Bias in random regret models due to formal and empirical comparison with random utility model measurement error

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Bias in random regret models due to measurement error: formal and empirical comparison with random utility model

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ABSTRACT
This study addresses the so-called uncertainty problem due to measurement error in random utility and random regret choice models. Based on formal analysis and empirical comparison, we provide new insights about the uncertainty problem in discrete choice modeling. First, we formally show how measurement error affects the random regret model differently from the random utility model. Then, random measurement error is introduced into level-of-service variables and the effect of measurement error is analyzed by comparing the estimated parameters of the concerned choice models, before and after introducing measurement error. We argue that although measurement error leads to biased estimation results in both types of models, uncertainty tends to accumulate in random regret models because this model involves a comparison of alternatives. Therefore, input uncertainty tends to lead to larger bias in random regret models. Moreover, since random regret models assume semi-compensatory decision processes, bias in random utility models is homogenous across individuals and alternatives, while bias in random regret models is heterogeneous. Several approaches are discussed to overcome this uncertainty problem in random regret models.

1. Motivation
The so-called uncertainty problem is an important issue in transportation studies (Ben-Akiva and Bierlaire 1999; Brownstone, Bunch, and Train 2000; Hensher and Greene 2003; Bhatta and Larsen 2011; Walker et al. 2011; Guevara and Polanco 2016). Researchers usually face many sources of uncertainty when they estimate discrete choice models to analyze travel behavior and predict travel demand. In an extension of random utility theory (Luce 1959), Manski (1973) identified four main sources of uncertainty: (1) measurement errors, (2) unobserved individual characteristics (so-called ‘unobserved taste variations’), (3) unobserved alternative attributes, and (4) proxy or instrumental variables. Those sources of uncertainty cause bias in model results and may thus prevent researchers formulating the right transport policy recommendations.
Measurement error is one of the main sources of uncertainty. It, for example, occurs when network models are used to measure level of service variables, such as travel time and costs, at the individual level. Since the level-of-service variables in network models are obtained from the centroids of two zones, this approach leads to imprecise results in representing individual behavior and introduces measurement error. As argued by McFadden (2001), these zone-based values from network models may cause systematic bias in disaggregate choice models.

Random utility models have dominated discrete choice modeling for several decades. These models are based on the principle that individuals choose an alternative such as to maximize their utility. The error terms in these models represent various sources of uncertainty. More recently, random regret choice models have been introduced in the travel behavior research community to provide an alternative to expected/random utility models (Chorus, Arentze, and Timmermans 2008a, 2008b; Chorus 2010; de Moraes Ramos, Daamen, and Hoogendoorn 2011; Chorus 2012b; Kaplan and Prato 2012; Chorus, Rose, and Hensher 2013; Hensher, Greene, and Chorus 2013; Boeri and Masiero 2014; Jang, Rasouli, and Timmermans 2016). Random regret models are based on the behavioral principle that individuals choose the alternative that minimizes their regret, where regret is defined as a function of attribute differences between the considered choice alternative and one or more non-chosen choice alternatives in an individual’s choice set. The model therefore assumes a semi-compensatory decision-making process. Unobserved regret is defined identically to (the negative of) unobserved utility. Standard regret models have been derived from the assumption that the error terms are identically and independently Gumbel distributed (IID) to obtain close form logistic expressions of choice probabilities.

The question addressed in this study is how measurement error may violate the IID assumption in regret models and the extent to which ignoring such a violation will bias parameter estimates. More specifically, the study is motivated by the thought that measurement error may differently affect random utility and random regret models because of their fundamental difference: the multinomial logit model is based on the behavioral postulate that individuals derive utility by processing the attributes of each choice alternative independently and separately, whereas regret-based choice models are based on the behavioral contention that individuals assess regret by systematically comparing choice alternatives. Unless we view these models as straightforward statistical models, these behavioral differences should be represented in the structure of the models. That is, the variance of the error terms in these models may thus be differently affected by the degree of uncertainty, causing different scale factors in these discrete choice models. First, compared to the (linear additive) random utility model, the comparison of alternatives in random regret models accumulates measurement error. This, in turn, may cause an increase in the variance of the error terms and a decreasing scale factor. Since we normally do not consider the change in scale factor, it may lead to larger bias in the estimated parameters of random regret models.

In this paper, we formally and empirically analyze how measurement error affects bias in regret-based and utility-based choice models. Following previous studies (Greene 2003; Gujarati 2003; Bhatta and Larsen 2011), we assume that measurement error is normally distributed and linearly incorporated in the level of service variables. Since to the best of our knowledge, to date random regret models have only been specified assuming IID error terms, in this study, we compare random regret models with the multinomial logit model, which is based on the same assumptions about the error terms.
The remainder of this paper is organized as follows. We will first formally analyze the influence of measurement error in both types of choice models. Next, we will complement the results from our formal analysis by providing empirical evidence using revealed choice data. Then, we will discuss how the bias can be expressed in the error terms to better represent true choice behavior. The paper is completed with a discussion of the results and avenues of future research.

2. Formal analysis

Consider a multi-alternatives choice set in which each of the choice alternatives \(i\) is characterized in terms of the two variables – travel time \(x_t\) and travel distance \(x_d\) – for each individual \(q\). Assume that the true utility is linear additive as formulated in Equation (1).

\[
U_{iq}^{\text{true}} = \beta_{t}^{\text{true}} x_{tiq} + \beta_{d}^{\text{true}} x_{djq} + \epsilon_{iq}^{\text{true}}.
\]  

In this paper, we assume that the error terms \(\epsilon_{iq}^{\text{true}}\) are identically and independently Gumbel distributed with a scale factor being equal to 1. The principle of utility maximization then results in the multinomial logit choice model.

Random regret models are based on the same assumptions about the error terms and the scale factor. Two basic types of random regret models have been introduced in the literature. The various versions of random regret models can be summarized as (Hensher, Rose, and Green, 2015; Rasouli and Timmermans 2016):

\[
\text{RR}_{\text{max}}^{\text{true}} = \max_{j \neq i} \left( \max \{0, \beta_{t}^{\text{true}} (x_{tjq} - x_{tiq})\} + \max \{0, \beta_{d}^{\text{true}} (x_{djq} - x_{djq})\} \right) + \epsilon_{iq}^{\text{true}},
\]

\[
\text{RR}_{\text{sum}}^{\text{true}} = \sum_{j \neq i} \left( \max \{0, \beta_{t}^{\text{true}} (x_{tjq} - x_{tjq})\} + \max \{0, \beta_{d}^{\text{true}} (x_{djq} - x_{djq})\} \right) + \epsilon_{iq}^{\text{true}},
\]

\[
\text{RR}_{\text{log}}^{\text{true}} = \sum_{j \neq i} \left[ \ln \left( 1 + \exp \left( \beta_{t}^{\text{true}} (x_{tjq} - x_{tjq}) \right) \right) + \ln \left( 1 + \exp \left( \beta_{d}^{\text{true}} (x_{djq} - x_{djq}) \right) \right) \right] + \epsilon_{iq}^{\text{true}},
\]

where \(j\) is the non-chosen alternative(s).

RRmax in Equation (2) is the original model specification proposed by Chorus, Arentze, and Timmermans (2008a). The behavioral rule underlying random regret models is that when people make a choice, they wish to avoid that the chosen alternative is outperformed by one or more other alternatives on one or more attributes (which would cause regret). To extend regret choice models from binary to multi-alternative choice models, Chrous, Arentze, and Timmermans (2008a) referred to Quiggin’s (1994) principle of Irrelevance of Statewise Dominated Alternatives (ISDA). This principle states that adding or removing non-best alternatives to/from a given choice set does not affect the amount of regret that is experienced or anticipated. In line with this principle, they assumed that the amount of regret individual \(q\) anticipates for the chosen alternative only depends on the attribute levels of the best non-chosen alternative.

The alternative assumption is that regret depends on the all non-chosen alternatives (RRsum in Equation (3)). Rasouli and Timmermans (2016) argued that the decision between RRmax and RRsum is an empirical matter, although it is difficult to believe that in large
choice sets, individuals compare all pairs of alternatives to mentally assess the degree of regret. Hensher, Rose, and Green (2015) argue that it may be affected by attribute variations.

Later, since the max operator causes a non-smooth likelihood function, Chorus (2010, 2012a) suggested a logarithmic approximation (RRlog in Equation (4)). RRlog is very similar to RRsum, and becomes asymptotically identical for larger attribute differences. The RRlog in Equation (4) still generates regret equal to $\ln(2) \approx 0.69$ when two alternatives are identical and thus have exactly the same attribute values. Therefore, Chorus (2014) proposed a modified version by subtracting $\ln(2)$ from RRlog, mentioning that choice probabilities and estimation outcomes remain unchanged by the correction term.

