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Detention Decisions for Empty Containers in the Hinterland Transportation System

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Abstract

In this paper, we study a hinterland empty container transportation system which consists of a sea container terminal and an inland container terminal. There are a hinterland container operator who is in charge of the hinterland container transportation and an ocean carrier who has an empty container depot at the sea container terminal. We utilize a two-stage game model to describe the ocean carrier's decision about the container's free detention time and the hinterland container operator's decision about the time when should an arrived empty container at the inland terminal be dispatched to the sea terminal. Optimal delivery policy of the empty container and the ocean carrier's optimal free detention time are derived. It is shown that the decentralized system does not guarantee system coordination all the time. The ocean carrier has incentive to integrate the hinterland transportation operation only if the hinterland area is not very short of empty containers.

Keywords: *Empty container repositioning; Detention; Ocean carrier; Hinterland container operator; Hinterland container transportation*

1 Introduction

With the development of the production regionalization and economic globalization, the international trade has been booming in the past few decades. The import and export freights in the international trade are mainly distributed by maritime transportation. The ocean container transportation is now dominating the maritime transportation system, with over 50% the value of the international trade cargo (Review of Maritime Transport, 2016). The performance of global supply chain depends crucially on the efficiency of the ocean container transportation system. In the global distribution channel, the consignor is the one who sends the cargo for ocean transportation, the consignee is the one who is receiving the cargo in the containers, the shipping liner (or ocean carrier) is the shipping company that is in charge of the cargo ocean transportation, and the container terminal operator is the company or organization that provides container operation services in the container terminal. There are two kinds of hinterland operation mode for containers, carrier haulage and merchant haulage. In the carrier haulage mode, the shipping liner is responsible for the hinterland transportation and storage and containers are under the shipping liner's control in the whole process. In the merchant haulage mode, there is a hinterland container operator who is in charge of the containerized freight transportation and operations in the hinterland. Under the merchant haulage mode, the shipping liner loses the control over the container in hinterland and will charge container charter fees from the hinterland container operator (so called demurrage and detention fees which will be defined later).

Participants in the global container supply chain, including the hinterland container operator, the shipping liner and the container terminal operator, are all trying to enhance the operation efficiency and the service level so as to pursuit high competitiveness. Operational approaches are emphasized and well developed, for example, container terminal crane scheduling and space allocation, collection and delivery arrangements of the hinterland container operator, and the ocean carrier's trade lane setting and route scheduling. Fransoo and Lee (2013) noted that there are many significant strategic problems that are underestimated, such as those concerning contracting, pricing, capacity development and risk management in the global distribution channel.

There are extensive studies about the empty container repositioning in both the seaborne level and the hinterland level. Seaborne empty container repositioning mainly involves the empty container sharing, leasing and transportation between different sea ports. The hinterland empty container repositioning includes the transportation and management of the empty containers in the hinterland so as to fulfill the empty container with the demand of export cargos and prevent unnecessary movements of the empty container in the hinterland.

In this paper, we consider a hinterland empty container transportation system in an export-oriented port area (for example, in the port area of China, the export cargo volume is higher than the import cargo volume and the empty container utilization in the hinterland is high) which consists of a sea container terminal and an inland container terminal. The ocean carrier has an empty container depot at the sea container terminal. There is a hinterland container operator who is in charge of the hinterland container transportation. When selecting port services, consignors and shipping liners are emphasizing the performance of the container supply chain rather than the port itself. The efficient hinterland accessibility is important in the port competition. Inland container terminals help to keep the hinterland accessibility of the seaports and provide a cost efficient and reliable alternative to truck transportation (Konings et al., 2013).

Once an imported container is unloaded from the vessel at the sea container terminal, the ocean carrier loses the control of the container and will start to charge demurrage fee and detention fee for the container (Deidda et al., 2008). Demurrage is related to the time range that a container spends at the sea container terminal, namely from the time an inbound container arrives at the sea terminal to the time it is moved out of the sea terminal by the hinterland container operator. Detention is related to the time range that a container spends in the hinterland, namely from the time it leaves the sea container terminal to the time it is sent back to the sea container terminal (the sent back container could be laden or empty). The ocean carrier will charge the consignee a demurrage fee per container per day if the demurrage exceeds a specific free time. The ocean carrier only counts the time the container spends in the sea container terminal without being affected by the ownership and operations in the sea container terminal. Similarly, a detention fee per container per day is charged by the ocean carrier if the detention exceeds a specific free time (so called free detention time). The detention and demurrage fee are also applied to the export process. These fees are not directly paid by the consignor or the consignee to the ocean carrier but paid through the hinterland container operator. Table 1 shows the demurrage and detention fees in the Port of Rotterdam. As shown in Table 1, to encourage the barge and train transport in the multi-modality hinterland transportation system, some ocean carriers provide longer free time for the detention and demurrage under these two transport modalities.

Table 1: The demurrage and detention free days for train or barge transport compared with truck transport in the Port of Rotterdam (Storm, 2011)

Shipping Line	**Demurrage Free Days**	**Detention Free Days**		**Combined Free Days**	
		Truck	**Train/Barge**	**Truck**	**Train/Barge**
Maersk	3	3	5		
MSC				3	7
CMA CGM				7	7
COSCO				7	7
Hapag Lloyd	5	2	6		
Evergreen	3	2	6		
APL	5	3	6		
CSCL	3	2	10		
Hanjin Shipping	4	5	5		
MOL	3	3	5		
Nyk Line	4	6	6		
OOCL	3	2	6		
CSAV	4	6	6		
Yang Ming	4	3	3		

The charging scheme that includes a "free-time" and an overtime-fee is also used by other players in the maritime transportation system, such as the sea container terminal operator. For the inbound containers that are unloaded from the vessels to the sea container terminal, the terminal operator will charge a storage fee per unit container per day if the container dwell time in the terminal exceeds a certain free-time (Yu et al. 2015; Kim and Kim, 2007). This kind of fee is charged by the sea container terminal operator for the occupation of storage space in the terminal yard. Differently, the demurrage and detention fees studied in our paper are charged by the ocean carrier for losing the control of containers.

After unloaded from vessels, inbound containers will be moved from the sea container terminal to the consignee in hinterland by the hinterland container operator. Becoming empty at the consignee, some empty containers are moved to and stored at the inland container terminal. Some empty containers will be moved back to the sea container terminal directly, especially for those which are close to the detention deadline. In this paper, we assume that the ocean carrier only holds empty container depot at the sea terminal. Namely, the detention time ends only when the container is sent back to the sea terminal.

We adopt a two-stage game model to study the ocean carrier's decision about the free detention time and the hinterland container operator's detention decision about the empty containers at the inland container terminal, namely, the time when the empty container arrived at the inland terminal is dispatched to the sea container terminal. For a specific empty container that arrives at the inland terminal, the hinterland container operator balances between the detention fee and the transportation cost to decide his best time to dispatch the empty container from the inland container terminal to the sea container terminal. We derive the system outcomes under the exponential distribution of the empty container called time by hinterland consignors. The system coordination of the decentralized transportation system is also analyzed. Through theoretical analysis and numerical study we mainly find that (1) the ocean carrier should try to centralize the system and take care of the multi-modal transportation system if the hinterland area is not very short of empty containers; (2) in the system that hinterland container operator takes care of the hinterland empty container transportation, the ocean carrier should not set detention fee arbitrarily high; (3) in the port area where the hinterland transportation system is weak such that containers spend much time in hinterland, the ocean carrier may get hurt in the detention profit.

