MASTER

Project risk budgeting using a VaR approach

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Project risk budgeting using a VaR approach

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Contents

Abstract

Risk management is an integral element of everyday business processes, with many companies designating specialists to cope with the appurtenant challenges. Products and projects becoming ever more complex, gave rise to so called Complex Project Based Organization (CPBO). Such organizations focus on developing large scale, complex and innovative systems at low volumes. Though risk management is a highly developed research domain, little attention has been devoted to this particular type of organizations. This thesis aims to formulate a risk management model, which is capable of estimating the monetary exposure of a project's specific risk portfolio. In order to deal with the characteristic of CPBOs having limited data available, this thesis applies the Value-at-Risk (VaR) principle. The thesis starts with a comprehensive literature review on the subject, which results in an overview of frequently occurring risk factors. Subsequently it identifies the mainstream risk management models which can be translated to the context of CPBO risk management. Via a numerical backtesting analysis it has been shown that Monte Carlo Simulation (MCS) is the most accurate risk portfolio VaR estimation method. Finally, a case study has been conducted to verify that MCS indeed outperforms the other methods and can lead to improved risk exposure estimates.

“If it was easy, someone else would have done it before you.“
My fellow colleagues

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Management summary

The increasing complexity of todays logistic solutions combined with ever more fierce competition has put more emphasis on companies to compose accurate project sales prices. An important cost driver of such prices is the risk budget. The goal of a risk budget is to account for any (un)foreseen risk factors that occur during project execution. As the pricing process becomes ever more complex and the required accuracy of such budgets keeps increasing, new challenges arise and so does the need for a sound risk management system. The current risk management literature is a well developed research domain, but typically assumes that data is abundantly available. The market in which logistic solutions companies operate on the other hand is one of low sales volumes, such that data is most often very scarce. This problem can be generalized to a much larger group of companies, referred to as Complex Project Based Organizations (CPBOs), of which many face the same problem. As such, this thesis will focus on constructing a risk management model which suits this purpose. To attain this goal, the model will incorporate a well known method in financial literature: Value-at-Risk (VaR), which will be used to estimate a risk profile's monetary exposure. In order to test the model, a case study will be conducted within a world leading logistic solutions company. The main research question of this thesis is defined as:

How can Project Based Complex Product System Organizations compose project specific risk budgets that incorporate the underlying characteristics and risk factors?

Model

We started the pursuit to construct a CPBO risk management model with a literature review. The goals of this review was twofold. First, we were interested to see whether researchers had previously developed a standard risk profile for so called Complex Product System (CoPS): highly complex and technologically advanced products. Whenever such profile exists, this would have been used to compose a risk profile, which forms the input of the risk management models. Unfortunately, due to the context dependency of CoPS, such basis profiles are impossible to compose. On the contrary literature did provide numerous frameworks to guide practitioners and researches in composing such a profile. The second goal of the literature review was to discover which risk management methods were most frequently applied when estimating financial portfolio VaRs. Though many different (and more advanced) methods exist, we were able to reduce them to three basic risk management methods: the Delta-Normal method, Historical Simulation and Monte Carlo Simulation. Subsequently we created three models, each incorporating its own VaR estimation method. Since no research has ever been conducted on this type of risk management models, we performed a theoretical backtest to check whether the performance of all three models is in line with the same models in financial literature. In doing so, we incorporated the Traffic Light backtest which has the main advantage that it accounts for any form of "bad luck" due to randomness. A backtesting procedure

uses historical data to train the model and another set of historical data to test whether the model is accurate. Since there is no scientific literature available which identifies what CPBO risk data looks like (i.e. what distributions they follow), we used case study data. For a set of five risk factors we took the data and fitted a distribution. To account for the fact that data was limited, and both fitting the distribution's parameters and testing their fit used the same data, we conducted an adjusted Kolmogorov-Smirnov test which accounts for this. Based on the fitted distributions, we generated a synthetic dataset which we used to conduct the theoretical backtesting procedure. From this backtest we concluded that the results were in line with the publications in financial literature. More specific, Monte Carlo Simulation is the best performing model, closely followed by Historical Simulation. The backtest discourages the use of a Delta-Normal model in the context of CPBO risk factors.

Subsequently we tested the risk management models within the case study company. We chose to test all three models, despite the prior results, to verify whether the theoretical outcomes were still valid. This decision is substantiated by the fact that in this application we did not generate synthetic data (i.e. data was scarce), and no scientific literature was available to substantiate on the model's expected behavior. Before backtesting the CPBO risk management models using case study data, we conducted a numerical study to determine the minimum number of data points required to properly estimate the risk portfolio's VaR. This again is important since scientific literature does not support us with an answer, and such a minimum is important since data is scarce meaning a lower bound might easily be violated. The study concluded that a risk portfolio composed of five correlated risk factors, which occur with an independent probability of 20% has a recommended minimum of 24 data points per risk factor and an absolute minimum of 10. This minima are important as we can use it to show that the CPBO risk management model indeed performs as required. An important assumption, that determines this minima, is that we allow residual budget of one risk factor to be used to compensate another factor's cost. We will refer to this as residual budget reallocation. With the theoretical basis covered we continued to the case study.

The case study company supplied us with a set of risk factor costs and a test set of 11 projects, for which we know the risk portfolio, sales price, initial budgets, the risk adjusted budgets, the budgets as sold and the realized costs. The first step in the case study was to determine which risk factors to include in the model, for which we used the following logic: Given the risk factors identified in the test set's risk profiles, create a subset of the risks which occur in the training set and have at least 10 data points. This resulted in a portfolio composed of six risk factors. Subsequently we computed the risk portfolio VaR estimate, using all three models, for all required portfolios. The results shown in Table 1 represent the estimated risk budgets in % of the sales price. Using the available test data, we computed the risk portfolio VaR estimations and hereafter determined the CPBO risk management model's budgets. The total overview of the testing phase has been summarized in Table 2.

Table 1: Model training - risk portfolio VaR estimates in % sales price

Portfolio	DN	HS	MCS
(1,1,1,1,1,1)	0.1296	0.0557	0.0491
(1,0,1,1,1,1)	0.1155	0.0516	0.0453
(1,0,1,1,0,0)	0.1031	0.0383	0.0346

Table 2: Descriptive statistics for risk estimation versus CPBO risk management models

$N = 11$, $N^{-} = 4$	Mean deficit	St. dev. deficit	Mean residual	St. dev. residual	RMSE	Minimum	Maximum	Sum of deficit	Sum of residual	Sum total
	Descriptive statistics risk estimation via Delta Normal method (in € million)									
Default	-3.425	3.204	1.668	1.599	4.219	-9.980	2.799	-30.828	3.336	-27.492
Adjusted default	-2.148	3.260	1.283	1.110	2.274	-5.907	3.085	-6.445	10.261	3.816
Non default	-1.501	1.778	0.689	0.769	1.519	-4.083	1.810	-6.004	4.820	-1.184
Delta Normal	-1.210	1.449	2.166	2.199	2.795	-2.234	5.213	-2.420	19.493	17.073
Historical Sim.	-1.899	1.922	1.181	1.226	1.796	-3.257	3.331	-3.797	10.625	6.828
Monte Carlo Sim.	-1.966	1.968	1.089	1.149	1.728	-3.358	3.145	-3.933	9.798	5.865

Results

From the case study backtest figures we can derive some key results. First, lets define the other budgets in Table 2. The default budget is the project's cost price. The adjusted default budget is the previous supplemented to account for project specific characteristics and risks. Lastly the non default budget is the budget as sold to the customer. This difference is determined by the sales department, based on amongst others their experience and insiders' knowledge. When focusing on the CPBO risk management models, we can conclude that the Delta Normal method performs worse compared to the other methods. This is in line with the theoretical studies and is therefore deemed unsuitable in the case of CPBO risk management. Both simulation methods on the contrary perform reasonably well given the limited training data, achieving a RMSE within the interval of both the adjusted- and non default budget. When comparing the CPBO risk management simulation models with the adjusted budgets, we conclude that both models outperform the current methodology. Both achieve a RMSE that is 24% and 21% lower compared to the adjusted budget's RMSE for the Historical- and Monte Carlo Simulation resp. For the CPBO risk management model that uses Monte Carlo Simulation we can conclude that it achieves a 53% greater net profit compared to the adjusted budget methodology. In addition, it also has a lower sum of residuals, which shows that it does not bluntly over inflate project budgets but does a better job at estimating them.

Conclusions & Recommendations

From this thesis we can conclude that the call for a risk management methods within the context of a CPBO can be answered by use of well-studied financial risk management models. We've seen that, even with little data, simulating risk portfolio VaR estimates yields fairly accurate results. Throughout the thesis we've encountered the effects of non-ideal data on simulation models. Therefore we stress the importance of carefully designing the data gathering process. For the case study company we embody this in the main recommendations:

- Improve risk registration; Make sure that every project produces the required risk registers. Additionally those registers should be checked for completeness, which is twofold. That is, instead of checking its existence it should be examined whether it at least aligns with the previously answered budgeting questions. Additionally, it is recommended that the current risk registers are standardized such that they always incorporate the risk factors currently adjusted for in the budgeting questions.
- Redesign the process of data administration; In order to open up the possibility of simulating risk budgets, the current process has to change. First of all one should start gathering the complete risk costs, instead of the costs exceeding the available budgets. Furthermore data should be gathered on activity level detail, rather than project level. This is particularly important since it enables simulating risk factors at a lower level, making sure risk supplements are correctly distributed.

List of Abbreviations

CI	Confidence Interval
cdf	Cumulative Distribution Function
CoPS	Complex Product Systems
CPBO	Project Based Complex Product Systems Organization
DN	Delta Normal approach
ecdf	Empirical Cumulative Distribution Function
HS	Historical simulation
i.i.d.	Independent and identically distributed
K-S test	KolmogorovSmirnov test
MCS	Monte Carlo simulation
ME	Mean Error
MPE	Mean Percentage Error
P&PV	Pricing and Proposal Verification
PBO	Project Based Organization
pdf	Probability Density Function
PSD	Positive Semi Definite
RMSE	Root Mean Squared Error
R&O	Risk and Opportunity register
r.v.	Random Variable
TU/e	Technical University of Eindhoven

Chapter 1

Introduction

Budgets are part of a company's everyday concern. Whether it are small companies or stock exchange listed enterprises, controlling costs is inescapable. Despite much emphasis has been placed upon compiling and allocating budgets, many companies still face the agonizing effects of not being able to manage their cash flows correctly. Take for example Toshiba's Westinghouse having a $6.3 billion write-down on its nuclear reactor business due to, among others, budget overruns (Clenfield et al., 2017). Or perhaps an example closer to home. The Dutch government exceeding the defense budget allocated for its new Joint Strike Fighters with € 500m (RTL Nieuws, 2016). With projects being ever more complex, promising opportunities in budgeting arise.

Throughout the years, a tremendous amount of research has been conducted with regards to risk management. The application of risk management has a wide spread, with the most well known area of implementation being financial institutions. Though the literature domain is very mature, it does have less developed sub domains, of which one is risk budgeting. Pearson (2002) defines risk budgeting as: a process of measuring and decomposing risk, using the measures in asset-allocation decisions, assigning portfolio managers risk budgets defined in terms of these measures, and using these risk budgets in monitoring the asset allocations and portfolio managers. This thesis will adopt Pearson's definition of risk budgeting, with the addition that it will focus on projects. That is, instead of measuring risk organization wide, it will concentrate on measuring risk on a project level. The goal of this thesis is to review, discuss and adapt risk management theories for Project Based Organization (PBOs) producing Complex Product System (CoPS) (hereafter simply referred to as CPBO). In this thesis we'll follow Ren and Yeo (2006) who define CoPS as "high cost, engineering-intensive products, systems, networks and constructs. They are business to business capital goods which form the backbone of modern economy and society". We argue that the latter research domain is an underexposed sub domain of the risk management literature which has a great potential to aid both small, medium and large enterprises.

1.1 Research motivation

In order to manage budgeting processes, a logistic solutions company (hereafter referred to as case study company) uses a pricing department. The department is, among other things, responsible for providing the sales department with tangible methods supporting them in determining the correct sales prices for new projects. One of the many subjects this department has to cope with is project risk. Examples of project risks are insufficient slack in project execution planning, high demanding customers, new product introductions, R&D projects, system requirements not being met or country specific conditions. In doing so, the company has created a so called R&O register (hereafter referred to as risk register) which is a tool to guide sales engineers in pricing project specific risk factors (hereafter referred to as risk factors). Given the ever increasing complexity and magnitude of projects, anticipating risks is becoming more complex too. Furthermore it is not only the occurrence which has to be estimated, but also the expected costs which requires extensive experience with and knowledge of amongst others the industry, client and country. With the company rapidly expanding its sales department due to the vast amount of available work, the number of newly acquired employees is growing accordingly. The case study company's problem can be generalized in a broader sense.

CPBOs worldwide struggle with the composition of risk budgets that are tailor made to their complex processes (Ivory and Alderman, 2005; Mayo, 2009) and would therefore benefit from a model which could estimate those costs given some confidence interval. Within conventional risk management models, such confidence intervals are frequently modeled via Value-at-Risk (VaR). VaR will ensure a confidence interval by computing the quantiles of some distribution, and will be incorporated into the CPBO risk management model.

Throughout literature, research has been conducted on identifying risk factors that play an important role in a project's risk profile, for many different disciplines (e.g. mega projects (Bosch-Rekveldt et al., 2011), IT projects (Hu et al., 2013) or Complex Product Systems (Jaber, 2016; Yeo and Ren, 2009)). Given that the latter studies are purely scientific, it would be worthwhile to compare and see if the risks of CPBOs in practice align with the available research outcomes. Yeo and Ren (2009) verify the complexity of CPBOs as they state: "In CoPS projects, adverse events often emerge as surprises. Complex products and integrated systems tend to exhibit nonlinear and emergent properties during production or implementation, as unforeseen and unexpected events and interactions often occur during design and systems engineering and integration phase." Moreover, given the business in which CPBOs operate: complex, large scale project, accumulating data might form a challenge. As such companies generally execute a few projects a year, gathering risk related data might be a time consuming process whereas risk management models typically rely on large datasets. From a research perspective it is interesting to see if risk management methods can achieve equal performance in a CPBO environment. Recognizing the prior needs, both in industry as in scientific research, substantiates the claim that a CPBO risk management model can be of great use within many corporations.

In line with these findings, this thesis aims to enrich the case study company's P&PV department with new insights into project risk budgeting. With respect to scientific literature, this research will first show if traditional risk management methods can be used in a CPBO environment and second report on their performance by means of a case study. For the case study company this entails that, through developing a new risk budgeting approach, the company should be able to create an insight into risks with a relative low occurrence but high monetary impact. Therefore the CPBO risk management model should contribute towards:

- Modeling the costs related to risk factors on a project level, taking into account characteristics such as the industry and project size.
- Gaining insightful information regarding the effects of (not) correctly pricing project risks.
- The trade off between higher levels of certainty and its effect on the respective budgets.

1.2 Thesis outline

The thesis will start with a comprehensive overview of the as-is situation in the next section, in which we'll describe the current risk management system and elaborate on the case study company's internal budgeting methods. Chapter 2 will give a concise overview of the currently available literature on risk management and its budgeting implications. With this chapter the reader will be made aware of both the available knowledge, the pro's and con's of the most popular models as well as the literature gaps. Some of the literature gaps will be answered throughout this thesis. Succeeding the literature review is the research layout in Chapter 3. In this chapter we'll present the research questions, -scope and -methodology. The research layout will form the backbone of the thesis. With the previous in mind, Chapter 4 will describe the mathematical models and its underlying components in detail. Furthermore we'll determine which risk management model should be preferred within a CPBO environment. To illustrate the power of risk budgeting models, a case study is conducted within Chapter 5. For the case study company we'll study subjects such as: the required data and the CPBO risk management model's performance. Some future practical applications will be introduced in Chapter 6, which will illustrate the CPBO risk management model's flexibility. Finally Chapter 7 will conclude the research findings, amongst others by answering all research questions and generalizing this to answer the composed literature gaps. This chapter ends with an overview of recommendations for the case study company.

1.3 As-is situation

The primary process of the case study company can be brought back to three main processes that are of interest for the CPBO risk management model: determining the risk budget, composing the sales price and project execution. Figure 1.1 shows a comprehensive overview of the risk management process, with black boxes representing the sales and project execution processes. The process starts with a project lead or tender at the sales department that needs a sales price. Using a pricing tool the sales engineer is able to construct the default budgets. Default budgets are the basis price of all individual components including direct labor etc. As an addition to the default budgets, the sales engineer composes a risk register. The register is a simple Excel template in which the sales engineer will identify possible risk factors, estimating their costs and assessing their probability of occurrence (all based on experience). Subsequently the Excel sheet multiplies the risk costs and their respective probabilities to compute the expected budget impact per risk factor. The sum of the latter computations is added to the default budgets as a risk budget.

Subsequently the sales engineer answers a set of questions, by which the default budgets are adjusted for project specific characteristics such as complexity and risks, which results in the adjusted budget. The latter adjustments are sets of scalar multiplications based on the P&PV's experience rather than data driven. The adjusted default budgets represent the

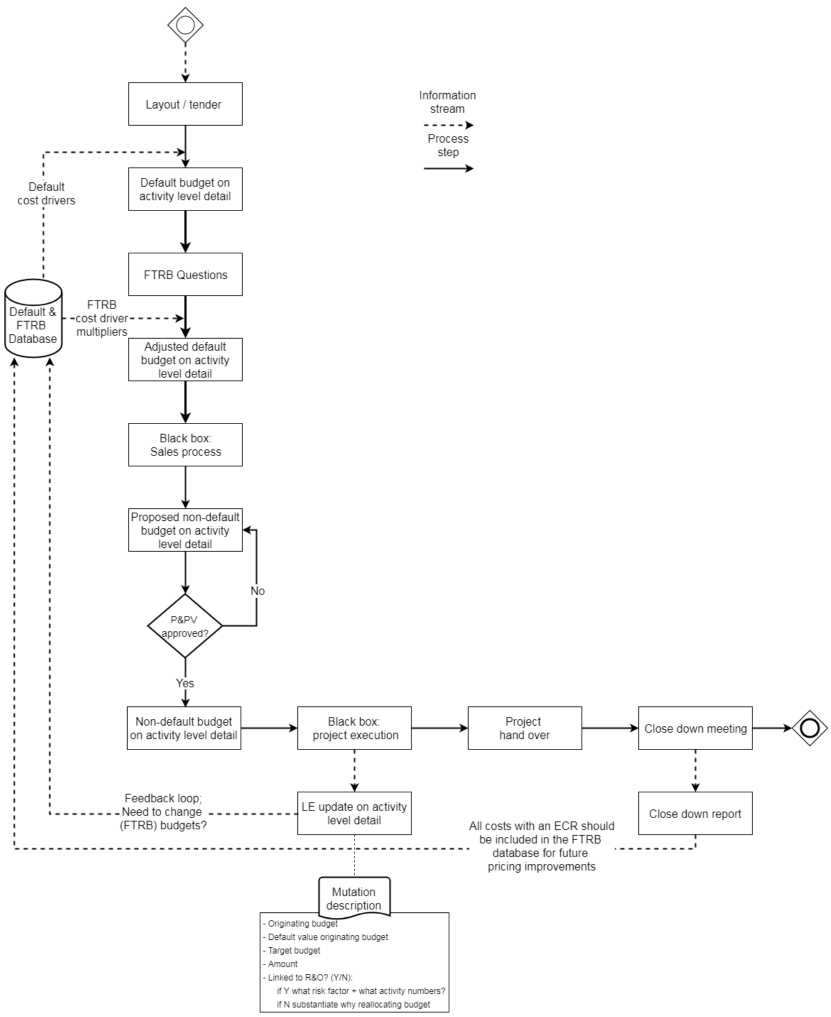

Figure 1.1: Flowchart of current risk management process

sales prices as advised by the P&PV department but are not final. The sales department will modify the adjusted budgets based on either extra information available or to maximize the probability of selling the project.

During project execution the project managers keep track of all budgets, and any budget mutation (or more specific for our case, budget shifts due to risk realizations) should be registered. Finally after project hand over, the project managers arrange a closing down meeting in which all risk factors that have realized a large amount of costs should be discussed. All project manager feedback regarding risk realizations should be captured and sent to the P&PV department to continuously improve the default and adjusted default budgets.

In practice we see that many parts of the primary process can be improved to some extent. When focusing on the sales engineers we see that lowering the sales price, without taking into account the increment in the risk portfolio's monetary impact, may in some situations lead to large budget overruns. Furthermore we see that estimating the costs and occurrence probability of risk factors in the risk registers is difficult and not based on any form of historical data. With respect to the project managers we observe that budget shifts due to realized risks are not documented properly, resulting in poor feedback. Finally with regards to the (automated electronic) administration of data, we see that the ERP system accumulates budget mutations*, and therefore the available data is of no use in terms of CPBO risk management. Since all information with respect to which budgets have been used for what risk factors is lost, we can't backtrack the total costs of risk factors. All the previous combined leads to inefficient continuous improvements of the default and adjusted default budgets. For more information on the internal accounting process and data issues see Appendix A.

*As an example, if period x had 2 risk budget mutations of -10 and -5 resp. the ERP system outputs: "a deduction of 15 from the risk budget in period x"

Chapter 2

Literature study

This literature review will give a comprehensive overview of the current risk management theories that might be applied in the context of CPBO risk budgeting. Before going into the models, there will be a concise introduction into risk management. Posterior to this introduction, the literature review will cover the three models most applied within the risk management literature domain: the Delta-Normal approach (DN), Historical Simulation (HS) and Monte Carlo Simulation (MCS) finally concluding how they could be used in the context of CPBOs. This review will end with the identification of existing literature gaps, which are translated into the academic contribution of this thesis.

2.1 Risk Management

Risk and more specific risk management are hot topics in modern academic literature (Garland, 2003). When turning to this one finds a tremendous amount of different interpretations of risk management, risk budgeting and notably risk itself. Therefore, before we can proceed let us give the definition of risk: "risk is the possibility of events that may cause adverse effects on project objective/performance" (Sato and Hirao, 2013). Note that this definition fits the context of a CPBO perfectly as:

1. It could be the case that a perceived risk does indeed damage performance (e.g. efficiency) but not objective. For example, when a certain process faces the occurrence of a risk, but was able to complete on time.
2. This definition emphasizes the fact that risks *may* cause adverse effects. Though a risk event might occur, the company might be able to resolve it before it does any damages to an objective or performance.

Risk management as a whole is concerned with improving the measurement and management of specific risks (Aebi et al., 2012), of which the measurement part will be of main concern throughout this thesis. Risk management has been used thoroughly throughout investment firms for years. Traditionally optimizing the selection of investment projects was a deterministic approach, not considering the stochastic nature of risk factors (Mosquera-López, 2015). Given the impact it can have on a company's profit, it developed into a well studied research domain. The most modern approach in selecting investment projects employs a stochastic model. Such a model, as introduced by Mosquera-López (2015), creates the opportunity to include investment specific risk factors in the go/no-go decision making process. To the best of our knowledge, such techniques, nor such concepts have ever been introduced to CPBOs, or more specific, to CPBO risk management.

In some sense, a CPBO makes an investment decision every time they accept or decline a new project. To add to this complexity, not only does a CPBO need to estimate the costs it will incur, but it also determines its own returns (by composing a quotation). When project complexity increases, so does the complexity of estimating costs, returns and monetary risks. In order to facilitate CPBOs in this decision making process, they could benefit from incorporating a stochastic model that estimates those underlying risks. In such an approach, a CPBO could evaluate the underlying risk factors of a project as a portfolio, making it eligible for portfolio (optimization) techniques. Given the many uncertainties in the sales phase, determining a sound quotation is a complex task. The major complexity lies in balancing the quotation such that one can make a profit on the one side, but is captivating for the customer on the other.

2.2 A project specific risk profile

The ever increasing complexity of R&D projects, complex systems and IT projects has resulted in a great amount of research interests. Over the years, many articles have been written that try to identify, model or simplify complexity in such endeavors. A subset of this literature domain focuses on risk management, which is what we'll focus on in this section. As is the case with the normal risk budgeting literature domain, there are no publications regarding the use of statistical methods to compose tailor made risk budgets for such complex systems.

Hillson and Simon (2012) refer to risk management both within PBO- as CPBO environment as: the core of modern project management and considered essential to successfully managing projects. Therefore it does not come as a surprise that many authors were aspired towards explaining the underlying concepts. One of such concepts that is well documented within literature is devoted to project risk factors. With a number of recent literature reviews at hand, one has the position to easily gain an understanding of the generic risk factors for a number of industries. Bosch-Rekveldt et al. (2011) conducted a literature review specifically focusing on factors that increase product complexity. In addition, they've conducted quantitative research to show which factors are encountered in practice (the top 5 of their research is enclosed in Table 2.1, the entire table of findings in Appendix B). Yeo and Ren (2009) used the same setup for their research, but did not restrain to finding the drivers of project complexity only, thus adding more content to the risk profile. The most common risks defined by Yeo and Ren (2009) are included in Table 2.1. Finally we partially include the work of Jaber (2016), as it adds some other - more practical - risk factors to the risk profile.

The authors have one unanimous reason for the fact that the lists of possible risk factors are particularly lengthy (49, 48 and 91 factors resp.). It's simply because there is no such thing as a standard risk profile for CPBOs. It is however, possible to construct such a profile when narrowing down to one specific industry, company or even product type. In order to guide practitioners in composing such a profile, all previous researchers proposed their own framework. By answering a set of questions or brainstorming about the applicability of all factors for a certain product one should be able to use the frameworks and construct a standard risk profile.