Very recently, Rasouli and Timmermans (2016) compared the three types of random regret models using a dedicated stated choice experiment. They concluded that for their data with small attribute differences RRmax shows better prediction power than the other models. Evidently, replicative research using different data sets is needed to judge the generalizability of their findings.

In the remainder of this paper, since it is difficult to mathematically define the effect of measurement error for the logarithmic and exponential functions in RRlog, we focus on the effect of measurement error in RRmax and RRsum. However, if we accept Chorus’ (2010, 2012a) argument that RRsum and RRlog are almost similar, the effect of measurement error may also be similar for both models.

Let us discuss how measurement error affects regret by simple example. Assume two alternative routes between an origin and a destination. The true travel time of route A is 5 min, while the travel time of route B is 10 min. Assume that the taste weight for it is $-1$ for all regret and utility models, and that the measurement error of travel time is normally distributed with variance of 1, fixed across alternatives and individuals.

Table 1(a) shows the change in the amount of regret and utility due to the introduction of measurement error in binary choice sets. Note that RRmax and RRsum differ by definition in multi-alternatives choice sets. This means that both regret models derive the same results in binary choice sets. In case of a linear additive utility function, measurement error changes the utility of the choice alternatives. The variation is constant across alternatives, meaning that the IID assumption still holds in the random utility model. However, the regret function of the random regret models (RRmax and RRsum) is differently affected by the introduction

### Table 1. Effect of measurement error.

<table>
<thead>
<tr>
<th></th>
<th>$U$</th>
<th>RR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(a) Binary choice set</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>True travel time</td>
<td>Route A: $-5 + \epsilon$</td>
<td>$0 + \epsilon$</td>
</tr>
<tr>
<td></td>
<td>Route B: $-10 + \epsilon$</td>
<td>$5 + \epsilon$</td>
</tr>
<tr>
<td>Travel time with measurement error</td>
<td>Route A: $-5 + (\epsilon + N(0,1))$</td>
<td>$0 + \epsilon$</td>
</tr>
<tr>
<td></td>
<td>Route B: $-10 + (\epsilon + N(0,1))$</td>
<td>$5 + (\epsilon + N(0,2))$</td>
</tr>
<tr>
<td><strong>(b) Multi-alternatives choice set</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>True travel time</td>
<td>Route A: $-5 + \epsilon$</td>
<td>$0 + \epsilon$</td>
</tr>
<tr>
<td></td>
<td>Route B: $-10 + \epsilon$</td>
<td>$5 + \epsilon$</td>
</tr>
<tr>
<td></td>
<td>Route C: $-15 + \epsilon$</td>
<td>$10 + \epsilon$</td>
</tr>
<tr>
<td>Travel time with measurement error</td>
<td>Route A: $-5 + (\epsilon + N(0,1))$</td>
<td>$0 + \epsilon$</td>
</tr>
<tr>
<td></td>
<td>Route B: $-10 + (\epsilon + N(0,1))$</td>
<td>$5 + (\epsilon + N(0,2))$</td>
</tr>
<tr>
<td></td>
<td>Route C: $-15 + (\epsilon + N(0,1))$</td>
<td>$10 + (\epsilon + N(0,2))$</td>
</tr>
</tbody>
</table>
of measurement error. The best alternative in the choice set (Route A) does not have any regret and is not affected by measurement error. However, the non-best alternative is highly affected by measurement error. The degree of variation is higher than in case of utility. This result raises concerns about the commonly made assumption underlying random regret models that the error terms are independently and identically distributed. According to the textbooks about discrete choice modeling (e.g. Ben-Akiva and Lerman 1985), choice behavior is decided by differences in the (deterministic) utility. However, it should not be forgotten that this property only holds if the IID assumption about error terms is valid. Therefore, the inherent comparison of choice alternatives implies that measurement error leads to non-IID error terms, and therefore choice probabilities in random regret models are not decided by differences in (deterministic) regret.

Consider additional route C with 15 min travel time in the choice set. Now both regret models (RRmax and \(m\)) produce different results. Table 1(b) shows that the choice alternatives according to the (linear additive) utility model still have identical variance due to measurement error, while the alternatives in both regret models exhibit non-identical variance. The difference is higher in RRsum than RRmax. More specifically, the variance of the worst alternative is increased most in RRsum. This implies that in the context of departure time and route choice decisions (many alternatives in the choice set), the non-identical variance of errors among alternatives may be very high in the RRsum model.

### 2.1. (Linear additive) random utility model

Assume measurement error occurs in travel time, and is fixed across alternatives and individuals.

\[
x_{itq} = x_{itq}^{true} + v_{it}, \quad v_{it} \sim N(0, \sigma_t^2). \tag{5}
\]

Substituting measurement error in travel time then gives

\[
U_{iq}^{true} = \beta_t^{true}(x_{itq}^{true} - v_{it}) + \beta_d^{true}x_{idq}^{true} + \varepsilon_{iq}^{true}. \tag{6}
\]

The identically and independently distributed (IID) error terms imply that the scale factor, which is inversely proportional to the variance of the error term, is normalized and assumed to be equal to one following the basic assumption of the random utility model (Ben-Akiva and Lerman 1985). Then, the variance of error terms is \(\pi^2/6\). However, since the measurement error is not a fixed value, but is represented as a distribution, it causes a change in the variance of the error term and scale factor as shown in the following equation:

\[
U_{iq}^{true} = \mu_{iq}(\beta_t^{true}x_{itq}^{true} + \beta_d^{true}x_{idq}^{true}) + (\varepsilon_{iq}^{true} - \beta_t^{true}v_{it}). \tag{7}
\]

By the rules of variance,

\[
\beta_t^{true}v_{it} \sim N(0, (\beta_t^{true})^2 \sigma_t^2). \tag{8}
\]

Since \(\varepsilon_{iq}^{true} - \beta_t^{true}v_{it}\) represents the difference between a Gumbel and a normal distribution, it does not follow an extreme value distribution. To overcome this difficulty, we use an approximation (e.g. Guevara and Ben-Akiva 2012) in which \(\varepsilon_{iq}^{true} - \beta_t^{true}v_{it}\) is Gumbel distributed, and the variance is a summation of variances in each component \((\varepsilon_{iq}^{true}, \beta_t^{true}v_{it})\). Thus, in this case, the variance is given by \(\pi^2/6 + (\beta_t^{true})^2 \sigma_t^2\).
Since the change in variance of the error terms due to measurement error in Equation (7) is generally not considered in the model estimation $\left( \varepsilon_{iq}^{true} - \beta_{t}^{true} \nu_{it} \Rightarrow \varepsilon_{iq}^{true} \right)$, the estimated parameters are biased $\left( \beta^{true} \Rightarrow \beta \right)$ as shown in Equation (7).

$$U_{iq} = \beta_{t} x_{itq} + \beta_{d} x_{idq}^{true} + \varepsilon_{iq}^{true}. \quad (9)$$

Likewise, if measurement error only occurs in travel distance $\left( \nu_{id} \right)$, and is fixed across alternatives and individuals:

$$x_{idq} = x_{idq}^{true} + \nu_{id}, \quad \nu_{id} \sim N(0, \sigma_{d}^{2}). \quad (10)$$

Then, the true utility can be generalized as

$$U_{iq}^{true} = \mu_{iq} (\beta_{t}^{true} x_{itq}^{true} + \beta_{d}^{true} x_{idq}^{true}) + (\varepsilon_{iq}^{true} - \beta_{d}^{true} \nu_{id}) \quad (11)$$

then, the variance of error terms is $\pi^{2}/6 + (\beta_{d}^{true})^{2} \sigma_{d}^{2}$.

The estimated parameters are generally biased by ignoring the change in variance in error terms due to measurement error:

$$U_{iq} = \beta_{t} x_{itq} + \beta_{d} x_{idq} + \varepsilon_{iq}. \quad (12)$$

### 2.2. Random regret models

#### 2.2.1. RRmax

In the case of regret, the model with measurement error in travel time can be represented as

$$RRmax_{iq}^{true} = \max_{j \neq i} \left\{ \max\left[0, \beta_{t}^{true} (x_{jtq}^{true} - x_{itq}^{true})\right] + \max\left[0, \beta_{d}^{true} (x_{jdq}^{true} - x_{idq}^{true})\right] \right\} + \varepsilon_{iq}^{true}. \quad (13)$$

While measurement error invariably affects the error terms in the linear additive utility, it differently affects the error terms in regret models due to the semi-compensatory decision rule represented by the max operator. Assume the sign of attribute-level differences and the best non-chosen alternative is not affected by measurement error. That is, measurement error is too small to change.