We summarize our contributions as follows:

(1) We investigate the empty container detention decision problem for both the ocean carrier and the hinterland container operator in the hinterland container transportation system. As far as we know, this is the first piece of research that explicitly studies the detention decisions in the hinterland container supply chain.

(2) We utilize the two-stage game framework to characterize the decision processes of the ocean carrier and the hinterland container operator. In the first stage, the ocean carrier determines and announces the free detention time. In the second stage, the hinterland container operator decides the best time to dispatch the empty container in the inland terminal back to the sea container terminal.

(3) We analyze the system coordination of the two-stage system and compare the ocean carrier's profits under the decentralized and centralized systems. In common prac-

tice, hinterland container operator takes care of the container operations in hinterland. Recent years, some ocean carriers try to integrate the hinterland container operations. For example, besides the ocean shipping services, Maersk also provides services of container terminal operation, hinterland transportation in some ports of the world. Our research results present suggestions for the ocean carrier who intends to integrate the hinterland container operations in the multimodal transportation.

The rest of this article is organized as follows. Section 2 discusses the related literature. In Section 3, we provide model basics and problem formulation. In Section 4, we study the two-stage game under the exponentially distributed empty container called time by the consignors. Section 5 analyzes the coordination of the two-stage system. In Section 6, we analyze the case when the ocean carrier is loss-averse. In Section 7, we use numerical studies to analyze our model results. Section 8 concludes the article. All proofs are provided in the Appendix for presentation elegance.

2 Literature Review

Two streams of research are involved with our study. The first stream is related to the detention and demurrage problem of containers in hinterland, and the second stream analyzes empty container repositioning problems. In the following, we discuss the results that are directly related to our research.

The concepts of detention and demurrage were firstly described from the perspective of legislation (Stewart and Scheinberg, 2002; Gottschalk, 1968). Schofiled (2013) provided detailed legislations about the demurrage and detention under different situations and cases. Storm (2011) firstly analyzed the demurrage and detention charges from the economic perspective by investigating the information sharing in the parties that participate in the hinterland transportation so as to control the demurrage and detention. Fazi (2014) proposed a linear programming formulation for the inbound container hinterland transportation concerning the demurrage and detention. The decision is about the end time of the demurrage (namely, the starting time of the detention) and the objective is to minimize the costs of demurrage, detention, delay to due dates and the holding cost of the container at the sea terminal. Numerical studies were conducted to analyze the model results. This is the first time that the demurrage and detention are modeled and studied in the operations research literature. Legros et al. (2017) modeled the empty container management problem of the hinterland container operator as a Morkov decision process using the waiting time of the oldest container as a decision variable. With the objective of minimizing the empty container holding cost and repositioning cost, they provided and proved a optimal threshold policy based on the age of the oldest empty container in the hinterland container operator's location. To simplify the analysis, they specifically analyzed two special cases of the detention fee, when the free detention time is zero and when the detention rate per unit time is infinite. Our paper studies the detention and demurrage problem from a different angle: we intend to derive the ocean carrier's free detention time setting and the hinterland container operator's decision about when to send the empty container back to the sea container terminal.

As an alternative to relieve the port congestion, improve throughput efficiency and lower the carbon emission, the inland container terminal is now attracting the Port Authorities' attention, especially those in Europe (Roso, 2007). Some researchers studied the functions and roles of the dry ports or inland container terminals (e.g., Roso et al., 2009; Cullinane and Wilmsmeier, 2011). Xie et al. (2017) studied the decentralized and centralized system in the hinterland intermodal transportation involving one sea port and one dry port. By analyzing the empty container inventory sharing game between the sea port and dry port, they designed a buy-back contract which depends on the initial empty container inventories. The existing researches ignore the detention decisions in the two-echelon hinterland container transportation system with one sea container terminal and one inland container terminal. Our researches fill this gap by investigating the incentive of the ocean carrier's hinterland empty container operation under the centralized manner.

Due to the international trade imbalance, empty container repositioning problem is now a major issue in the container supply chain management. Lee and Song (2017) pointed out empty container repositioning is a major topic in the six planning issues in maritime container transportation. Empty container repositioning occurs at both global and regional level (Legros et al., 2017). Global empty container repositioning involves container sharing and movements between different sea ports around the world, repositioning empty containers from the port which is accumulated with empty containers to the port which is in shortage of empty containers. We refer to extensive studies about the global empty container repositioning problem (Song and Dong, 2015; Song and Carter, 2009; Li et al., 2007, Zheng et al., 2015; Lam et al., 2007). Regional level (or hinterland) empty container repositioning includes two approaches, sending the empty container in hinterland back to the sea port for global repositioning or reload the empty container with export cargoes in hinterland. Jula et al. (2006) analyzed these two approaches of the hinterland container operators as depot-direct and street-turn. In the street-turn mode, the empty container will be sent to the hinterland consignor by the hinterland container operators to load export cargos. In the depot-direct mode, the hinterland container operator will send the empty containers back to the sea port rather than keep on storing them in the inland terminals. Furió et al. (2013) proposed two different integer programming models to make inland empty container assignment under the empty container repositioning with and without street-turn. They considered different objectives: land movements, containers usage, storage yard usage, etc. Deidda et al. (2008) studied both the truck routing problem and the container allocation problem in the hinterland empty container repositioning with street-turn strategy. They compared their results with a real shipping company's practice and showed that their optimization model could produce significantly better solution for the truck routings. Bernat et al. (2016) proposed stochastic review policies to involve emissions, realistic maintenance and street-turn in the empty container allocation scheme. Yang (2015) utilized a two-stage optimization model to characterize the sea-rail multimodal transportation problem in terms of the slot allocation in the contract market, empty container allocation as well as the pricing and inventory control for the shipping liner. An et al. (2015) studied the hinterland empty container repositioning problem from the angle of the inland waterway shipping. Hjortnaes et al. (2017) studied the empty container repositioning cost minimization problem concerning the repair operations.

Besides the above mentioned repositioning scheme, technical solutions are also used in practice to relieve the empty container problem, e.g., the foldable containers. Shintani et al. (2010) studied the empty container repositioning optimization in the hinterland by integer programming and showed that the use of foldable containers could significantly reduce the repositioning cost. Myung and Moon (2014) proposed a network flow model to study the seaborne empty container repositioning by considering both the standard and foldable containers. Zheng et al. (2016) used a two-stage optimization method to study the perceived container leasing prices in the liner shipping network design problem involving the foldable containers.

Note that the existing research in the literature about the hinterland empty container repositioning mainly focuses on the container allocation, truck routing and network design. The demurrage and detention concepts exist in both practice and the legislation, but are not well studied in the operations research literature. Our paper contributes to the literature by initiating the study of a hinterland empty container repositioning problem involving the consideration of both the detention fee and the street-turn strategy.

3 Problem Formulation

In this section, we present the basic model settings. Assume that there are a sea container terminal and an inland container terminal in a simple hinterland container transportation system. The ocean carrier holds an empty container depot at the sea container terminal. Although in reality there exists the case that the ocean carrier also holds empty container depot in the inland container terminal, our paper applies to the situation that the empty container depot is at the sea terminal. Namely, the detention time ends only when the container is sent back to the sea terminal. The hinterland container operator is the central planner of the hinterland container operation. After the vessels arrive at the sea terminal, inbound containers are unloaded from the vessels, moved by the sea terminal's internal trucks to the storage yard and then transferred by yard cranes to planned storage locations. Inbound containers stay in the sea terminal yard until required by consignees. The consignee usually consigns a hinterland container operator to collect the container by truck, train or barge. Then the inbound container will be moved to an inland terminal where it will change transportation modality to truck if necessary. After the hinterland container operator sends the import container to the consignee by truck, the cargos inside are unloaded and the empty container will be moved by the hinterland container operator to the inland terminal by truck. For the empty containers in the inland terminal, they may be moved to the hinterland consignors again to load export cargos (street-turn repositioning) or be dispatched back to the sea terminal for other use (depot-direct repositioning).

