observed angle-dependent variations remain within a factor of $\approx2$ and exhibit no clear systematic dependence on the detection angle. The latter indicates that the observed variations are the result of random, rather than systematic measurement errors, and thus do not reflect a systematic issue with the data analysis method. This suggests that taking an average of the data measured at different detection angles provides a more reliable result than data from a single detection angle measurement. We thus average the mean-square displacements obtained at all accessed angles, using the inverse of the estimated error bars as weights in the averaging procedure. Using this averaged data to calculate the viscoelastic response of the sample we obtain an averaged viscoelastic response, shown in Fig. 5(d), where we plot the storage modulus $G'$ (solid squares) and the loss modulus $G''$ (open squares) as a function of frequency $\omega$. The oscillations observed in the data at frequencies between of $\approx30$–1000 Hz [as indicated by the gray box in Fig. 5(d)] are likely the result of a mechanical disturbance or vibration that we could not eliminate in our experimental light scattering setup on this highly out-of-equilibrium sample. The effect can be directly observed in the measured correlation functions at the corresponding time scales, as seen in Fig. 5(a). For mechanically weaker samples we have not observed these types of oscillations in our setup. As a result of the importance of the time-derivative of $\langle \Delta r^2(t) \rangle$ in determining the viscoelastic moduli, the oscillations in the $g_1(t)$ data are amplified in the corresponding viscoelastic moduli. The frequency range where we do not trust the microrheology data due to a mechanical vibration; the resultant fluctuations are modest for $G'$, but significant for $G''$. Comparing the averaged moduli to results from conventional oscillatory rheology, with $G'(\omega)$ shown as solid black circles and $G''(\omega)$ shown as open black circles, we observe good agreement.

IV. CONCLUSIONS

We have developed a simple method for properly interpreting dynamic light scattering data from highly multiple scattering samples using a standard dynamic light scattering setup with a cylindrical sample geometry. By performing ideal random-walk simulations within a cylindrical geometry we predict the path length distribution $P(s)$ of photons...
Passing through the sample cell. This enables us to extend
the use of DWS measurements to standard dynamic light
scattering instruments. The method can be applied in the
context of micro rheology, where the dynamics of embedded
tracer particles are used to access the frequency-dependent
viscoelastic response of soft materials.

The main strength of our approach, besides not requiring
a dedicated instrument, lies in the fact that by varying the
detection angle we can access a wide range of different effect-
tive sample geometries with different average path lengths,
using one single cylindrical sample cell. This variation of
the detection angle is analogous to performing a series of
conventional DWS measurements using a series of sample
cells with varying thickness or with varying tracer particle
concentrations.

In summary, besides validating our method using a series
of tracer particle suspensions in water, we have illustrated the
usefulness of our method for DWS-based micro rheology on a
soft solid, gelatin, for which we obtain good agreement with
macroscopic oscillatory rheology experiments. Data recorded
for different detection angles enable an important consistency
check for the microrheology measurements and our results il-
lustrate that the accuracy of such micro rheology measure-
ments can be improved by averaging over measurements obtained at
different detection angles.

ACKNOWLEDGMENTS

The work of F.A. and H.M.W. forms part of the research
programme of the Dutch Polymer Institute (DPI), Project
No. 738. J.M. and H.M.W. are also grateful to the Royal
Society for travel support (Grant No. IE111253). We thank
the Institute for Complex Molecular Systems (ICMS) at
Eindhoven University of Technology for support.

APPENDIX A: ERROR ANALYSIS FOR DWS
AND MICRORHEOLOGY DATA

In the following we provide a brief analysis aimed at
estimating the relative experimental errors associated with our
DWS-based micro rheology measurements.

In DWS, the dynamics of tracer particles is quantified
in terms of temporal autocorrelation functions, from which
MSD-based microrheology measurements.

are introduced via the relative error in the intensity ratio
\[ Y = \frac{1}{T} \] between the time-averaged and the ensemble-averaged
scattering intensities.

1. Statistical errors in the intensity correlation function

Given a measurement duration \( T \), the statistical error
in the intensity correlation function \( g_2(t) \) at lag time \( t \) can be expressed as
\[ \Delta g_2(t) \approx 6 \frac{t}{T} \sqrt{ \frac{T}{T} }. \] (A1)

Given the Siegert relation \( g_1(t) = \sqrt{g_2(t)} - T \), this translates to an error in the field correlation function \( g_1(t) \) as
\[ \Delta g_1(t) \approx \frac{3}{g_1(t)} \frac{t}{T} \sqrt{ \frac{T}{T} }. \] (A2)

For the purposes of this error analysis, we assume a
simplified relationship between the field correlation function
\( g_1(t) \) and the mean-square displacement \( \text{MSD}(t) \),
\( g_1(t) \approx e^{-k_0^2/3 \text{MSD}(t)} \), with \( k_0 \) the wave vector of the laser light, \( \delta \)
the average path length of photons, and \( l^* \) the transport mean
free path.