If the value of the multiplication of the parameter and the true attribute-level difference is positive $\beta_{t}^{true} (x_{jtq}^{true} - x_{itq}^{true}) > 0$, then the true attribute-level regret is also positive $\max\left[0, \beta_{t}^{true} (x_{jtq}^{true} - x_{itq}^{true})\right] > 0$, and the error terms in the regret model are fully affected by measurement error. This means that the multiplication of the parameter and attribute-level difference with measurement error is still positive $\beta_{t}^{true} (x_{jtq}^{true} - x_{itq}^{true}) > 0$, and that the value of attribute-level regret differs from true attribute-level regret by measurement error $\beta_{t}^{true} (\nu_{jt} - \nu_{it})$.

The subtraction of two normal distributions is another normal distribution, and the variance is the sum of both variances $\nu_{jt} - \nu_{it} \sim N(0, \sigma_{t}^{2} + \sigma_{t}^{2})$. Therefore, the change in error
same as in the case of positive attribute-level regret. The difference with measurement error can be positive or negative. If positive, the bias is the error term due to measurement error under this condition. This means that the error terms in the original regret model are not affected by measurement error. Therefore, Equation (12) can be generalized to attribute-level regret would be positive with a 50% probability, or negative with a 50% probability. If the value of the multiplication of the parameter and the true attribute-level difference is zero, then the true attribute-level regret becomes zero because max[0, βttrue(xjftq - xiitq)] = 0. Assuming that the value of measurement error difference is negligible compared to the attribute value difference multiplication of the parameter and true attribute-level difference with measurement error would be negative βttrue(xjftq - xiitq) < 0, the attribute-level regret with measurement error is still zero: max[0, βttrue(xjftq - xiitq)] = 0. This means that the error terms in the original regret model are not affected by measurement error under this condition.

\[
RR_{\text{max}}^{\text{true}}_{iqt} = \mu_{iq} \cdot \left( \max_{j \neq i} \left[ \max[0, \beta_t^{\text{true}}(x_{jftq} - xiitq)] + \max[0, \beta_d^{\text{true}}(x_{jdtq} - x_{idtq})] \right] + \{\varepsilon_{iq}^{\text{true}} - \beta_t^{\text{true}}(v_{jt} - v_{it}) \} \right),
\]

(14)

where

\[
\beta_t^{\text{true}}(v_{jt} - v_{it}) \sim N(0, (\beta_t^{\text{true}})^2 \cdot 2\sigma_t^2).
\]

Therefore, the error term with measurement error εiqtrue - βttrue(vjt - vit) is approximately Gumbel distributed with zero mean and variance π^2/6 + (βttrue)^2 * 2σ_t^2. Thus, if the value of the multiplication of the parameter and the true attribute-level difference is positive, the bias in the original random regret model due to measurement error is smaller than the bias in the linear additive random utility model.

If the value of the multiplication of the parameter and true attribute-level difference is negative βttrue(xjftq - xiitq) < 0, then the true attribute-level regret becomes zero because max[0, βttrue(xjftq - xiitq)] = 0. Assuming that the value of measurement error difference is negligible compared to the attribute value difference multiplication of the parameter and attribute-level difference with measurement error would be negative βttrue(xjftq - xiitq) < 0, the attribute-level regret with measurement error is still zero: max[0, βttrue(xjftq - xiitq)] = 0. This means that the error terms in the original regret model are not affected by measurement error under this condition.

\[
RR_{\text{max}}^{\text{true}}_{iqt} = \mu_{iq} \cdot \left( \max_{j \neq i} \left[ \max[0, \beta_t^{\text{true}}(x_{jftq} - xiitq)] + \max[0, \beta_d^{\text{true}}(x_{jdtq} - x_{idtq})] \right] + \varepsilon_{iq}^{\text{true}} \right),
\]

(15)

The error term εiqtrue is still Gumbel distributed with zero mean and variance π^2/6. Thus, if the value of the multiplication of the parameter and the true attribute-level difference is negative, the bias in the original random regret model due to measurement error is smaller than the bias in random utility models.

If the value of the multiplication of the parameter and true attribute-level difference is zero βttrue(xjftq - xiitq) = 0, which can only happen if the parameter is zero and/or the two choice alternatives have the same travel times, then the true attribute-level regret is also zero max[0, βttrue(xjftq - xiitq)] = 0. The multiplication of the parameter and attribute-level difference with measurement error can be positive or negative. If positive, the bias is the same as in the case of positive attribute-level regret [εiqtrue ⇒ εiqtrue - βttrue(vjt - vit); βtrue ⇒ β]. If negative, there would be no bias because attribute-level regret is still zero [εiqtrue ⇒ εiqtrue; βtrue ⇒ βtrue]. Since the normal distribution is symmetric around the zero mean, attribute-level regret would be positive with a 50% probability, or negative with a 50% probability. Therefore, Equation (12) can be generalized to

\[
RR_{\text{max}}^{\text{true}}_{iqt} = \mu_{iq} \cdot \left( \max_{j \neq i} \left[ \max[0, \beta_t^{\text{true}}(x_{jftq} - xiitq)] + \max[0, \beta_d^{\text{true}}(x_{jdtq} - x_{idtq})] \right] + \left\{ \varepsilon_{iq}^{\text{true}} - \beta_t^{\text{true}}(v_{jt} - v_{it}) \cdot \frac{1}{2} \right\} \right),
\]

(16)
where
\[ \beta_t^{\text{true}}(v_{jt} - v_{it}) * \frac{1}{2} \sim N(0, (\beta_t^{\text{true}})^2 \sigma^2_t). \]

Therefore, the error term with measurement error \( \epsilon_{iq}^{\text{true}} - \beta_t^{\text{true}}(v_{jt} - v_{it}) * 1/2 \) is approximately Gumbel distributed with zero mean and variance \( \pi^2/6 + (\beta_t^{\text{true}})^2 \sigma^2_t \). Thus, if the parameter is equal to zero and/or the choice alternatives have identical travel times, the bias due to measurement error is the same in random regret models as in the corresponding linear additive random utility model.

As shown above, the variance of the error terms in the regret model depends on the true attribute-level regret. This can be summarized as

\[
\text{RRmax}_{iq}^{\text{true}} = \mu_{iq} (\max_{j \neq i} [\max[0, \beta_t^{\text{true}}(x_{jq} - x_{iq})] + \max[0, \beta_d^{\text{true}}(x_{jdq} - x_{idq})]]) + \epsilon_{iq}^{\text{true}},
\]

if \( \max_{j \neq i} [\beta_t^{\text{true}}(x_{jq} - x_{iq})] > 0 \), \( \epsilon_{iq} = \epsilon_{iq}^{\text{true}} - \beta_t^{\text{true}}(v_{jt} - v_{it}) \),

if \( \max_{j \neq i} [\beta_t^{\text{true}}(x_{jq} - x_{iq})] = 0 \), \( \epsilon_{iq} = \epsilon_{iq}^{\text{true}} - \beta_t^{\text{true}}(v_{jt} - v_{it}) * \frac{1}{2}, \)

if \( \max_{j \neq i} [\beta_t^{\text{true}}(x_{jq} - x_{iq})] < 0 \), \( \epsilon_{iq} = \epsilon_{iq}^{\text{true}}. \)

The variation of error terms implies heterogeneous error terms across individuals and alternatives in the regret model under measurement error. This means that the assumption of independently and identically distributed error terms is no longer valid when measurement error occurs.

The bias in estimated parameters occurs because the change in the variance of the error terms due to measurement error is ignored:

\[
\text{RRmax}_{iq} = \max_{j \neq i} [\max[0, \beta_t^{\text{true}}(x_{jq} - x_{iq})] + \max[0, \beta_d^{\text{true}}(x_{jdq} - x_{idq})]]) + \epsilon_{iq}^{\text{true}}.
\]

Likewise, if measurement error only occurs in travel distance, the true regret model with measurement error can be generalized according to the following equation:

\[
\text{RRmax}_{iq}^{\text{true}} = \mu_{iq} (\max_{j \neq i} [\max[0, \beta_t^{\text{true}}(x_{jq} - x_{iq})] + \max[0, \beta_d^{\text{true}}(x_{jdq} - x_{idq})]]) + \epsilon_{iq}^{\text{true}},
\]

if \( \max_{j \neq i} [\beta_d^{\text{true}}(x_{jdq} - x_{idq})] > 0 \), \( \epsilon_{iq} = \epsilon_{iq}^{\text{true}} - \beta_d^{\text{true}}(v_{jd} - v_{id}) \),

if \( \max_{j \neq i} [\beta_d^{\text{true}}(x_{jdq} - x_{idq})] = 0 \), \( \epsilon_{iq} = \epsilon_{iq}^{\text{true}} - \beta_d^{\text{true}}(v_{jd} - v_{id}) * \frac{1}{2}, \)

if \( \max_{j \neq i} [\beta_d^{\text{true}}(x_{jdq} - x_{idq})] < 0 \), \( \epsilon_{iq} = \epsilon_{iq}^{\text{true}}. \)