In this paper, we aim to adopt a two-stage game model to study the ocean carrier's decision about the free detention time and the hinterland container operator's decision about the time when the empty container arrived at the inland container terminal is dispatched to the sea terminal. The hinterland container operator plays as a middle man between the ocean carrier and the consignee. Rather than interacting with the ocean carrier directly, the consignee pays the detention fee through the hinterland container operator to the ocean carrier. The detention fee is first paid by the hinterland container operator to the ocean carrier and the hinterland container operator will charge the consignee for it later (sometimes the time interval between these two payments could be as long as half a year). Considering the time value of the money he pays, the hinterland container operator has incentive to reduce the detention payment to the ocean carrier. As soon as the empty container arrives at the sea terminal, its detention time ends. The hinterland container operator (he) faces the tradeoff between the detention fee and the transportation cost of moving the empty container between the inland container terminal and the sea container terminal. If he dispatches a specific empty container to the sea terminal too early, the detention fee he pays is low. But the consignor may need this empty container later and it needs to be moved back to the hinterland from the sea terminal. Therefore, the randomness comes from the time that the consignor calls this empty container for export cargo transportation.

In our research, we treat the detention free time as the decision variable of the ocean carrier and take the detention fee rate as given. Practically, the detention free times of an ocean carrier to different consignees or different hinterland container operators may be different, depending on many factors such as relationship, contract and barging power, etc. While, the detention fee rate of a specific ocean carrier is usually unified. Therefore, in our one-carrier one-hinterland-container-operator model, we only focus on the optimal value of the detention free time. The decision of the unified detention fee for multiple consignees (or multiple hinterland container operators) could be a future research topic.

For a certain empty container in the inland terminal, the time when the consignor calls it in the future is random and follows a certain distribution. Although a container may be called by the consignor any time after it entering the hinterland, we only consider the called time of the empty container that has already arrived at the inland terminal in our model. We study one type of empty containers (e.g., 20ft or 40 ft). The proposed model is for a certain empty container which represents for a type of empty containers with similar physical and utilization characteristics, with a known distribution of its called time by consignors (obtained, e.g., from historical data). The empty containers are often sent back to the inland container terminal from consignees by truck intermittently. The hinterland container operator's problem is a tactical problem to determine a guideline for setting an average dispatching time for a type of empty containers arrived. There are lots of consignors in the hinterland who call the empty containers in the inland container terminal for export cargo transportation. By estimating the historical data of the empty containers' called time by consignors, we could obtain the called time distributions of a certain type of empty containers. Different types of empty containers may have different distributions of called time by the consignors. Although in the on-line decision process, the decision makers' information about the empty container's possible called time will be more and more accurate with time proceeding, as an off-line decision problem here, our model treats the empty container called time distribution to be independent of the decision time.

We emphasize the situation in the export-oriented hinterland area where empty containers are in shortage and the empty container inventory level in the inland container terminal is not high or even very low. In this case, the hinterland container operator's transportation cost and detention cost matter. Therefore, we model the hinterland container operator's decision from the dispatching time angle in this paper, although we find that the time-based dispatching scheme does not necessarily guarantee the inventory optimized solution. In our model, we focus on the empty container's dispatching time to the sea container terminal. From the operation point of view, this time-based empty container dispatching scheme is easier to control than the inventory-based empty container dispatching scheme.

3.1 Assumptions and notations

The mathematical models in this article are formulated given the following assumptions.

Assumption 1. It is assumed that there is an inland container terminal where the transportation modality is changed. Inbound containers are moved from the sea container terminal by train or barge to the inland container terminal where the modality is changed to truck.

Assumption 2. For simplicity of analysis, we assume that the transportation cost of an empty container between the sea terminal and the inland terminal is fixed, independent of the utilization of the barge or train. Namely, the economic of scale in transportation volume is ignored here.

Assumption 3. After its arrival, an empty container could be transported from the sea terminal or the inland terminal any time as the hinterland container operator likes.

Assumption 4. The cargo loading and unloading time at the consignee/consignor are neglected.

We now describe the notations in our models:

s: the detention fee rate per container per day.

a_1: the transportation time between the inland container terminal and the sea container terminal.

a_2: the transportation time between the inland terminal and the consignor. Here we use the average distance between the inland container terminal and the potential consignors to estimate the transportation time. Taking the average of the transportation times is reasonable because the potential consignors are unknown at the time of the inland container operator's decision.

τ: the arrival time of the empty container at the inland container terminal. It is assumed that $\tau \geq a_1 + 2a_2$. We define time 0 as the time when the imported laden container leaves the sea container terminal.

T: the time when the consignor calls the empty container at the inland container terminal. From the hinterland container operator's point of view, T is a random variable and its probability density function is denoted as $f(T)$, we have $T \geq \tau$.

M: the transportation cost of moving the container between the inland container terminal and the sea container terminal.

N: the transportation cost of moving the container between the inland terminal and the consignor. Similar to a_2, N is the average transportation cost of between the consignors and the inland container terminal.

h: the ocean carrier's holding cost per unit time for storing the empty container at the depot of the sea container terminal.

p: the opportunity cost per container of the ocean carrier for sending the empty container to the hinterland consignor to satisfy the export cargo business.

v: the revenue of the hinterland container operator by providing services for the consignee and the consignor for one round of imported and exported container operations.

The decision variables are:

F: the free detention time which is decided by the ocean carrier. It is assumed that $F \geq 2a_1 + 2a_2$. Namely, the free detention time is longer than the total pure transportation time of the container in the hinterland. This assumption is fair and consistent with the practice cases in the port area that is export-oriented and empty containers are in shortage. In this situation, consignees and consignors are pushed by the hinterland container operators and ocean carrier to vacate and return the empty containers soon. Not much time is wasted besides the pure transportation time and container loading and unloading time.

t: the time the hinterland container operator plans to dispatch the empty container to the sea container terminal from the inland container terminal. $t \geq \tau$.

3.2 Model basics

The event sequence is defined as follows. Firstly, the ocean carrier decides the free detention time, F. Then, the hinterland container operator decides the time t, that he plans to dispatch the empty container to the sea terminal from the inland terminal when the empty container arrives at the inland terminal. At last, the consignor calls for the empty container. If the empty container has already leaved the inland terminal when the consignor calls it, then the hinterland container operator needs to move it from the sea terminal back to the inland terminal where it is transported by truck to the consignor. Otherwise, the hinterland container operator will move the empty container directly from the inland container terminal to the consignor when it is called by the consignor.

We use a two-stage game model to characterize the ocean carrier's decision about the free detention time and the hinterland container operator's decision about the planned time that the empty container is dispatched to the sea container terminal. Based on the event sequence described above, we use backward deduction to solve this two-stage game model. Given the ocean carrier's decision about the free detention time, we first derive the hinterland container operator's optimal decision about the dispatching time of the empty container as function of the free detention time. Then, we derive the ocean carrier's optimal decision about the free detention time.