Table 2.1: Elements underlying project complexity and uncertainty

Source	Category	Element
1	Knowledge	Experience with technology
1	Number of partners, contractors, suppliers	Number of stakeholders
1	Project management process	Number of different project management methods and tools
1	Project management process	The largeness of the project scope
1	Risk management process	No early or clear identification of risks
1	Skills	Resource and skills availability
1	Variety of perspectives	Variety of stakeholders perspectives
2	Culture	Fear-based culture, poor risk awareness
2	Culture	Lack of assumptions testing and learning
2	Leadership	Lack of top management support and priority
2	Organization structure	Weakness in matrix structure (wrongly structured PBO)
2	Project management process	Ad Hoc or inadequate project planning and control systems
2	Stakeholder coalition	Lack of user involvement and inputs in defining requirements in design phase
2	Stakeholder coalition	Poor relationship with client or customers
2	System design	Ill defined product/system requirements & functionalities
3	Number of partners, contractors, suppliers	Level of involvement/dependence subcontractor
3	System design	Number of subsystems/integration complexity

Sources: [1]: Bosch-Rekveldt et al. (2011) [2]: Yeo and Ren (2009) [3]: Jaber (2016)

2.3 Portfolio Theory

The classical risk budgeting literature mainly focuses on portfolio optimization. Therefore, in order to apply some of the well known risk management theories in CPBO risk budgeting, the portfolio theory model should be elaborated on and adapted. At the base of every risk management technique is the portfolio which, in the mainstream financial literature, exists of a combination of amongst others bonds and stocks. A portfolio can hold a multiple of each asset and every asset has its own risks. The main goal of risk management is twofold. The first goal is to identify the risks, their costs and as such the portfolio Value-at-Risk (VaR). Second, portfolio optimization theories aim to diversify the portfolio, minimizing the overall monetary risk for the investor.

In CPBO risk budgeting, one does not have different asset classes, but shares the same goal of identifying and minimizing the overall risk of a project. Therefore instead of separating a portfolio in bonds and stocks, each having their own risk profile, CPBO risk budgeting will express a project's overall risk by dividing it into different risk factors. As with the original model, every risk factor has its own risk profile. The ultimate goal is to optimize a portfolio of risk factors, such that the company's monetary exposure to risk is minimized. This translation can be graphically represented as in Figure 2.1. As mentioned before, classical portfolio theory allows the practitioner to hold a multiple of stocks or bonds. In the case of CPBO risk budgeting this is impossible, as the portfolio is a set of identified risk factors. Summarizing the above, a CPBO constructs a portfolio every time they accept a new project, by identifying all possible risk factors. In addition to identification, CPBOs also have to

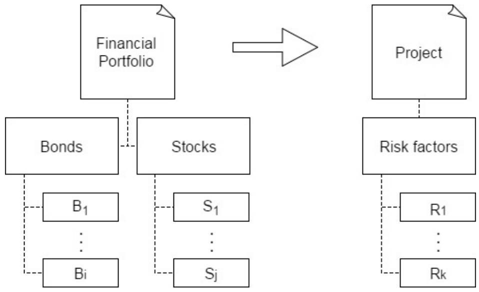

Figure 2.1: Graphical representation of the translation between financial- and project portfolio's

estimate what costs they're likely to incur given the characteristics of the portfolio (based on experience). As such, CPBOs could benefit from incorporating a stochastic model that approximates underlying risk factors of a project.

2.4 Value-at-Risk

The term Value-at-Risk (VaR) is widely used throughout scientific literature as it's an easy to understand concept that computes the maximum loss of a portfolio given a certain confidence level. Many researchers have applied either the VaR or Conditional Value-at-Risk (CVaR) methods to risk budgeting models, making it the most popular method to express the expected portfolio loss due to risk. Given a portfolio, computing the VaR requires just two steps:

1. The portfolio's distribution has to be estimated. This can be done in many different ways, e.g. in closed form when assuming all assets are normally distributed or via simulation.
2. The VaR has to be computed, which is done by computing the quantile of the distribution.

Though the concept of VaR has many different textual interpretations, the underlying math is identical. In principal it's just the β-quantile of a cumulative distribution function (cdf). The mathematical expression of the VaR in risk management was first defined by Rockafellar and Uryasev (1999), and will be transformed for CPBO risk management purposes and explained in Chapter 4. Throughout the thesis, β will represent the confidence level of VaR models.

From the literature review we expect one important downside to incorporating CVaR in CPBO risk budgeting, which is the fact that the way one models the tail behavior will have significant influence on the magnitude of the budget (Mazaheri, 2008). Therefore VaR might be better for portfolio optimization, when good models for the tails are not available (Stan et al., 2010), though it will limit the possibilities w.r.t. portfolio optimization.

2.5 Risk management models

Traditionally risk management literature focuses on financial institutions. Therefore, when looking into which models would suit CPBO risk budgeting, we can distinguish two groups: change based and value based models. Change based models, like the immensely popular

GARCH and RiskMetrics method base their calculations on time series of price changes. That is, change based models construct the costs function between time τ and $\tau + 1$, for a large number of consecutive time periods, in order to estimate the expected cost of a risk factor in the future. CPBO risk budgeting operates in a context where a risk either occurs or not, and its occurrence won't be undone (i.e. the value becoming zero or positive again) as would e.g. a stock's value. As a results, those models will not qualify and therefore will not be included in this thesis with one exception. Though historical simulation was essentially created to work with time series, it can be redefined to fit CPBO risk budgeting and will therefore be included in the thesis.

The second group, value based models, base their computations on actual outcomes (i.e. actual costs incurred due to some risk factor) and therefore directly qualify to be used in CPBO risk budgeting. Value based models focus on estimating a distribution which explains the data well in order to predict future expected costs of a risk portfolio. Figure 2.2 shows an overview of the five mainstream risk management theories classified per group (see Li et al. (2012) for a comprehensive overview of others).

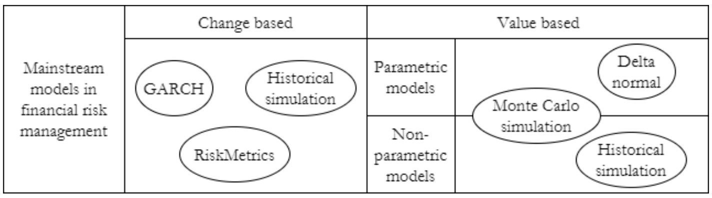

Figure 2.2: Mainstream risk management models categorized

Within the value based models, one can distinguish between two different approaches of acquiring the risk distributions: parametric- and non-parametric models. When assumptions are made with respect to the cost distribution one refers to a parametric model. Instead a researcher can choose to define the cost distribution based on empirical data, classifying it as a non-parametric model. Looking back at the models we previously discussed, we can subdivide them as: one parametric method (Delta-Normal), one non-parametric methods (Historical Simulation) and a hybrid or semi-parametric method (Monte Carlo Simulation). One obvious reason for choosing non-parametric models in risk management are the characteristics specific to financial market data, who contradict normality assumptions. We will now discuss all three models in more detail.

2.5.1 Delta-normal Method

As the name suggests, the basis of delta-normal (DN) methods evolves around the assumption that all risk factors are normally distributed. Though some authors deprecate using the DN model, a study by Kondapaneni (2005) showed that 42% of all British banks apply the method, showing its popularity. Over time, a large number of models have been derived from the DN method, i.e. the delta-gamma or delta-gamma-theta method, who focus on improving the model's accuracy. For our model we will incorporate the version that assumes normal distributions. In the next chapter we'll cover two methods in depth to show the difference in the model's underlying assumptions. Both methods follow the same sequence of operations. They compute the monetary exposure of separate risk factors, assuming they are normally distributed. Subsequently they use the fact that a summation of normal distributions also follows a normal distribution to compute the portfolio VaR. Finally the DN methods corrects for any form of relation between the risks in the portfolio (correlations) to find the risk portfolio VaR estimate with confidence level β. In order to implement the delta-normal method, the following parameters need to be estimated: the monetary exposure per risk factor and the correlation matrix.

The DN method knows many different implementations. Even though the fundamentals stay the same, choosing the most appropriate version might significantly improve the ease of practical implementation. The first method, as proposed by Mausser and Rosen (1998), works by multiplying the expected costs of a portfolio with the adjusted covariance matrix. One of the disadvantages of using covariances however, is that it's influenced by scale (Serfling, 2014). In practice this means that whenever different risk factors are of different scale, so is their covariance, making it difficult to compare. The answer to this comes from Čorkalo (2011), who proposed to use the correlation between risk factors (which is not affected by scale) instead. To account for this change, they use the β-VaR of the individual risk factors instead of their expected costs. However the traditional methods solely revolves around price change data (e.g. of stocks, bonds or commodities), this can easily be translated into CPBO risk budgeting without changing the model. To do so, one computes the monetary exposure for risk factors based on their historical values, rather than on change values.

Common to all DN methods is the usage of the covariance or correlation matrix, who represents the interdependence between certain risk factors and scales the VaR accordingly. An important side note to this is that, whenever a covariance or correlation of two risk factors is 0, this does not mean they are independent. Rather they are linearly independent (Serfling, 2014), there can still be some nonlinear dependency between risk factors (e.g. of quadratic form) which can't be modeled via the DN method. Furthermore, due to the nature of risk factor data (i.e. limited occurrence), creating the covariance matrix requires a great amount of data. That is, in order to tell something meaningful about the relationship between two variables, the data points need to be taken from the same project, at the same time, for a multiple of observations. In order to construct the entire covariance matrix, one needs to gather the simultaneous data points of all possible combinations of risk factors.

Pros of the delta-normal method:
One of the biggest advantages of the DN method over any other model is its clarity and efficiency. The model has a closed form solution and is very easy to understand making computations insightful. Because of the latter, sensitivity analysis is easy to compute. Also optimization techniques under budget constraints can easily be performed. Another advantage of its clarity is that the model is easy to explain to (senior) management.

Cons of the delta-normal method:
Fundamental to this method is the fact that it *assumes* normal distributed risk. Whenever the *actual* risk follow a different distribution, extra error will be incurred. Another arguably deal breaking con of DN methods is the fact that it does not correctly model tail behavior of risk distributions. Many risk factors are said to have fat tailed distributions, which is being discarded (and thus underestimated) by this method. This model is thus very dependent on the curvature of the functions.

2.5.2 Historical simulation

Li et al. (2012) define Historical Simulation (HS) as: the procedure for predicting VaR by "simulating" (or rather constructing) the empirical cumulative distribution function (ecdf) of asset returns over time. In CPBO risk management we will be modeling the risk costs rather than asset returns. This method thanks his popularity to its swiftness and clarity (Linsmeier and Pearson, 1996), consisting of only three concise steps.

The first step in HS is generating a large amount of random scenarios. This is done by choosing historical data for every risk factor at random, and iterating this for a large amount of iterations. Secondly one evaluates the scenarios with the current portfolio, simulating the portfolio's probability density function (pdf). Finally, the VaR will be set equal to an amount that is exceeded only $(1 - \beta)\%$ of all instances.

The primary assumption of this method is that the distribution of the risk factors in the portfolio is constant over the sample period (Danielsson and De Vries, 2000). Though historical simulation is hugely popular under practitioners, it disregards years of scientific research on conditional asset return models (e.g. GARCH) (Christoffersen and Pelletier, 2004). Since this is a non-parametric approach, no parameters need to be estimated.

Pros of the Historical Simulation method:
The major advantage of HS is the fact that it's easy to understand and implement, doesn't require much computational effort and (if good data is available) is an accurate method too. The accuracy of HS has been tested very frequently, with multiple authors emphasizing that it indeed is reliable (Huang, 2000). Another great feature of historical data is that it captures underlying risk probabilities without any form of parameterization, ensuring no systematic risk.

Cons of the Historical Simulation method:
The most commonly denoted downside of using HS is the fact that it is very dependent on the available historical data. Pritsker (2001) suggested that in the case of computing VaRs related to stock portfolios, 500 days of stock value data may not be enough to accurately compute 1% VaR at a 10-day horizon, concluding that large data sets are a necessity. Furthermore do those data sets need to contain sufficient extreme observations to be able to correctly construct any fat tails of the distribution. Finally multiple researchers (e.g. Li et al. (2012); Población García (2017)) mention another limitation of this method, namely: the assumption of independent and identically distributed (i.i.d.) risk factors, which might not be the case.

2.5.3 Monte Carlo simulation

The Monte Carlo Simulation (MCS) method is the second basic methodology for non-parametric approaches and has been widely used throughout the years. Some aspects of MCS look similar to historical simulation. Both methods repeatedly re-evaluate the value of a portfolio (Pearson, 2002), using hypothetical new values of the underlying risk factors that determine the portfolio value. MCS differentiates itself from HS because it chooses a statistical distribution that is believed to adequately capture or approximate the possible changes in the risk factors based on the available data (Linsmeier and Pearson, 1996).

MCS simulates the market factors that determine the portfolio value rather than the entire portfolio value at once. Thereafter MCS creates numerous replications which represent hypothetical changes in the underlying risk factors, by taking a large amount of random samples from each risk factor's statistical distribution. Subsequently, MCS evaluates the scenario's with the current portfolio, simulating the portfolio's pdf. It is this distribution that will be used to compute the VaR. Although MCS is considered a non-parametric method, it much better could be classified as a semi-parametric method. That is, it does require parameter estimation albeit based on data rather than on an abstract level. In order to implement the MCS method, the following parameters need to be estimated: the probability density function that models risk factor k best: f_k and its associated quantile function $F_k^{-1}(\beta)$.

MCS is arguably one of the most applied techniques in the entire risk management domain. Its capability to simulate complex systems quickly outweighs the computational burden. In the case of CPBO risk budgeting, MCS grants the possibility to work with small datasets and simulate potential non-linear relationships (when modeled) between a set of risk factors. The process of executing a MCS however is more tricky than one might think, especially in the case when little data is available. One of the consequences of having a small dataset is the fact that one won't be able to split in a training and test set. As a result, when fitting a distribution using the MCS, a standard Kolmogorov Smirnov (KS) test will have less predictive power (i.e. it overestimates the p-value) (Khamis, 2000). In the proposed case study application the distribution parameters will be estimated based on a small dataset, thus a standard KS test won't suffice. Therefore an improved KS test (including its own MCS), as introduced by Clauset et al. (2009), will be conducted.

Pros of the Monte Carlo Simulation method:
The primary advantage of using MCS in projects is that it is an extremely powerful tool when trying to understand and quantify the potential effects of uncertainty of the project (Kwak and Ingall, 2007). Not only is it able to model non-linear and path dependent payoff functions (Li et al., 2012), it also bases its approximation on historical data without making abstract assumptions of the underlying risk distribution, thus limiting systematic risk (since estimation errors will still be incurred). In addition to the prior, it's also able to generate random values outside the range of the available historical data. One of the most important advantages in terms of CPBO risk management is the fact that MCS can easily simulate correlated portfolios. The most applied technique for simulating correlated portfolios is by use of copulas.

Cons of the Monte Carlo Simulation method:
Simulation, though very effective, comes with a few drawbacks. The one most commonly denoted throughout scientific literature is its computational cost. Furthermore MCS requires very intensive simulation to achieve risk measure predictions with acceptable numerical errors (Montagna et al., 2007). Both Gudmundsson (2013) and Tuffin (2008) showed that Monte Carlo Simulation requires a tremendous amount of iterations to compute reliable estimators for events that have a small probability of occurrence, thus making it less fit for rare event simulation without modifications. Finally, Monte Carlo simulation is very dependent on both the availability of data and the constructed underlying model. If the project model or network is lacking, the simulation will not reflect real world activities accurately (Kwak and Ingall, 2007).

Copulas

As discussed in the previous section, MCS can be used to simulate a range of possible portfolio outcomes given a project and its risk factors. In the most simplistic case of every risk factor being independent this will not be a problem. However in order to better model real world scenarios, we want to be able to model linear dependences within risk factors when they arise. An approach that fits this purpose and is frequently applied within the risk management domain is called copulas. Embrechts et al. (2001) emphasize that copulas can increase the quality of risk management models as it creates the possibility to loosen normality assumptions and simplifies the combination of independently computed VaRs. Some well-known copulas are the: gaussian-, t- and archimedean family copulas, of which the gaussian is the standard go to copula in risk management literature (Murphy and Murphy, 2006). Bloomfield (2013) defines a copula as: "the joint distribution of random variables $U_1, U_2, \ldots, U_p$, each of which is marginally uniformly distributed as $\mathcal{U}(0,1)$. By Sklar's Theorem we can describe the joint distribution of $C_1, C_2, \ldots, C_{\tilde{k}}$ by the marginal distributions $F_k(c_k)$ and the copula C'". The proof comes from Bloomfield (2013) and Meucci (2011) for the case all distributions are continuous (which is true for our application) and will be discussed in Chapter 4.

Whenever necessary we will use copulas to compute correlated random portfolios by means of MCS. The copulas enriches us with the capability to do so, by means of a simple random number generator and a copula cdf. In applying this method one will need to make assumptions on the model's target distributions to find easy solutions for the copula's cdf. Whenever those need to be relaxed, the copula cdf can be found via the maximum likelihood method. The copula uses correlation matrices when generating random numbers. Since correlation matrices are required to be positive semidefinite (PSD), we should check whether this condition is satisfied whenever making use of copulas. One method that qualifies for this goal is called the eigenvalue decomposition, which we will substantiate on in Chapter 4.

2.5.4 Conclusions on VaR estimation methods

The preceding literature review compares the performance of three frequently occurring risk management methods. All models have their own specific (dis)advantages over the others, which have been summarized in Table 2.2. From the preceding literature review we can generalize a set of modeling steps which form the support of all methods:

1. Determine the risk factor's statistical distributions (either via assumptions or fitting)
2. Compute monetary exposure per risk factor (by use of closed form expressions or simulation)
3. Evaluate monetary exposure with current risk portfolio
4. Estimate the risk portfolio's VaR

Table 2.2: Risk management method pros and cons

Method	Advantage	Disadvantage
Delta-normal	Ease of implementation. Computational efficiency. Closed form solution.	Normal distribution is known to not properly model leptokurtic tails of financial data.
Historical simulation	Frequently tested to be an accurate method, which requires little computational effort. Non-parametric thus no systematic risk	Very dependent on both the quality and quantity of the underlying data. Needs "extreme" observations in order to model leptokurtic tails.
Monte Carlo simulation	Known to be a good method to quantify the potential of uncertainty. Is able to model non-linear functions. Semi-parametric method thus little systematic risk. Can simulate values outside the range of the underlying dataset	Computationally intensive, especially when little numerical error is required. Very dependent on underlying model and quantity of the data.

2.6 Backtesting VaR models

A lot of research attention has been devoted to backtesting different risk management methods. It should be noted however, that there are very limited publications that compare all three methods used in this thesis. Furthermore, most publications on backtests tend to investigate more advanced applications, (e.g. including Extreme Value Theory (EVT) (Fernandez, 2005), kernel functions (Huang, 2000) or copulas (Andersen and Pedersen, 2010)). The small number of publications found for this thesis conclude that the best performing VaR model is situation dependent. That is, the best performing method changes with the underlying data and practitioner's goals. The most important situational variables that seem to have an impact on the method of choice are the amount of data used in the VaR computation, the distribution and volatility of that data and the required confidence level. Moreover, some authors show that when one needs to model the tail behavior of some portfolio's, this has important implications for the method of choice.

Table 2.3: Backtesting results from literature

		Performance		
Source	Included methods	Best	Worst	Note
Rank (2002)	DN, HS & MCS	HS	DN	
Danielsson and De Vries (2000)	DN, HS,& MCS+EVT	MCS+EVT	DN	
Pritsker (2001)	HS & Filtered HS	Filtered HS	HS	Shows how bad HS can be if conditional volatility changes, advises the use of models like MCS
Hendricks (1996)	DN, HS, Moving Average			Conclude that "best model" is scenario dependent, though all models perform sufficiently well for the 95% quantile. Only 1 version of HS worked for 99% quantile.
Lechner and Ovaert (2010)	DN & advanced models			Conclude that "best model" is scenario dependent and DN will underestimate fat tailed risk factors
Van Den Goorbergh and Vlaar (1999)	DN & HS	HS	DN	Results show that, for lower confidence both measures underestimate the VaR

Table 2.3 shows a comprehensive overview of the results derived from literature. With this in mind there are two important questions that need to be answered before choosing the preferred CPBO risk management model:

1. What distributions fit CPBO risk factors best? And how does this impact the portfolio distribution?
2. What confidence level do we want our risk management model to achieve?

Since the available literature does not provide a ready to use answer, we'll investigate which method fits the CPBO context best. To do so, we'll conduct our own backtest, taking into account the previous questions. We will start by finding out how to model the CPBO risk factors based on real company data. Subsequently we perform the backtest based on the Basel Committee on Banking Supervision traffic light approach, by which we decide which method suits the purpose of CPBO risk management best. In Section 5.2 we elaborate on the effect of the confidence level on the risk portfolio VaR estimate.

2.7 Literature gaps

Conclusion literature review

With this section the literature review concludes its findings and therefrom defines the identified literature gaps. Throughout this review one has seen that tremendous work has been devoted to risk management. The thing less developed in the examined literature domain is project risk budgeting. It seems that, though project management is a very evolving research domain, not many articles have been devoted to understanding the underlying effects of individual risk drivers on project costs.

Since the introduction of Value at Risk concepts, both researchers and practitioners constructed easy to implement but sound estimators for the expectation of a risk portfolio. It is unfortunate however that those measures haven't led to a better understanding of risk specific behavior. With the recent publication of Mosquera-López (2015), VaR measures have been shown to offer valuable information to e.g. decision makers or project managers. However there can be a great downside to incorporating VaR methods in project pricing tools. Mazaheri (2008) showed that in the case of over constraining VaR models (or being too conservative, i.e. for high β's), this may lead to a significant price increase. In turn this can result in unrealistic price markups, unrealistic quotations to customers and even worse in lost sales due to project rejection. As a result of the previous literature review we can conclude that the CPBO risk management will be enhanced if combined with Value at Risk concepts. From this point of view, several literature gaps (i.e. questions that have not or insufficiently been covered in the scientific risk management domain) have been constructed, which will be commented on hereafter.

Literature gaps

1. Incorporate VaR measures into CPBO risk budgeting
Given their estimating power, Value at Risk measures are specifically well suited for budgeting purposes. That is at least under the assumption that probability density functions are known. Converting Value at Risk measures to price markups is straightforward, leading to a transparent pricing process. Application could be two ways. First it can be done by setting a desired confidence level. Secondly a maximum budget can be set after which one can compute the maximum confidence level that goes with it (i.e. compute what % risk is at stake).

2. The effect of extreme value theory in CPBO risk budgeting
Extreme value theory (EVT) has been integrated vastly within financial risk management theories. Therefore it's interesting to test whether this still works for CPBO risk budgeting. The research should have two important conclusions: the first being if the effect of combining both theories yields the same effect (i.e. gives insight in the extreme values of the project's risk portfolio). The second interesting insight derived from this research is to what extent incorporating EVT increases the project risk budget and if it would still be feasible to include this budget in sales prices.

3. Defining common probability distributions for CPBO risk budgeting
Throughout literature there are no identification of common statistical distributions that occur in CPBO risk budgeting. Therefore, it's interesting to see whether those are in line with the ones found in normal risk management practices. Furthermore there is no information available on the minimum number of data points required to accurately fit probability distributions to a risk factor's behavior.

4. Investigate the effect of copulas in a CPBO risk budgeting application
From the classical risk management literature we've seen the effect of correlation and therewith diversification. When thinking from the practical CPBO risk budgeting perspective, it is obvious that certain risk factors correlate. Therefore it would be valuable to investigate the effect of using a copula to accurately model correlations in Monte Carlo- or historical simulation.

Chapter 3

Research layout

This chapter will start by covering the research questions in Section 3.1, by illustrating the practical or scientific relevance. Subsequently in Section 3.2 we narrow down the scope of this thesis, by identifying which steps of the risk management process will be covered. Finally Section 3.3 gives a brief explanation of the research methodology, and the model this thesis will adhere to.

3.1 Research questions

The primary goal of this thesis is to explore what risk management theories can be applied to estimating project specific risk for CPBOs. In order to guide this thesis towards it's objective, a set of research questions has been composed. The main research question is formulated as:

How can Project Based Complex Product System Organizations compose project specific risk budgets that incorporate the underlying characteristics and risk factors?

The following sub research questions will structure the thesis and ensure proper substantiation of its findings:

1. Can we define a template for CPBO's project risk factors, based on literature and verified in practice?
The case study company is very interested in the question if a generic risk profile for CoPS exists in literature. Therefore the first sub research question is to research whether such a profile exists. If so, this might be a proper starting point for further investigation into the effects of such risk factors within the case study company. Moreover it might be useful if literature describes common relationships between such risk factors. This could be especially useful during the modeling stage.

2. What are the most applied risk budgeting methods in financial literature, and which can be applied in the context of CPBOs?
With this research question we aim to investigate which are the go-to risk management models in literature as well as study if the same models can be applied within CPBO risk management models.

3. Does the method choice influence the conclusion on the preferred risk management model?
The goals of the third sub research question is to test whether there is a substantial difference between the outcomes from the risk management methods found by the previous question. If so, we will determine which method is should be preferred in the case of CPBO risk management.