Because the change in variance in error terms due to measurement error is not considered, the estimated parameters are biased.

\[
\text{RRmax}_{iq} = \max_{j \neq i} [\max[0, \beta_t^{\text{true}}(x_{jq} - x_{iq})] + \max[0, \beta_d^{\text{true}}(x_{jdq} - x_{idq})]]) + \epsilon_{iq}^{\text{true}}.
\]
2.2.2. RRsum

The RRsum model with measurement error in travel time can then be expressed as

\[
\text{RRsum}_{iq}^{\text{true}} = \sum_{j \neq i} \left\{ \max[0, \beta_t^{\text{true}}(x_{j1tq} - x_{itq})] + \max[0, \beta_d^{\text{true}}(x_{j2dq} - x_{idq})] \right\} \\
+ \epsilon_{iq}^{\text{true}}. \tag{21}
\]

Assume three alternatives in the choice set with labeling chosen alternative as \(i\) and two non-chosen ones as \(j_1\) and \(j_2\). If the value of the multiplication of the parameter and the true attribute-level difference is positive with respect to the comparison to both non-chosen alternatives \(\beta_t^{\text{true}}(x_{j1tq} - x_{itq}) > 0\) and \(\beta_t^{\text{true}}(x_{j2tq} - x_{itq}) > 0\), then the true attribute-level regrets are also positive \(\max[0, \beta_t^{\text{true}}(x_{j1tq} - x_{itq})] > 0\) and \(\max[0, \beta_t^{\text{true}}(x_{j2tq} - x_{itq})] > 0\), and the error terms in the regret model are fully affected by measurement error. This indicates that the value of attribute-level regret differs from true attribute-level regret by measurement error \(\beta_t^{\text{true}}(v_{j1t} - v_{it}) + \beta_t^{\text{true}}(v_{j2t} - v_{it})\). Therefore, the change in error term due to measurement error when the true attribute-level regret is positive can be generalized as expressed in Equation (22).

\[
\text{RRsum}_{iq}^{\text{true}} = \mu_{iq} \left[ \sum_{j \neq j_1, j_2} \left\{ \max[0, \beta_t^{\text{true}}(x_{j1tq} - x_{itq})] + \max[0, \beta_d^{\text{true}}(x_{j2dq} - x_{idq})] \right\} \\
+ \epsilon_{iq}^{\text{true}} - \beta_t^{\text{true}}(v_{j1t} - v_{it}) - \beta_t^{\text{true}}(v_{j2t} - v_{it}) \right], \tag{22}
\]

where

\[
[\beta_t^{\text{true}}(v_{j1t} - v_{it}) + \beta_t^{\text{true}}(v_{j2t} - v_{it})] \sim N(0, (\beta_t^{\text{true}})^2 \cdot 4\sigma_t^2). \]

If the value of the multiplication of the parameter and the true attribute-level difference is positive from the comparison with non-chosen alternative \(j_1(\beta_t^{\text{true}}(x_{j1tq} - x_{itq}) > 0)\), and zero from the comparison with non-chosen alternative \(j_2(\beta_t^{\text{true}}(x_{j2tq} - x_{itq}) = 0)\), then the error terms in RRsum can be expressed as

\[
\text{RRsum}_{iq}^{\text{true}} = \mu_{iq} \left[ \sum_{j \neq j_1, j_2} \left\{ \max[0, \beta_t^{\text{true}}(x_{j1tq} - x_{itq})] + \max[0, \beta_d^{\text{true}}(x_{j2dq} - x_{idq})] \right\} \\
+ \left\{ \epsilon_{iq}^{\text{true}} - \beta_t^{\text{true}}(v_{j1t} - v_{it}) + \beta_t^{\text{true}}(v_{j2t} - v_{it}) \cdot \frac{1}{2} \right\} \right], \tag{23}
\]

where

\[
\beta_t^{\text{true}}(v_{j1t} - v_{it}) \sim N(0, (\beta_t^{\text{true}})^2 \cdot 3\sigma_t^2). \]

If the value of the multiplication of the parameter and the true attribute-level difference is positive from the comparison with non-chosen alternative \(j_1(\beta_t^{\text{true}}(x_{j1tq} - x_{itq}) > 0)\),
and negative from the comparison with non-chosen alternative $j_2$ ($\beta_{t}^{\text{true}}(x_{jtq}^{\text{true}} - x_{itq}^{\text{true}}) < 0$), then the error terms in the regret model are only affected by measurement error from the comparison with non-chosen alternative $j_1$. Equation (23) shows the change in error term due to measurement error in RRsum.

$$RRsum_{iq}^{\text{true}} = \mu_{iq} \times \left( \sum_{j=1}^{j_2} \left[ \max[0, \beta_{t}^{\text{true}}(x_{jtq} - x_{itq})] + \max[0, \beta_{d}^{\text{true}}(x_{jq} - x_{idq})] \right] \right)$$

$$+ \{ \varepsilon_{iq}^{\text{true}} - \beta_{t}^{\text{true}}(v_{jt} - v_{it}) \}$$

(24)

where

$$\beta_{t}^{\text{true}}(v_{jt} - v_{it}) \sim N(0, (\beta_{t}^{\text{true}})^2 * 2\sigma_t^2).$$

If the multiplication of the parameter and the true attribute-level difference is zero from the comparison with non-chosen alternative $j_1$ ($\beta_{t}^{\text{true}}(x_{jtq}^{\text{true}} - x_{itq}^{\text{true}}) = 0$), and positive from the comparison with non-chosen alternative $j_2$ ($\beta_{t}^{\text{true}}(x_{jtq}^{\text{true}} - x_{itq}^{\text{true}}) > 0$), the error terms affected by measurement error is

$$RRsum_{iq}^{\text{true}} = \mu_{iq} \times \left( \sum_{j=1}^{j_2} \left[ \max[0, \beta_{t}^{\text{true}}(x_{jtq} - x_{itq})] + \max[0, \beta_{d}^{\text{true}}(x_{jq} - x_{idq})] \right] \right)$$

$$+ \{ \varepsilon_{iq}^{\text{true}} - \left[ \beta_{t}^{\text{true}}(v_{jt} - v_{it}) * \frac{1}{2} + \beta_{t}^{\text{true}}(v_{jt} - v_{it}) \right] \}$$

(25)

where

$$\left[ \beta_{t}^{\text{true}}(v_{jt} - v_{it}) * \frac{1}{2} + \beta_{t}^{\text{true}}(v_{jt} - v_{it}) \right] \sim N(0, (\beta_{t}^{\text{true}})^2 * 3\sigma_t^2).$$

If both multiplications of the parameter and the true attribute-level difference are zero from the comparison with non-chosen alternatives ($\beta_{t}^{\text{true}}(x_{jtq}^{\text{true}} - x_{itq}^{\text{true}}) = 0$) and ($\beta_{t}^{\text{true}}(x_{jtq}^{\text{true}} - x_{itq}^{\text{true}}) = 0$), then the error terms in RRsum can be expressed as

$$RRsum_{iq}^{\text{true}} = \mu_{iq} \times \left( \sum_{j=1}^{j_2} \left[ \max[0, \beta_{t}^{\text{true}}(x_{jtq} - x_{itq})] + \max[0, \beta_{d}^{\text{true}}(x_{jq} - x_{idq})] \right] \right)$$

$$+ \{ \varepsilon_{iq}^{\text{true}} - \left[ \beta_{t}^{\text{true}}(v_{jt} - v_{it}) * \frac{1}{2} + \beta_{t}^{\text{true}}(v_{jt} - v_{it}) * \frac{1}{2} \right] \}$$

(26)

where

$$\left[ \beta_{t}^{\text{true}}(v_{jt} - v_{it}) * \frac{1}{2} + \beta_{t}^{\text{true}}(v_{jt} - v_{it}) * \frac{1}{2} \right] \sim N(0, (\beta_{t}^{\text{true}})^2 * 2\sigma_t^2).$$

If the multiplication of the parameter and the true attribute-level difference is zero from the comparison with non-chosen alternative $j_1$ ($\beta_{t}^{\text{true}}(x_{jtq}^{\text{true}} - x_{itq}^{\text{true}}) = 0$), and negative from
the comparison with non-chosen alternative \( j_2(\beta_t^{true}(x_{j2tq}^{true} - x_{itq}^{true}) < 0) \), the error terms affected by measurement error is

\[
RRsum_{iq}^{true} = \mu_{iq} \star \left( \sum_{j=j_1, j_2} \{ \max[0, \beta_t^{true}(x_{jtq}^{true} - x_{itq}^{true})] + \max[0, \beta_d^{true}(x_{jdq}^{true} - x_{idq}^{true})] \} \right)
+ \left\{ \varepsilon_{iq}^{true} - \beta_t^{true}(v_{jtq} - v_{itq}) \right\} * \frac{1}{2},
\]