(1) The objective function of the hinterland container operator:

Based on the assumptions and notations defined above, we provide the objective function of the hinterland container operator.

$$\begin{aligned}\max_{t\geq\tau} \quad \Pi(t) = \quad & v - \int_{\tau}^{t} \left[s(T+2a_2+a_1-F)^+ + 2N + M\right] f(T)dT \\ & - \int_{t}^{\infty} \left[s(t+a_1-F)^+ + 2N + 3M\right] f(T)dT. \qquad (3.1)\end{aligned}$$

In the objective function, the hinterland container operator intends to maximize his profit, which is the revenue minus the detention fee and the transportation cost. As

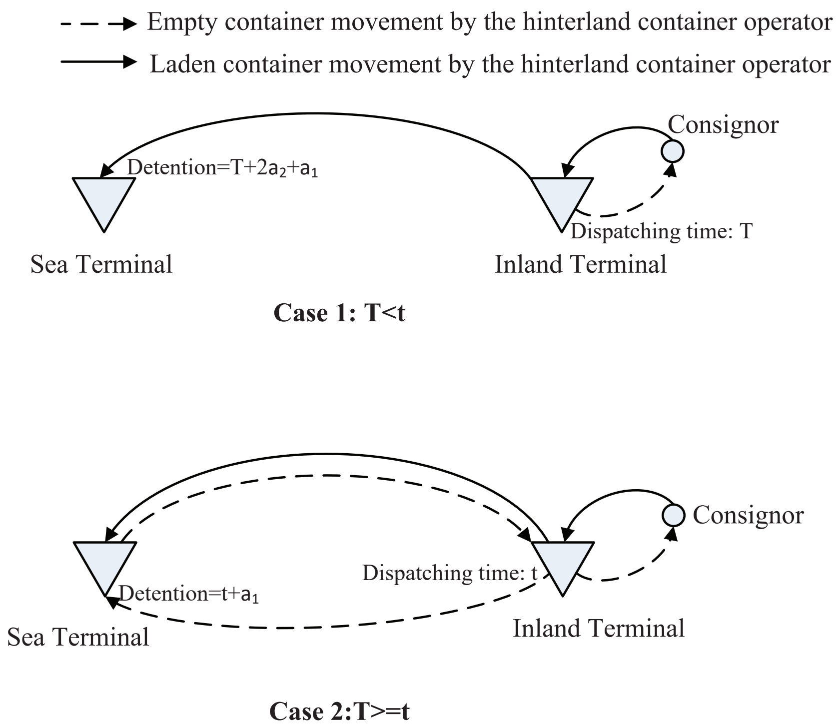


Figure 1: The two cases of the detention

shown in Figure 1, if $T \geq t$, the consignor calls the empty container after it is dispatched to the sea terminal from the inland terminal by the hinterland container operator. In this case, the detention is $t+a_1$ and the detention fee is $s(t+a_1-F)^+$. While, the hinterland container operator needs to move the empty container back to the inland terminal from where the container is moved to the consignor. And after the consignor fills the empty container, it is moved by the hinterland container operator to the sea terminal through the inland terminal. Therefore, the total transportation cost of the hinterland container operator is $3M+2N$. The time spent on this export business round-trip is $2a_1+2a_2$ which is shorter than the free detention time F. If $T<t$, the empty container is still in the inland terminal when the consignor calls it. The hinterland container operator will move the empty container out of the inland terminal at time T and dispatch it to the consignor. After it is filled, he will move it to the sea terminal. Therefore, in this case, the transportation cost of the hinterland container operator is $M+2N$, the detention is $T+2a_2+a_1$, and the detention fee is $s(T+2a_2+a_1-F)^+$. These two cases are illustrated in Figure 1.

We can rewrite Equation (3.1) as:

$$\Pi(t)=\begin{cases}\Pi_1(t)=v-\int_\tau^t(M+2N)f(T)dT-\int_t^\infty(3M+2N)f(T)dT,\\ \qquad\qquad if\quad t\le F-2a_2-a_1;\\ \Pi_2(t)=v-\int_\tau^t\left[s(T+2a_2+a_1-F)+M+2N\right]f(T)dT\\ \quad-\int_t^\infty(3M+2N)f(T)dT,\\ \qquad\qquad if\quad F-2a_2-a_1<t\le F-a_1,\tau\ge F-2a_2-a_1;\\ \Pi_3(t)=v-\int_{F-2a_2-a_1}^t s(T+2a_2+a_1-F)f(T)dT+\int_\tau^t(M+2N)f(T)dT\\ \quad-\int_t^\infty(3M+2N)f(T)dT,\\ \qquad\qquad if\quad F-2a_2-a_1<t\le F-a_1,\tau<F-2a_2-a_1;\\ \Pi_4(t)=v-\int_\tau^t\left[s(T+2a_2+a_1-F)+M+2N\right]f(T)dT\\ \quad-\int_t^\infty\left[s(t+a_1-F)+3M+2N\right]f(T)dT,\\ \qquad\qquad if\quad t>F-a_1,\tau\ge F-2a_2-a_1;\\ \Pi_5(t)=v-\int_{F-2a_2-a_1}^t s(T+2a_2+a_1-F)f(T)dT+\int_\tau^t(M+2N)f(T)dT\\ \quad-\int_t^\infty\left[s(t+a_1-F)+3M+2N\right]f(T)dT,\\ \qquad\qquad if\quad t>F-a_1,\tau<F-2a_2-a_1.\end{cases}\tag{3.2}$$

It is easy to find that $\Pi_1(t)$ is increasing in t. Therefore, in the following analysis we only focus on the properties of $\Pi_2(t)$, $\Pi_3(t)$, $\Pi_4(t)$ and $\Pi_5(t)$.

(2) The objective function of the ocean carrier:

The ocean carrier intends to maximize his profit by setting a proper detention free time:

$$\max_{F\ge 2a_1+2a_2}\qquad \widetilde{\Pi}(F)=\int_\tau^t rs(T+2a_2+a_1-F)^+f(T)dT+\int_t^\infty rs(t+a_1-F)^+f(T)dT$$
$$-h\int_t^\infty(T-t-a_1)^+f(T)dT-\int_t^\infty pf(T)dT.\tag{3.3}$$

We use "$\sim$" to denote the ocean carrier. Here, $0\le r\le 1$ is the weight of the revenue which denotes the ocean carrier's emphasizing on the revenue. The ocean carrier's profit equals the income from the detention fee minus the holding cost of the empty container at the sea terminal depot, and the opportunity cost of using this empty container for export cargo business in the hinterland. Rather than keeping the empty container in hinterland, the ocean carrier could make money by using the container elsewhere in his system, such as deep sea inter-port empty container repositioning, empty container leasing, etc. Therefore, the value of the opportunity cost p could be set by estimating all the possible cases in which the ocean carrier could make money rather than sending the empty container back to the hinterland. By adjusting the value of r, the ocean carrier could control the detention time of the container. In some cases, the ocean carrier emphasizes on the income from the detention (e.g., r is set large) and really make money from the detention (e.g., according to our consulting information, Mearsk earns a lot from the hinterland detention charge on the military cargo transportation in USA).