4. What is the minimum number of data required to accurately model a risk factor?
Throughout literature there is no such thing as "the minimum amount" of data points required to model lets say an exponential distribution. In order to guide CPBOs in this question, we'll conduct a numerical study. The goal of this study is twofold. First, we will determine the minimum number of data points required to correctly model a known risk factor portfolio. Second, we test if diversification has an effect on this minimum (by adding more risks to the portfolio).

5. Is a risk management model able to accurately model risk profiles of complex projects?
The last sub research question concerns the accuracy of the designed CPBO risk management model. By use of case study data, we will show if such a model is able to appropriately capture the complexity and project specific characteristics of CoPS.

3.2 Scope

In order to manage stakeholder expectation, we will clearly define the scope of this thesis. From the as-is situation we can extract five process steps which offer an opportunity for improvement. Those are: estimation of risk costs, estimation of risk occurrence probability, budget revision due to complexity and risks, budget revision due to sales objective and feedback of realized risks. The main focus of this thesis will be on the estimation of risk costs and the budget revision due to complexity and risks. By use of currently available risk management methods we will show the potential improvements. As for the budget revision due to sales objective we will first demonstrate how to combine those revisions with a risk management model and thereafter demonstrate an optimization for the trade off between sales price and probability of selling a project. The importance of proper feedback will be discussed only in the case study company's recommendations. The only thing this study will not take into account is the estimation of risk occurrence probabilities, which will be proposed as future research. The reasons for this are both time constraints and data on the matter being unavailable.

3.3 Methodology

There are many ways to categorize different research problems, each having their own optimal research methodology. This thesis will adhere the definitions of van Aken et al. (2012) and therewith define our problem setting as a field problem, i.e. a problem situation in which an important stakeholder feels the as-is state can and should be improved. Given the focus of this thesis evolves around a business phenomenon not yet discussed within the scientific literature, van Aken et al. (2012) advise to follow the reflective redesign process steps, see Figure 3.1, which are based on the well-known problem solving cycle of Dewey. As with many research methodologies, we will also use an iterative method to construct the model. The model's iterations can for instance increase or fine-tune the model's performance, increase its alignment with the current business problem or address follow-up questions from the case study company. The last step of the reflective design method is to translate the knowledge gathered from the pilot implementation into new academic insights and describe future research subjects that might follow-up the current research.

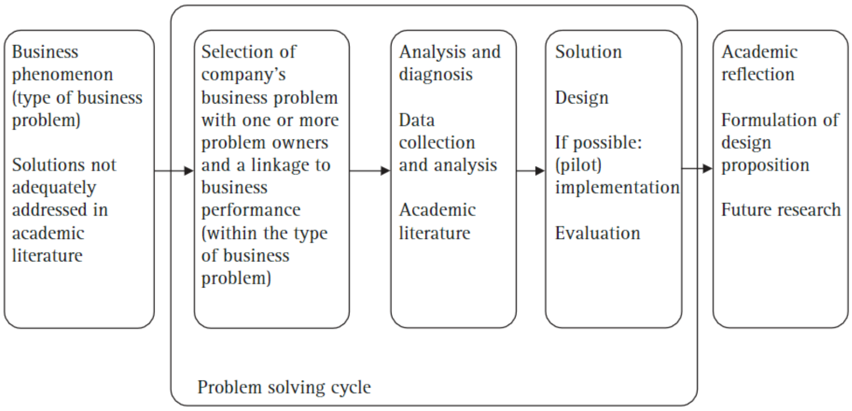

Figure 3.1: Reflective redesign steps, adapted from van Aken et al. (2012)

Chapter 4

Estimating risk budgets

This chapter will start by explaining the elements important to CPBO risk management model. Starting from a deterministic point of view, the model will end in stochastic form in Section 4.1. In the next section we'll elaborate on how we will be using the different risk management methods seen in the literature review within the CPBO risk management model. Following the model's construction we'll identify which currently available risk management model will theoretically be best suited for CPBO risk budgeting, by use of a backtesting study in Section 4.3.

4.1 Formulating the CPBO risk budgeting model

Before one can tend to analyzing project risk portfolios, some CPBO characteristics have to be included into the mathematical model. We'll start by examining a deterministic approach which forms the basis in all three frameworks adopted in Section 2.2 of the literature review on standard risk profiles. The largest difference between CPBO- and standard financial risk management models is the probability of occurrence. More specific that is, financial assets either increase or decrease in value and therefore by definition a change occurs. In the case of project risk factors one might simply not appear during a project's execution, in fact risk factors are more likely to not be realized, meaning their cost value is zero. We will use the vector r to represent if a certain risk factor has been identified for the project (i.e. should be budgeted for). More specific, if risk factor k has been identified, we have $r_k = 1$. The project specific budgets is build from two components: the base cost price and the risk supplement. The base cost price of a projects will be denoted in the mathematical model as b. This budget is considered out of scope for this thesis as it represents the cost price of the model including some overhead supplement and is not adjusted to account for risk. The second portion of the budget, the risk supplement, will be determined by use of the CPBO risk management model.

Nomenclature:

- b' Risk adjusted budget
- K Set of all risk factors
- k Unique risk factor, i.e. $k \in K$
- $\tilde{k}$ Number of elements in K
- r $\tilde{k}$ x 1 vector specifying which risk factors to include, i.e. $r_k \in \{0,1\}\ \forall\ k \in$ K
- $\bar{c}$ $\tilde{k}$ x 1 vector holding all expected cost values per risk factor
- b Base cost price of the project

Then the basic CPBO risk budgeting model can be formulated as:

$$b' = b + \sum_{\forall k \in K} r_k * \bar{c}_k \tag{4.1}$$

Would a company use such an approach however, it in essence disregard the situations that inflict the most damage. That is, the situations in which the costs are e.g. 2 standard deviations above the mean. Furthermore, this approach would not guarantee any type of confidence interval wherein the company is sure to cover all costs. The next step is to translate model (4.1) to a stochastic model. Instead of directly incorporating the expected cost values of risk factors ($\bar{c}_k$), we introduce two random variables: O_k which represents the probability of risk factor k occurring and C_k representing the cost of that risk factor. Throughout the thesis we'll model O_k as a Bernoulli trial with probability p_k, which is denoted as a binomial distribution with $n = 1$ and probability of success being p_k. The cost random variable will be discussed further on in this chapter. Since this is a stochastic model, it allows for computing confidence intervals which ensure that the random variables are within a certain range. The stochastic risk management model, is then formulated as:

Additional nomenclature:

O_k	Random variable for the occurrence of risk factor k, modeled as $\text{B}(1, p_k)$ $\forall\ k \in K$
p_k	Probability of occurrence for risk factor k $\forall\ k \in K$
C_k	Random variable for the cost of risk factor k $\forall\ k \in K$

$$b' = b + \sum_{\forall k \in K} r_k * O_k * C_k \tag{4.2}$$

The model proposed in (4.2) opens up the possibility of including risk factor specific distributions, creating a tailor made model for every project. Furthermore it would be possible to expand this model to adapt for the specific industry the project is in or the project size. In the case where data regarding O_k and C_k is abundant, it is possible to estimate the distributions for the random variables with high precision. In the context of CPBOs however, this is most often not the case. Some reasons for this might be 1) the fact that different projects executed by the company are fundamentally different such that they (and their risk factors) can't be compared, 2) the industry wherein projects are executed changes which impacts the risk factors or 3) the amount of projects conducted each year is simply small, such that data gathering progresses very slowly. As a results, we propose to use a method often applied within traditional risk management models: Value-at-Risk (VaR) to estimate the quantities of O_k and C_k.

In essence, a VaR measure is a quantile estimation of a (joint) probability distribution. In the case of CPBO risk management, the β-VaR measure assures that the risk supplement will be sufficient in $\beta\%$ of all projects. We incorporate the β-VaR in the CPBO risk management model, which will be the backbone of this thesis, as follows:

Additional nomenclature:

$\alpha_\beta(r)$	VaR measure of the risk portfolio given r
β	Confidence level

$$b' = b + \alpha_\beta(r) \tag{4.3}$$

Where

$$\alpha_\beta(r) = \min\{\alpha \in \mathbb{R} : \mathbb{P}[\sum_{\forall k \in K} r_k * O_k * C_k \leq \alpha] \geq \beta\} \tag{4.4}$$

In scientific sense, using VaR measures is limiting however. When one tries to incorporate them in portfolio optimization techniques its mathematical shortcomings will soon come to light, e.g. it being non-convex and non-subadditive (Artzner et al., 1999) and it having multiple local extrema when optimizing scenario's (Uryasev, 2000). As a result many authors pledged to switch to a more robust version known as Conditional VaR (Rockafellar and Uryasev, 1999). The β-CVaR for the CPBO risk management model can be defined as:

Additional nomenclature:
$\phi_\beta(r)$ CVaR measure of the risk portfolio given r

$$\phi_\beta(r) = \mathbb{E}[\sum_{\forall k \in K} r_k * O_k * C_k \geq \alpha_\beta(r)] \tag{4.5}$$

The practical difference between VaR and CVaR is easy to explain. Instead of looking at the expected minimum cost for a given confidence level β, CVaR looks at the average expected cost *beyond* that confidence level. Thus CVaR gives better insight, as it gives an average expected cost instead of a range of potential costs (VaR). Therefore it increases the confidence level, at the cost of potentially overpricing the risk markup. On the contrary, Daníelsson et al. (2005) showed that, though mathematically proven to be incorrect, VaR behaves sub-additive in real world applications where distributions have heavy tails. From this we conclude that, whenever our case study data turns out to be heavy tailed, VaR could be a viable option over CVaR. Lastly, in the case all risk factors follow a normal distribution, subadditivity is also achieved (Artzner et al., 1999; Tsay, 2013). Given that we will not be optimizing risk portfolios in this thesis, backed by the foregoing arguments and the fact that CVaR might overestimate in the case limited data of the distribution's tails is available (Mazaheri, 2008) we adopt the VaR method in the CPBO risk management model. As such, this thesis we will use the VaR formula in all computations unless stated otherwise. Both in the example below as in Chapter 5 we'll see the effects of choosing the CVaR measure over the VaR and the implications this brings for a CPBO. Next, let us show a small example which will illustrate all variables of the CPBO risk management model.

Example:
Consider a project with two risk factors for which we want to compute the budget using a confidence level (β) of 80% for the VaR calculations. We know that the base budget (b) is equal to €10,000 and assume that both risk factors are independent and normally distributed with $\mu_1 = 1,000$, $\mu_2 = 500$, $\sigma_1 = 100$, $\sigma_2 = 50$ (all in €) resp. We define our project specific risk portfolio as $r = (1,1)$ since both risks have been identified for this example project. Lastly we assume both risk factors will occur during project execution, such that $O_k \sim B(1,1)\ \forall\, k \in K$. We now have all the information necessary to compute the VaR and CVaR, as done below:

$$\begin{aligned} 80\%\text{-VaR} &= \alpha_{80\%}(r) \\ &= \min\{\alpha \in \mathbb{R} : \mathbb{P}[\sum_{\forall k \in K} r_k * O_k * C_k \leq \alpha] \geq 80\%\} \\ &= €1,594 \end{aligned}$$

Which can easily be verified numerically as:

$$f_1 \sim \mathcal{N}(\mu_1,\ \sigma_1) + f_2 \sim \mathcal{N}(\mu_2,\ \sigma_2) = f_{1+2} \sim \mathcal{N}(\mu_1 + \mu_1,\ \sqrt{\sigma_1^2 + \sigma_2^2})$$

Taking the one-sided 80% quantile then simply gets us:

$$\begin{aligned}
80\%\text{-VaR} &= (\mu_1 + \mu_2) + z_{0.8} * \sqrt{\sigma_1^2 + \sigma_2^2} \\
&= (1,000 + 500) + 0.84 * \sqrt{100^2 + 50^2} \\
&= €1,594
\end{aligned}$$

From this the CVaR is simply the mean of all occurrences after the VaR, or:

$$\begin{aligned}
80\%\text{-CVaR} &= \phi_{80\%}(r) \\
&= \mathbb{E}[\sum_{\forall k \in K} r_k * O_k * C_k \geq 1,594] \\
&= €1,656
\end{aligned}$$

Such that the total budget using the CPBO risk management model is:

$$\begin{aligned}
b'_{\alpha_{80\%}(r)} &= b + \alpha_\beta(r) = €10,000 + €1,594 = €11,594 \\
b'_{\phi_{80\%}(r)} &= b + \phi_\beta(r) = €10,000 + €1,656 = €11,656
\end{aligned}$$

For the example we can conclude that the CVaR risk supplement is 3.89% higher compared to the portfolio VaR estimate.

4.2 Portfolio VaR estimation methods

In the literature review we've discussed the risk management methods which we'll study in this thesis. In this section we'll elaborate on how to mathematically incorporate those methods within the CPBO risk management model, starting with the DN method. Though the DN method knowns many slightly different implementations, we will highlight two of them, which both have appealing properties. Subsequently we look at both HS and MCS, for the latter we elaborate on the copula.

4.2.1 Incorporating the Delta Normal method

The first DN method, proposed by Mausser and Rosen (1998), might be interesting as it assumes that the risk factor costs are joint normally distributed with zero mean. This might be applied in CPBOs that want to model both risks as opportunities (e.g. under- and overestimations of the portfolio cost). An example might be the fact that exchange rates can move in both directions. The model works as follows:

Additional nomenclature:

C	Joint distribution of all risk factors in K
$\mathcal{N}(\mu, \mathbf{\Sigma})$	Multivariate normal distribution with mean vector μ and covariance matrix $\mathbf{\Sigma}$
$\mathbf{\Sigma}$	Covariance matrix of risk factor costs
$\mathbf{\Sigma}^*$	Scaled covariance matrix of risk factor costs
z_α	One sided standard normal z-score for α probability in the tail area
$m(r)$	$1 \times \tilde{k}$ VaR map given portfolio r

Assume there exists a set of project risk factors whose costs are joint normally distributed with zero mean. That is, $C \sim \mathcal{N}(0, \mathbf{\Sigma})$, where $\mathbf{\Sigma}$ is the covariance matrix of the risk factor costs. The next step is to scale the covariance matrix with the squared one sided standard z-score in the tail area for $\alpha = 1 - \beta$.

$$\mathbf{\Sigma}^* = z_\alpha^2 \mathbf{\Sigma}$$

The final step before computing the portfolio VaR estimate, is to compute the portfolio's monetary exposure, for which the DN method uses a so called VaR map. The VaR map is a vector holding the mean costs per risk factor*, that is: $m(r) = (r_1\mu_1, r_2\mu_2, \cdots, r_{\tilde{k}}\mu_{\tilde{k}})$. Finally the portfolio's β-VaR can be computed as:

$$\alpha_\beta(r) = \sqrt{m(r)\mathbf{\Sigma}^* m(r)^T} \tag{4.6}$$

The second DN method we will consider, proposed by Čorkalo (2011), follows the same approach with two fundamental differences: the first one being that it computes the β-quantile for each risk factor's distribution individually and the second it uses the correlation- instead of the covariance matrix.

Additional nomenclature:

$\boldsymbol{\rho}$	Correlation matrix of risk factors
f_k	Probability density function of the costs of risk factor k, $\forall\ k \in$ K
F_k	Cumulative distribution function of risk factor k, $\forall\ k \in$ K
$F_k^{-1}(\beta)$	β-Quantile function of F_k, $\forall\ k \in$ K
$\mathcal{N}(\mu, \sigma)$	Normal distribution with mean μ and standard deviation σ

Assume there exists a set of risk factors (K) whose costs follow normal distributions with mean μ_k and standard deviation σ_k: $f_k \sim \mathcal{N}(\mu_k, \sigma_k), \forall k \in K$, with correlation matrix $\boldsymbol{\rho}$. Let us now construct the VaR map which, for this method, exists of the β-quantiles of the risk factors in K.

$$m(r) = [r_1 F_1^{-1}(\beta), r_2 F_2^{-1}(\beta), \cdots, r_{\tilde{k}} F_{\tilde{k}}^{-1}(\beta)]$$

Then the portfolio's β-VaR can be computed as:

$$\alpha_\beta(r) = \sqrt{m(r)\boldsymbol{\rho} m(r)^T} \tag{4.7}$$

When using the correlation matrix, one should check whether it's positive semi definite (PSD). One way to check for positive semi definiteness (since a correlation matrix is symmetric) is

*In the original model the VaR map represents the sum of monetary exposure per risk factor resulting from holding an asset in the portfolio. Since in CPBO risk management the portfolio itself represents risk factors, this simplifies to a vector of all mean costs (as the effect of a risk factor triggering the realization of another risk factor is modeled via the covariance matrix).

by use of eigenvalue decomposition. First one computes the eigenvalue decomposition of the correlation matrix. If none of the eigenvalues are negative, the matrix is PSD. Whenever the correlation matrix is not PSD we will use the identity matrix and therefore in some way penalize this method for being incapable of correctly estimating the portfolio VaR.

For the remainder of this thesis we will adopt the second DN method. Reasons for this include the fact that, for the case study company, we will not be modeling opportunities regarding risk, and second because this method is expected to perform better in the case of scarce data. We expect higher performance because the second method uses correlations over covariances. More specific, in the case data is scarce, it might be difficult to find accurate estimators for the mean and variance of the risk factors. As a result, we might be forced to define relations via expert opinion. We argue that experts might be better of estimating relations on a scale of -1 to 1, instead of via covariances.

Incorporating the Historical Simulation method

The concept of HS is rather simple and has already be discussed in Chapter 2. The method of HS will use a random sample of historical data to "simulate" the possible outcomes based on the current risk portfolio, and can mathematically be expressed as follows:

Additional nomenclature:

$\boldsymbol{U}$ $n \times \tilde{k}$ matrix with randomly chosen historical data points

$\hat{F}_y$ Empirical cumulative distribution function of the data in vector y

First one generates n random numbers with replacement representing historical data points in the database, for every risk factor k. This results in the $n \times \tilde{k}$ matrix of randomly chosen historical data $\boldsymbol{U}$, or simulation sample paths. As an example, if risk factor k has five data points available: (100, 200, 300, 400, 500), and the random numbers are (1, 2, 4, 1), column k of matrix $\boldsymbol{U}$ would be $(100, 200, 400, 100)^T$. Subsequently the current portfolio is evaluated for all sample paths, to form the risk portfolio's simulated possible outcomes:

$$y = \boldsymbol{U} r$$

Such that y is a column vector of n possible portfolio outcomes. Lastly one uses the empirical cumulative distribution function (ecdf) $\hat{F}_y$ of the random risk portfolio costs in y, to compute the β-VaR by using the quantile function:

$$\begin{aligned} \alpha_\beta(r) &= \hat{F}_y^{-1}(\beta) \\ &= \min\{\alpha \in y : \hat{F}_y(\alpha) \geq \beta\} \end{aligned} \tag{4.8}$$

Incorporating the Monte Carlo simulation method

The fundamental difference between HS and MCS is the method of generating the random portfolio outcomes. Therefore, instead of composing $\boldsymbol{U}$ via historical data, MCS fits a distribution to each risk factor's data: f'_k. Subsequently it generates random numbers based on this distribution. The procedure of computing the MCS VaR goes as follows:

Additional nomenclature:

f'_k Monte Carlo simulation fitted pdf for risk factor k, $\forall\ k \in K$

For every risk factor, fit the most accurate distribution on all of its available historical data. Subsequently compose $\boldsymbol{U}$ by sampling n random numbers from those distributions, and compute y in the same manner as for HS. From this data compute the risk portfolio VaR, again via the same procedure as seen in HS.

The important difference, on top of the ones substantiated in the literature review, is MCS's capability to model correlations by means of copulas. Whenever we compute correlated portfolios in the remainder of this thesis, we'll incorporate a copula. This extension is expected to result in a more accurate CPBO risk management model as we can now model (linearly) correlated risk factors which might occur in real world scenarios. The copula method, for the case all distributions are continuous (which is true for our application), can mathematically be expressed as follows (Bloomfield, 2013; Meucci, 2011):

Additional nomenclature:

F_k^{-1}	Inverse function of F_k such that: $F_k[F_k^{-1}(u)] = u$, $\forall\ 0 \leq u \leq 1$.
$\bar{f}_k$	Target probability density function
$\bar{F}_k$	Target cumulative distribution function
C'	Cumulative distribution function of the copula
$\mathcal{U}(0,1)$	Uniform distribution with support [0,1]
U_k	Uniformly distributed random variable

When we feed a random variable (r.v.) C_k through its own cumulative distribution function (cdf), we transform it to the so called grade of C_k†, which is uniformly distributed on the interval [0,1]:

$$U_k = F_k(C_k) \implies U_k \sim \mathcal{U}(0,1) \tag{4.9}$$

Which is proven with the following simple example:

$$\begin{aligned} \mathbb{P}(U_k \leq u) &= \mathbb{P}(F_k(C_k) \leq u) \\ &= \mathbb{P}[C_k \leq F_k^{-1}(u)] \\ &= F_k[F_k^{-1}(u)] \\ &= u \end{aligned} \tag{4.10}$$

From this observation Meucci (2011) concludes that when one has an arbitrary continuous target distribution and a uniform random variable, this can be transformed into the target distribution as:

$$\left.\begin{matrix} \bar{f}_k \\ U_k \sim \mathcal{U}(0,1) \end{matrix}\right\} \implies C_k \text{ is equivalent to } \bar{F}_k^{-1}(U_k) \text{ and } \bar{F}_k^{-1}(U_k) \sim \bar{f}_k \tag{4.11}$$

†Meucci (2011) defines the Grade of C_k as: "A very special transformed random variable, distributed uniform on the unit interval regardless of the original distribution".

The next step is combining the previous in constructing the actual copula. Meucci (2011) conceptually described the copula as the missing information that you need when constructing a joint distribution from marginals. By Sklar's theorem we can construct the copula's cdf as:

$$\begin{aligned} F_U(u) &= \mathbb{P}(U_1 \leq u_1, U_2 \leq u_2, \cdots, U_{\tilde{k}} \leq u_{\tilde{k}}) \\ &= \mathbb{P}(F_{C_1} \leq u_1, F_{C_2} \leq u_2, \cdots, F_{C_{\tilde{k}}} \leq u_{\tilde{k}}) \\ &= \mathbb{P}(C_1 \leq \bar{F}_{C_1}^{-1}(u_1), C_2 \leq \bar{F}_{C_2}^{-1}(u_2), \cdots, C_{\tilde{k}} \leq \bar{F}_{C_{\tilde{k}}}^{-1}(u_{\tilde{k}})) \\ &= F_C[\bar{F}_{C_1}^{-1}(u_1), \bar{F}_{C_2}^{-1}(u_2), \cdots, \bar{F}_{C_{\tilde{k}}}^{-1}(u_{\tilde{k}})] \end{aligned} \tag{4.12}$$

Finally, writing C' as the cdf of the copula, we get the standard form:

$$\begin{aligned} F(c_1, c_2, \ldots, c_{\tilde{k}}) &= \mathbb{P}(C_1 \leq c_1, C_2 \leq c_2, \cdots, C_{\tilde{k}} \leq c_{\tilde{k}}) \\ &= \mathbb{P}[U_1 \leq F_1(c_1), U_2 \leq F_2(c_2), \cdots, U_{\tilde{k}} \leq F_{\tilde{k}}(c_{\tilde{k}})] \\ &= C'[F_1(c_1), F_2(c_2), \cdots, F_{\tilde{k}}(c_{\tilde{k}})] \end{aligned} \tag{4.13}$$

We will use these results to compute correlated random portfolios by means of MCS. The copula method as described above enriches us with the capability to do so, by means of a random number generator and a copula cdf. In our model we'll go with the Gaussian copula, meaning we use a multivariate normal random number generator to compute our correlated random numbers. This implies that we assume the correlation between risk factors can be modeled via a normal distribution, and is in line with numerous publications on risk management models. Since a multivariate normal distribution uses a correlation matrix, we should once again check whether it's PSD. In doing so we'll use the eigenvalue decomposition again and use the identity matrix if violated.

4.2.2 Capturing project specific characteristics

The most significant difference between the traditional risk management and CPBO risk management models are the portfolio assets. In the primary case, the assets are stocks or bonds which all have some sort of return distribution. Though there might be a large difference between the types or behavior of those assets, in some way they all are financial positions. The assets underlying a CPBO risk management portfolio are the complete opposite as they can be fundamentally different (e.g. hedging for exchange rates vs. costs due to insufficient slack in the project's execution planning). This flexibility in risk management models is what it makes especially suited to model CPBO project specific budgets. More specific, it allows companies to incorporate project specific characteristics in the portfolio, therefore creating tailor made risk budgets. There is however one notable disadvantage to this application, which is the model's underlying data.

As discussed in Chapter 2, most risk management models require large amounts of data to construct accurate VaR estimates. Due to the nature of CPBOs - complex, large scale and expensive projects - this might not be readily available and gathering it might take a lot of. time. Therefore we propose two methods of composing a model's underlying data: one for companies who have sufficient data at hand and one for those with scarce availability. We'll discuss the question of "how much data is sufficient" in Chapter 5.