(27)

where

\[
\beta_t^{true}(v_{jtq} - v_{itq}) \sim N(0, (\beta_t^{true})^2 * \sigma_t^2).
\]

If the multiplication of the parameter and the true attribute-level difference is negative from the comparison with non-chosen alternative \( j_1(\beta_t^{true}(x_{j1tq}^{true} - x_{itq}^{true}) < 0) \), and zero from the comparison with non-chosen alternative \( j_2(\beta_t^{true}(x_{j2tq}^{true} - x_{itq}^{true}) = 0) \), then the error terms in \( RRsum \) can be expressed as

\[
RRsum_{iq}^{true} = \mu_{iq} \star \left( \sum_{j=j_1, j_2} \{ \max[0, \beta_t^{true}(x_{jtq}^{true} - x_{itq}^{true})] + \max[0, \beta_d^{true}(x_{jdq}^{true} - x_{idq}^{true})] \} \right)
+ \left\{ \varepsilon_{iq}^{true} - \beta_t^{true}(v_{jtq} - v_{itq}) \right\},
\]

(28)

where

\[
\beta_t^{true}(v_{jtq} - v_{itq}) \sim N(0, (\beta_t^{true})^2 * 2\sigma_t^2).
\]

If both multiplications of the parameter and the true attribute-level difference are negative \((\beta_t^{true}(x_{j1tq}^{true} - x_{itq}^{true}) < 0) \) and \((\beta_t^{true}(x_{j2tq}^{true} - x_{itq}^{true}) < 0) \), the error terms in \( RRsum \) are

\[
RRsum_{iq}^{true} = \mu_{iq} \star \left( \sum_{j=j_1, j_2} \{ \max[0, \beta_t^{true}(x_{jtq}^{true} - x_{itq}^{true})] + \max[0, \beta_d^{true}(x_{jdq}^{true} - x_{idq}^{true})] \} \right)
+ \left\{ \varepsilon_{iq}^{true} - \beta_t^{true}(v_{jtq} - v_{itq}) \right\} * \frac{1}{2},
\]

(29)

where

\[
\beta_t^{true}(v_{jtq} - v_{itq}) \sim N(0, (\beta_t^{true})^2 * \sigma_t^2).
\]
not affected by measurement error as shown in the following equation:

\[
RR_{\text{sum}} = \mu_{iq} * \left( \sum_{j=1,j \neq 2} \{ \max[0, \beta_t^{true}(x_{j1q} - x_{i1q})] + \max[0, \beta_{d}^{true}(x_{j2q}^{true} - x_{i2q}^{true})] \} \right) \\
+ \varepsilon_{iq}^{true}.
\]

As shown above, the variance of the error terms in the regret model depends on the true attribute-level regret. This can be summarized as

\[
RR_{\text{sum}} = \mu_{iq} * \sum_{j=1,j \neq 2} \{ \max[0, \beta_t^{true}(x_{j1q} - x_{i1q})] + \max[0, \beta_{d}^{true}(x_{j2q}^{true} - x_{i2q}^{true})] \} \\
+ \varepsilon_{iq}.
\]

if \( \beta_t^{true}(x_{j1q} - x_{i1q}) > 0 \) and \( \beta_t^{true}(x_{j2q}^{true} - x_{i2q}^{true}) > 0 \),

\[\varepsilon_{iq} = \varepsilon_{iq}^{true} - [\beta_t^{true}(x_{j1q} - x_{i1q}) + \beta_t^{true}(x_{j2q}^{true} - x_{i2q}^{true})],\]

if \( \beta_t^{true}(x_{j1q} - x_{i1q}) > 0 \) and \( \beta_t^{true}(x_{j2q}^{true} - x_{i2q}^{true}) = 0 \),

\[\varepsilon_{iq} = \varepsilon_{iq}^{true} - [\beta_t^{true}(x_{j1q} - x_{i1q}) + \beta_t^{true}(x_{j2q}^{true} - x_{i2q}^{true})],\]

if \( \beta_t^{true}(x_{j1q} - x_{i1q}) > 0 \) and \( \beta_t^{true}(x_{j2q}^{true} - x_{i2q}^{true}) < 0 \),

\[\varepsilon_{iq} = \varepsilon_{iq}^{true} - [\beta_t^{true}(x_{j1q} - x_{i1q}) + \beta_t^{true}(x_{j2q}^{true} - x_{i2q}^{true})],\]

if \( \beta_t^{true}(x_{j1q} - x_{i1q}) = 0 \) and \( \beta_t^{true}(x_{j2q}^{true} - x_{i2q}^{true}) > 0 \),

\[\varepsilon_{iq} = \varepsilon_{iq}^{true} - [\beta_t^{true}(x_{j1q} - x_{i1q}) + \beta_t^{true}(x_{j2q}^{true} - x_{i2q}^{true})],\]

if \( \beta_t^{true}(x_{j1q} - x_{i1q}) = 0 \) and \( \beta_t^{true}(x_{j2q}^{true} - x_{i2q}^{true}) = 0 \),

\[\varepsilon_{iq} = \varepsilon_{iq}^{true} - [\beta_t^{true}(x_{j1q} - x_{i1q}) + \beta_t^{true}(x_{j2q}^{true} - x_{i2q}^{true})].\]

The bias in estimated parameters occurs because the change in the variance of the error terms due to measurement error is ignored:

\[
RR_{\text{sum}} = \sum_{j \neq i} \left\{ \max[0, \beta_t^{true}(x_{j1q} - x_{i1q})] + \max[0, \beta_{d}^{true}(x_{j2q}^{true} - x_{i2q}^{true})] \right\} + \varepsilon_{iq}^{true}.
\]
Likewise, if measurement error only occurs in travel distance, the true regret model with measurement error can be generalized according to Equation (33).

\[
RRsum_{iq}^{\text{true}} = \mu_{iq} \ast \sum_{j \neq i} \{ \max[0, \beta_i^{\text{true}}(x_{ijq}^{\text{true}} - x_{iq}^{\text{true}})] + \max[0, \beta_d^{\text{true}}(x_{ijdq} - x_{idq}^{\text{true}})] \} + \varepsilon_{iq},
\]

if \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) > 0 \) and \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) > 0 \),
\[
\varepsilon_{iq} = \varepsilon_{iq}^{\text{true}} - [\beta_d^{\text{true}}(v_{jd} - v_{id}) + \beta_d^{\text{true}}(v_{jd} - v_{id})],
\]

if \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) > 0 \) and \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) = 0 \),
\[
\varepsilon_{iq} = \varepsilon_{iq}^{\text{true}} - [\beta_d^{\text{true}}(v_{jd} - v_{id}) + \beta_d^{\text{true}}(v_{jd} - v_{id}) \ast \frac{1}{2}],
\]

if \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) > 0 \) and \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) < 0 \),
\[
\varepsilon_{iq} = \varepsilon_{iq}^{\text{true}} - \beta_d^{\text{true}}(v_{jd} - v_{id}),
\]

if \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) = 0 \) and \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) > 0 \),
\[
\varepsilon_{iq} = \varepsilon_{iq}^{\text{true}} - [\beta_d^{\text{true}}(v_{jd} - v_{id}) \ast \frac{1}{2} + \beta_d^{\text{true}}(v_{jd} - v_{id})],
\]

if \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) = 0 \) and \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) = 0 \),
\[
\varepsilon_{iq} = \varepsilon_{iq}^{\text{true}} - [\beta_d^{\text{true}}(v_{jd} - v_{id}) \ast \frac{1}{2} + \beta_d^{\text{true}}(v_{jd} - v_{id}) \ast \frac{1}{2}],
\]

if \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) = 0 \) and \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) < 0 \),
\[
\varepsilon_{iq} = \varepsilon_{iq}^{\text{true}} - \beta_d^{\text{true}}(v_{jd} - v_{id}) \ast \frac{1}{2},
\]

if \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) < 0 \) and \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) > 0 \),
\[
\varepsilon_{iq} = \varepsilon_{iq}^{\text{true}} - \beta_d^{\text{true}}(v_{jd} - v_{id}),
\]

if \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) < 0 \) and \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) = 0 \),
\[
\varepsilon_{iq} = \varepsilon_{iq}^{\text{true}} - \beta_d^{\text{true}}(v_{jd} - v_{id}) \ast \frac{1}{2},
\]

if \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) < 0 \) and \( \beta_d^{\text{true}}(x_{ijdq}^{\text{true}} - x_{idq}^{\text{true}}) < 0 \),
\[
\varepsilon_{iq} = \varepsilon_{iq}^{\text{true}} - \beta_d^{\text{true}}(v_{jd} - v_{id}) \ast \frac{1}{2},
\]

Thus, the results of our formal analysis indicate that the basic assumption of independently and identically distributed (IID) error terms among alternatives and individuals can be formally valid in (linear additive) random utility models when measurement error occurs. However, the assumption is difficult to justify for random regret models because for these models it can be logically deducted that error terms are heterogeneous across alternatives and individuals due to the semi-compensatory decision rule and the comparison of alternatives.