We can rewrite Equation (3.3) to:

$$\widetilde{\Pi}(F) = \begin{cases} \widetilde{\Pi}_1(F) = -h\int_{t+a_1}^{\infty}(T-t-a_1)f(T)dT - \int_t^{\infty} pf(T)dT, \quad if \quad t \leq F-2a_2-a_1; \\ \widetilde{\Pi}_2(F) = \int_{\tau}^{t} rs(T+2a_2+a_1-F)f(T)dT - h\int_{t+a_1}^{\infty}(T-t-a_1)f(T)dT \\ \quad -\int_t^{\infty} pf(T)dT, \quad if \quad F-2a_2-a_1 < t \leq F-a_1, \tau \geq F-2a_2-a_1; \\ \widetilde{\Pi}_3(F) = \int_{F-2a_2-a_1}^{t} rs(T+2a_2+a_1-F)f(T)dT - h\int_{t+a_1}^{\infty}(T-t-a_1)f(T)dT \\ \quad -\int_t^{\infty} pf(T)dT, \quad if \quad F-2a_2-a_1 < t \leq F-a_1, \tau < F-2a_2-a_1; \\ \widetilde{\Pi}_4(F) = \int_{\tau}^{t} rs(T+2a_2+a_1-F)f(T)dT + \int_t^{\infty} rs(t+a_1-F)f(T)dT \\ \quad -h\int_{t+a_1}^{\infty}(T-t-a_1)f(T)dT - \int_t^{\infty} pf(T)dT, \quad if \quad t > F-a_1, \tau \geq F-2a_2-a_1; \\ \widetilde{\Pi}_5(F) = \int_{F-2a_2-a_1}^{t} rs(T+2a_2+a_1-F)f(T)dT + \int_t^{\infty} rs(t+a_1-F)f(T)dT \\ \quad -h\int_{t+a_1}^{\infty}(T-t-a_1)f(T)dT - \int_t^{\infty} pf(T)dT, \quad if \quad t > F-a_1, \tau < F-2a_2-a_1. \end{cases} \tag{3.4}$$

Based on the sequence of event defined above, we use backward deduction to solve this two-stage game model. Given the ocean carrier's free detention time, we first derive the optimal time that the hinterland container operator plans to dispatch the empty container to the sea terminal from the inland terminal. After obtaining the empty container dispatched time as a function of the ocean carrier's free detention time, we then derive the ocean carrier's optimal free detention time in equilibrium.

In the following section, we investigate the two-stage game model under exponentially distributed empty container called time. The empty container called time by consignors is mainly affected by the demand of empty containers in hinterland. Nowadays, the demand in container transportation increases and consignors want their cargoes to be delivered faster (Bouchery et al., 2015). Especially in the export-oriented port area which is in shortage of empty containers, empty containers are likely called by consignors shortly after arriving at the inland container terminal. The empty container's probability of being called by consignors decreases with time proceeding. Therefore, exponential distribution is a proper estimation for the empty container called time.

4 Exponentially distributed empty container called time

When the empty container called time by the consignor follows an exponential distribution, we have $f(T) = \lambda e^{-\lambda(T-\tau)}, T \in [\tau, \infty)$. We get the hinterland container operator's profit function as follows:

$$\Pi^E(t) = \begin{cases} \Pi_1^E(t) = v - \int_\tau^t (M+2N)\lambda e^{-\lambda(T-\tau)} dT - \int_t^\infty (3M+2N)\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad if \quad t \le F - 2a_2 - a_1; \\ \Pi_2^E(t) = v - \int_\tau^t \left[s(T+2a_2+a_1-F)+M+2N\right]\lambda e^{-\lambda(T-\tau)} dT \\ \quad - \int_t^\infty (3M+2N)\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad if \quad F-2a_2-a_1 < t \le F-a_1, \tau \ge F-2a_2-a_1; \\ \Pi_3^E(t) = v - \int_{F-2a_2-a_1}^t s(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)} dT - \int_\tau^t (M+2N)\lambda e^{-\lambda(T-\tau)} dT \\ \quad - \int_t^\infty (3M+2N)\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad if \quad F-2a_2-a_1 < t \le F-a_1, \tau < F-2a_2-a_1; \\ \Pi_4^E(t) = v - \int_\tau^t \left[s(T+2a_2+a_1-F)+M+2N\right]\lambda e^{-\lambda(T-\tau)} dT \\ \quad - \int_t^\infty \left[s(t+a_1-F)+3M+2N\right]\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad if \quad t > F-a_1, \tau \ge F-2a_2-a_1; \\ \Pi_5^E(t) = v - \int_{F-2a_2-a_1}^t s(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)} dT - \int_\tau^t (M+2N)\lambda e^{-\lambda(T-\tau)} dT \\ \quad - \int_t^\infty \left[s(t+a_1-F)+3M+2N\right]\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad if \quad t > F-a_1, \tau < F-2a_2-a_1. \end{cases} \tag{4.1}$$

Here, we use "E" to denote the exponentially distributed empty container called time. By analyzing $\Pi_2^E(t)$, $\Pi_3^E(t)$, $\Pi_4^E(t)$, $\Pi_5^E(t)$, we have the following propositions.

Proposition 1. *In the case of exponentially distributed empty container called time,*
(1) If $F - 2a_2 - a_1 + 2M/s \le F - a_1$, $\Pi_2^E(t)$ and $\Pi_3^E(t)$ are quasi-concave functions in their feasible regions, and $t = F - 2a_2 - a_1 + 2M/s$ is the maximizer;
(2) If $F - 2a_2 - a_1 + 2M/s > F - a_1$, $\Pi_2^E(t)$ and $\Pi_3^E(t)$ are increasing functions in their feasible regions, and $t = F - a_1$ is the maximizer.

Proposition 2. *In the case of exponentially distributed empty container called time,*
(1) If $M > a_2 s + s/2\lambda$, $\Pi_4^E(t)$ and $\Pi_5^E(t)$ are increasing functions in their feasible regions;
(2) If $M = a_2 s + s/2\lambda$, $\Pi_4^E(t)$ and $\Pi_5^E(t)$ are independent of t;
(3) If $M < a_2 s + s/2\lambda$, $\Pi_4^E(t)$ and $\Pi_5^E(t)$ are decreasing functions in their feasible regions.

From Propositions 1 and 2, we can summarize that in the case of exponentially distributed empty container called time, the optimal strategy of the hinterland container operator is,

$$t^*(F) = \begin{cases} \infty, & if\ M > a_2 s + s/2\lambda; \\ F - 2a_2 - a_1 + 2M/s, & if\ M < a_2 s,\ \tau \le F - 2a_2 - a_1 + 2M/s; \\ \tau, & if\ M < a_2 s,\ \tau > F - 2a_2 - a_1 + 2M/s; \\ & if\ a_2 s \le M \le a_2 s + s/2\lambda,\ \tau > F - a_1; \\ F - a_1, & if\ a_2 s \le M \le a_2 s + s/2\lambda,\ \tau \le F - a_1. \end{cases}$$