In case the company has sufficient data at hand, it's advised to create different partitions based on the segregation desired. As an example, take a company which operates in two industries and distinguishes between small, medium and large projects. The company knows that both industries have their own set of potential risks, but the same risks occur for all project sizes. The company should start by creating two partitions, one per industry. Subsequently it should explore whether the magnitude of the costs per risk factor differ per project size. If this is the case, it might be best to further divide the data based on project size (e.g. via sales value). If not the case, the data can potentially be scaled (e.g. by sales value again) and used for all size categories. Either way the company would end up with two or six datasets per risk factor, depending on the latter, which form the model's input. This is the easiest and arguably most accurate model possible, being a direct translation from the original way of working in financial risk management models.

The second scenario, limited availability and/or slow gathering of data, will be less accurate and is advised to only be used until sufficient data is available (see Chapter 5 for the minimum number required). Instead of creating multiple datasets that individually represent a project characteristic this method will aggregate all data into one. That is, first you scale all available data (e.g. via sales value) per risk factor. The second step is to create a mapping of scalar values that represent the cost behavior of a certain risk factor, given the size of the project. Lastly one can re-scale the input value from the scaled dataset into project specific risk factors. Figure 4.1 illustrates the mapping for a company active in one industry, identifying three risk factors and making distinction between small, medium, large and major projects. One of the major drawbacks is that one treats all projects as equals such that on average they approximate the real world scenarios, thus creating inaccuracy. Furthermore, scaling might be tricky when the costs don't scale with the project's sales value and it's more difficult to spot outliers in the dataset. Also in the case a certain subset deviates a lot from the others this behavior might be lost due to scaling, making it difficult to model.

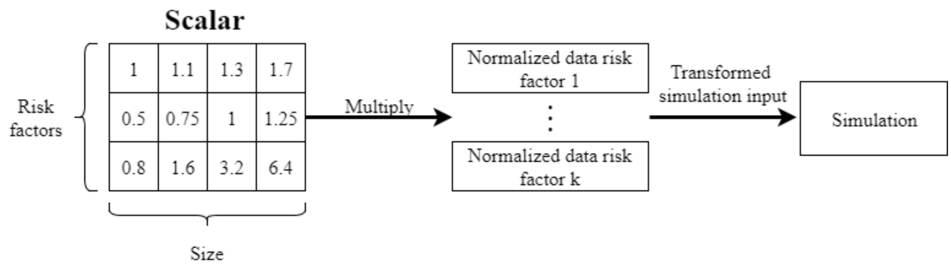

Figure 4.1: Stylized model of data scaled by project characteristics

To strengthen our proposition we will now have a look at the performance of both methods based on a small example. We define D as the set of all industries and J the set of all sizes. Take a company with four size categories, operating in three industries which uses a portfolio of three risk factors. Assume that all risk factors always occur, the impact increases with project size and the impact differs per industry. Furthermore we assume that all three risk factors follow normal distributions with scaling means $10,000 \cdot s_j \cdot i_d$, $20,000 \cdot s_j \cdot i_d$ and $8,000 \cdot s_j \cdot i_d$ resp. and all distributions have a standard deviation of 2,000. In this, s_j represents the project size and i_d represents the industry type. We use the scaling factors as shown in Table 4.1.

For this example we will compute the MCS VaR and test, based on an independent dataset, if the method indeed achieves a (1-β)% error margin. We estimate the portfolio VaR separately for every combination between risk factors and industry types. The simulation starts off by generating 100 and 50 random numbers per risk factor, per project size and per industry for the training and test set respectively. Subsequently we assemble the combined data source by randomly choosing $100/12 \approx 9$ data points from the previously generated initial data source (note this represents the fact that the second method should be used if less data is available). From this we compute the MCS risk portfolio VaR using a 80% confidence level and backtest if the method achieves the supposed error margin of $(1 - 0.8) * 50 = 10$ exceedances.

From the results in Table 4.2 we learn that the first method, separating data into project specific bins, indeed yields more accurate VaR estimations. We see that all backtesting procedures revolve around the expected error margin of 10 exceedances. On the other hand we also see that the performance in the second scenario fluctuates a lot. Using the same scaled data for all project characteristic combinations results in 5 under- and 7 overestimations of the model, with non of the models being as accurate as in the first scenario. From this small simulation study we conclude that indeed our proposition to use separate data bins over one large dataset is justified.

Table 4.1: Database layout - Simulation parameters

Size	s_j	Industry	i_d
Small	1	1	0.9
Medium	1.2	2	1.8
Large	1.5	3	1.2
Major	2.5		

Table 4.2: Database layout - Simulation result - # errors

	Separated data source			Combined data source		
	i_1	i_2	i_3	i_1	i_2	i_3
Small	10.064	10.046	10.306	15.676	8.270	13.110
Medium	10.208	9.848	10.264	14.020	6.030	11.022
Large	10.000	9.866	9.964	11.466	3.590	8.178
Major	10.032	9.970	10.116	5.474	0.500	2.778

4.3 Backtesting risk management methods

In this section we will execute a backtesting procedure to verify which estimation method performs best, in a CPBO context. As scientific literature does not provide us with information on which distributions frequently occur in this context, we'll use the case study data. This will ensure that we backtest the methods on realistic distributions. Les us now consider the risk data made available by the case study company. The risk factors underlying the data are all very operationally oriented risks (e.g. delays in execution or exceeding travel budgets) but have been converted to euro values (if necessary) and scaled (via sales value), creating the possibility to compare them and making it easy to use in a modeling approach. There are two drawbacks to the available data however: the sample sizes are very small (26, 20, 18, 17, 14 respectively) and the data has been gathered from projects who significantly differ in size. The result of this, as we'll see when fitting distributions, are large fitting errors (up to 20% in the worst case). The implication of this is that, when modeling the portfolio using the fitted distributions, this merely represents an approximation of the real world scenario rather than being a trustworthy model.

Since there are no hints in scientific literature on which distributions fit certain risk factors well, we try a set of common distributions seen throughout risk management literature (e.g. as defined by Chatterjee (2014)) to see which one fits best (referred to as set of suggested distributions). In order to estimate the distribution that most accurately expresses the risk factors' behavior in the best possible way, this thesis will adapt a combination of the Maximum Likelihood Estimation technique (MLE), corrected Akaike Information Criterion (AICc) and Kolmogorov-Smirnov (KS) test. At this point we can decide which distributions will be used to approximate the risk factors. The distributions resulting from this procedure will be used to generate random data which we will be using in the backtesting procedure.

4.3.1 Approximating risk factors

First, by use of optimizing the MLE, we will find the optimal fit for every suggested distribution. Choosing the MLE method is substantiated by the fact that it's said to be the best method for parameter estimation by many statisticians (Myung, 2003).

Subsequently every optimal suggested distribution will be tested on the likelihood of its fit, by use of a visual method (ecdf error plot) and statistical method (KS test). The ecdf error plot emphasizes a poor fit by showing the difference between the ecdf and fitted distribution. The KS test will take a random sample and determine the likelihood that the sample follows the suggested distribution, based on the distance between the suggested distribution and sample ecdf. The output parameters of the KS test are the p-value and D-statistic. The D-statistic represents the maximum distance measured between the suggested distribution and sample ecdf, whereas the p-value represents the likelihood that one doesn't observe a higher D-statistic for other KS samples. We accept the KS test when the p-value is greater than the α-threshold. Though there is not one optimal threshold, we will follow the advice of Krawczyk (2015) and Fisher (1926), who states "If p is between 0.1 and 0.9 there is certainly no reason to suspect the hypothesis tested. If it is below 0.02 it is strongly indicated that the hypothesis fails to account for the whole of the facts. We shall not often be astray if we draw a conventional line at 0.05", by setting $\alpha = 0.05$. Throughout literature it has been argued that the KS test's p-value is overestimated in the case that the suggested parameters are estimated from the same dataset as the one used to compute the test statistic (Keutelian, 1991), and a MCS should be used to improve its accuracy (Stephens, 1974; Khamis, 2000; Clauset et al., 2009). Therefore, this thesis will incorporate the MCS as proposed by Clauset et al. (2009), which takes the inverse cdf of the fitted distribution to create synthetic datasets which will individually be submitted to a KS test. We refer to this as being synthetic datasets since they're generated from a hypothetical distribution rather than being data that is actually available. Subsequently the overall performance of the fitted distribution, for all synthetic datasets, will determine the final conclusion with regards to the hypothesis being accepted or rejected (see Appendix C for the algorithms). The results of the adjusted KS test is a subset of the suggested distributions who potentially fit the underlying data, denoted as I.

Finally, to make a well substantiated decision between the remainder of the suggested distributions, we will look for the most accurate one to represent the risk factor. To do so, we will compare their performance based on the AICc score, which is a sequel to the AIC norm. The AICc adds another penalization term for datasets that have a small sample size compared to the number of parameters and should be used until $n/\kappa > 40$ (Burnham and Anderson, 2002).

$$\text{AIC} = 2\kappa - 2\log(\mathcal{L}(\hat{\theta}|y)) \tag{4.14}$$

$$\text{AICc} = \text{AIC} + \frac{2\kappa(\kappa+1)}{n-\kappa-1} \tag{4.15}$$

Where κ is the distribution's number of parameters, $\mathcal{L}(\hat{\theta}|y))$ is the MLE for the fitted distribution and n is the number of available data points.

Once we know the AICc for every distribution in set I, we compute its AICc difference score: Δ_i which makes for easy comparison. Burnham and Anderson (2002) have defined a simplistic rule that all pdfs with $\Delta_i \geq 2$ should be excluded as they have too little empirical support for the model. See Table 4.3 for a complete overview of the implication of the Δ_i scores. The Δ_i is defined as:

$$\text{AICc difference} = \Delta_i = \text{AICc}_i - \text{AICc}_{min}, \forall\ i \in I \tag{4.16}$$

Where AICc_{min} represents the smallest AICc score of all suggested distributions, i.e.:

$$\text{AICc}_{min} = \min_{\forall\ i \in I} \text{AICc}_i. \tag{4.17}$$

Burnham and Anderson (2002) stress the importance of showing the AICc differences for every fitting procedure as it quickly substantiates the proportional probability of a model other than the best performing one being the correct model.

Table 4.3: Implication of AICc difference relative to the empirical support of model i. Adapted from Burnham and Anderson (2002)

Δ_i	Level of Empirical Support of Model i
0-2	Substantial
4-7	Considerably less
> 10	Essentially none.

We will now go through the fitting procedure for the first risk factor 1. We start by fitting every suggested distribution to the available data of this risk factor. Using the *fitdist* function of Matlab R2017a, we find the results, sorted on AICc, as shown in Table 4.4.

Table 4.4: Fitting parameters for f_1 per distribution

Risk factor 1											
N = 26	Likelihood estimations					Parameter estimates					
Distribution Name	NlogL	BIC	AIC	AICc	Δ_i	Name	Value	Name	Value	Name	Value
Exponential	-64.71	-126.17	-127.43	-127.26	0.00	Mu	0.0305				
Beta	-65.74	-124.97	-127.49	-126.97	0.29	alpha	0.7140	beta	22.7269		
Gamma	-65.69	-124.86	-127.38	-126.85	0.40	Shape	0.7285	Scale	0.0419		
Weibull	-65.66	-124.80	-127.31	-126.79	0.47	Scale	0.0274	Shape	0.8135		
Lognormal	-64.68	-122.84	-125.35	-124.83	2.43	Log location	-4.3140	Log scale	1.5323		
Generalized pareto	-65.78	-121.79	-125.56	-124.47	2.79	Shape	0.3221	Scale	0.0212	Threshold	0.0008
Inverse gaussian	-62.81	-119.10	-121.62	-121.10	6.16	Scale	0.0305	Shape	0.0051		
Generalized extreme value	-62.52	-115.26	-119.04	-117.94	9.31	Shape	1.1367	Scale	0.0104	Location	0.0076
Normal	-51.70	-96.89	-99.40	-98.88	28.38	Location	0.0305	Scale	0.0338		
t-locationscale	-52.72	-95.67	-99.45	-98.36	28.90	Location	0.0238	Scale	0.0250	DoF	4.3237
Extreme value	-44.95	-83.38	-85.89	-85.37	41.89	Location	0.0490	Scale	0.0421		

The second stage of this fitting procedure, testing the likelihood of the suggested distributions, starts with visual inspection of two plots: 1. the pdf on the data and 2. the cdf fit on the ecdf including its corresponding error plot. Both Figure 4.2 and Figure 4.3 show the plots for the top 5 best fits (based on AICc). Using both figures we can conclude that the suggested distributions either tend to fit the body or tail well, but not both. Even with the spiking error margin of 15% the exponential distribution has the best overall fit. Next we will conduct the adjusted KS test to assess whether the suggested distributions might fit the data. For every suggested distribution we formulate the following null and alternative hypothesis:

H_0: The data follows the suggested distribution
H_a: The data does not follow the suggested distribution

We reject H_a, and therewith conclude that the data *might* follow the suggested distribution, if $p \geq 0.05$. Table 4.5 shows the outcome of all adjusted KS tests. Finally we determine which distribution fits the data best, by choosing the distribution which minimizes the AICc on the condition that $\Delta_i \leq 2$ and $p_i \geq 0.05$. The procedure results in the following distributions for the five risk factors (see Appendix D for all tables and figures):

$$f_1 \sim \text{Exp}(0.0305)$$
$$f_2 \sim \text{Exp}(0.0110)$$
$$f_3 \sim \text{IG}(0.0064,\ 0.0034)$$
$$f_4 \sim \text{IG}(0.0191,\ 0.0049)$$
$$f_5 \sim \text{Exp}(0.0171)$$

Table 4.5: p-value Adjusted KS test f_1

Distribution Name	p-value
Exponential	0.1410
Beta	0.5528
Gamma	0.4892
Weibull	0.2919
Lognormal	0.0593
Generalized pareto	0.1549
Inverse gaussian	0.0060
Generalized extreme value	0.0143
Normal	0.0050
t-locationscale	0.0020
Extreme value	0.0000

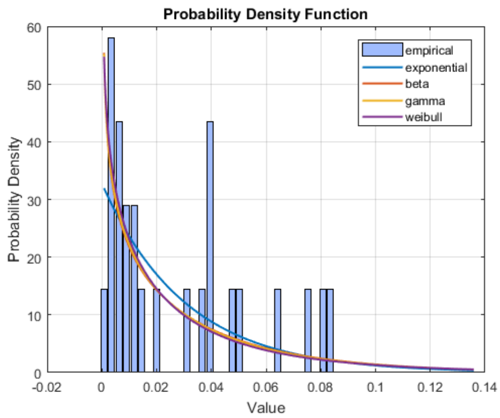

Figure 4.2: Top 5 Probability density function fits for f_1 based on AICc

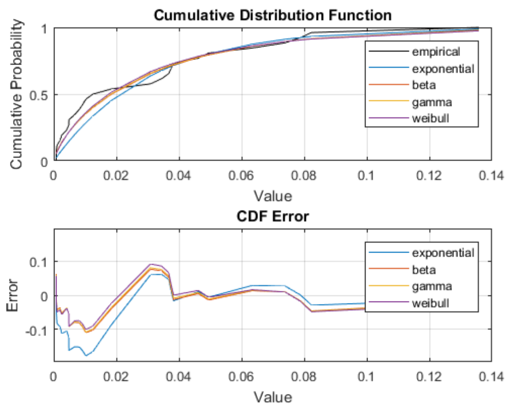

Figure 4.3: Top 5 Cumulative distribution function fits for F_1 based on AICc

4.3.2 Backtesting risk management methods in the CPBO context

With the portfolio's underlying distributions estimated, we can now continue to the actual backtest. Simply put, a backtest uses historical data to compute and compare the accuracy of a risk portfolio VaR estimate. Let us first elaborate on the assumptions we need to make in order to execute the backtesting procedure.

Assumptions

In order to properly conduct the backtest we will make four assumptions, of which three are data related and one is model related. In the introduction of this section we introduced the fact that we will use data provided by the case study company in order to test the risk management model. The first assumption will substantiate on how we'll be using this data. The second and third assumptions were necessary since the available data is limiting our ability to test some properties of the data. Finally the last assumptions revolves around how we define an exceedance.

1. All risk factors exactly follow a known distribution
Since the case study company can't provide us with sufficient data for both model creation and testing, we had to make this assumption for model testing purposes. We estimated the distribution of the available risk data, per risk factor and subsequently generate random numbers from this distribution. We then assume that the generated numbers are actual realizations of a certain risk. Note that in the case of correlated portfolio's we've also incorporated the same correlation in generating the random realizations. An important implication of this assumption is that the models created in this thesis will most likely be more accurate compared to the same model being tested on actual data. Therefore the conclusions derived from this thesis (e.g. on performance or minimum observations) should merely be seen as a lower bound.

2. Risk factor correlation can be modeled using a Gaussian copula
An important property of the CPBO risk management model is it's ability to model correlations between certain risk factors or characteristics. Literature distinguishes multiple methods to model such correlations, of which a frequent one are copulas. Copulas can be further divided into several subsets, but we will assume that the Gaussian copula (which is frequently used in other scientific research) is able to properly model the correlation of our underlying risk factors. This assumption has to be made since the case study company does not have sufficient data to test for different methods.

3. Missing correlations are complemented via expert opinion
Based on the available case study data, we were able to compute (approximate) three correlations. The other correlations were estimated via expert opinion, and we assume those are accurate. Since such correlations might not be very obvious, and experts had a hard time estimating them, we used the following approach: First the experts had to choose between either positive or negative correlation. Thereafter they could classify this as small, medium, large or exact (i.e. one-to-one ratio). Using the scales of (Cohen, 1992) we translated those estimations to correlations values of 0.1 (small), 0.3 (medium), 0.5 (large) and 1 (exact).

4. Residual budgets for one risk factor will be used to cover others
The simulation will propose an aggregated risk portfolio value, whereas in practice such budgets will be allocated per risk factor. We make this assumption to simplify evaluating the

model's performance. This assumptions will not yield any complications in practice as the project realization will accumulate all costs and budgets anyway. Note that this also follows the concept of reallocating budgets which has been discussed throughout the thesis.

Backtesting analysis

When backtesting we use two datasets. The first set is referred to as the train set which, in this backtest, consists of 100 synthetic data points. In this, a data points is defined as a single risk factor cost for a specific project (such that our train set is a $\tilde{k}$ by 100 matrix). Again, we refer to the data points as being synthetic since they will be generated using an approximated distribution instead of being actual data. The first step in the backtest is to compute the VaR, which is based on this train set, using all three methods discussed in the thesis. Secondly we have the test set, which consists of 250 synthetic data points. Based on the test set we compute the number of exceedances, where we define an exceedance as the case in which the risk portfolio VaR estimate was insufficient to cover the risk portfolio's value for the simulated project. In the case of an exceedance, the company will incur more costs than it's willing to and will therefore decrease a project's profit margin.

As mentioned before, the methodology we will use is called the traffic light method and is the advised methodology by the Basel Committee on Banking Supervision (1996). This method owes its name to the underlying categories in which a VaR method can be ascribed to, which are green, yellow and red. Instead of computing the maximum number of allowed exceedances using the portfolio distribution, the traffic light method uses a binomial distribution. Whenever a backtest results in a number of exceedances which is ≤ the 95% quantile of the binomial cdf, the model falls within the green zone. The yellow zone covers all exceedances up to and including the 99.99% quantile and the red zone all thereafter. The upper bounds of the green and yellow zone can be mathematically expressed as:

$$\begin{aligned} \text{Green zone upper bound} &= \max\{n \in \mathbb{Z} : \mathbb{P}[X \leq n] \leq 95\%\} \\ \text{Yellow zone upper bound} &= \max\{n \in \mathbb{Z} : \mathbb{P}[X \leq n] \leq 99.99\%\} \end{aligned} \tag{4.18}$$

Where X represents the number of exceedances. Whenever a VaR method is backtested and falls within the red category, it is highly unlikely that the method models the real world scenario properly, and therefore should not be used. As for the yellow category this will not be as straightforward to conclude. The Basel Committee on Banking Supervision (1996) states that methods who fall within the lower spectrum of the yellow category might correctly model the real world scenario, but its parameters should be tuned. The higher the number of exceedances in the yellow zone, the less likely the method is correct. In order to convert a yellow zone method to the green zone, the Basel Committee on Banking Supervision (1996) proposes to use a scalar value. Such a value should be compute per individual portfolio distribution. Therefore we will only do this if the preferred VaR method should fall in the yellow zone. One can compute the probability of n exceedances, given confidence level β as follows:

$$\mathbb{P}[X = n] = \binom{m}{n}(1-\beta)^n \beta^{(m-n)} \tag{4.19}$$

Where m is the number of simulated risk portfolio values, n is the number exceedances to compute the probability for and β is the confidence level.

For this backtesting procedure we will take a 95% confidence level and simulate 250 risk portfolio values, resulting in the following distribution:

$$\mathbb{P}[X = n] = \binom{250}{n}(0,05)^n \, 0,95^{(250-n)} \tag{4.20}$$

Therefore the probability that there are exactly 0 exceedances in 250 projects, using a 95% VaR, is equal to:

$$\mathbb{P}[X = 0] = \binom{250}{0}(0,05)^0 \, 0,95^{(250)} = 2,6971\text{E}^{-06} \tag{4.21}$$

Repeating the latter computation for all n up to the 99.99% quantile results in the probabilities as in Table 4.6. From this table we can conclude that whenever the backtest for any of the VaR methods has up to 17 or 26 exceedances it is allocated to the green and yellow category respectively and all other fall within the red zone. Note that because of using the binomial distribution, the Basel Committee allows for a greater number of exceedances. More specific, for $\beta = 0.95$ and $m = 250$ the Basel Committee allows 17 (see Table 4.6) instead of $m(1-\beta) = 250 * 0.05 = 12.5$ exceedances. This should account for any negative random events on the one hand, but still prove the method correctly models the real world situation on the other. In terms of statistics, this method balances the probability of a rejecting an accurate model (type 1 error) and probability of accepting an erroneous model (type 2 error).

Table 4.6: Backtesting probabilities overview

n	$\mathbb{P}[X = n]$	$\mathbb{P}[X \leq n]$	n	$\mathbb{P}[X = n]$	$\mathbb{P}[X \leq n]$	n	$\mathbb{P}[X = n]$	$\mathbb{P}[X \leq n]$
0	2.6971 e-06	2.6971 e-06	10	0.0963	0.2909	20	0.0123	0.9851
1	3.5489 e-05	3.8186 e-05	11	0.1106	0.4016	21	0.0071	0.9922
2	0.0002	0.0003	12	0.1160	0.5175	22	0.0039	0.9961
3	0.0010	0.0013	13	0.1117	0.6293	23	0.0020	0.9981
4	0.0033	0.0046	14	0.0996	0.7288	24	0.0010	0.9991
5	0.0085	0.0131	15	0.0824	0.8113	25	0.0005	0.9996
6	0.0183	0.0314	16	0.0637	0.8750	**26**	0.0002	**0.9998**
7	0.0336	0.0650	**17**	0.0462	**0.9212**	27	0.0001	0.99993
8	0.0537	0.1186	18	0.0315	0.9526			
9	0.0760	0.1946	19	0.0202	0.9729			
Green zone: $n \leq 17$			Yellow zone: $n \leq 26$			Red zone: $n > 26$		

For the previously composed project risk portfolio we compute the VaR estimates using the algorithms implemented in Matlab R2017a (see Appendix C for the required algorithms), and therewith find the figures as shown in Table 4.7. Note that we initialize the model by generating 100 random samples of the portfolio and use this as available data underlying the VaR models. Furthermore we set the number of simulation sample paths for both HS and MCS to 3,000. In order to simplify the backtesting procedure we made two assumptions:

1. All portfolio risk factors are independent (both frequency and severity)
2. We assume all risk factors appear in all projects

Though both assumptions can be relaxed (and will be in the final model), this drastically decreases the amount of sample paths required to get an accurate projection of the possible outcomes. Before we simulate the realized project risk portfolio outcomes, and assign each VaR method a category, we need to define the indicator function used for counting the number of exceedances:

$$I_{t,z} = \begin{cases} 1 \text{ if } \gamma_z > \alpha_{\beta,t}(r) \\ 0 \text{ if } \gamma_z \leq \alpha_{\beta,t}(r) \end{cases} \tag{4.22}$$

Where γ_z represents the project risk portfolio value for simulation z, T is the set of all VaR estimation methods and $\alpha_{\beta,t}(r)$ represents the VaR estimate using method t and confidence level β. Using the indicator function we can find the total number of exceedances for $\alpha_{\beta,t}(r)$, as this is the summation of the indicator function over all z:

$$X_t = \sum_{z=1}^{m} I_{t,z}, \forall\ t \in T \tag{4.23}$$

After simulating 250 random risk portfolio values, we compute the exceedances per method X_t. To minimize the effect of randomness on the classification procedure, we repeat the backtest procedure 5,000 times. Finally, by use of the threshold values in Table 4.6 we classify the methods as in Table 4.7. From our backtest we learn that both the HS as MCS method qualify to be used in CPBO project risk management. That is, they are both classified in the green zone. The fact that the DN method does not qualify is a logical effect of the risk factors not being approximated appropriately by normal distributions. More specific, since the portfolio is composed of exponential and inverse Gaussian distributions (and the normal distribution does not approximate any of those two) it was not expected to perform well. Both HS and MCS were expected to perform well as they simulate a (empirical) distribution of the portfolio. As in line with the theory, the MCS does outperform HS, probably since it's better able to model values outside the dataset. Given it has the lowest mean and standard deviation of all, we choose the MCS method as the main VaR method for the remainder of this thesis and the case study.