We can then express the mean-square displacement as
\( \text{MSD}(t) \approx 3l^* \ln(g_1(t))/(k_0^2 \delta) \). With \( \partial \text{MSD}/\partial g_1 \approx \frac{3}{g_1 \ln(g_1)} \approx \text{MSD}/(g_1 \ln(g_1)) \) and \( \Delta g_1 \approx \frac{3}{g_1 \sqrt{T}} \), we can now express the
total relative error in the complex modulus as
\[ \Delta G^*/G^* \approx \frac{\Delta \text{MSD}}{\text{MSD}} \approx \frac{1}{\text{MSD}} \frac{\partial \text{MSD}}{\partial g_1} \Delta g_1 \approx \frac{1}{g_1 \ln(g_1)} \frac{3}{g_1 \sqrt{T}}. \] (A3)

2. Errors introduced during the Pusey averaging procedure

For nonergodic samples, additional errors are introduced
via the Pusey averaging procedure used to estimate ensemble-
averaged temporal autocorrelation functions. These errors
are introduced via the relative error in the intensity ratio
\[ Y = \frac{1}{T} \] between the time-averaged and the ensemble-averaged
scattering intensities.

\[ g_1(t) = \frac{Y - 1}{Y} + \frac{1}{Y} [\tilde{g}_2(t) - \sigma^2]^2, \] (A5)

where \( \tilde{g}_2(t) = 1 + \frac{g_1(t) - 1}{\beta} \) is the time-averaged intensity au-
tocorrelation function normalized by the coherence factor
\( \beta \), and \( \sigma^2 = \tilde{g}_2(t) - \sigma^2 \) characterizes the short-time intercept of
\( \tilde{g}_2(t) \). This yields the second term in Eq. (A4) as
\( \frac{\partial}{\partial \beta} \approx \frac{1}{Y} [1 - \sqrt{\tilde{g}_2 - \sigma^2}], \) and with \( \sqrt{\tilde{g}_2} - \sigma^2 \approx 1 + Y g_1 - Y, \) this
results in
\[ \frac{\partial g_1}{\partial Y} \approx 1 - g_1. \] (A6)

The total relative error introduced by the intensity ratio \( Y \) is thus
\[ \Delta \text{MSD} \approx \frac{1 - g_1}{g_1 \ln(g_1)} \frac{\Delta Y}{Y}. \] (A7)

Plotting the function \( f(x) = \frac{1-x}{x \ln(x)} \), we observe \(|f(x)| \approx 1\) for values of \( x \) close to 1. For our solidlike samples, \( g_1 \) is close to 1 at long times, and thus \(|\frac{1-g_1}{g_1 \ln(g_1)}| \approx 1\); therefore, in this case we can make the simple approximation \( \Delta \text{MSD} \approx \frac{\Delta Y}{Y} \).

3. Total estimated error

The total estimated error as a result of the above two main sources of error can then be written as
\[ \Delta G^*/G^* \approx \frac{\Delta \text{MSD}}{\text{MSD}} \approx \frac{3}{g_1^2 \ln(g_1)} \sqrt{\frac{T}{T}} + \frac{1 - g_1}{g_1 \ln(g_1)} \frac{\Delta Y}{Y}. \] (A8)

Of these two contributions to the experimental error, the latter usually dominates, provided that the measurement duration is sufficiently long, such that the factor \( \sqrt{T} \) becomes small enough. Nevertheless, for \( g_1(t) \) sufficiently close to 1, the term \( \frac{1}{\ln(g_1)} \) would become very large, and eventually lead to the first term becoming dominant.

Indeed, we expect the errors to be highest for cases where \( g_1(t) \) is very close to zero (as a result of the factor \( \frac{3}{g_1^2 \ln(g_1)} \)) or \( \frac{1 - g_1}{g_1 \ln(g_1)} \), respectively), or very close to unity (as a result of the factor \( \frac{1}{\ln(g_1)} \)).

APPENDIX B: CODE FOR DATA ANALYSIS OF DWS IN A CYLINDRICAL CELL

1. Codes for analyzing DWS data in a cylindrical cell

The complete numerical codes used for analyzing dynamic light scattering data using the approach outlined here can be obtained from our bitbucket repository [27] or by request via email.