3. Empirical evidence

Having examined the issue of uncertainty formally, we continue by investigating empirically how measurement error causes bias in estimated parameters of both the utility-based MNL and original regret-based discrete choice models. Following previous research (Bhatta and Larsen 2011), we assume the raw revealed choice data are the true data excluding any
bias, and generated random measurement error in one or more variables. Also the variance of measurement error is assumed to correspond to the portion of the variance of each variable. In total, five scenarios exist for the standard deviation of the measurement error, ranging from 10% to 50% of the standard deviation of each variable at 10% intervals. The standard deviation of travel time is 7.52, and is larger than the standard deviation of travel distance (7.13). To moderate the effect of randomness, we generated 10 normalized random numbers, and used the average value. Parameters estimated from the basic (assumed true) data and from the data containing measurement error were compared to analyze the bias in the estimated parameters.

### 3.1. Data

The analyses are based on the 2009 MON data (Mobiliteit Onderzoek Nederlands – the Dutch National Travel Survey). The survey was administered to representative residents of the Netherlands to collect data about their daily travel behavior. More specifically, we used the sub-data set from the Province of Noord Brabant, which included 1158 respondents. The data include two levels of service variables (travel time and travel distance) for three mode choice alternatives (car, bike, and walk). The estimated models thus predict the probability of transportation mode choice as a function of these levels of service variables. The travel time ranges from 1 to 134 min, while travel distance varies from 1 to 91 km.

### 3.2. Estimated bias

The estimation results for the linear additive random utility model and the both random regret models ($RR_{max}$ and $RR_{sum}$), assumed to represent the true model, are shown in Table 2. All coefficients are statistically significant at the 95% confidence level and their sign is in the anticipated direction.

#### 3.2.1. Measurement error in travel time

First, we generated measurement error in travel time only. The bias in both parameters due to measurement error in the travel time variable is presented in Figure 1. The bias in parameter for travel time (Figure 1(a)) shows that the bias in all the random utility and random regret models increases with an increasing standard deviation of the measurement error. When the variance of measurement error is small (Scenarios 1), the bias in both parameters is also small and similar in size. However, as the variance of the measurement error increases, the parameters of the random regret model are increasingly more biased. In scenario 5 (50% variance for the travel time variable), while the parameter for travel time in the linear additive random utility model is only downward biased about 20%, it is almost

<table>
<thead>
<tr>
<th>Mode choice</th>
<th>$U$</th>
<th>$RR_{max}$</th>
<th>$RR_{sum}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (t-value)</td>
<td>$-0.1143 (-15.90)$</td>
<td>$-0.1152 (-14.97)$</td>
<td>$-0.0614 (-14.10)$</td>
</tr>
<tr>
<td>Distance (t-value)</td>
<td>$-0.5724 (-14.26)$</td>
<td>$-0.5222 (-13.01)$</td>
<td>$-0.325 (-12.56)$</td>
</tr>
<tr>
<td>$\rho^2$</td>
<td>0.220</td>
<td>0.202</td>
<td>0.204</td>
</tr>
<tr>
<td>Final likelihood</td>
<td>$-992.114$</td>
<td>$-1015.838$</td>
<td>$-1012.287$</td>
</tr>
</tbody>
</table>
30% in RRmax, even 40% in RRsum. This empirical evidence shows that the bias due to measurement error in the linear additive random utility model and random regret models may differ. This is similar in the bias in travel distance (Figure 1(b)).

Theoretically, the degree of bias should be the same for both variables. However, bias largely occurs in the parameter for travel time. This may reflect a correlation between measurement error (in the time variable) and the distance variable and/or correlation between time and distance variables. The coefficient of correlation between the two variables is 0.451, which is substantial.

Figure 1. Bias in parameters due to measurement error in travel time: normalized scale factor. (a) Travel time; (b) travel distance.
3.2.2. Measurement error in travel distance
In a second analysis, measurement error was generated in travel distance only. Since the variance of the distance variable is smaller than the variance in the travel time variable, the standard deviation of measurement error is smaller compared to the standard deviation reported in the previous section. Figure 2 shows the bias in both parameters due to measurement error in the travel distance variable. It reveals that the bias in both models increases with increasing standard deviation of the measurement error. Since the variance

Figure 2. Bias in parameters due to measurement error in travel distance: normalized scale factor. (a) Travel time; (b) travel distance.
and value of the parameter for the travel distance variable are smaller compared to the travel time variable, the bias is also smaller.

The bias in the parameters is larger again in the random regret models, compared to the linear additive random utility model. Also, $RRsum$ still shows a higher bias than $RRmax$.

### 3.2.3. Mean elasticity

Table 3 shows the transition of mean elasticity of attributes by measurement error in all random utility and random regret models. In the raw data (assumed without measurement error), when travel time for an alternative is increased with one unit, the choice probability is reduced by 1.56% on average according to the linear additive random utility model, while it is 1.72% for the $RRmax$, and 1.93% for the $RRsum$. In the case of an increasing travel distance of one unit for an alternative, the probability of choosing the alternative reduces with 1.92% according to the linear additive random utility model. This value is higher in the $RRmax$ (2.15%) and $RRsum$ (2.30%). After occurrence of measurement error, the transition is increasing. The higher variance of measurement error leads to the higher decline of choice probability: When the variance of measurement error is 50% of variance of variable, the decline of choice probability is 2.32% in the linear additive random utility model, 2.66% in $RRmax$, and 3.17% in the $RRsum$. Also, it is 2.53% decrease in the linear additive random utility model, 2.86% decrease in the $RRmax$, and 3.42% in $RRsum$ for travel distance.

Table 4 shows the details of the changes in predictive performance of each model due to the introduction of measurement error. The higher the variance of measurement error, the lower the value of final log-likelihood value in the linear additive random utility model and both random regret models. That is, as the variance of measurement error is increasing, the predictive power of the models becomes worse.

### 4. Scaling approach

The results of the empirical analyses thus confirm our theoretical contention that measurement error introduces bias in the linear additive random utility model and both random
regret models (RRmax and RRsum), but that bias is higher in random regret models. As shown in the formal analysis, while the variance of the error terms in the linear additive random utility model is formally the same across alternatives and individuals, it differs in the random regret models because of comparison of alternatives and semi-compensatory decision rule leading to non-identical error terms in the random regret models. Moreover, based on the empirical data used, the bias differs between the linear additive random utility and the random regret models: the bias is larger in the random regret models. Therefore, in this section, we allow varying scale factors in the models, applying a homogeneous and heterogeneous approach to reflect bias due to measurement error.

Since the variance of the error term is inversely proportional to the square of the scale factor, the change in variance can be expressed as a change of the scale factor. As we discussed in the formal analysis section, we assume the difference between a Gumbel and a normal distribution is Gumbel distributed, and the variance of the error terms is increased. Then, the scale factor is decreased, as shown in Equations (34) and (35). If

\[ \frac{1}{\text{VAR}(\varepsilon_{iq})} = (\mu_{iq})^2 : \frac{1}{\text{VAR}(\varepsilon_{iq})} \]  

then

\[ \mu_{iq} = \sqrt{\frac{\text{VAR}(\varepsilon_{iq})}{\text{VAR}(\varepsilon_{true})}}. \]  

### 4.1. Homogeneous scale parameters in both models

First, since the error terms in the random regret model are classically assumed to be independently identically (negative) Gumbel distributed, as in the linear additive random utility model, we assume that the bias due to measurement error in the random regret models is the same as in the linear additive random utility model. Therefore, the scale parameters are homogeneous in the random regret model across individuals and alternatives.