Table 4.7: VaR model exceedances and classification

		Number of exceedances		
Model	VaR	Mean	Standard deviation	Classification
DN	0.1263	38.9416	5.7503	Red
HS	0.1712	14.6890	3.7072	Green
MCS	0.1780	12.7850	3.4496	Green

The conclusions found in this chapter are all based on the assumption that sufficient data is available. In the context of normal risk management models (e.g. financial institutions) this might be valid. However, in the specific case we're looking for this might not be true. Therefore, in the next chapter, we will look at a case study and see whether we will find the same conclusions, based on real company data. Some main difference with respect to the data in comparison with both theoretical and financial market data, is the fact that it might be contaminated, scarce and subjective to inaccuracy if gathered by hand.

Chapter 5

Case study: Putting a risk budgeting model to work

In this chapter we will test whether the previously found theoretical outcomes also hold within a case study company. This is specifically important since there is no scientific literature available to substantiate which method to incorporate in the CPBO risk management model. The most important difference with the previous chapter is the fact that the available training and testing data will be of lower quality (i.e. no longer sampled from a statistical distribution when backtesting, but realized costs instead), which is expected to decrease the CPBO risk management model's prediction accuracy. Lastly we'll use this case study to conclude whether the CPBO risk management model yields more accurate results compared to the current risk management system in place. It should be noted that the four assumptions, as seen in the previous chapter, still apply. In Section 5.3 we will no longer generate synthetic data, such that we lot go assumption 1. An important difference between this chapter and the previous is the fact that we allow risk factors to be correlated. In doing so, we will add the following assumption:

5. Risk occurrence is uncorrelated, even if risk factors are correlated

This assumption has to be made since we're unable to determine correlations between occurrences based on case study data. We assume that, even if risk factors are correlated, their occurrence is independent. More specific, we assume that O_k is independent even if C_k is not. We know that this assumption violates the real world scenarios, but data on risk occurrences can't be acquired.

This chapter starts by determining the minimum number of data points required per risk factor, in order to properly estimate their VaRs in Section 5.1. The confidence level parameter of the CPBO risk management model is the only parameter available for the user to change. As such, we'll briefly elaborate on the impact this parameter has in Section 5.2. Finally we compute and backtest the risk portfolio VaR estimates in Section 5.3 and elaborate on how to incorporate this into the sales price in Section 5.4.

5.1 Minimum data requirements

Given the limited number of projects a typical CPBO completes a year, data gathering can be a time-consuming process. Combining this with the necessity to accurately estimate project risk, it's interesting to know the minimum amount of data required before one can incorporate the model designed in Chapter 4. Furthermore, since the case study company can only provide us with risk data of limited sample sizes, we can use this simulation study to test whether we can safely use the data to substantiate our research findings. The observation threshold is a very influential parameter in any risk budgeting model, as it is closely related to the modeling accuracy. That is, setting the threshold too low yields inaccurate models. Given the implications of such a threshold, it's surprising to see that literature does not provide ready to use, approximate threshold numbers. A rule of thumb that is commonly referred to is the "30 samples rule", but this has been debunked by Cohen (1990) a long time ago. Smith and Wells (2006) showed that for the normal- and uniform distribution sample sizes of 15 and 50 resp. will suffice. Since this thesis will take on a portfolio containing different risk factors, each following their own specific distribution, we cannot determine the threshold based on the existing scientific literature. In order to guide CPBOs in determining such a threshold, three types of simulations have been conducted, each type having a small and large portfolio. The first simulation focuses on an independent risk portfolio, the second addresses correlated portfolios and the last one takes the portfolios as used by the case study company, which are correlated and incorporate the probability of a risk factor occurring. For this analysis we will only compute the MCS VaR, since we want to obtain the absolute minimum number of necessary data points. Since MCS has been shown to be the most accurate method on the previous chapter it makes sense to use this method.

Since no sufficient data can be provided by the case study company we will follow the following procedure: We simulate a number of random portfolio values from the distributions fitted to the risk factors (as shown in Chapter 4 for risk factor 1). Subsequently we divide this set into two subsets: the training and test set. We use the training data to compute the MCS VaR and backtest this value with the second subset, using the traffic light backtest as seen in Chapter 4. Note that, since all data is randomly generated (i.e. synthetic), we expect the model to be more accurate than would be the case when real data were used. One of the factors underlying this expectation is the fact that we assume risk factors to follow a certain (fixed) distribution, whereas this would not be true in a real world application.

The first type of simulation takes on an independent portfolio with a different number of risk factors. The first simulates a portfolio with two independent risk factors, the second a portfolio composed of five. We expect that due to amongst others residual budget reallocation, the number of required observations should decrease. We test for n ranging from 5 to 100, and iterate a large number of times to account for any form of randomness in the simulation. The simulation results have been summarized in Figure 5.1 and Figure 5.2. From those figures we learn that both simulations seem to operate within their limits when the number of available data points (i.e. risk costs realized) is above 15 per risk factor, i.e $n^* = 15$ for both simulations. Though the minimum number of observations is equal, we see a significant difference in the simulation's 95% confidence interval, in particular for lower n. A lower confidence interval indicates that the VaR handles extreme outcomes better, which might be due to budget reallocation. More specific that is, in the case a risk factor does not (fully) occur, the residual budget can be reallocated to other risk factors. Therefore we argue that this effect might only be observable in the case that risk factor values are of the same magnitude.

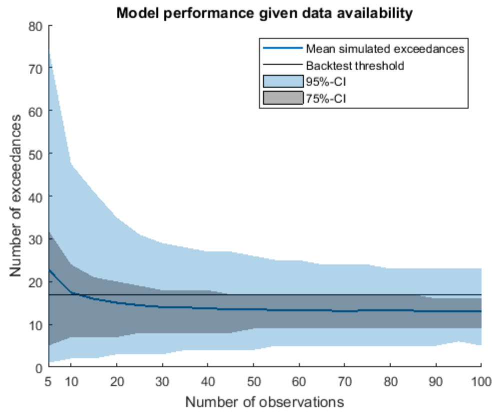

Figure 5.1: Performance portfolio of two independent risks

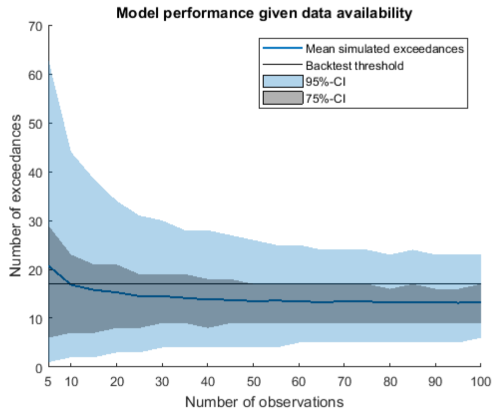

Figure 5.2: Performance portfolio of five independent risks

We repeat the simulation, using correlated risk factors instead. Correlations have been estimated from data where possible, and complemented with expert opinion. Even though correlation has a damping effect on diversification (Blumenthal, 2015), and thus on residual budget reallocation, we expect this effect to be limited for our portfolio since most correlations are small. Furthermore we expect that the addition of risk factors results in a lower n^*, since it should create more residual budget which can be used for other risk realizations. From the simulation (figures added to Appendix E) we can conclude that our assumptions were indeed correct. The correlated risk portfolio with two risk factors needed at least 25 data points to be within acceptable range, whereas the larger portfolio needed only 22. We expect that the larger n^* can be explained by the fact that the model needs to estimate the correlations. Once again we can observe smaller confidence intervals for the larger portfolio, albeit to a lesser extent. The overall confidence intervals are wider compared to the first simulation results, which might be explained by the correlation's effect on the risk factor's residual budget reallocation.

Lastly we change the simulation setup to include the probability of a risk occurring: O_k. In line with this change, we'll also incorporate this probability into the backtesting procedure. This inclusion however requires a modeling decision. That is, since we allow risk factors to not occur, i.e. their cost being zero for a given project, there exists the scenario that no risk factors occur and the realized risk portfolio cost is zero*. Logically, allowing those projects to be in the test set will increase the model's performance (see Appendix E for a plot which allows such portfolio's in the test set). Since the goal of a risk management model is to hedge against risks that actually occur, we make the modeling decision not to include zero cost realized projects into the test set. This means that we use a worst case scenario (i.e. all projects always have some form of risk occurrence) to determine the minimum number of required data points. As such, when a company employs the model with this number of minimum data requirements, it's unlikely to result in underestimations due to the model itself.

*If $p_k = 0.2 \forall k \in K$ and $\tilde{k} = 5$ then the probability of the portfolio value being zero equals $(1-0.2)^{\tilde{k}} = 0.8^5 \approx$ 33%

As a result of incorporating the occurrence probabilities, we expect two things to happen. First we expect the VaR estimates to go down, since the lower bound of the data's interval is extended to zero. Second we expect the number of required data points per risk factor to increase. That is, since a risk factor will not occur with probability $1 - p_k$, we will have less information available with respect to the portfolio values to model our VaR estimates from.

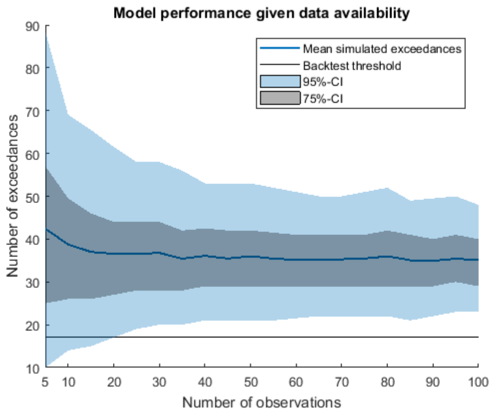

Figure 5.3: Performance portfolio of two correlated risks including occurrence probability

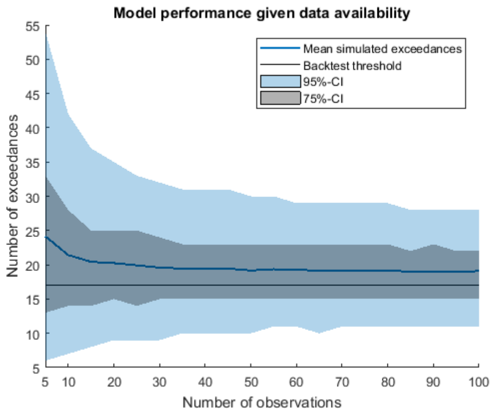

Figure 5.4: Performance portfolio of five correlated risks including occurrence probability

After conducting the simulation we can conclude that our expectations were correct (see Figure 5.3 and 5.4), but came with a surprise. First of all let us discuss the unexpected outcomes in terms of n^*. Based on the simulation outcomes we conclude that incorporating small probabilities results in consistent underestimation of the risk portfolios VaR, where we expected it to only increase n^*. In terms of the small portfolio (Figure 5.3) we see that the number of exceedances converges to approximately 36 exceedances, whereas the large portfolio (Figure 5.4) converges to approx. 20 exceedances. In terms of the n^* we conclude that both models are unable to achieve the required accuracy, regardless of the number of available data points. That is, in the case we limit our test set to positive real numbers. We therefore set the optima for both portfolios equal to the first number n which is equal to the converged values of both simulations (36 and 20 resp.), rounded up to the nearest integer. This results in the recommended number of data points of $n^* = 34$ and $n^* = 24$ for the small and large portfolio resp. Note that we refer to this as recommended rather than minimum since they are based on the worst case scenario. When we conduct the same simulation, but this time allow zero valued portfolios we find the absolute minimum number of required data points, being $n^- = 11$ and $n^- = 10$ for the small and large portfolio resp. (see Appendix E for the simulation figures). As was in line with our expectations, both risk portfolio VaR estimates did indeed decreased with approx 58% and 65% for the small and large portfolio resp. This can be explained by the fact that non-occurring risk factors don't generate costs, i.e. portfolio values now range from 0 in the case $o_k = 0 \ \forall k \in K$, which occurs in roughly 33% of all cases.

We can observe a large difference between the small and large portfolio's performance, with the latter having a 84% lower deviation when converged. Again we expect the explanation to be the budget reallocation. The effect of this can be explained by the following example: The first column of Table 5.1 shows, for a hypothetical project, how the budget is allocated prior

Table 5.1: Example of budget reallocation

k	Allocated budget	Realized costs	Deviation	
1	100	80	20	
2	0	50	-50	
3	0	0	0	
4	75	25	50	
5	25	40	-15	+
Sum	200	195	5	

to project execution. From the second column we see that, at the end of the project, some risks were less expensive as anticipated (risk factor 1 and 4), some did indeed not occur (risk factor 3) and others were underestimated (risk factor 5) or not anticipated for at all (risk factor 2). Even though there was some error in estimating the risk factor costs, the overall portfolio outcome is positive and so would have been the backtest.

From this simulation we've learned that it becomes more difficult to estimate the risk factors themselves (i.e. less information is available since risk factors might not occur) when including O_k. As a result of including the occurrence probability the simulated risk portfolio values can become zero, thus distorting the risk portfolio VaR estimate. This so called zero inflated data results in the risk portfolio VaR being underestimated in the case test data is restricted to positive real numbers. Lastly we can see from the simulation results that the confidence intervals are vastly more narrow, when little data is available (with a maximum decrease on the 95%-CI upper bound of 36% at $n = 10$ for the last simulation). This effect diminishes however when more data becomes available.

We conclude this section with a concise overview of the required minimum number of data points per model in Table 5.2. We will refer to this table when implementing the CPBO risk management model within the case study company. In this section we've established two important insights: the first being that reallocating residual budgets does have a positive effect on a risk factor portfolio outcomes and second being that including small occurrence probabilities results in structural underestimates of the risk portfolio VaR estimate.

Table 5.2: Minimum data requirements per CPBO model

Size	$k = 2$			$k = 5$		
Model	n^*	CI_{lb}	CI_{ub}	n^*	CI_{lb}	CI_{ub}
Independent	15	7	21	15	7	21
Dependent	25	9	21	22	8	22
Dependent + probability	$34^\dagger$	28	43	$24^\dagger$	15	25

CI represents 75% CI of exceedances at n^*

†first n equal to simulation convergence

Table 5.3: Risk portfolio VaR estimate per simulation

Size	$k = 2$	$k = 5$
Model	VaR	VaR
Independent	0.1005	0.1821
Correlated	0.1059	0.1997
Correlated + probability	0.0447†	0.0707†

† Simulation converges to this value

5.2 Modeling impact of confidence level

A portfolio's VaR estimation depends heavily on the confidence level parameter β. Furthermore, since VaR represents the quantile estimation, the underlying data structure determines its behavior. Given the small occurrence probabilities encountered in the CPBO context, we observe that data is heavily right skewed (see Figure 5.5). In terms of realizations this means that we expect most risk portfolio outcomes to be low values since the probabilities that multiple risk factors don't occur is fairly large. One of the major consequences of this is the fact that VaR estimations might significantly differ between two fairly close confidence levels. Figure 5.6 shows the relation between β and the portfolio VaR estimate. One can see that in the lower regions, i.e. $\beta \leq 0.7$ the slopes are quite flat, meaning a small increase in the portfolio's VaR estimate covers a significantly larger range of probable risk portfolio outcomes. Getting into the higher confidence levels, the slope steepens rather quickly, which is in line with the stretching tail of the histogram in Figure 5.5. From this section we've learned that, when incorporating a CPBO risk management model one has to pay attention when choosing a certain confidence level. Bluntly choosing a large threshold might feel like a logical or safe choice, but will result in substantially higher portfolio VaR estimates.

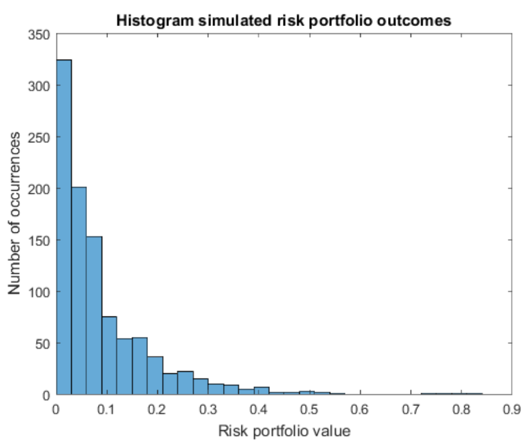

Figure 5.5: Histogram simulated portfolio outcomes

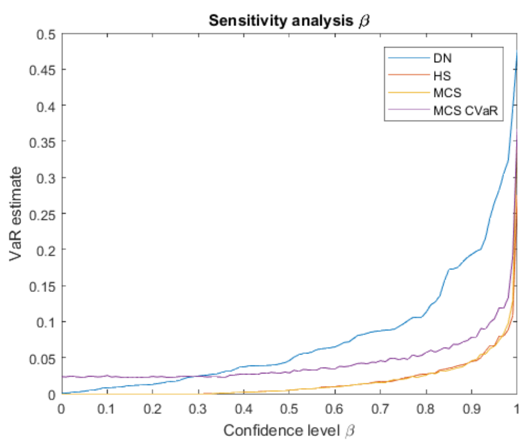

Figure 5.6: Sensitivity analysis β parameter

5.3 Backtesting CPBO risk management model within case study company

In this section we'll incorporate the CPBO risk management model within a case study company. For a small number of projects we have been given the risk portfolio, their initial budgets and the realized costs after project completion (referred to as project data). The only data related problem we need to address before moving forward is the sample size of the project data. That is, we were only able to acquire the data of 11 projects, which will be a problem when testing the model (i.e. prove the model's reliability and validity). An importance difference between this section, and the analysis done before is the fact that we will not generate any form of synthetic data. As a result, we have less data available to train the model such that the model in this section is a genuine representation of the CPBO risk management model's performance. In terms of assumptions we let of assumption 1, and keep the others.

Before moving on to training the model, we should explain the origin and format of the available training data. Let us start by given a few definitions which might be confusing. The case study company uses three different kinds of budgets: default-, adjusted default- and non default budgets which have been defined in Table 5.4.

Table 5.4: Budget definitions

Budget	Description
Default	The default budget is the project's cost price
Adjusted default	The adjusted budget is the default budget, supplemented with extra budget that should hedge against project specific characteristics and relevant risk factors. The CPBO risk management model should outperform this method at minimum.
Non default	The non default budget is the budget incorporated in the sales price. The difference between the adjusted- and non default budget is determined by the sales department and is a combination of experience (e.g. with the customer), insiders' knowledge, hunches and a discount factor.

Lastly we elaborate on the current methodology of gather data. This is important since the case study company gathers all budget overruns rather than the total costs of a risk factor. That is, the available training data is the difference between the realized budget and the non default budget. As an example, if the budget for risk factor 1 is 80 and the realized cost is 100, the available data for risk factor 1 is 20. As a result, the available training data describes the expected difference between realized costs and non default budgets rather than the full risk costs. Under normal circumstances one would compute the latter, but this is impossible as was discussed here, in Section 1.3 and Appendix A. In the model testing phase we'll see the effect of this data gathering methodology. Furthermore we see that the available data in the training set specifically revolves around large projects in one industry, whereas the available test set (project data) revolves around small and large projects in multiple industries. As such, we will scale the training data by the sales value of its project. The simulated risk portfolio VaR then represents a % which we multiply with the sales price of the project under evaluation. The method of computing the final budget will be addressed in Section 5.3.2.

5.3.1 Model training

The training procedure of the CPBO model exists of three steps. First, we need to compose a list of risk factors that will be included in the model, second we need to find the distributions that fits those risk factors best and third we have to compute the risk portfolio VaR estimate for all available risk portfolios (R). Given the limited availability of data, this case study will choose the risk factors based on the following logic: Given the risk factors identified in the 11 available risk profiles, create a subset from the risks which occur in our cost database and have at least 10 data points. The resulting risk factors are the same as previously estimated in Chapter 4 plus one ($C_6 \sim \text{Exp}(0.0079)$, see Appendix D for the fitting details), whereas the threshold value of 10 data points has been established in Section 5.1. We use the absolute minimum number of required data points instead of the recommended as this would've led to only 1 risk factor qualifying. However we don't expect this to be a problem, since our test set is fairly balanced (35% under- vs. 65% overestimated projects). Finally, it should be mentioned that the data used for training and testing the case study CPBO risk management model is independent (i.e. there aren't any overlapping projects). We initialize O_k as independent Bernoulli trials with $p_k = 0.2 \ \forall \ k \in K$ such that all risk factors have an equal probability of occurring in our portfolio's. We model a project's risk portfolio by use of r. Lastly we will set the confidence level equal to $\beta = 90\%$. Finally, when using the copula, we will use the same correlation matrix as used in Chapter 4. The following example shows the computation of the risk portfolio VaR estimate for the first portfolio composition: $r = (1, 1, 1, 1, 1, 1)$. As before, we denote the VaR estimates per risk management method as $\alpha_{\beta,t}(r)$, where $T = (\text{DN}, \text{HS}, \text{MCS})$. The data can be summarized as follows:

$$\tilde{k} = 6, \quad r = (1,1,1,1,1,1), \quad O_k \sim B(1, 0.2) \ \forall \ k \in K, \quad \begin{aligned} &C_1 \sim \text{Exp}(0.0305) \\ &C_2 \sim \text{Exp}(0.0110) \\ &C_3 \sim \text{IG}(0.0064,\ 0.0034) \\ &C_4 \sim \text{IG}(0.0191,\ 0.0049) \\ &C_5 \sim \text{Exp}(0.0171) \\ &C_6 \sim \text{Exp}(0.0079) \end{aligned} \tag{5.1}$$

First we compute the Delta Normal VaR estimate $\alpha_{\beta,DN}(r)$. We start of by computing the VaR map of the risk portfolio, assuming all data is normally distributed. That is, we first fit a normal distribution to the available training data for each risk factor and subsequently compute the quantile functions:

$$\begin{aligned} m(r) &= [r_1 F_1^{-1}(\beta), r_2 F_2^{-1}(\beta), \cdots, r_{\tilde{k}} F_{\tilde{k}}^{-1}(\beta)] \\ &= [F_1^{-1}(0,9), F_2^{-1}(0,9), \cdots, F_6^{-1}(0,9)] \\ &= [0.0738,\ 0.0246,\ 0.0166,\ 0.0623,\ 0.0404,\ 0.0190] \end{aligned}$$

With the correlation matrix being:

$$\boldsymbol{\rho} = \begin{bmatrix} 1 & 0.67 & -0.04 & 0.1 & 0 & 0.1 \\ 0.67 & 1 & 0.018 & 0.1 & 0 & 0.1 \\ -0.04 & 0.018 & 1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 1 & 0 & 0.1 \\ 0 & 0 & 0.1 & 0 & 1 & 0 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0 & 1 \end{bmatrix}$$

Which results in the risk portfolio VaR estimate:

$$\alpha_{0.9,DN}(r) = \sqrt{m(r)\boldsymbol{\rho}m(r)^T} = 0.1296\% \tag{5.2}$$

Second we will show the intermediary steps of the computation of $\alpha_{\beta,HS}(r)$. We start of by generating 3,000 x $\tilde{k}$ random numbers with replacement. For every risk factor we use a different interval to generate the random numbers, such that the interval is [1, # data points available for risk factor k]. Based on the random numbers we construct the random portfolios $\boldsymbol{U}$ and evaluate this with the current portfolio r, to get vector y. From this we compute $\alpha_{\beta,HS}(r)$ as follows:

$$\alpha_{0.9,HS}(r) = \hat{F}_y^{-1}(0.9) = \min\{\alpha \in y : \hat{F}_y(\alpha) \geq 0.9\} = 0.05570\%^\dagger \tag{5.3}$$

Finally we compute the MCS VaR, i.e. $\alpha_{\beta,MCS}(r)$. The MCS method follows the same intermediary steps as the HS method. Instead of generating random numbers it samples 3,000 random numbers from the distributions of C_k to construct $\boldsymbol{U}$. We again evaluate $\boldsymbol{U}$ with the current portfolio r to find the VaR estimate:

$$\alpha_{0.9,MCS}(r) = \hat{F}_y^{-1}(0.9) = \min\{\alpha \in y : \hat{F}_y(\alpha) \geq 0.9\} = 0.04907\%^\dagger \tag{5.4}$$

We repeat this procedure for all relevant portfolio's in R. The results of the model training phase are summarized in Table 5.5.

Table 5.5: Model training - risk portfolio VaR estimates in % sales price

	$\alpha_{0.9,t}(r)$		
r	DN	HS	MCS
(1,1,1,1,1,1)	0.1296	0.0557	0.0491
(1,0,1,1,1,1)	0.1155	0.0516	0.0453
(1,0,1,1,0,0)	0.1031	0.0383	0.0346

5.3.2 Model testing

Based on the available project data (i.e. their different budgets and realized outcome), we can determine the performance of the previously trained CPBO risk management model. Since we simulate the expected difference between the non default budget and the realized outcomes, we compute the simulated budget for project p using risk management method t as follows:

$$\hat{b}'_{p,t} = \text{Simulated budget}_{p,t} = \text{non default budget}_p + \alpha_{\beta,t}(r_p) \times sv_p, \forall\ t \in T,\ p \in P \tag{5.5}$$

Where r_p refers to the risk portfolio of project p, sv_p represents the sales value of project p and P is the set of all available projects. We compute the risk portfolio VaR estimates per risk management method t, to find the descriptive statistics as shown in Table 5.6. The project data consists of eleven project from which four were underestimated (i.e. the difference

†This value is attained after iterating a large number of times and taking the mean, to overcome any form of randomness due to the simulation.

between non default and realized budgets is negative). The Root Mean Squared Error (RMSE) will be our performance measure and is expressed as follows:

$$\text{RMSE}_t = \sqrt{\frac{1}{\tilde{p}} \sum_{\forall p \in P} (\tilde{b'}_p - \hat{b'}_{p,t})^2}, \forall\ t \in T \tag{5.6}$$

Where $\tilde{b'}_p$ represents the realized portfolio cost and $\tilde{p}$ is the number of projects in P. We choose the RMSE over, for instance, the mean absolute error, as it tends to punish more heavily the larger the error gets. This is specifically important in the case where a company should include such risk portfolio VaR estimates in the sales price or when it favors modeling accuracy of the extremes.