2. Code for calculating the path length distributions

The main code for calculating the path length distribution \( P(s) \) is a simple random-walk simulation with step length \( l^* \), as described in the main text. Below is the C-code that accomplishes this task; we keep track of the results in the output array \( y \), which for each bin, corresponding to a segment of detection angle and a segment of path length, keeps track of the number of photons exiting the cell within the corresponding angle range and within the corresponding path length range. Angular bins evenly divide the angular space between 0° and 180°; we usually choose 180 bins of 1° width. Path length bins are also linearly spaced, with 300 bins total and the 100th bin corresponding to a path length of \( (L(\theta)/l^*)^2 \) in units of \( l^* \), where \( L(\theta) \) is the distance between the entry point and the exit point of the simulated photon, as a function of the detection angle \( \theta \).

Listing of “pathlengthsCy1.c”:

```c
#include "mex.h"
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
#include <math.h>

void pathlengthsCy1(double y[], double x[], size_t mrows, size_t ncols)
{
    int ii,m,n,step,anglebin,Nbins,N_iter,index1,index2,index3;
    double xx,yy,zz,dxx,dyy,dzz,lstar,LL,pi,deltaz,weight,dist;
    lstar=x[0];
    Nbins=x[1];
    N_iter=x[2];
    pi=3.14159265359;
    srand(rand() ^ time(NULL));
    for(m=0;m<mrows;m++)
    {
        for(n=0;n<ncols;n++)
        {
            index1=(m % mrows )+mrows *n;
            y[index1]=0;
        }
    }
    for(n=0;n<ncols;n++)
    {
```
\[ \text{dist} = \sqrt{(-1 - \cos(\pi/N_{\text{bins}}))(1 - \cos(\pi/N_{\text{bins}})) + \sin(\pi/N_{\text{bins}})\sin(\pi/N_{\text{bins}})}; \]

\[ y_{\text{index1}} = 2 + m_{\text{rows}}; \]

\[ y_{\text{index1}+1} = \text{ceil}(\text{dist}^2/l_{\text{star}}); \]

\[ \text{for}(i=0; i < N_{\text{iter}}; i++) \{ \]
\[ xx = -1 + l_{\text{star}}; \]
\[ yy = 0; zz = 0; \]
\[ \text{step} = 1; \]
\[ \text{while } ((xx^2 + yy^2) < 1) \{ \]
\[ dxx = 2; dyy = 2; dzz = 2; \]
\[ \text{while } (dxx^2 + dyy^2 + dzz^2 > 1) \{ \]
\[ dxx = 2 \times (\text{double}(\text{rand}()) / (\text{double}(\text{RAND\_MAX})) - 1; \]
\[ dyy = 2 \times (\text{double}(\text{rand}()) / (\text{double}(\text{RAND\_MAX})) - 1; \]
\[ dzz = 2 \times (\text{double}(\text{rand}()) / (\text{double}(\text{RAND\_MAX})) - 1; \]
\[ \} \]
\[ L_{\text{L}} = \sqrt{dxx^2 + dyy^2 + dzz^2}; \]
\[ dxx = dxx/L_{\text{L}}; dyy = dyy/L_{\text{L}}; dzz = dzz/L_{\text{L}}; \]
\[ xx = xx + dxx; yy = yy + dyy; zz = zz + dzz; \]
\[ \text{step}++; \]
\[ \} \]
\[ L_{\text{L}} = \sqrt{xx^2 + yy^2}; \]
\[ xx = xx/L_{\text{L}}; yy = yy/L_{\text{L}}; zz = zz/L_{\text{L}}; \]
\[ \text{anglebin} = \text{ceil}(\arccos(xx)/(\pi/N_{\text{bins}} - 1)); \]
\[ \text{index1} = m_{\text{rows}} + \text{anglebin}; \]
\[ \text{deltaz} = \pi/2/N_{\text{bins}}; \]
\[ \text{weight} = \text{erf}(3 \times \text{deltaz}/L_{\text{L}}/\sqrt{\text{step}}); \]
\[ \text{if } (\text{weight} == 0) \text{ mexPrintf(''erf: %f
'', weight); \}
\[ y_{\text{index1}} = y_{\text{index1}+1} + \text{weight}; \]
\[ \text{if } (y_{\text{index2}} > 0) \{
\[ y_{\text{index3}} = \text{floor}(y_{\text{index2}}); \]
\[ y_{\text{index3}} = y_{\text{index3}+1} + \text{weight}; \]
\[ \} \]
\[ \} \]
\[ \} \]
\[ \} \]