#### 4.1.1. Homogeneous change of scale factor by measurement error

In the case of measurement error occurring in travel time only, the variance of the error terms in the linear additive random utility model changes from \( \pi^2/6 \) to \( \pi^2/6 + (\beta_{true}^{t})^2 \sigma_{t}^2 \) as shown in the formal analysis. Therefore, the changed scale factor can be formulated as

\[ \mu_{iq} = \sqrt{\frac{\text{VAR}(\varepsilon_{iq})}{\text{VAR}(\varepsilon_{true})}} = \sqrt{\frac{\pi^2/6}{\pi^2/6 + (\beta_{true}^{t})^2 \sigma_{t}^2}}. \]  

If measurement error only occurs in travel distance, the changed scale can be computed, following the same logic, using Equation (35).

\[ \mu_{iq} = \sqrt{\frac{\text{VAR}(\varepsilon_{iq})}{\text{VAR}(\varepsilon_{true})}} = \sqrt{\frac{\pi^2/6}{\pi^2/6 + (\beta_{true}^{d})^2 \sigma_{d}^2}}. \]
Likewise, if measurement error occurs in both variables, the variance of the error terms equals $\pi^2/6 + (\beta_t^{\text{true}})^2 \sigma_t^2 + (\beta_d^{\text{true}})^2 \sigma_d^2$, the changed scale factor can be derived as

$$\mu_{iq} = \sqrt{\frac{\text{VAR}(\varepsilon_{iq}^{\text{true}})}{\text{VAR}(\varepsilon_{iq})}} = \sqrt{\frac{\pi^2}{6} + (\beta_t^{\text{true}})^2 \sigma_t^2 + (\beta_d^{\text{true}})^2 \sigma_d^2}.$$  

(38)

These changes in the scale factor in Equations (36)–(38) are applied to the MNL linear additive random utility model and the random regret models.

4.1.2. Measurement error in travel time

Based on the data with measurement error in travel time, the scale factor was calculated using Equation (35). Figure 3 shows the bias in the models when considering the change in the scale factor. Compared to Figure 1, Figure 3 shows that the bias decreases substantially in all models and all scenarios when taking the change in the scale factor into account. For the linear additive random utility model, the bias is less than 10% across all scenarios. The parameters are slightly upward biased when the standard deviation of measurement error is small. Bias increases with an increasing standard deviation of measurement error.

For the random regret models, while the bias is close to zero when the standard deviation of measurement error is small, it is increasing with an increasing standard deviation of the measurement error. The bias is much higher in RRsum than RRmax. All parameters are downward biased. Theoretically, after introducing the variance of scale factors, the bias should be zero and the value of the estimated parameters should be equal to the true parameters. The difference may be caused by correlations between the variables, and the approximation of the scale factor due to the mixture of the Gumbel and Normal distribution. In addition, we assumed the base model is the true model meaning that the input data are unbiased, but the input data may already be biased.

4.1.3. Measurement error in travel distance

We estimated parameters, taking into account the scale factor, using Equation (36), with measurement error in travel distance, and compared the results with the parameters based on the normalized scale factor. As shown in Figure 4, we can closely approximate the true parameter in the linear additive random utility model by considering the change in the scale factor in the estimation process. The bias is only around 5%. In case of the random regret models, the estimated parameters are also close to the true parameters when the standard measurement error is small (Scenarios 1 and 2). However, the bias is increasing when the standard deviation of measurement error becomes larger (Scenario 5). Still the bias is larger in RRsum than RRmax. This implies that the heterogeneity (non-identicalness) between error terms is higher in RRsum.

4.2. Heterogeneous scale parameters in the random regret model

The results of the previous theoretical and empirical analyses suggest that the introduction of heterogeneous error terms in random regret models across alternatives and individuals may be needed to correct for bias due to measurement error. In this section, we report the estimation results allowing for heterogeneous scale parameter and compare the resulting bias in the random regret model against the homogeneous case.
4.2.1. Heterogeneous change of the scale factor due to measurement error

If measurement error only occurs in travel time, based on Equations (17) and (31), the scale factor is equal to

$$\mu_{iq} = \sqrt{\frac{\text{VAR}(\varepsilon_{iq}^{\text{true}})}{\text{VAR}(\varepsilon_{iq})}} = \sqrt{\frac{\pi^2}{6} \frac{\text{VAR}(\varepsilon_{iq})}{\text{VAR}(\varepsilon_{iq})}}$$

for the RRmax:

$$\text{if } \max_{j \neq i}[p_{t}^{\text{true}}(x_{tq}^{\text{true}} - x_{tq}^{\text{true}})] > 0, \quad \text{VAR}(\varepsilon_{iq}) = \frac{\pi^2}{6} + (\beta_{t}^{\text{true}})^2 \cdot 2\sigma_t^2,$$
Figure 4. Bias in parameters due to measurement error in travel distance: homogeneous scale factor. (a) Travel time; (b) travel distance.

\[
\text{if } \max_{i \neq i'} [\beta^\text{true}_i (x^\text{true}_{it} - x^\text{true}_{iq})] = 0, \quad \text{VAR}(\epsilon_{iq}) = \frac{\pi^2}{6} + (\beta^\text{true}_t)^2 \sigma^2_t,
\]

\[
\text{if } \max_{i \neq i'} [\beta^\text{true}_i (x^\text{true}_{it} - x^\text{true}_{iq})] < 0, \quad \text{VAR}(\epsilon_{iq}) = \frac{\pi^2}{6}
\]

for the RRsum:

\[
\text{if } \beta^\text{true}_t (x^\text{true}_{j1} - x^\text{true}_{it}) > 0 \text{ and } \beta^\text{true}_t (x^\text{true}_{j2} - x^\text{true}_{it}) > 0, \quad \text{VAR}(\epsilon_{iq}) = \frac{\pi^2}{6} + (\beta^\text{true}_t)^2 \cdot 4 \sigma^2_t,
\]
if $\beta_t^{true}(x_{j1tq} - x_{itq}) > 0$ and $\beta_t^{true}(x_{j2tq} - x_{itq}) = 0,$

VAR($\varepsilon_{iq}$) = $\pi^2/6 + (\beta_t^{true})^2 \times 3\sigma_t^2$,

if $\beta_t^{true}(x_{j1tq} - x_{itq}) > 0$ and $\beta_t^{true}(x_{j2tq} - x_{itq}) < 0,$

VAR($\varepsilon_{iq}$) = $\pi^2/6 + (\beta_t^{true})^2 \times 2\sigma_t^2$,

if $\beta_t^{true}(x_{j1tq} - x_{itq}) = 0$ and $\beta_t^{true}(x_{j2tq} - x_{itq}) > 0,$

VAR($\varepsilon_{iq}$) = $\pi^2/6 + (\beta_t^{true})^2 \times 3\sigma_t^2$,

Figure 5. Bias in parameters due to measurement error in travel time: homogeneous and heterogeneous scale factor in regret. (a) RRmax: travel time; (b) RRmax: travel distance; (c) RRsum: travel time; and (d) RRsum: travel distance.
Figure 5. Continued.

If $\beta_{t}^{true}(x_{j1tq}^{true} - x_{itq}^{true}) = 0$ and $\beta_{t}^{true}(x_{j2tq}^{true} - x_{itq}^{true}) = 0,$

\[
\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_{t}^{true})^2 \ast 2\sigma_t^2,
\]

If $\beta_{t}^{true}(x_{j1tq}^{true} - x_{itq}^{true}) = 0$ and $\beta_{t}^{true}(x_{j2tq}^{true} - x_{itq}^{true}) < 0,$

\[
\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_{t}^{true})^2 \ast \sigma_t^2,
\]

If $\beta_{t}^{true}(x_{j1tq}^{true} - x_{itq}^{true}) < 0$ and $\beta_{t}^{true}(x_{j2tq}^{true} - x_{itq}^{true}) > 0,$

\[
\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_{t}^{true})^2 \ast 2\sigma_t^2.
\]
if $\beta_t^{\text{true}}(x_{i1}^{\text{true}} - x_{itq}^{\text{true}}) < 0$ and $\rho_t^{\text{true}}(x_{i2}^{\text{true}} - x_{itq}^{\text{true}}) = 0$,

$$\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_t^{\text{true}})^2 \sigma_t^2,$$

if $\beta_t^{\text{true}}(x_{i1}^{\text{true}} - x_{itq}^{\text{true}}) < 0$ and $\rho_t^{\text{true}}(x_{i2}^{\text{true}} - x_{itq}^{\text{true}}) < 0$,

$$\text{VAR}(\varepsilon_{iq}) = \pi^2/6. \tag{39}$$

Likewise, based on measurement error only in travel distance, using Equations (19) and (33), the scale factor is

$$\mu_{iq} = \sqrt{\frac{\text{VAR}(\varepsilon_{iq}^{\text{true}})}{\text{VAR}(\varepsilon_{iq})}} = \sqrt{\frac{\pi^2}{6 \text{VAR}(\varepsilon_{iq})}}$$

for the RRmax:

if $\max_{j \neq i}[\beta_t^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}})] > 0$, $\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_d^{\text{true}})^2 \sigma_d^2$,