Table 5.6: Descriptive statistics for risk estimation versus CPBO risk management models

	Descriptive statistics risk estimation via Delta Normal method (in € million)									
$N = 11$, $N^- = 4$	Mean deficit	St. dev. deficit	Mean residual	St. dev. residual	RMSE	Minimum	Maximum	Sum of deficit	Sum of residual	Sum total
Default	-3.425	3.204	1.668	1.599	4.219	-9.980	2.799	-30.828	3.336	-27.492
Adjusted default	-2.148	3.260	1.283	1.110	2.274	-5.907	3.085	-6.445	10.261	3.816
Non default	-1.501	1.778	0.689	0.769	1.519	-4.083	1.810	-6.004	4.820	-1.184
Delta Normal	-1.210	1.449	2.166	2.199	2.795	-2.234	5.213	-2.420	19.493	17.073
Historical Sim.	-1.899	1.922	1.181	1.226	1.796	-3.257	3.331	-3.797	10.625	6.828
Monte Carlo Sim.	-1.966	1.968	1.089	1.149	1.728	-3.358	3.145	-3.933	9.798	5.865

Before going into the simulation results we will define the two possible outcomes. The first scenario is when the risk budget is underestimated (i.e. an exceedance occurred) which will result in a budget deficit. The second scenario will be referred to as having a budget residual (i.e. the portfolio value was overestimated). With respect to the first scenario we see that the DN method achieves a considerable lower mean value compared to the adjusted- and non default budgets. One simple explanation for this is the fact that this method overestimates all portfolio's. This claim can be substantiated by the observation that it also results in a total sum of all budget deviations being almost 350% greater than the total sum of the adjusted default method. The mean deficit results from both the HS and MCS are very close and lay on the interval between the results of the adjusted- and non default method. The MCS estimate results in the largest mean deficit, which is to be expected since it incorporates the lowest risk portfolio VaR estimates (see Table 5.5). With respect to the second scenario, the overestimated risk portfolios, we see the results behaving more or less the same. The DN method once again results in great overestimations, whereas the HS and MCS are within the interval of the adjusted- and non default method. As was the case for the exceedance scenario, both HS and MCS tend towards the results of the adjusted default budgets.

When comparing the simulated budgets with current risk adjusted default budgets, we can conclude that both the HS and MCS methods outperform the current methodology. Both models achieve a lower RMSE of 21% and 24% for the HS and MCS method resp. The latter can possible be explained by the fact that it makes smarter use of the limited data that's available. If we for example look at the MCS's total sum, we observe that it's approx. 53% greater compared to the adjusted default's total sum but at the same time achieves an approx. 4.5% smaller sum of residuals. From this we conclude that it indeed does a better job at estimating the expected costs rather than just increasing the budgets. The latter statement cannot be made for the HS method. On the contrary, HS and MCS both achieve lower maximum losses and greatly reduced sums of losses (approx 41% and 39% resp. compared to the adjusted default method)

Based on the RMSE scores, we can conclude that the non default model performs best. This might be explained by the insiders' knowledge and sales engineers' experience being incorporated in the budgets. With respect to the simulated budgets, we observe that the MCS risk portfolio VaR estimate performs best, being 13.76% higher compared to the non default's RMSE. We should note however that the test data is somewhat biased, as it represents more over- than underestimated projects. This enhances the default budget's performance since it tends to be the lowest budget in almost all cases.

Finally we focus on the budget coverage ratio. We define the budget coverage ratio as the percentage of total projects whose costs are completely covered by the available budget. From Table 5.7 we conclude that none of the three simulation methods achieved a budget coverage ratio equal or greater than the desired confidence level, which was set to 90%. Nevertheless did all three methods outperform the current adjusted- and non default budgets. Reasons for not achieving the desired ratio might be that costs do not perfectly scale with the sales value (which was our method of scaling the training data), the sample sizes being too small or the model achieving structural underestimations. Whenever the CPBO risk management model would be incorporated using a large amount of training and test data, and it would still result in an underestimation, one could apply the same scaling methods as referred to in Chapter 6.

Table 5.7: Portfolio realization coverage

β = 0.9	# deficit	# residual	% coverage
Default	9	2	18
Adjusted default	3	8	73
Non default	4	7	64
Simulation - DN	2	9	82
Simulation - HS	2	9	82
Simulation - MCS	2	9	82

The simulation outcomes are satisfactory, but have to be taken with a grain of salt due to the sample size. We expect the performance to improve when either more data will be added to the cost database or more project data comes available. With regard to the cost database we expect that the extra data will mainly fall within the interquartile range of the data, since it already incorporates some extreme values. This would in turn result in both the mean of the cost data and the median of the simulated portfolio values shifting to the left, resulting in lower risk portfolio VaR estimates. Furthermore this should improve estimation of correlations, such that expert opinions can be substituted by data driven values. With more project data becoming available we expect the reliability and validity to improve accordingly. As a results one should be able to gather more meaningful (both in term of effectiveness as trustworthiness) information about the CPBO risk management model's performance. Finally we see another opportunity in the form of enriching the cost database. In the current setting, it only holds the negative outcomes of projects (i.e. risk occurred that were underestimated). Including other scenario's e.g. when risks did not occur or when risks occurred but there was sufficient (or surplus) budget available will enrich the database. This should result in better models for the risks factors or could be used to validate occurrence probabilities.

5.4 Incorporating risk budgets in sales prices

The last step is incorporating the implemented CPBO risk management model into the current pricing process of new projects. That is, the risk budgeting step which is a sequel of the CPBO risk management model. There are however some important considerations to be made before bluntly translating a portfolio VaR estimate to a sales value, of which one is the β parameter. Throughout this thesis we've seen a few examples w.r.t. the effect of this parameter and learned that it can result in tremendous overestimations. Such overestimations will in turn greatly increase the sales price and therewith decrease the probability of actually selling the project or tender offer. Figure 5.7 shows where and how in the pricing process to incorporate the CPBO risk management model to price the project's risk portfolio.

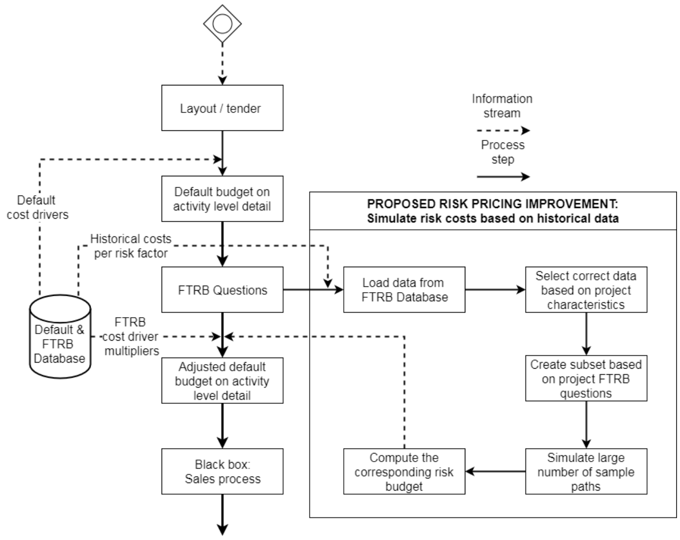

Figure 5.7: Proposed risk budget simulation process step

Assuming that reliable data is available, incorporating the process steps as shown in Figure 5.7 will result in more accurate adjusted default budgets. In the case abundant data is available and backtesting shows the model is accurate, the VaR estimates can directly be incorporated into the budgets. On the other hand, when data is not sufficiently at hand, we've seen in this chapter that one can still acquire more accurate risk adjusted budgets. It is proposed that the case study company starts simulating the risk portfolio VaR estimates, and uses those instead of adjusted budgets. Subsequently, those budgets can be given as input to the sales process, such that they can be improved, e.g. via insiders' knowledge.

In order to be able to deploy the CPBO risk management model the company should however start gathering risk related data on activity level[‡] rather than project level. Since the activity level budgets are critical during project execution, one can't simply disregard this. Whenever such data is available, the CPBO risk management model could be used to model the risk factor's behavior on activity level, thus opening up for risk budgeting inside the case study company. Whenever the company chooses not to incorporate such a change to the data gathering process, it has to use some sort of division scheme to transform the adjusted risk budget (on project level) to budgets on activity level. An example of one such scheme could be via the current budgetting rules, but it should be stated that this will most likely reduce the model's risk estimation accuracy.

[‡]A project is administrated by use of activity numbers. Every activity number is a cost item belonging to a project (e.g. mechanical equipment, electrical installation hours, project management and of course project risk) and basically determines the amount of money can be spend on an activity.

Chapter 6

Future CPBO model extensions

6.1 Optimal risk adjusted discount factor

When taking a closer look at the pricing process of CPBOs one sees that there are more factors at play than the product itself (e.g. R&D costs, market circumstances or competition). One of such factors is the customer, and more specific, the willingness to pay for a certain project. More often than not, large scale projects will be bought via a tender process. In such a process, multiple companies offer their view on the best solution based on the customer's requirements (i.e. their tender). The customer scores all tenders on a set of criteria and the best performing company will get the job and there is no room to negotiate. One obviously very important criterion is the sales price. A major concern of the case study company's sales department is that pricing risks (and more specifically taxing this to the customer) will reduce their probability of selling the project. As a result, the sales department will often apply some form of discount before handing in the tender offer, which creates opportunities for the CPBO risk management model.

In order for the CPBO risk management model to be of added value, we need to assume that the discount factor is either applied to the risk supplement or both the risk supplement and profit margin. We will assume the second for this example. If we define d as the discount factor, this can be expressed as follows:

$$b' = b + (1 - d)(\alpha_\beta(r) \times sv + e \times sv) \tag{6.1}$$

Where sv represents the sales value of the project and e is the profit margin in % of the sales value. Understanding the impact of a certain discount factor, on the project's expected risk coverage* is very difficult, as this is a nonlinear relationship. Using the CPBO risk management model one can create a plot which visualizes this for a specific project, as seen in Figure 6.1. Visualizing trade off functions can enhance the sales engineer's understanding of the implications of discount factors. Furthermore this opens up the possibility for (senior) management to very quickly get an overview of the amount of risk taken, given a certain discount factor.

Let us now continue with the actual minimization problem that computes the optimal discount factor d^*. By defining an optimization problem one can maximize the expected

*As was the case in Chapter 5 we define risk coverage as the ability of the model to cover all risk related costs by use of the risk supplement.

profit ($v(d)$) by changing the discount factor. When the discount factor is being increased, it will increase the probability of winning the tender process, but will also increase the project's expected risk portfolio cost. The function we will use to model the selling probability (γ) merely serves as an example. Let us have a look at the following optimization problem, which maximizes the difference between EBIT and risk coverage, by choosing the discount factor.

Nomenclature:

X	EBIT in euro
Y	Project risk budget in euro
γ	Probability of selling the project
δ	Proportion of risk not covered
d	Discount rate in % of the sales price
r	Risk portfolio
$\alpha_\beta(r)$	Project portfolio VaR in % of the sales price using a β confidence level
e	Profit margin in % of the sales price
$F^{-1}(\alpha)$	Quantile function of the risk portfolio distribution evaluated at α

$$\max_d \quad v(d) = \gamma(1-d)X - (1-\delta^*)Y \tag{6.2}$$

$$\text{s.t.} \quad \gamma = 0.5^{(1-d)^2} \tag{6.3}$$

$$\delta^* = \max\{\delta \in [0,1] : F^{-1}(\delta) \le (1-d)\alpha_\beta(r)\} \tag{6.4}$$

$$d \in [0,1] \tag{6.5}$$

$$X = sv \times e \tag{6.6}$$

$$Y = sv \times \alpha_\beta(r) \tag{6.7}$$

The model works as follows: the objective function, which one wants to maximize, is a combination of the expected EBIT minus the expected risk portfolio cost. The probability of selling the project is captured in Equation 6.3 and can take on values between 0.5 and 1. The last part in the optimization model that needs elaboration is Equation 6.4. This function computes the maximum δ such that the inverse of the distribution underlying the CPBO risk management model at that point is smaller than or equal to the available risk supplement. The risk supplement is linearly scaled with the discount rate.

Solving the previous model is rather straightforward, as we will do for the following theoretical example. Say we have a very risky project with a sales value of € 30m and EBIT of 8%. For this project we identify all risk factors available in the portfolio and compute the portfolio VaR at 4.9%. We then define the optimization problem as:

$$\begin{aligned} \max_d \quad & v(d) = 4,042,548\gamma(1-d) - 2,400,000(1-\delta^*) \\ \text{s.t.} \quad & \gamma = 0.5^{(1-d)^2} \\ & \delta^* = \max\{\delta \in [0,1] : F^{-1}(\delta) \le 4.9\%(1-d)\} \\ & d \in [0,1] \end{aligned}$$

Which results in:

$$d^* = 12,40\% \qquad v(d^*) = €1,121,482 \qquad \gamma^* = 58,75\% \qquad \delta^* = 7,72\%$$

From the objective function plot in Figure 6.2 we see that the discount factor has an important impact on a project's expected profit. When increasing the discount factor, one sees that up to the optimal point, the increase in expected profit outweighs the increment of expected risk costs. After the optimal point however we see that the weight of the expected profit drastically decreases. This makes sense, since we're evaluating a very risky project, which means that the underlying portfolio distribution has fat tails.

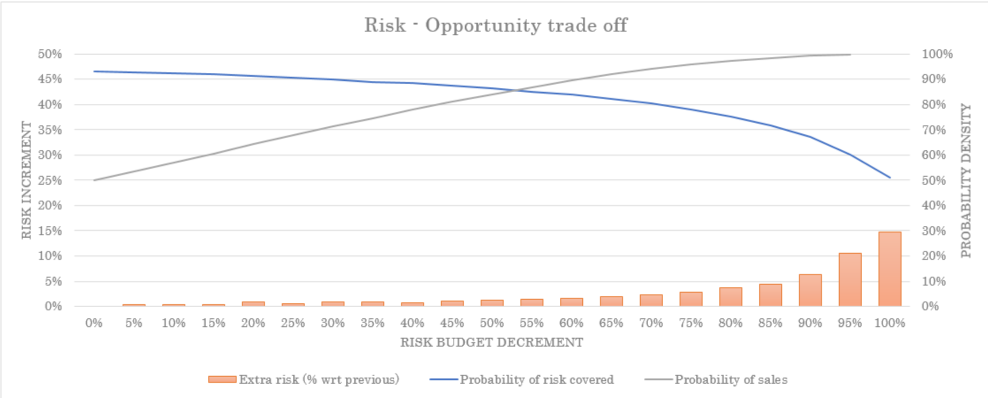

Figure 6.1: Risk-Opportunity trade off

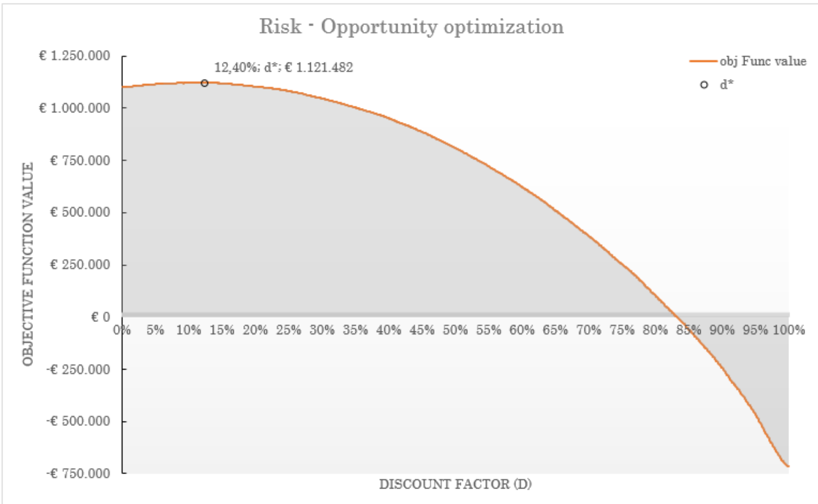

Figure 6.2: Risk-Opportunity objective maximization

6.2 Improving occurrence probabilities by use of classification models

One of the fundamentals of the CPBO risk management model are the occurrence probabilities. Throughout the thesis we've seen that such probabilities can have a large impact on VaR estimates, and therefore proper approximation of such probabilities can be of great importance. In recent publications it has been shown that Artificial Neural Networks can very well be used for this purpose. As an example of a publication that perfectly qualifies for this purpose, one could tend to Syimun et al. (2014). The authors study the use of Artificial Neural Networks for construction projects, estimating the probability of occurrence for certain risk

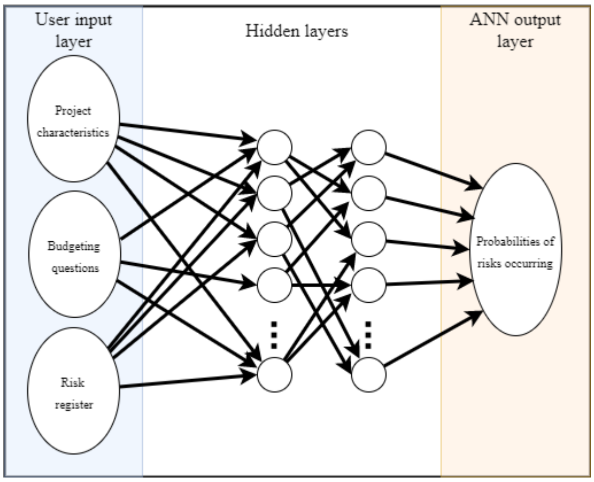

Figure 6.3: Example Artificial Neural Network for occurrence probabilities

factors. Using those estimated project risk probabilities instead of sales engineer's estimation might yield a significant improvement of the CPBO risk management model. Figure 6.3 shows an example Artificial Neural Network including the company's input layers.

6.3 Increasing accuracy with importance sampling

The current CPBO risk management model transforms risk portfolio cost values into an ecdf to compute its VaR. Given the nature of risks, they only occur in small amounts, this method might result in an unbalanced dataset[†]. That is, if for a large number of projects a risk does not occur (i.e. its cost are zero) this will result in zero-inflated data. This data might in turn lead to underestimations of the VaR (specifically in the tail regions). Scientific literature on risk management has found a way to overcome this hurdle by use of importance sampling.

The importance sampling technique increases the probability of all values except zero occurring, in essence overestimating the VaR. Subsequently it corrects for this, achieving a more accurate VaR than before. Importance sampling has been incorporated in many risk management model and has shown to be a great improvement in case of zero-inflated data (Brechmann et al., 2014; Targino et al., 2015) but is mainly applied in combination with extreme value theory (Bardou et al., 2009; Merino and Nyfeler, 2004; Rubino et al., 2009).

6.4 Impact and effect of diversification in CPBO risk management models

In this thesis we've established that, in the case of the case study company, one needs a recommended 24 data points per risk factor in order to properly establish risk portfolio VaR estimates. Within the same analysis, we've hinted on the proposed effect of budget reallocation on the risk portfolio outcomes. We argued that the effect shown in this thesis exists because risk factor costs are of same magnitude (i.e. their costs, if realized, are approximately equal). It will be interesting to study the VaR estimate's behavior when either further increasing the number of risk factors or the dimension of costs.

[†]Remember that approx. 33% of the portfolios in the case study's simulation had a value of 0

Chapter 7

Conclusion and recommendations

Conclusion

With this chapter we will end the master thesis and conclude on our research findings. Throughout the thesis we've learned that CPBO risk management models can be both very powerful as very tricky. Building a model is not only situationally dependent but also depends on the underlying data structures (i.e. risk distributions) and intended usage. We started this thesis with the question: "*How can Project Based Complex Product System Organizations compose project specific risk budgets that incorporate the underlying characteristics and risk factors?*" and divided this to a number of sub-research questions to guide us in finding the answer. We started with the very practical question: is there such thing as a standard risk profile for Complex Product System projects. It turns out that, though researchers have tried to formulate it, it is too much context depending. There are however multiple frameworks that could lead a company to constructing a risk profile, based on amongst other things the characteristics of their projects.

Subsequently we've conducted a literature review to answer the second sub research question: "*What are the most applied risk budgeting methods in financial literature, and which can be applied in the context of CPBOs?*". We've seen that a tremendous amount of research attention has been devoted towards risk management models, but not so much in the context of CPBOs. Popular risk management models are the Delta Normal, Historical simulation and Monte Carlo simulation methods. In more recent publications we have seen however that many authors nowadays focus on more advanced versions of the latter models, which overcome the main disadvantages of the models. In the case of delta normal methods, research has not progressed too much. This can mainly be explained by the fact that the assumptions underlying this model do not allow for much room to improve. Both simulation methods have gotten a lot of research attention, mainly focusing on improving the estimation quality and incorporating extreme value theorems. We tested all three methods in the context of CPBOs, to answer the third research questions:"*Does the method choice influence the conclusion on the preferred risk management model?*". From those tests we found that, based on the Basel Committee on Banking Supervision (1996) backtest, both simulation methods qualify for use in CPBO risk management models, whereas the Delta Normal method does not. Based on the backtesting performance we've chosen Monte Carlo simulation to be the risk management model of choice.

Whilst building and initializing the model we've learned a great deal about it's underlying structures and assumptions. Using this acquired knowledge we answered the fourth sub research question: "*What is the minimum number of data required to accurately model a risk factor?*". We've seen that the performance of the CPBO risk management model can be very dependent on a number of parameters, of which the most important ones were the confidence level β and the number of underlying data points. By backtesting the risk portfolio VaR estimations of a portfolio holding five risk factors, we've established the recommended number of data points per risk factor the case study company needs in order to properly model a risk factor, which turned out to be 24. Subsequently we've briefly shown the impact of the β parameter on the estimation of the risk portfolio VaR. By explaining the relationship between this parameter and the underlying data we showed the (possibly negative) effect of bluntly choosing high β values.

Subsequently we've backtested three CPBO risk management models using case study data. Every model used a different risk management method to estimate the risk portfolio VaR, which was either the Delta Normal, Historical Simulation or Monte Carlo Simulation method. It is by use of this case study that we answer the fifth sub research question: "*Is a risk management model able to accurately model risk profiles of complex projects?*". Even though the sample sizes of the available training and testing data was very small, we see some promising results. First off, we've tested all three risk portfolio VaR estimation methods to conclude that the performance is in line with the theoretical backtest. More specific, MCS outperformed both HS and the DN method, based on the Root Mean Squared Error (RMSE) of the estimated and realized project results. Hereafter we showed that the CPBO risk management model using MCS was able to find more accurate risk portfolio VaR estimates than the current adjusted budgets. That is, the model achieved a 24% lower RMSE compared to the adjusted budget method. At the same time it decreased the sum of deficits by 39% whilst increasing the total profit of the test set by 53%. In addition to this, we've showed that none of the three methods achieved the desired confidence level of 90% as all methods only achieved 82%. Unfortunately, due to the small sample size, we're unable to conclude whether this is caused by the model or the data. Lastly we discussed how to translate risk portfolio VaR estimates to sales price markups, without inflating prices. Since the available data is limited we proposed to use the CPBO risk management's budget as input for the sales phase. It is expected that, when the initial budget is more accurate, the accuracy of the non default budget will also increase. We concluded that, in order for the case study company to deploy the new model, risk costs should be gathered in full and on activity level detail rather than project level. Whenever more data becomes available it, theoretically will be possible to use the CPBO risk management budget instead of the non default. Nevertheless, manual input of insiders' knowledge might always be a beneficial addition.

In addition to the CPBO risk management model we've addressed some extensions based on practical situations encountered within the case study company. First we've addressed how to optimize the trade off function between hedging against risk costs on the one hand and increasing the probability of selling a project on the other by giving a discount. Secondly we've briefly elaborated on improving the estimation of occurrence probabilities, which has important implications for the CPBO risk management model. Based on a recent study we've see that Artificial Neural Networks can very accurately estimate such probabilities, which might greatly improve the model's accuracy. Following this extension we've seen another which focuses on improving the CPBO risk management model's accuracy, as we argued that it will be worthwhile to incorporate importance sampling technique into the model. Finally the last proposed extension of the model is to investigate the effect of residual budget reallocation on portfolio's with costs of different magnitudes. It would be interesting to see whether the same effects (i.e. lower exceedances, more narrow confidence intervals and lower minimum required data points) still hold when there is less residual budget to re-allocate.

We end this conclusion with a word of caution regarding the research outcomes. The availability of quantitative data has been an issue throughout the thesis. As such, some backtesting procedures had to be conducted based on synthetic data. Even though we've done our utmost best to account for this wherever possible, the outcomes of this thesis might be subjected to inaccuracy.

Recommendations

The concepts and models adopted in the CPBO risk management model are all highly data dependent. Whilst incorporating the model within the case study company this happened to be a problem. Though a lot of data was thought to be available, it actually was not or it had been collected in a mismatching form. On top of this, there was insufficient data available to transform the mismatching data to a usable format. That is, without making a tremendous amount of assumptions that would have impacted the model's validity. Therefore the decision has been made to work with particularly small sample sizes, which results in the recommendations mainly revolving around the case study company's risk related data.

