

# Maximum likelihood estimation in dynamic networks with rank-reduced noise

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# Maximum likelihood estimation in dynamic networks with rank-reduced noise

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## Introduction

Dynamic networks are rising in popularity as tool to model systems.

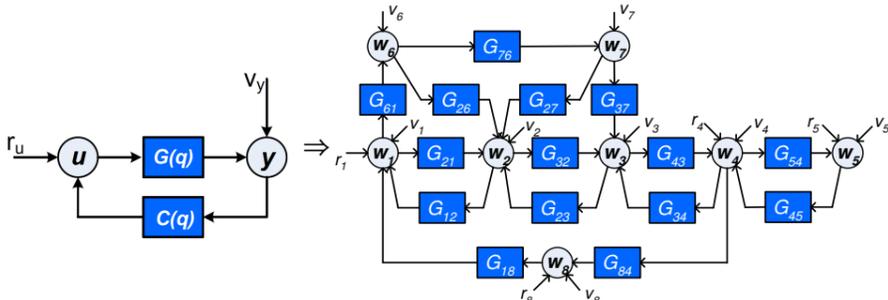


Figure 1: From classic models (left) to Dynamic networks (right).

With many measurements, the noise can be generated by fewer sources than nodes, i.e. noise is *rank-reduced*.

## Maximum likelihood

Assuming initial conditions are 0, and  $e(t)$  is Gaussian, the maximum likelihood estimate can be obtained.

**Result: Constrained optimization results in the Maximum Likelihood estimate<sup>1</sup>**

$$\hat{\theta}_{ML} = \arg \min_{\theta} \det \left( \frac{1}{N} \sum_{t=1}^N \varepsilon_a(t, \theta) \varepsilon_a^T(t, \theta) \right)$$

$$\text{subject to } \frac{1}{N} \sum_{t=1}^N \varepsilon^T(t, \theta) \begin{bmatrix} \Gamma^T(\theta) \\ -I \end{bmatrix} \begin{bmatrix} \Gamma(\theta) & -I \end{bmatrix} \varepsilon(t, \theta) = 0.$$

A consistent (but not minimum variance) estimate can be obtained by<sup>2</sup>

$$\hat{\theta}_{WLS} = \arg \min_{\theta} \frac{1}{N} \sum_{t=1}^N \varepsilon^T(t, \theta) Q \varepsilon(t, \theta).$$

## Variance

Typical variance expressions are invalid for constrained criteria. For analysis use a new parameter  $\rho$  that always satisfies the constraint.

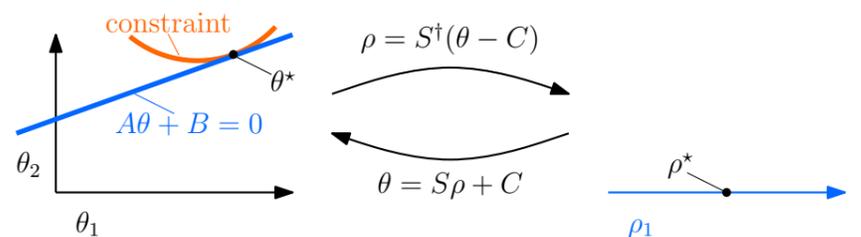


Figure 4: Original parameter space with constraint (orange), approximation of constraint (blue), and a mapping (defined by  $A(S\rho + C) + B = 0 \forall \rho$ ) to an unconstrained lower dimensional space.

An unconstrained criterion where  $\theta$  is substituted for  $\rho$  can be solved. The  $\rho$  has distribution

$$\sqrt{N}(\hat{\rho} - \rho^*) \sim \mathcal{N}(0, P_{\rho}),$$

where the variance has some lower bound  $P_{\rho} \geq P_{\rho}^0$ . Via the reverse mapping above we obtain the variance and lower bound on  $\theta$ .

**Result: The variance of  $\hat{\theta}$  is<sup>1</sup>  $P_{\theta} \geq P_{\theta}^0 = S P_{\rho}^0 S^T$ .**

## References

- H.H.M. Weerts, P.M.J. Van den Hof, A. Dankers. Identification of linear dynamic networks with rank-reduced noise, *In preparation*.
- H.H.M. Weerts, P.M.J. Van den Hof, A. Dankers. Identification of dynamic networks with rank-reduced process noise, *IFAC WC*, 2017.

**Goal: Obtain maximum likelihood estimates of the network, and provide a lower bound on the variance.**

A special structure can result in a variance-free estimate, which is a special case of the maximum likelihood estimate.

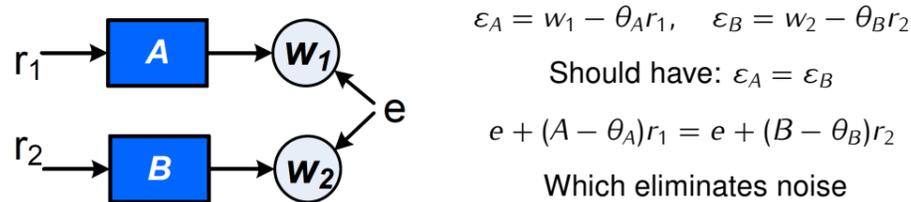


Figure 2: Example of rank-reduced identification. Parameters  $\theta_A = A$  and  $\theta_B = B$  can be determined variance-free.

## The model

We assume that the first  $p$  nodes are affected by a full rank noise, and the other noise is fully dependent on this first part.

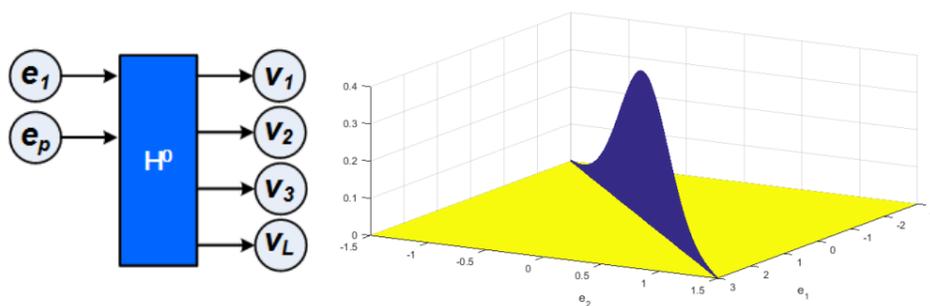


Figure 3: Rank-reduced noise model.

A prediction error is defined, with a full rank and a dependent part

$$\varepsilon(t, \theta_0) = \begin{bmatrix} \varepsilon_a(t, \theta_0) \\ \varepsilon_b(t, \theta_0) \end{bmatrix} = \begin{bmatrix} I \\ \Gamma \end{bmatrix} e(t) =: \check{\varepsilon}(t).$$

The innovation is white noise  $e(t)$  which is mapped to higher dimensional space.