We now derive the optimal decision of the ocean carrier about the free detention time. When the empty container called time follows an exponential distribution, we have the objective function of the ocean carrier:

$$\max_{F \ge 2a_1 = 2a_2} \quad \widetilde{\Pi}^E(F) = \int_\tau^t rs(T + 2a_2 + a_1 - F)^+ \lambda e^{-\lambda(T-\tau)} dT + \int_t^\infty rs(t + a_1 - F)^+ \lambda e^{-\lambda(T-\tau)} dT$$
$$-h \int_t^\infty (T - t - a_1)^+ \lambda e^{-\lambda(T-\tau)} dT - \int_t^\infty p\lambda e^{-\lambda(T-\tau)} dT. \tag{4.2}$$

we can rewrite it to:

$$\widetilde{\Pi}^E(F) = \begin{cases} \widetilde{\Pi}_1^E(F) = -h \int_{t+a_1}^\infty (T - t - a_1)\lambda e^{-\lambda(T-\tau)} dT - \int_t^\infty p\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad if\ t \le F - 2a_2 - a_1; \\ \widetilde{\Pi}_2^E(F) = \int_\tau^t rs(T + 2a_2 + a_1 - F)\lambda e^{-\lambda(T-\tau)} dT \\ \qquad -h \int_{t+a_1}^\infty (T - t - a_1)\lambda e^{-\lambda(T-\tau)} dT - \int_t^\infty p\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad if\ F - 2a_2 - a_1 < t \le F - a_1, \tau \ge F - 2a_2 - a_1; \\ \widetilde{\Pi}_3^E(F) = \int_{F-2a_2-a_1}^t rs(T + 2a_2 + a_1 - F)\lambda e^{-\lambda(T-\tau)} dT \\ \qquad -h \int_{t+a_1}^\infty (T - t - a_1)\lambda e^{-\lambda(T-\tau)} dT - \int_t^\infty p\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad if\quad F - 2a_2 - a_1 < t \le F - a_1, \tau < F - 2a_2 - a_1; \\ \widetilde{\Pi}_4^E(F) = \int_\tau^t rs(T + 2a_2 + a_1 - F)\lambda e^{-\lambda(T-\tau)} dT + \int_t^\infty rs(t + a_1 - F)\lambda e^{-\lambda(T-\tau)} dT \\ \qquad -h \int_{t+a_1}^\infty (T - t - a_1)\lambda e^{-\lambda(T-\tau)} dT - \int_t^\infty p\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad if\quad t > F - a_1, \tau \ge F - 2a_2 - a_1; \\ \widetilde{\Pi}_5^E(F) = \int_{F-2a_2-a_1}^t rs(T + 2a_2 + a_1 - F)\lambda e^{-\lambda(T-\tau)} dT + \int_t^\infty rs(t + a_1 - F)\lambda e^{-\lambda(T-\tau)} dT \\ \qquad -h \int_{t+a_1}^\infty (T - t - a_1)\lambda e^{-\lambda(T-\tau)} dT - \int_t^\infty p\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad if\quad t > F - a_1, \tau < F - 2a_2 - a_1. \end{cases}$$

Based on the optimal strategy of the hinterland container operator, we can get the best response function of the ocean carrier as follows:

$$\widetilde{\Pi}^E(F) = \begin{cases} \widetilde{\Pi}_4^E(F)|_{t=\infty} = \int_\tau^\infty rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT, \\ \qquad\qquad if\ M > a_2 s + s/2\lambda,\ \tau \geq F-2a_2-a_1; \\ \widetilde{\Pi}_5^E(F)|_{t=\infty} = \int_{F-2a_2-a_1}^\infty rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT, \\ \qquad\qquad if\ M > a_2 s + s/2\lambda,\ \tau < F-2a_2-a_1; \\ \widetilde{\Pi}_2^E(F)|_{t=F-2a_2-a_1+2M/s} = \int_\tau^{F-2a_2-a_1+2M/s} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT \\ \quad -h\int_{F-2a_2+2M/s}^\infty (T-F+2a_2-2M/s)\lambda e^{-\lambda(T-\tau)}dT - \int_{F-2a_2-a_1+2M/s}^\infty p\lambda e^{-\lambda(T-\tau)}dT, \\ \qquad\qquad if\ M < a_2 s,\ F-2a_2-a_1 \leq \tau \leq F-2a_2-a_1+2M/s; \\ \widetilde{\Pi}_3^E(F)|_{t=F-2a_2-a_1+2M/s} = \int_{F-2a_2-a_1}^{F-2a_2-a_1+2M/s} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT \\ \quad -h\int_{F-2a_2+2M/s}^\infty (T-F+2a_2-2M/s)\lambda e^{-\lambda(T-\tau)}dT - \int_{F-2a_2-a_1+2M/s}^\infty p\lambda e^{-\lambda(T-\tau)}dT, \\ \qquad\qquad if\ M < a_2 s,\ \tau < F-2a_2-a_1; \\ \widetilde{\Pi}_2^E(F)|_{t=\tau} = -h\int_{\tau+a_1}^\infty (T-\tau-a_1)\lambda e^{-\lambda(T-\tau)}dT - \int_\tau^\infty p\lambda e^{-\lambda(T-\tau)}dT, \\ \qquad\qquad if\ M < a_2 s,\ F-2a_2-a_1+2M/s < \tau \leq F-a_1; \\ \widetilde{\Pi}_4^E(F)|_{t=\tau} = \int_\tau^\infty rs(\tau+a_1-F)\lambda e^{-\lambda(T-\tau)}dT - h\int_{\tau+a_1}^\infty (T-\tau-a_1)\lambda e^{-\lambda(T-\tau)}dT \\ \quad - \int_\tau^\infty p\lambda e^{-\lambda(T-\tau)}dT, \\ \qquad\qquad if\ M < a_2 s, \tau > F-a_1, \\ \qquad\qquad if\ a_2 s \leq M \leq a_2 s + s/2\lambda, \tau > F-a_1; \\ \widetilde{\Pi}_2^E(F)|_{t=F-a_1} = \int_\tau^{F-a_1} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT - h\int_F^\infty (T-F)\lambda e^{-\lambda(T-\tau)}dT \\ \quad - \int_{F-a_1}^\infty p\lambda e^{-\lambda(T-\tau)}dT, \quad if\ a_2 s \leq M \leq a_2 s + s/2\lambda, F-2a_2-a_1 \leq \tau \leq F-a_1; \\ \widetilde{\Pi}_3^E(F)|_{t=F-a_1} = \int_{F-2a_2-a_1}^{F-a_1} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT - h\int_F^\infty (T-F)\lambda e^{-\lambda(T-\tau)}dT \\ \quad - \int_{F-a_1}^\infty p\lambda e^{-\lambda(T-\tau)}dT, \quad if\ a_2 s \leq M \leq a_2 s + s/2\lambda, \tau < F-2a_2-a_1. \end{cases}$$

By analyzing the property of $\widetilde{\Pi}^E(F)$, we can get the following proposition.

Proposition 3. *In the case of exponentially distributed empty container called time,*
(1) The best response functions $\widetilde{\Pi}_4^E(F)|_{t=\infty}$, $\widetilde{\Pi}_5^E(F)|_{t=\infty}$, $\widetilde{\Pi}_2^E(F)|_{t=F-2a_2-a_1+2M/s}$, $\widetilde{\Pi}_4^E(F)|_{t=\tau}$, $\widetilde{\Pi}_2^E(F)|_{t=F-a_1}$ *decrease in* F.
(2) The best response function $\widetilde{\Pi}_2^E(F)|_{t=\tau}$ *is independent of* F.
(3) If $B \geq 0$, *the best response function* $\widetilde{\Pi}_3^E(F)|_{t=F-2a_2-a_1+2M/s}$ *increases with* F*; otherwise, the best response function* $\widetilde{\Pi}_3^E(F)|_{t=F-2a_2-a_1+2M/s}$ *decreases with* F*. Here,* $B = rs - rse^{2M\lambda/s} + 2rM\lambda + he^{-\lambda a_1} + p\lambda$.
(4) If $C \geq 0$, *the best response function* $\widetilde{\Pi}_3^E(F)|_{t=F-a_1}$ *increases with* F*; otherwise, the best response function* $\widetilde{\Pi}_3^E(F)|_{t=F-a_1}$ *decreases with* F*. Here,* $C = rs - rse^{2a_2\lambda} + 2rsa_2\lambda + he^{-\lambda a_1} + p\lambda$.