1. Improve risk registration
The risk registers form one of the pillars of any risk management activity within the company. That is, they are the primary source with respect to the identification of a risk factor, including its estimated occurrence probability and severity. Therefore in order for a company to incorporate a risk management system it's of great importance to fully control this process. It is recommended that the risk registers are checked for completeness, which is twofold. First it should be checked if a risk register exists and second it should be checked whether it aligns with the budgeting questions from the calculation tool. Furthermore we recommend, though it's known to be very difficult, to standardize the risk registers. One possible way might be to split the risk register in mandatory and non-compulsory fields. The mandatory fields should align with the budgeting questions answered in the sales phase. The non-compulsory fields can be used by the sales engineer to incorporate any insiders' knowledge into the risk budget. The previous should open up the possibility to create feedback between the sales engineer's estimations, the budgeting rules and actual realizations, whilst on the other hand safeguarding against incorrect risk information. Lastly there should be a centralized location to store all risk registers in order to simplify the data gathering process.

2. Redesign the process of data administration
Based on the functional specification of the risk management model, it is recommended to change which data is being administrated, and how it is. Changing which data will be gathered is important such that the risk management model can achieve the required specifications, of which the main goal is to forecast the monetary value of a tending project's risk portfolio. In order to accomplish this goal, the company should start administrating the full costs of a risk factor, rather than the costs exceeding a certain budget, as is currently the case. This recommendation should safeguard both the accuracy and validity of the model. It's recommended to require the project manager to specify if a budget mutation has a relation with

an (un)identified risk factor. When such a change is made in the budget mutation process, one will create a constant flow of risk cost and cash flow related information, decreasing the likelihood that such data goes missing. Furthermore this opens up the possibility to backtrack expenditures when desired. In order to minimize the amount of administrative burden for the project managers, it's best to combine this with a standardized risk register. Additionally one can choose to make this a mandatory action, such that in the case no standardized risk caused the mutation, a user input is required. This recommendation can fairly easily be implemented in the current process of updating the latest estimates of project budgets. Since the project managers already submit an overview of the revised budgets from time to time, this could be seamlessly incorporated and verified by the responsible financial controller.

Another important addition to this recommendation is the fact that data should be gathered at a lower level. That is, in the current practice the company gathers risk related data on project level. On the contrary they compute budgets on activity number level. In order to avoid any complications when deploying the CPBO risk management model, risk related data should be gathered on the same level. Whenever the company chooses not to, they will inevitably need to use some division scheme in order to translate simulated portfolio costs to individual activity levels. This will most likely lead to a higher RMSE and decreased modeling accuracy.

Finally it makes sense to think about what project specific data one needs to gather to allow for classification models in the future. The company acknowledges the need to improve the estimation of occurrence probabilities. In order to build an extensive database for future reference, the company should start thinking about which data to store today. Important factors to store have been identified with the company throughout the thesis. Such data are e.g. the project's characteristics (e.g. industry, customer, country, system layout and time to tender deadline), the risk registers, the fact if risk have been mitigated and, if applicable, the final (and complete) overview of costs of realized risks.

3. Guard for data quality and completeness
Given the internal accounting method, it's fairly easy for a project manager to keep certain costs "under the radar". As a result it's very difficult to judge, from a data perspective, if it's representing the entire costs of a risk factor. The company should therefore be careful during the first stages of incorporating a risk management model and continue backtesting the model' s reliability and validity. The implementation of this recommendation could go hand in hand with the prior. That is, whenever the budget mutations have been submitted to the financial controller, he or she is responsible for verifying quality and completeness. Since the controller is familiar with the project, this should lead to both valid and reliable risk data.

4. Instigate feedback initiatives
In theory, every project should have a so called closing down meeting. During such a meeting the project teams would discuss on (large) failures and successes and create lessons learned. The feedback from such an initiative should lead to continuous improvements in pricing-, sales-, execution- and other business processes, during all stages of a project. Unfortunately, given the time pressure, such meetings are a rarity nowadays. The absence of a complete feedback loop negatively influences the ability to improve the pricing accuracy of risk factors. As such it's recommended that the company should restore the last step of their primary process to ensure continuous improvement.

5. *Copy best practices from one business unit to the others*
From the pricing department's perspective it can be concluded that one business unit in particular is performing very well with regards to both data gathering as continuous improvement initiatives. Moreover, the continuous improvement tool that is in development within this business unit is particularly well suited to incorporate accumulation of the data required by the risk management model. More specific, the current tool as developed does identify the entire cost of risks that have occurred during project executing, but it's not linked to the (standardize) risk register. It's recommended that the pricing department works together with this business unit to expand their continuous improvement tool such that it automatically stores all required data (i.e. realized risk factor costs on activity level detail). It's expected that this in turn can lead to a constant flow of information with regards to pricing risk factors. When the tool turns out to be successful, the company should distribute the continuous improvement tool over all business units.

Conclusions on literature gaps

In the beginning of this thesis we conducted a literature review which led to a number of questions which were unanswered within the current scientific literature. Throughout this thesis we've discussed* three of such questions, which we will now concisely substantiate on. We argued that it would be interesting to see whether VaR methods could be incorporated within CPBO risk budgeting. Moreover, VaR methods had the potential to create transparent pricing processes. We've seen that indeed VaR methods can be included and, in the case of MCS, can result in accurate estimates. Moreover, since MCS VaR computes a price markup per risk portfolio, this is indeed a transparent process which will lead to constant risk markups rather than budgets that change due to the practitioner's preference.

Secondly there is currently no scientific literature available which identifies common distribution functions that model both PBO or CPBO risk factors well. In this thesis we've tested whether the CPBO risk factor distributions could be modeled by the same distributions often found in financial risk management literature. It turns out that four out of six of the risk factors we've tested followed an Exponential distribution and two followed an Inverse Gaussian distribution. Given the small sample sizes of our training data however, we can't conclude those distributions were good estimators.

Finally we investigated the effect of incorporating copulas into the CPBO risk management model. We showed that modeling correlations had a negative effect on the number of data points required to accurately estimate the risk portfolio VaR (i.e. the minimum increased). The explanation comes from the fact that positive correlation results in less residual budgets, such that its harder to compensate for exceptionally high costs or other risk factors occurring.

Lastly we would like to end by generalizing the CPBO risk management model. This model has shown to be capable of estimating risk portfolio VaRs with decent accuracy whilst having very limited data. Therefore, it might just as well work in any context where data isn't abundant. Some examples of such industries are startups, small business or infrastructural mega projects.

*We refer to discussing rather than answering the literature gaps, since all analysis was conducted using limited sample sizes.

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Appendix A

Data preparation case study

The goal of this appendix is to guide the reader in how to prepare the model's underlying data. To accomplish this, the context of the case study company is described here, such that future practitioners might relate to this for their own application.

To ensure that the mathematical model accurately represents the real world situation, extra care was given to preparing the input data. Initially it was argued by the company that the available data, in the form of predicted estimates, could be used as model input. That is, the sales engineer's estimations of possible (and most likely probable) risk factors for a given project and their estimated adhering costs. One of the most important downsides of using estimations as model input are the fact that they are most likely inaccurate, subjective and scenario dependent. Therefore, in order to increase the model's validity, the decision has been made not to use this data. Instead, data on realized projects should be gathered, which brought some challenges. Before going into those, some background information regarding the company's (internal) accounting methods should be provided.

Internal accounting process

A project is administrated by use of activity numbers. Every activity number is a cost item belonging to that project (e.g. mechanical equipment, electrical installation hours, project management and of course project risk, also referred to as contingency). In the sales phase, sales engineers will determine all costs related to a project. Subsequently after the project has been sold, the accumulation of all predetermined costs is called the "as sold cost amount". During the execution phase, project managers will update those cost estimations per activity number on a periodical basis (e.g. every month), which is called the "Latest Estimate (LE) cost amount". All those transactions are entered into the ERP system, under the so called LE ledger*. The mutations are either positive or negative and connections between budget shifts are not specified. Note that not specifying such connections is specifically important since project managers have the possibility to re-allocate budgets thus potentially creating arbitrage situations, which result in incorrect data. Finally there is a database called the feedback loop (FBL), which is used for periodic reporting. More specific, this database holds a concise overview of a project's financial KPIs. The FBL shows among others: the as sold-, LE and (eventually) actual cost amounts per period. The data is imported from the ERP

*This is an internal ledger, used for project management, not for financial reporting system and accumulates all data specified per activity number. Note that the actual costs are simply all costs made up to project completion.

Data related issues

With the previous in mind, lets now focus on the challenges of using actual over estimated data for the case study company:

1. The only (financial) data available regarding budgets (gathered in a routinely matter), is the deviation between the accumulated as sold cost amount and accumulated actual cost amount†;
2. A small database exists which stored cost overruns for a specific industry within the company. This data however, was not fit for estimation risk budgets of new projects due to a number of reasons;
3. In theory, every budget transaction from the contingency budget to any activity number should be textually specified in a LE mutation. However, in practice, such specifications were barely available. Budgets were reallocated without specification, such that it is impossible to determine wherefore it has been used post completion;
4. No overview exists which specifies the occurred risk factors per project;
5. Since the ERP system accumulates budget mutations per activity number over some time horizon, it is impossible to reconstruct the cash flows by hand.

As a result of this, there is no available data for the case study company that satisfies all quality requirements. Moreover, since there are no records of budget reallocations nor specifications of budget decrements, there is no possible way of reconstructing the realized costs due to some risk factor in a robust way. A small relief came from a process improvement employee which gathered -and investigated- data for a business unit within the case study company. Since this data is well prepared and satisfies the quality requirements, it will be used as input for the model. The only downside are the small sample sizes, which unfortunately can't be overcome. Moreover, this data represents the losses on top of the risk budget. As such it can't be used to estimate the initial risk budgets. The main reasons for this is the fact that we simply can't trace back what budget cash flows were allocated to certain risk factors (see point 3). Therefore this database represents just half the actual costs of the risk factor.

The previous substantiation should show practitioners that it might be more challenging to find correct data than expected. First one needs to know exactly what is to be simulated, from which one can reason the required data. It is of great importance to double check both if your data is indeed fully representing the objective (i.e. need more sources to complete the picture?) and if this data is indeed trustworthy (or are there any arbitrage possibilities). If not both these requirements are met, any mathematical model build based on such data will be subjective to inaccuracy and modeling errors.

For the case study company we can conclude that the available dataset, under some modeling conditions, satisfies our conditions. By analyzing the data we were able to clean it (i.e. delete noise). Hereafter we've scaled the remaining data using appropriate scales (e.g. via sales price). Scaling was important as else the data did not allow for fair comparison. As

†The difference between the as sold cost amount and actual cost amount is called the deviation and is internally associated with budget over- and underruns. However, this deviations solely tells whether sufficient budgets were calculated, not whether or which risks occurred.

a result the data's nature (cost preceding the allocated budget rather than the entire costs of a risk factor) we are only able to simulate the expected budget shortage rather than the expected costs of the entire risk portfolio. When comparing performance we'll look at the difference between the realized and non default budget + simulates shortages. Obviously, this is not the implementation as intended when constructing the CPBO risk management model.

Appendix B

Risk factors defined throughout literature

The following table of elements has been adapted from Bosch-Rekveldt et al. (2011):

Elements defined	Mentioned in how many interviews?	Mentioned in how many cases?
Experience with technology	17	6
Variety of stakeholders perspectives	16	6
Number of different project management methods and tools	15	6
Resource and skills availability	15	6
Number of stakeholders	14	6
Contract types	13	6
Uncertainties in scope	13	5
Experience with parties involved	13	5
Interrelations between technical processes	12	6
Newness of technology (world-wide)	12	6
Trust in contractor	11	6
HSSE issues	11	5
Cooperation JV partner	11	5
Trust in project team	10	5
Political influence	10	5
Company internal support	9	4
Number of different norms and standards	8	5
Number of different nationalities	8	5
Dependencies on other stakeholders	8	5
Level of competition	6	5
Environmental risks	6	4
Technical risks	6	4
Variety of tasks	6	4
Uncertainty in technical methods	6	3
Number of different languages	6	3
Interference with existing site	6	3
Number of tasks	6	3
Goal alignment	5	4
Number of locations	5	4
Scope largeness	5	3
Size of site area	5	3
Internal strategic pressure	5	3
Dependencies between tasks	4	4
Size of project team	4	4
Quality requirements	4	3
Number of financial resources	4	3
Project drive	4	2
Weather conditions	3	3
Remoteness of location	3	3
Organizational risks	3	2
Size in CAPEX	3	2
Number of different disciplines	3	2
Overlapping office hours	3	2
Experience in the country	3	2
Size in engineering hours	2	2
Union power	2	2
Required local content	2	1
Clarity of goals	1	1
Stability project environment	1	1

Appendix C

Algorithms VaR methods

Throughout this appendix we'll treat boldfaced variables $\boldsymbol{x}$ as matrices and normal font variables x as either a vector or scalar.

Delta-Normal method

Let $F_k \sim N(\mu_k,\ \sigma_k)$ be a normal distribution fitted to the values in column k of matrix **data**, and therefore F_k^{-1} be the inverse normal distribution. Then the β-quantile of such a distribution in the right tail is equal to $F_k^{-1}(\beta)$.

input:

data:= $n \times \tilde{k}$ matrix holding n data points for all k risk factors in the portfolio
r:= $1 \times \tilde{k}$ binary vector identifying risk factors in project portfolio
β:= confidence parameter

Result:

Value-at-Risk using the Delta-Normal method

Algorithm;

$$m_k \leftarrow \begin{cases} F_k^{-1}(\beta) & \text{if } r(k) = 1 \\ 0 & \text{Otherwise} \end{cases} \quad \forall\ k \in K$$

$$\boldsymbol{\rho} \leftarrow \begin{cases} \text{Portfolio correlation matrix of returns} & \text{if } \boldsymbol{\rho} \text{ is PSD} \\ \text{Identity matrix} & \text{Otherwise} \end{cases}$$

$$\text{VaR} = \sqrt{\boldsymbol{m} \cdot \boldsymbol{\rho} \cdot \boldsymbol{m}^T}$$

Algorithm 1: Algorithm delta-normal method

Historical simulation

Let $\hat{F}_y$ be the ecdf fitted to the data in vector y, and therefore $\hat{F}_y^{-1}$ be its inverse cdf. Then the β-quantile of such a distribution in the right tail is equal to $\hat{F}_y^{-1}(\beta)$. Furthermore let $\mathbf{data(\boldsymbol{u})}$ represent the values in **data** on the positions of $\boldsymbol{u}$, where the first column of $\boldsymbol{u}$ represents the positions in the first column of matrix **data** randomly selected by the simulation. Finally `.*` represents the pointwise multiplication operation.

input:

data:= n × $\tilde{k}$ matrix holding n data points for all k risk factors in the portfolio

b:= $\tilde{k}$ × 1 vector with Bernoulli trials representing the probability of occurrence for every risk factor. If unidentified in portfolio this is a Bernoulli trial (n = 1, p = 0)

β:= confidence parameter

simSize:= the number of simulation iterations

type:= binary variable identifying whether data input is correlated, 1 if true

Result:

Value-at-Risk using historical simulation method

Algorithm;

$$\boldsymbol{u} \leftarrow \begin{cases} \text{1 x simSize random number vector} & \text{if type == 1} \\ \tilde{k} \times \text{simSize random number matrix} & \text{Otherwise} \end{cases}$$

$\boldsymbol{b'} \leftarrow \tilde{k} \times$ simSize random Bernoulli trials using b

$\boldsymbol{y} = \mathbf{data(\boldsymbol{u})}$.* $\boldsymbol{b'}$

y = sum($\boldsymbol{y}$,2)

VaR $\leftarrow \hat{F}_y^{-1}(\beta)$

Algorithm 2: Algorithm Historical simulation method

Monte Carlo simulation

Let $\hat{F}_y$ be the ecdf fitted to the data in vector y, and therefore $\hat{F}_y^{-1}$ be its inverse cdf. Then the β-quantile of such a distribution in the right tail is equal to $\hat{F}_y^{-1}(\beta)$. Furthermore let $.*$ represents the pointwise multiplication operation.

input:

data:= $n \times \tilde{k}$ matrix holding n data points for all k risk factors in the portfolio

b:= $\tilde{k} \times 1$ vector with Bernoulli trials representing the probability of occurrence for every risk factor. If unidentified in portfolio this is a Bernoulli trial (n = 1, p = 0)

β:= confidence parameter

simSize:= the number of simulation iterations

type:= binary variable identifying whether data input is correlated, 1 if true

distSet:= vector holding names of optimal distributions for every risk factor

Result:

Value-at-Risk using Monte Carlo simulation method

Algorithm;

$$\boldsymbol{\rho} \leftarrow \begin{cases} \text{correlation matrix of } \textbf{data} & \text{if it's PSD and type == 1} \\ \text{Identity matrix} & \text{Otherwise} \end{cases}$$

y = copula(**data**, $\boldsymbol{\rho}$, b, simSize, type, distSet)

VaR ← $\hat{F}_y^{-1}(\beta)$

Algorithm 3: Algorithm Monte Carlo simulation method

Copula function

Let F^n represents the standard cumulative normal distribution and $\mathcal{N}(0, \mathbf{\Sigma})$ represent a multivariate normal distribution with zero vector mean and covariance matrix $\mathbf{\Sigma}$. Furthermore let u^i represent the i^{th} column of matrix $\boldsymbol{u}$. Finally $.*$ represents the pointwise multiplication operation.

input:

data:= $n \times \tilde{k}$ matrix holding n data points for all k risk factors in the portfolio

$\boldsymbol{\rho}$:= correlation matrix corresponding to **data** (All-ones matrix if type was 0)

b:= $\tilde{k} \times 1$ vector with Bernoulli trials representing the probability of occurrence for every risk factor. If unidentified in portfolio this is a Bernoulli trial (n = 1, p = 0)

simSize:= the number of simulation iterations

distSet:= vector holding names of optimal distributions for every risk factor

Result:

simsize x 1 vector of simulated correlated risk portfolio values

Algorithm;

$\boldsymbol{r} \leftarrow$ simSize x $\tilde{k}$ random numbers from $\mathcal{N}(0, \boldsymbol{\rho})$

$\boldsymbol{u} \leftarrow F^n(\boldsymbol{r})$

for $i = 1 : \tilde{k}$ **do**

    $f_i \leftarrow$ fit pdf distSet(i) using MLE to values of column i in matrix **data**

    bin $\leftarrow$ generate simSize random numbers from $b(i)$

    $y^i \leftarrow F_i^{-1}(u^i).*$ bin

end

$y = \text{sum}(\boldsymbol{y}, 2)$ return $[y]$

Algorithm 4: Algorithm copula function

function bestFitDist

input:

data := n x 1 vector holding n data points for a specific risk factor

suggestedDistSet := vector holding names of all suggested distributions

Result:

Best fitting probability density function for the provided data vector

Algorithm;

for $i = 1 : \tilde{k}$ **do**

    Fit all distributions in suggestedDistSet to the data vector **data** using MLE;

    Compute AICc and Δ_i;

    Sort AICc lowest to highest;

    foreach *distribution with* $\Delta_i \leq 2$ **do**

        [p, h] = call adjustedKsTest;

        if h ~= 0 **then**

            try next distribution

        else

            $f_i \leftarrow$ first distribution with h == 0 and $\Delta_i \leq 2$;

            exit for loop

        end

    end

end

return $[f_i, p]$

Algorithm 5: Algorithm bestFitDist function

function adjustedKsTest

Let F be the cumulative distribution function given as input to this function. Then F^{-1} is the inverse of that cdf. This Kolmogorov-Smirnov test will analyze the following hypothesis:
H_0: The data vector follows the requested distribution.
H_a: The data vector does not follow the requested distribution.

input:

  data := n x 1 vector holding n data points for a specific risk factor
  distName := the name of the pdf to test
  distParam := vector holding the parameters of the pdf to test
  simSize:= the number of simulation iterations
  α:= significance parameter (default $\alpha = 0.05$)

Result:

p-value and hypothesis test result of a simulated KS-test

Algorithm
D_{init} ← perform KS-test on F and **data** and store the KS test statistic
while $i \leq simSize$ **do**
  r ← generate n uniform random numbers
  $x_{syn} \leftarrow F_i^{-1}(1-r)$
  F_{syn} ← fit distribution F using MLE to the data of x_{syn}
  $D_{syn,i}$ ← perform KS-test on F_{syn} and x_{syn} and store the KS test statistic
end
$p = \frac{1}{simSize}\sum_{j=1}^{simSize}\delta_j$, with $\delta_j = \begin{cases} 1 & D_{syn,j} > D_{init} \\ 0 & \text{otherwise} \end{cases}$;
if $p > \alpha$ **then**
  Succeeded to reject H_a;
  $h \leftarrow 0$;
else
  Failed to reject H_a;
  $h \leftarrow 1$;
end
return $[h, p]$

Algorithm 6: Algorithm adjusted KS-test, based on the application to power-law distributions of Clauset et al. (2009)

Minimum number of observations

Let $\hat{F}_y$ be the ecdf fitted to the data in vector y, and therefore $\hat{F}_y^{-1}$ be its inverse cdf. Then the β-quantile of such a distribution in the right tail is equal to $\hat{F}_y^{-1}(\beta)$. Finally let data^i represent the i^{th} column of matrix **data**.

input:

b:= $\tilde{k}\times 1$ vector with Bernoulli trials representing the probability of occurrence for every risk factor. If unidentified in portfolio this is a Bernoulli trial (n = 1, p = 0)

β:= confidence parameter

simSize:= the number of simulation iterations

type:= binary variable identifying whether data input is correlated, 1 if true

distSet:= vector holding names of optimal distributions and it's parameters for every risk factor

Result:

Required minimum number of observations to compute VaR including confidence level

Algorithm;
for $n = 5 : 5 : 100$ **do**
    for $j = 1 : simSize$ **do**
        $r \leftarrow$ generate $n \times \tilde{k}$ (correlated) uniform random numbers
        for $i = 1 : \tilde{k}$ **do**
            $f_i \leftarrow$ fit pdf distSet(i) using its parameters
            data^i = $F_i^{-1}(1-r)$
        end
        $$\boldsymbol{\rho} \leftarrow \begin{cases} \text{correlation matrix of } \textbf{data} & \text{if it's PSD and type} == 1 \\ \text{Identity matrix} & \text{Otherwise} \end{cases}$$
        y = copula(**data**, $\boldsymbol{\rho}$, b, simSize, type, distSet)
        VaR $\leftarrow$ $\hat{F}_y^{-1}(\beta)$
        $u \leftarrow$ generate 250 (correlated) uniform random numbers
        for $i = 1 : \tilde{k}$ **do**
            data_backtest^i = $F_i^{-1}(1-u)$
        end
        Compute descriptive statistics
    end
    Compute means of descriptive statistics
end
$n^* = \{\min n \mid X(n) \le 17\}$
$n^*_{ci} = \{\min n \mid X(n) \le 17, CI_{75} \le 17\}$

Algorithm 7: Algorithm required minimum number of observations

Appendix D

Figures statistical distribution fit risk factors

Data for risk factor C_1

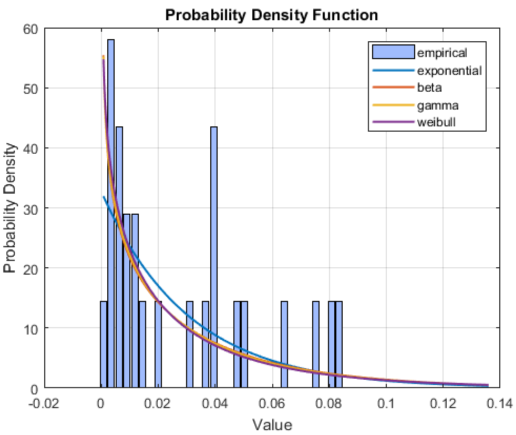

Figure D.1: Top 5 Probability density function fits for C_1 based on AICc

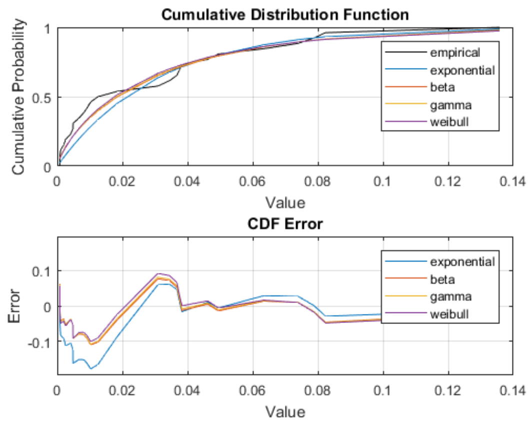

Figure D.2: Top 5 Cumulative distribution function fits for C_1 based on AICc

Risk factor 1

N = 26	Likelihood estimations					Parameter estimates					
Distribution Name	NlogL	BIC	AIC	AICc	Δ_i	Name	Value	Name	Value	Name	Value
Exponential	-64,71	-126,17	-127,43	-127,26	0,00	Mu	0,0305				
Beta	-65,74	-124,97	-127,49	-126,97	0,29	alpha	0,7140	beta	22,7269		
Gamma	-65,69	-124,86	-127,38	-126,85	0,40	Shape	0,7285	Scale	0,0419		
Weibull	-65,66	-124,80	-127,31	-126,79	0,47	Scale	0,0274	Shape	0,8135		
Lognormal	-64,68	-122,84	-125,35	-124,83	2,43	Log location	-4,3140	Log scale	1,5323		
Generalized pareto	-65,78	-121,79	-125,56	-124,47	2,79	Shape	0,3221	Scale	0,0212	Threshold	0,0008
Inverse gaussian	-62,81	-119,10	-121,62	-121,10	6,16	Scale	0,0305	Shape	0,0051		
Generalized extreme value	-62,52	-115,26	-119,04	-117,94	9,31	Shape	1,1367	Scale	0,0104	Location	0,0076
Normal	-51,70	-96,89	-99,40	-98,88	28,38	Location	0,0305	Scale	0,0338		
t-locationscale	-52,72	-95,67	-99,45	-98,36	28,90	Location	0,0238	Scale	0,0250	DoF	4,3237
Extreme value	-44,95	-83,38	-85,89	-85,37	41,89	Location	0,0490	Scale	0,0421		

