Lazy motion planning for robotic manipulators

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1 Introduction

Robotic manipulators are making a shift towards mobile bases in both industry and domestic environments, which puts high demands on efficient use of the robot’s limited energy resources. In this work, the problem of reducing energy usage of a robot manipulator during a task is investigated. We achieve this by allowing the robot to rest on a surface, i.e., by making it lazy. Energy reduction is often formulated as an optimization problem, which due to the high dimensionality and non-linearity of robot manipulators is often computationally expensive. This is avoided by separating the task into two parts and subsequently using feedback linearization and optimization on each part.

2 Approach

Our approach begins with the insight that the original, switched, nonlinear problem consists of two distinct phases, as shown for a planar manipulator with two revolute joints in Figure 1. In phase 1 the manipulator has an unconstrained motion as shown in Figure 1a, until it reaches the surface as in Figure 1b, which we call the switch configuration. In phase 2 manipulator moves from the switch configuration until the final configuration, constrained by the surface.

![Figure 1: Configurations of a planar manipulator](image)

2.1 Feedback Linearization

For phase 1 the dynamics of an unconstrained, fully actuated, rigid robotic manipulator with \( n \in \mathbb{R} \) joints are given by [1]:

\[
M(q)\ddot{q} + C(q, \dot{q}) + g(q) = \tau_m, \tag{1}
\]

where \( q(t), \dot{q}(t), \ddot{q}(t) \in \mathbb{R}^n \), denote the joint positions, velocities and accelerations, respectively. \( M(q) \in \mathbb{R}^{n \times n} \), \( C(q, \dot{q}) \in \mathbb{R}^n \) and \( g(q) \in \mathbb{R}^n \) are the inertia matrix, centripetal and Coriolis vector, and the gravity vector, respectively. The torques acting on the manipulator consist of the motor torques, \( \tau_m(t) \in \mathbb{R}^n \).

Using the following feedback linearization (FBL) law with \( u \in \mathbb{R}^n \) a virtual input and

\[
\tau_m = M_f(b)\dot{q} + C(q, \dot{q}) + g_f(q) + u \tag{2}
\]

\[
M_f(q) = M(q) - \tilde{M} \in \mathbb{R}^{n \times n} \tag{3}
\]

\[
g_f(q) = g(q) - \tilde{g} \in \mathbb{R}^n \tag{4}
\]

chosen so that by combining (1)-(4) we arrive at a linear synthesis of phase 1:

\[
\tilde{M}\ddot{q} + \tilde{g} = u \tag{5}
\]

with

\[
\tilde{M} = M(q_L), \quad \tilde{g} = g(q_L) \tag{6}
\]

and \( q_L \) being the configuration in which the dynamics are feedback linearized.

2.2 Optimization

An optimization problem that reduces energy use is formulated as

\[
\min_u \int_0^{t_f} u^T Rd\dot{t} \tag{7a}
\]

subject to

\[
\tilde{M}\ddot{q} + \tilde{g} = u \tag{7b}
\]

\[
q(0) = q_0, \quad q(t_s) = q_s \tag{7c}
\]

\[
\dot{q}(0) = \dot{q}_0, \quad \dot{q}(t_s) = \dot{q}_s \tag{7d}
\]

where it is assumed that the begin configuration, \( q_0 \), and velocity, \( \dot{q}_0 \in \mathbb{R}^n \), are known and the switch configuration \( q_s \), and velocity, \( \dot{q}_s \in \mathbb{R}^n \) are to be determined. Since the dynamics are linear and the objective function is convex, the problem can be solved using Pontryagin’s Minimum Principle [2], resulting in the optimal input \( u^* \) as a function of the initial and switch conditions.

By following a similar approach for the second phase, we have optimal inputs for both phases as a function of the initial, switch and final conditions, where the initial and final conditions are assumed to be known, leaving only the switching conditions to be determined. This is done by solving a nonlinear optimization with \( 2(n-1) + 1 \) variables using a nonlinear solver.

References