From the properties described in the above proposition, we have the optimal detention free time of the ocean carrier:

$$F^* = \begin{cases} \bar{F}, & if\ M < a_2 s, B > 0, \widetilde{\Pi}_4^E(F = 2a_1 + 2a_2)|_{t=\tau} < \widetilde{\Pi}_3^E(F = \bar{F})|_{t=\bar{F}-2a_2-a_1+2M/s}; \\ & if\ a_2 s \le M \le a_2 s + s/2\lambda, C > 0, \widetilde{\Pi}_4^E(F = 2a_1 + 2a_2)|_{t=\tau} < \widetilde{\Pi}_3^E(F = \bar{F})|_{t=\bar{F}-a_1}; \\ 2a_1 + 2a_2, & if\ M > a_2 s + s/2\lambda; \\ & if\ M < a_2 s, B \le 0; \\ & if\ M < a_2 s, B > 0, \widetilde{\Pi}_4^E(F = 2a_1 + 2a_2)|_{t=\tau} \ge \widetilde{\Pi}_3^E(F = \bar{F})|_{t=\bar{F}-2a_2-a_1+2M/s}; \\ & if\ a_2 s \le M \le a_2 s + s/2\lambda, C \le 0; \\ & if\ a_2 s \le M \le a_2 s + s/2\lambda, C > 0, \widetilde{\Pi}_4^E(F = 2a_1 + 2a_2)|_{t=\tau} \ge \widetilde{\Pi}_3^E(F = \bar{F})|_{t=\bar{F}-a_1}. \end{cases}$$

Here, $\bar{F}$ is the maximal value of the the free detention time, $\bar{F} > \tau + 2a_2 + a_1$.

From the optimal strategies of the hinterland container operator and the ocean carrier, we could find the following insights.

When $M < a_2 s, B > 0, \widetilde{\Pi}_4^E(F = 2a_1 + 2a_2)|_{t=\tau} < \widetilde{\Pi}_3^E(F = \bar{F})|_{t=\bar{F}-2a_2-a_1+2M/s}$ (or $a_2 s \le M \le a_2 s + s/2\lambda, C > 0, \widetilde{\Pi}_4^E(F = 2a_1 + 2a_2)|_{t=\tau} < \widetilde{\Pi}_3^E(F = \bar{F})|_{t=\bar{F}-a_1}$), the detention fee matters and the holding and opportunity cost of the ocean carrier are relatively high. In these case, $F^* = \bar{F}$ and $\tau < F^* - a_1$, which indicates that the empty container arrives at the inland terminal early. The hinterland container operator will move the empty container from the inland terminal at time $F - 2a_2 - a_1 + 2M/s$ (or at $F - a_1$) to the sea terminal such that the free detention time is exactly enjoyed and the ocean carrier has no income. Knowing about this, the ocean carrier will set the optimal free detention time as long as possible ($F^* = \bar{F}$) so as to minimize the holding and opportunity costs. Because the longer the F, the shorter time the empty container stays in the sea terminal.

When $M > a_2 s + s/2\lambda$, then $F^* = 2a_1 + 2a_2$, namely, the transportation cost between the sea terminal and inland terminal matters. In this case, the hinterland container operator will keep the empty container at the inland terminal till it is called by the consignor and move it back to the sea terminal after it is filled by the consignor. Knowing about this, the ocean carrier will set the free detention time F as short as possible so as to maximize his income from the detention.

In other cases, the detention fee matters and the holding and opportunity cost are not very high. In these cases, $F^* = 2a_1 + 2a_2$ and $\tau \ge F^* - a_1$, which implies that the empty container arrives at the inland terminal relatively late. The hinterland container operator will move the empty container from the inland terminal to the sea terminal as soon as it arrives at the inland terminal ($t^* = \tau$). Knowing about this, the ocean carrier will set the optimal free detention time as short as possible ($F^* = 2a_1 + 2a_2$) so as to maximize his income from the detention.

Based on the optimal decisions of the ocean carrier and the hinterland container operator, we can summarize their objective functions in equilibrium as follows.

The hinterland container operator's objective function in equilibrium is:

$$\Pi^E(t^*) = \begin{cases} v - \int_{\tau}^{\infty}[s(T-a_1)+M+2N]\lambda e^{-\lambda(T-\tau)}dT, \\ \quad if\ M > a_2 s + s/2\lambda; \\ v - \int_{\tau}^{\infty}[s(\tau-a_1-2a_2)+3M+2N]\lambda e^{-\lambda(T-\tau)}dT, \\ \quad if\ M < a_2 s, B \le 0; \\ \quad if\ M < a_2 s, B > 0, \widetilde{\Pi}_4^E(F=2a_1+2a_2)|_{t=\tau} \ge \widetilde{\Pi}_3^E(F=\bar{F})|_{t=\bar{F}-2a_2-a_1+2M/s}; \\ \quad if\ a_2 s \le M \le a_2 s + s/2\lambda, C \le 0; \\ \quad if\ a_2 s \le M \le a_2 s + s/2\lambda, C > 0, \widetilde{\Pi}_4^E(F=2a_1+2a_2)|_{t=\tau} \ge \widetilde{\Pi}_3^E(F=\bar{F})|_{t=\bar{F}-a_1}; \\ v - \int_{\bar{F}-2a_2-a_1}^{\bar{F}-2a_2-a_1+2M/s}[s(T+2a_2+a_1-\bar{F})]\lambda e^{-\lambda(T-\tau)}dT \\ \quad - \int_{\tau}^{\bar{F}-2a_2-a_1+2M/s}(M+2N)\lambda e^{-\lambda(T-\tau)}dT - \int_{\bar{F}-2a_2-a_1+2M/s}^{\infty}(3M+2N)\lambda e^{-\lambda(T-\tau)}dT, \\ \quad if\ M < a_2 s, B > 0, \widetilde{\Pi}_4^E(F=2a_1+2a_2)|_{t=\tau} < \widetilde{\Pi}_3^E(F=\bar{F})|_{t=\bar{F}-2a_2-a_1+2M/s}; \\ v - \int_{\bar{F}-2a_2-a_1}^{\bar{F}-a_1}[s(T+2a_2+a_1-\bar{F})]\lambda e^{-\lambda(T-\tau)}dT \\ \quad - \int_{\tau}^{\bar{F}-a_1}(M+2N)\lambda e^{-\lambda(T-\tau)}dT - \int_{\bar{F}-a_1}^{\infty}(3M+2N)\lambda e^{-\lambda(T-\tau)}dT, \\ \quad if\ a_2 s \le M \le a_2 s + s/2\lambda, C > 0, \widetilde{\Pi}_4^E(F=2a_1+2a_2)|_{t=\tau} < \widetilde{\Pi}_3^E(F=\bar{F})|_{t=\bar{F}-a_1}. \end{cases}$$