if $\max_{j \neq i}[\beta_t^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}})] = 0$, $\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_d^{\text{true}})^2 \sigma_d^2$,

if $\max_{j \neq i}[\beta_t^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}})] < 0$, $\text{VAR}(\varepsilon_{iq}) = \pi^2/6$ for the RRsum:

if $\beta_d^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}}) > 0$ and $\beta_t^{\text{true}}(x_{i2}^{\text{true}} - x_{idq}^{\text{true}}) > 0$,

$$\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_d^{\text{true}})^2 \sigma_d^2,$$

if $\beta_d^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}}) > 0$ and $\beta_t^{\text{true}}(x_{i2}^{\text{true}} - x_{idq}^{\text{true}}) = 0$,

$$\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_d^{\text{true}})^2 \sigma_d^2,$$

if $\beta_d^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}}) > 0$ and $\beta_t^{\text{true}}(x_{i2}^{\text{true}} - x_{idq}^{\text{true}}) < 0$,

$$\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_d^{\text{true}})^2 3\sigma_d^2,$$

if $\beta_d^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}}) = 0$ and $\beta_t^{\text{true}}(x_{i2}^{\text{true}} - x_{idq}^{\text{true}}) > 0$,

$$\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_d^{\text{true}})^2 \sigma_d^2,$$

if $\beta_d^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}}) = 0$ and $\beta_t^{\text{true}}(x_{i2}^{\text{true}} - x_{idq}^{\text{true}}) = 0$,

$$\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_d^{\text{true}})^2 \sigma_d^2,$$

if $\beta_d^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}}) = 0$ and $\beta_t^{\text{true}}(x_{i2}^{\text{true}} - x_{idq}^{\text{true}}) < 0$,

$$\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_d^{\text{true}})^2 \sigma_d^2,$$

if $\beta_d^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}}) < 0$ and $\beta_t^{\text{true}}(x_{i2}^{\text{true}} - x_{idq}^{\text{true}}) > 0$,

$$\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_d^{\text{true}})^2 \sigma_d^2,$$

if $\beta_d^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}}) < 0$ and $\beta_t^{\text{true}}(x_{i2}^{\text{true}} - x_{idq}^{\text{true}}) = 0$,

$$\text{VAR}(\varepsilon_{iq}) = \pi^2/6 + (\beta_d^{\text{true}})^2 \sigma_d^2,$$

if $\beta_d^{\text{true}}(x_{i1}^{\text{true}} - x_{idq}^{\text{true}}) < 0$ and $\beta_t^{\text{true}}(x_{i2}^{\text{true}} - x_{idq}^{\text{true}}) < 0$,

$$\text{VAR}(\varepsilon_{iq}) = \pi^2/6. \tag{40}$$
4.2.2. Measurement error in travel time

Similar to the analyses reported in the previous sections, we first report the case in which measurement error occurs only in travel time. The comparison of bias in the random regret model between the models with a homogeneous (Equation (36)) and heterogeneous scale factor (Equation (39)) is shown in Figure 5. When the standard deviation of measurement error is small, the estimated parameters with a heterogeneous scale factor are similar to the parameters with homogenous error terms, and close to the true parameters. However,

**Figure 6.** Bias in parameters due to measurement error in travel distance: homogeneous and heterogeneous scale factor in regret. (a) RRmax: travel time; (b) RRmax: travel distance; (c) RRsum: travel time; and (d) RRsum: travel distance.
when the measurement error becomes larger, the estimated parameters with a heterogeneous scale factor are still close to the true parameters (the maximum bias is around 10%), whereas we saw evidence of increasing bias for the model with a homogenous scale factor.

4.2.3. Measurement error in travel distance
Next, we examined the case of measurement error in travel distance only. Figure 6 shows the difference in bias in the random regret model allowing for respectively a homogeneous

**Figure 6.** Continued.
(Equation (37)) and heterogeneous (Equation (40)) scale factor. Since the standard deviation of travel distance is smaller than the standard deviation of travel time, the bias is also smaller. Therefore, the bias is within 10% in all scenarios for both assumptions about the scale factor in the random regret model. However, allowing for heterogeneous scale factors, the bias is smaller when the standard deviation of measurement error is larger.

By comparing bias in the parameters of the random regret models based on respectively homogeneous and heterogeneous error terms, our findings suggest that bias increases with increasing measurement error in case of a homogenous scale factor but not that much once scale parameter is estimated in a heterogeneous way, i.e. introducing heterogeneous scale factors reduced the bias. The bias in the random regret models with heterogeneous scale factors is around 5%, which is in the same order of magnitude as for the linear additive random utility model with a homogenous scale factor. Therefore, we argue that the error terms in the linear additive random utility model are still independently identically Gumbel distributed, even though measurement error occurs. However, the error terms in the random regret model are no longer independently and identically Gumbel distributed. The measurement error causes heterogeneity into the random regret models. Moreover, the heterogeneous is higher when the amount of regret is defined against all non-chosen alternatives (RRsum), compared to when the amount of regret is defined only against the best non-chosen alternative (RRmax).

5. Conclusions and discussion

Random regret choice models have recently been introduced in the travel behavior research community as an alternative to random utility models. Their popularity has rapidly increased and evidence of their outperformance in some choice contexts has been documented. Reactions of conference audiences and reviewers suggest a need for a more fundamental discussion of the difference between random utility and random regret models. Recent theoretical progress and empirical evidence supports the relevance of elaborating regret-based choice models in travel behavior research and critically examining common practice.

The current paper has addressed a commonly made assumption in the specification of the random regret models, which has hitherto gone untested. Commonly made assumptions regarding the error terms of the regret function in the estimation of these models have not received much critical reflection. Rather, mainly for ease of estimation, it has been commonly and conveniently assumed that the negative of errors are independently and identically Gumbel distributed, following the assumptions underlying the multinomial logit model.

In this paper, we critically assess the validity of this assumption, both theoretically and empirically, for the case of measurement error. We have shown that the assumption of independently and identically distributed error terms in the original random regret model should be more critically assessed compared to the linear additive random utility model in the sense that while the bias due to measurement error introduced in the linear additive random utility model is homogeneous, the bias in the random regret models is heterogeneous. More specifically, the heterogeneity among individuals is higher when regret is defined against all non-chosen alternatives, compared to the situation that regret is defined only against the best non-chosen alternative. This result seems reasonable considering the
different decision processes in the random utility and random regret models. While the decision process in the multinomial logit model is based on the assumption that individuals derive a utility by processing the attributes of each choice alternative independently and separately, the decision process in the original random regret model assumes that individuals assess regret by systematically comparing choice alternatives. Moreover, while utility is generated based on a full-compensatory decision rule, regret is generated through a semi-compensatory decision rule represented by the max operator (regret is only generated when the chosen alternative is inferior to the non-chosen alternative).

The implications of this finding for travel demand forecasting are straightforward. In case researchers are willing to assume that measurement error is negligible, common practice in the assumptions about the distributions of the random components of the regret function is defendable. However, if these assumptions are not warranted, the variance of the errors of the choice alternatives of the random regret models will be heterogeneous not only across alternatives but also across individuals with respect to their choice set composition. Consequently, the assumptions of independently and identically Gumbel distributed error terms, currently applied in random regret models, may misrepresent individual’s behavior, and might lead to false policy recommendations. Thus, if researchers have reason to believe that measurement error is substantial or wish to avoid the rigorous assumptions of IID distributed error terms to start with, the use of heterogeneous scale factors in estimating the parameters of the random regret models is advisable to avoid biased estimates.

To complete this paper, some limitations should be made explicit. First, the current study is based on the original random regret models, in detail, RRmax and RRsum. Our assumption is that the kind of results, obtained for the RRsum, is similar to the smooth version of regret function (RRlog) when we accept the argument that the RRlog and RRsum are almost similar (Chorus 2010, 2012a). However, formal and empirical analyses are still needed for RRlog. A similar analysis is warranted for the logarithmic approximation (van Cranenburgh, Guevara, and Chorus 2015) of regret choice models. Second, a single study can just provide limited evidence. Although not very popular, replicative studies using different data sets and different choice problems are needed to better understand the importance of using more sophisticated specifications and estimation methods. Thirdly, the current study is based on the assumption of normally distributed error terms. Some researchers will object to this distribution for domain-restricted variables, which strictly speaking is justified although sometimes ignorable in practice. It would be interesting in further research to replicate the current study for the asymmetric case of measurement error and corresponding distributions of error terms. Finally, it would be relevant to examine any effects of endogeneity (Hensher, Rose, and Greeen 2015) in random regret choice models.

Disclosure statement
No potential conflict of interest was reported by the author(s).

References