Data for risk factor C_2

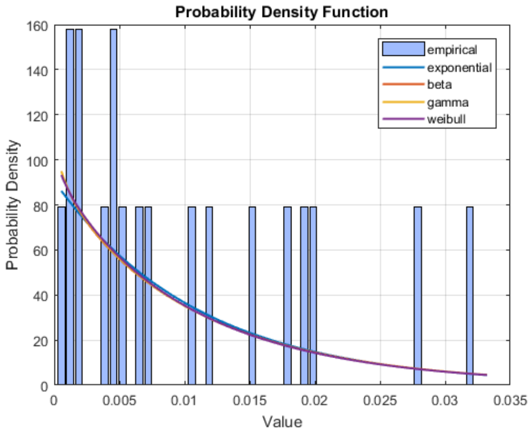

Figure D.3: Top 5 Probability density function fits for C_2 based on AICc

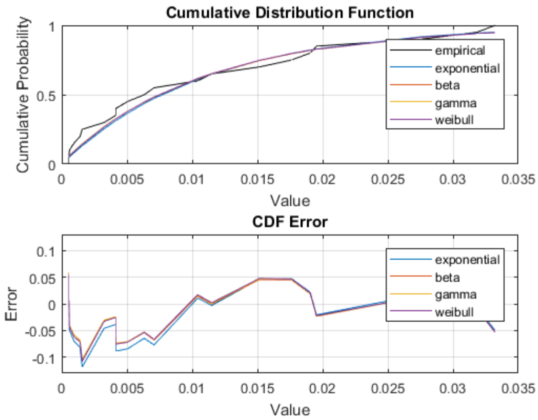

Figure D.4: Top 5 Cumulative distribution function fits for C_2 based on AICc

Risk factor 2

N = 20	Likelihood estimations					Parameter estimates					
Distribution Name	NlogL	BIC	AIC	AICc	Δ_i	Name	Value	Name	Value	Name	Value
Exponential	-70,13	-137,27	-138,26	-138,04	0,00	Mu	0,0110				
Beta	-70,18	-134,38	-136,37	-135,66	2,38	alpha	0,9266	beta	83,1219		
Gamma	-70,16	-134,34	-136,33	-135,62	2,42	Shape	0,9330	Scale	0,0118		
Weibull	-70,15	-134,31	-136,31	-135,60	2,44	Scale	0,0109	Shape	0,9644		
Generalized pareto	-71,17	-133,35	-136,33	-134,83	3,21	Shape	-0,0586	Scale	0,0111	Threshold	0,0006
Lognormal	-69,26	-132,53	-134,52	-133,82	4,23	Log location	-5,1304	Log scale	1,3145		
Inverse gaussian	-68,55	-131,11	-133,10	-132,39	5,65	Scale	0,0110	Shape	0,0034		
Generalized extreme value	-67,82	-126,64	-129,63	-128,13	9,91	Shape	0,7574	Scale	0,0047	Location	0,0041
Normal	-63,09	-120,19	-122,18	-121,47	16,57	Location	0,0110	Scale	0,0106		
t-locationscale	-63,10	-117,22	-120,21	-118,71	19,34	Location	0,0110	Scale	0,0103	DoF	4959233
Extreme value	-59,84	-113,68	-115,67	-114,96	23,08	Location	0,0166	Scale	0,0113		

Data for risk factor C_3

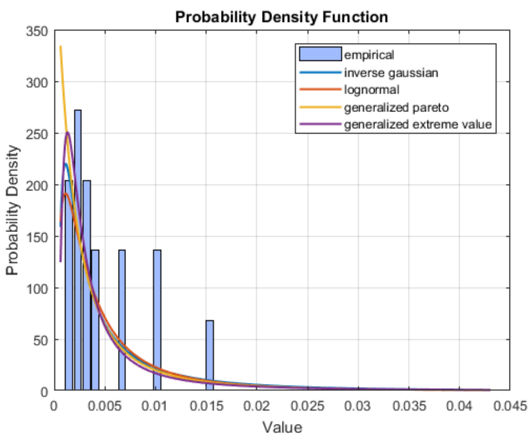

Figure D.5: Top 5 Probability density function fits for C_3 based on AICc

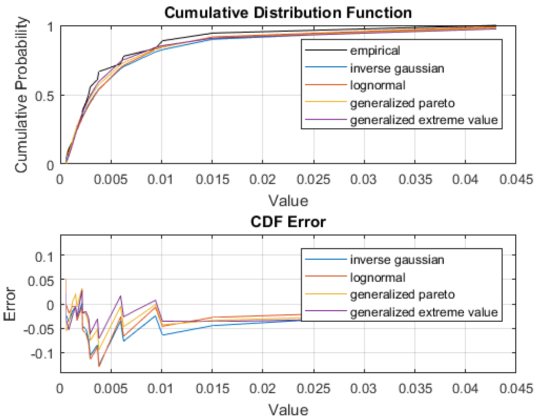

Figure D.6: Top 5 Cumulative distribution function fits for C_3 based on AICc

Risk factor 3

N = 18	Likelihood estimations					Parameter estimates					
Distribution Name	NlogL	BIC	AIC	AICc	Δ_i	Name	Value	Name	Value	Name	Value
Inverse gaussian	-76,37	-146,96	-148,74	-147,94	0,00	Scale	0,0064	Shape	0,0034		
Lognormal	-75,87	-145,97	-147,75	-146,95	1,00	Log location	-5,6683	Log scale	1,0639		
Generalized pareto	-77,26	-145,84	-148,52	-146,80	1,14	Shape	0,5203	Scale	0,0030	Threshold	0,0006
Generalized extreme value	-76,43	-144,20	-146,87	-145,15	2,79	Shape	0,7935	Scale	0,0019	Location	0,0022
Exponential	-72,85	-142,81	-143,70	-143,45	4,49	Mu	0,0064				
Weibull	-73,16	-140,53	-142,31	-141,51	6,43	Scale	0,0059	Shape	0,8836		
Gamma	-72,88	-139,97	-141,75	-140,95	6,99	Shape	0,9366	Scale	0,0069		
Beta	-72,81	-139,84	-141,62	-140,82	7,12	alpha	0,9268	beta	142,7165		
t-locationscale	-71,23	-133,80	-136,47	-134,75	13,19	Location	0,0024	Scale	0,0013	DoF	0,8878
Normal	-58,03	-110,27	-112,05	-111,25	36,69	Location	0,0064	Scale	0,0099		
Extreme value	-50,46	-95,14	-96,92	-96,12	51,82	Location	0,0123	Scale	0,0151		

Data for risk factor C_4

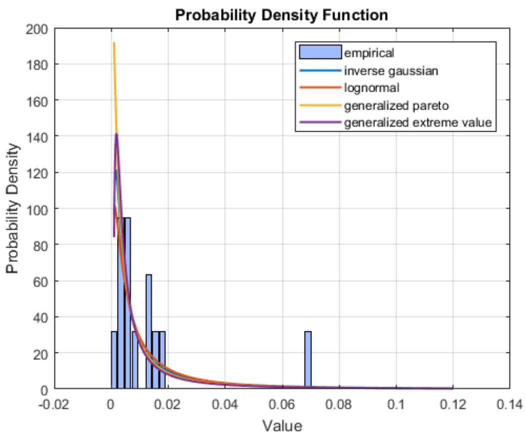

Figure D.7: Top 5 Probability density function fits for C_4 based on AICc

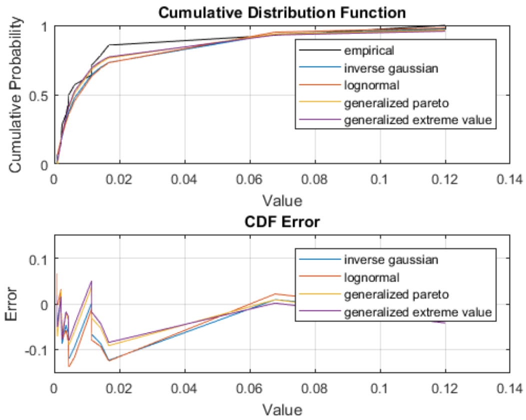

Figure D.8: Top 5 Cumulative distribution function fits for C_4 based on AICc

Risk factor 4

N = 14	Likelihood estimations					Parameter estimates					
Distribution Name	NlogL	BIC	AIC	AICc	Δ_i	Name	Value	Name	Value	Name	Value
Inverse gaussian	-46,25	-87,21	-88,49	-87,40	0,00	Scale	0,0191	Shape	0,0049		
Lognormal	-45,35	-85,41	-86,69	-85,60	1,80	Log location	-4,9243	Log scale	1,3528		
Generalized pareto	-46,77	-85,61	-87,53	-85,13	2,27	Shape	0,9172	Scale	0,0052	Threshold	0,0010
Generalized extreme value	-46,16	-84,41	-86,32	-83,92	3,48	Shape	1,1034	Scale	0,0041	Location	0,0039
Weibull	-43,30	-81,33	-82,61	-81,52	5,88	Scale	0,0145	Shape	0,7115		
Exponential	-41,38	-80,13	-80,77	-80,43	6,97	Mu	0,0191				
Gamma	-42,55	-79,83	-81,11	-80,02	7,38	Shape	0,6330	Scale	0,0302		
Beta	-42,44	-79,60	-80,88	-79,79	7,61	alpha	0,6169	beta	31,1822		
t-locationscale	-40,79	-73,67	-75,59	-73,19	14,21	Location	0,0037	Scale	0,0024	DoF	0,6578
Normal	-28,12	-50,96	-52,24	-51,15	36,25	Location	0,0191	Scale	0,0337		
Extreme value	-23,40	-41,52	-42,80	-41,71	45,69	Location	0,0383	Scale	0,0453		

Data for risk factor C_5

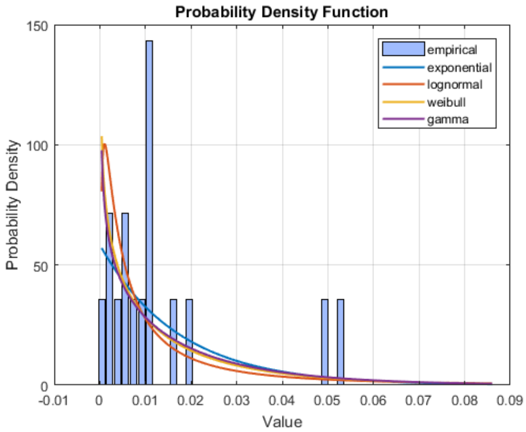

Figure D.9: Top 5 Probability density function fits for C_5 based on AICc

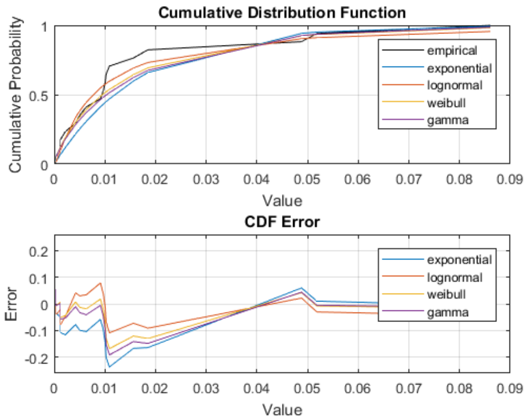

Figure D.10: Top 5 Cumulative distribution function fits for C_5 based on AICc

Risk factor 5

N = 17	Likelihood estimations					Parameter estimates					
Distribution Name	NlogL	BIC	AIC	AICc	Δ_i	Name	Value	Name	Value	Name	Value
Exponential	-52,14	-101,44	-102,27	-102,01	0,00	Mu	0,0171				
Lognormal	-53,26	-100,85	-102,51	-101,66	0,35	Log location	-4,8723	Log scale	1,4190		
Weibull	-52,94	-100,21	-101,88	-101,02	0,99	Scale	0,0150	Shape	0,8035		
Gamma	-52,69	-99,71	-101,37	-100,52	1,49	Shape	0,7441	Scale	0,0230		
Beta	-52,64	-99,62	-101,28	-100,43	1,58	alpha	0,7317	beta	41,8266		
Generalized pareto	-53,88	-99,26	-101,76	-99,92	2,09	Shape	0,4841	Scale	0,0095	Threshold	0,0004
Inverse gaussian	-52,01	-98,36	-100,02	-99,17	2,84	Scale	0,0171	Shape	0,0035		
Generalized extreme value	-52,55	-96,60	-99,10	-97,25	4,75	Shape	0,8360	Scale	0,0059	Location	0,0051
t-locationscale	-47,92	-87,34	-89,84	-87,99	14,02	Location	0,0075	Scale	0,0051	DoF	1,0704
Normal	-40,32	-74,97	-76,64	-75,78	26,23	Location	0,0171	Scale	0,0233		
Extreme value	-35,19	-64,72	-66,39	-65,53	36,48	Location	0,0302	Scale	0,0301		

Data for risk factor C_6

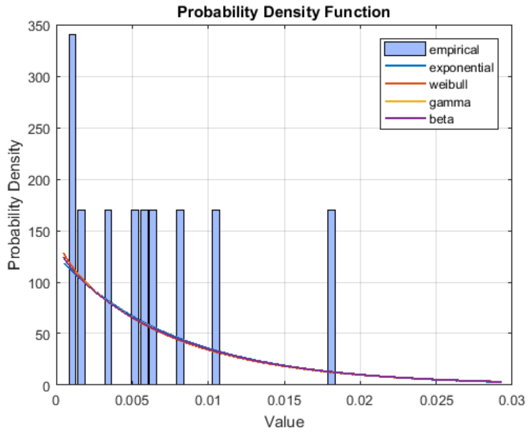

Figure D.11: Top 5 Probability density function fits for C_6 based on AICc

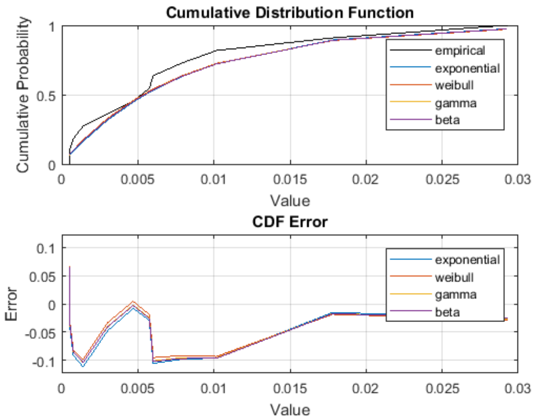

Figure D.12: Top 5 Cumulative distribution function fits for C_6 based on AICc

Risk factor 6

N = 11	Likelihood estimations					Parameter estimates					
Distribution Name	NlogL	BIC	AIC	AICc	Δ_i	Name	Value	Name	Value	Name	Value
Exponential	-42,21	-82,01	-82,41	-81,97	0,00	Mu	0,0079				
Weibull	-42,22	-79,64	-80,44	-78,94	3,03	Scale	0,0077	Shape	0,9616		
Gamma	-42,21	-79,63	-80,42	-78,92	3,05	Shape	0,9647	Scale	0,0077		
Beta	-42,21	-79,62	-80,42	-78,92	3,05	alpha	0,9585	beta	0,9647		
Lognormal	-42,07	-79,34	-80,14	-78,64	3,33	Log location	-5,437	Log scale	0,9585		
Inverse Gaussian	-41,83	-78,85	-79,65	-78,15	3,82	Scale	0,0079	Shape	-5,437		
Generalized Pareto	-43,02	-78,84	-80,04	-76,61	5,36	Shape	0,1951	Scale	0,0079	Threshold	0,0004
Generalized Extreme Value	-41,45	-75,70	-76,90	-73,47	8,50	Shape	0,6442	Scale	0,1951	Location	0,0031
Normal	-37,12	-69,45	-70,24	-68,74	13,23	Location	0,0079	Scale	0,6442		
t-locationscale	-38,74	-70,29	-71,48	-68,05	13,91	Location	0,0048	Scale	0,0079	DoF	1,6553
Extreme Value	-34,35	-63,90	-64,70	-63,20	18,77	Location	0,0125	Scale	0,0048		

Adjusted KS-test statistics

Table D.1: p-value Adjusted KS test C_1

Distribution Name	p- value
Exponential	0,1410
Beta	0,5528
Gamma	0,4892
Weibull	0,2919
Lognormal	0,0593
Generalized pareto	0,1549
Inverse gaussian	0,0060
Generalized extreme value	0,0143
Normal	0,0050
t-locationscale	0,0020
Extreme value	0,0000

Table D.2: p-value Adjusted KS-test C_2

Distribution Name	p- value
Exponential	0.8161
Beta	0.8517
Gamma	0.8530
Weibull	0.7921
Generalized pareto	0.2909
Lognormal	0.6661
Inverse gaussian	0.1646
Generalized extreme value	0.4329
Normal	0.0397
t-locationscale	0.0203
Extreme value	0.0120

Table D.3: p-value Adjusted KS test C_3

Distribution Name	p- value
Inverse gaussian	0.7271
Lognormal	0.5731
Generalized pareto	0.9967
Generalized extreme value	0.9727
Exponential	0.1156
Weibull	0.1496
Gamma	0.0573
Beta	0.0533
t-locationscale	0.0387
Normal	0.0007
Extreme value	0,0000

Table D.4: p-value Adjusted KS-test C_4

Distribution Name	p- value
Inverse gaussian	0,9034
Lognormal	0,6485
Generalized pareto	0,982
Generalized extreme value	0,7428
Weibull	0,2046
Exponential	0,027
Gamma	0,075
Beta	0,0457
t-locationscale	0,0183
Normal	0
Extreme value	0

Table D.5: p-value Adjusted KS test C_5

Distribution Name	p- value
Exponential	0.0796
Lognormal	0.5405
Weibull	0.2196
Gamma	0.1333
Beta	0.1243
Generalized pareto	0.7514
Inverse gaussian	0.0660
Generalized extreme value	0.2929
t-locationscale	0.0690
Normal	0.0003
Extreme value	0

Table D.6: p-value Adjusted KS-test C_6

Distribution Name	p- value
Exponential	0.9893
Weibull	0.9947
Gamma	0.9860
Beta	0.9903
Lognormal	0.6055
Inverse Gaussian	0.2069
Generalized Pareto	0.8900
Generalized Extreme Value	0.5628
Normal	0.0953
t-locationscale	0.2323
Extreme Value	0.0207

Appendix E

Figures minimum observation simulation

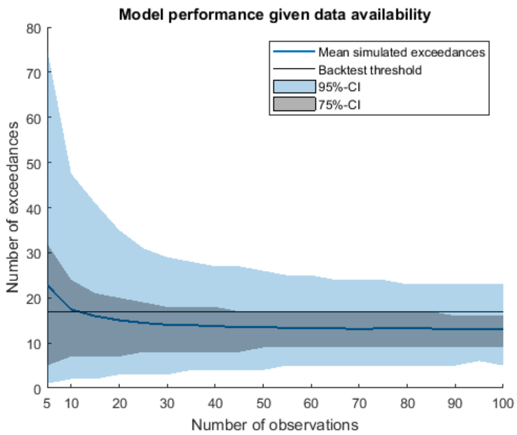

Figure E.1: Performance portfolio of two independent risks

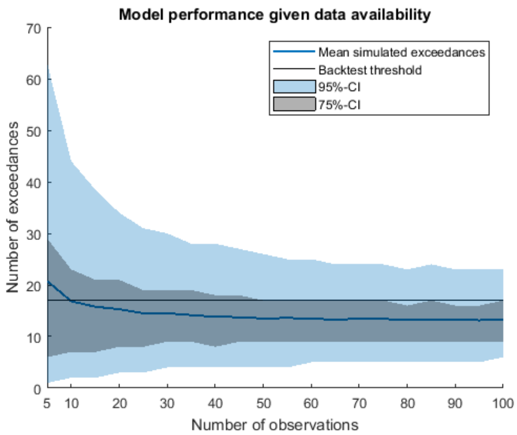

Figure E.2: Performance portfolio of five independent risks

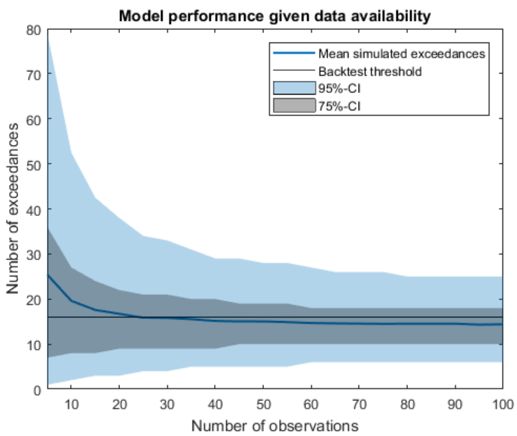

Figure E.3: Performance portfolio of two dependent risks

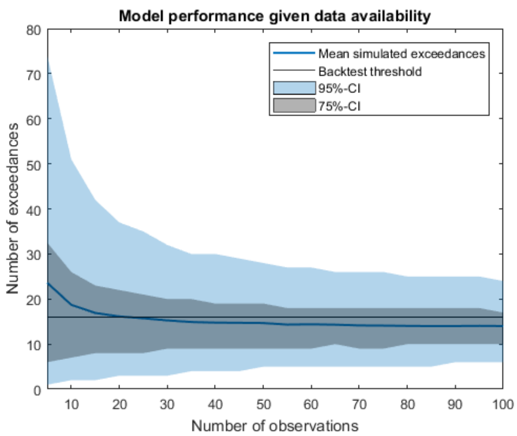

Figure E.4: Performance portfolio of five dependent risks

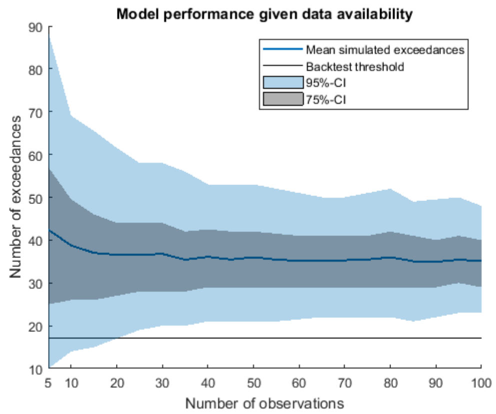

Figure E.5: Performance portfolio of two correlated risks including occurrence probability

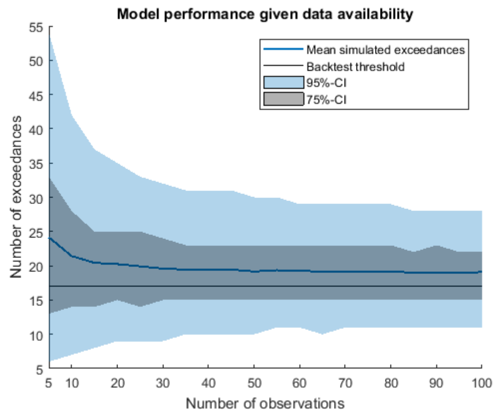

Figure E.6: Performance portfolio of five correlated risks including occurrence probability

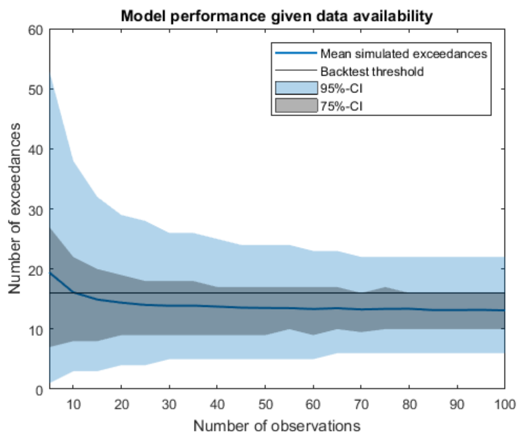

Figure E.7: Performance portfolio of two correlated risks including occurrence and allowing zero test data

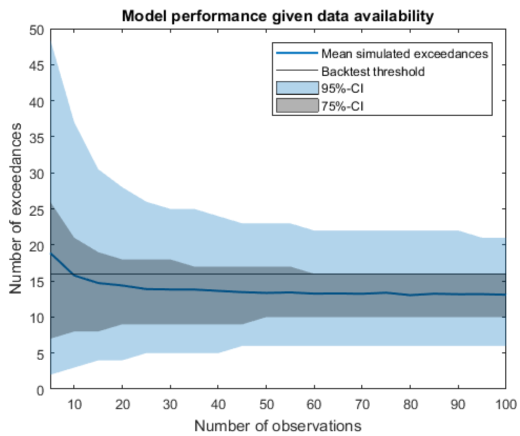

Figure E.8: Performance portfolio of five correlated risks including occurrence and allowing zero test data