The ocean carrier's objective function in equilibrium is:

$$\widetilde{\Pi}^E(F^*) = \begin{cases} \int_{\tau}^{\infty} rs(T-a_1)\lambda e^{-\lambda(T-\tau)}dT, \\ \quad if\ M > a_2 s + s/2\lambda; \\ \int_{\tau}^{\infty} rs(\tau-a_1-2a_2)\lambda e^{-\lambda(T-\tau)}dT - h\int_{\tau+a_1}^{\infty}(T-\tau-a_1)\lambda e^{-\lambda(T-\tau)}dT \\ \quad - \int_{\tau}^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\ \quad if\ M < a_2 s, B \le 0; \\ \quad if\ M < a_2 s, B > 0, \widetilde{\Pi}_4^E(F=2a_1+2a_2)|_{t=\tau} \ge \widetilde{\Pi}_3^E(F=\bar{F})|_{t=\bar{F}-2a_2-a_1+2M/s}; \\ \quad if\ a_2 s \le M \le a_2 s + s/2\lambda, C \le 0; \\ \quad if\ a_2 s \le M \le a_2 s + s/2\lambda, C > 0, \widetilde{\Pi}_4^E(F=2a_1+2a_2)|_{t=\tau} \ge \widetilde{\Pi}_3^E(F=\bar{F})|_{t=\bar{F}-a_1}; \\ \int_{\bar{F}-2a_2-a_1}^{\bar{F}-2a_2-a_1+2M/s}[rs(T+2a_2+a_1-\bar{F})]\lambda e^{-\lambda(T-\tau)}dT \\ \quad -h\int_{\bar{F}-2a_2+2M/s}^{\infty}(T-\bar{F}+2a_2-2M/s)\lambda e^{-\lambda(T-\tau)}dT - \int_{\bar{F}-2a_2-a_1+2M/s}^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\ \quad if\ M < a_2 s, B > 0, \widetilde{\Pi}_4^E(F=2a_1+2a_2)|_{t=\tau} < \widetilde{\Pi}_3^E(F=\bar{F})|_{t=\bar{F}-2a_2-a_1+2M/s}; \\ \int_{\bar{F}-2a_2-a_1}^{\bar{F}-a_1}[rs(T+2a_2+a_1-\bar{F})]\lambda e^{-\lambda(T-\tau)}dT \\ \quad -h\int_{\bar{F}}^{\infty}(T-\bar{F})\lambda e^{-\lambda(T-\tau)}dT - \int_{\bar{F}-a_1}^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\ \quad if\ a_2 s \le M \le a_2 s + s/2\lambda, C > 0, \widetilde{\Pi}_4^E(F=2a_1+2a_2)|_{t=\tau} < \widetilde{\Pi}_3^E(F=\bar{F})|_{t=\bar{F}-a_1}. \end{cases}$$

It is easy to verify that $\frac{\partial \Pi^E(t^*)}{\partial s} < 0$ and $\frac{\partial \widetilde{\Pi}^E(F^*)}{\partial s} > 0$. Namely, the equilibrium objective value of the hinterland container operator decreases with the detention fee and the ocean carrier's equilibrium objective value increases with the detention fee. This result is reasonable and intuitive.

5 System Coordination Analysis

Till now, we have analyzed the two-stage game model involving the decisions of the ocean carrier and the hinterland container operator. In the first stage, the ocean carrier decides the free detention time by considering the hinterland container operator's best response. In the second stage, the hinterland container operator determines the best time to dispatch the empty container from the inland container terminal to the sea terminal. This two-stage process is so called merchant haulage mode, in which the hinterland container operator is in charge of the hinterland container operations. As described before, in the carrier haulage mode (centralized situation), the shipping liner is responsible for the hinterland container operations. In this section, we intend to study the carrier haulage mode so as to investigate whether the merchant haulage mode (decentralized situation) reaches system coordination.

In the centralized situation (carrier haulage mode), there is a unique decision maker who is in charge of the container operations in the hinterland and determines the best time to dispatch the empty container from the inland container terminal to the sea terminal. In practice we can assume this central planner is the ocean carrier who also controls the container operations in the hinterland. In this case, the central planner intends to maximize the profit, which is the revenue minus the transportation cost, empty container holding cost at the sea terminal and the opportunity cost of sending the empty container to the hinterland from the sea terminal.

In this section, we let $r = 1$. We denote the empty container dispatching time to the sea container terminal from the inland container terminal by the central planner as t_C. Here, the subscript "C" is utilized to denote the centralized system. The objective function of the central planner is to maximize his profit:

$$\begin{aligned}\Pi^C(t_C) = \quad & v - \int_\tau^t (M+2N)f(T)dT - \int_t^\infty (3M+2N)f(T)dT \\ & - \int_t^\infty h(T-t-a_1)^+ f(T)dT - \int_t^\infty pf(T)dT.\end{aligned} \tag{5.1}$$

In the above objective function, the second and third parts are the transportation costs of the central planner in the hinterland (refer to Figure 1). The forth part is the holding cost of the empty container at the sea terminal depot and the last part is the opportunity cost of sending the empty container back to the hinterland from the sea terminal. We can rewrite the above equation into:

$$\begin{aligned}\Pi^C(t_C) = \quad & v - \int_{\tau}^{t}(M+2N)f(T)dT - \int_{t}^{\infty}(3M+2N+p)f(T)dT \\ & - \int_{t+a_1}^{\infty} h(T-t-a_1)f(T)dT.\end{aligned} \tag{5.2}$$

Under the exponentially distributed container called time, we rewrite the objective function into:

$$\begin{aligned}\Pi^{CE}(t_C) = \quad & v - \int_{\tau}^{t}(M+2N)\lambda e^{-\lambda(T-\tau)}dT - \int_{t}^{\infty}(3M+2N+p)\lambda e^{-\lambda(T-\tau)}dT \\ & - \int_{t+a_1}^{\infty} h(T-t-a_1)\lambda e^{-\lambda(T-\tau)}dT.\end{aligned}$$

Here, the superscript "E" denotes exponential distributed container called time by the consignor. It is easy to derive that $\Pi^{CE}(t_C)$ decreases with t_C. Therefore, we have $t_C^* = \infty$. Namely, the central planner should keep the empty container at the inland terminal rather than dispatching it back to the sea terminal. In this case, the total profit of the central planner is $\Pi^{CE}(t_C^*) = v - M - 2N$, the revenue minus the transportation cost between the inland terminal and the sea terminal and the round-trip transportation cost between the inland terminal and the consignor.

Having derived the optimal decisions of the central planner under the centralized situation, we now check whether the two-stage decision system we studied reaches system coordination. Namely, we investigate whether the two-stage decision system produces the best global solution. We compare the system profit under centralized and decentralized situations.

For the exponential distribution case, given other parameters, it is easy to verify that if $M > a_2 s + s/2\lambda$, then the empty container will be kept in the inland terminal and the decentralized system reaches coordination (the system profit difference between centralized and decentralized system is zero).

We use computational study to check how system coordination changes with the parameters under the exponential distributed container called time. In the computational experiment, we set $a_1 = 2\ days$, $a_2 = 0.5\ days$, $M = \$50$, $N = \$10$, $\lambda = 5$, $s = \$260/day$, $\bar{F} = 10\ days$, $\tau = Day\ 4$, $h = \$10/day$, $p = \$200$, $r = 1$, $v = \$200$. Since the system reaches coordination when $M > a_2 s + s/2\lambda$, we focus on the case when $M \leq a_2 s + s/2\lambda$.

The computational result is shown in Figure 2. As shown in Figure 2, the more likely the empty container is called by the consignor shortly after it arrives at the inland terminal (namely λ is larger), the smaller the system profit difference between the centralized and decentralized system. When λ is large enough, the consignor calls the empty container shortly after it arrives at the inland terminal. In this case, the empty container is still at the inland terminal when it is called and the total profit of the decentralized system is $v - M - 2N$ which implies the system coordination.

In summary, the coordination of the decentralized system depends on the characteristics of the parameter. In the export-oriented hinterland area where there are much more exported cargos than imported cargos (e.g., the situation in the port hinterlands of China) and empty containers at the hinterland inland terminals are likely to be called by the consignors shortly (namely λ is large), the two-stage system is coordinated. Therefore, in the port area which is not very short of empty containers, the ocean carrier has incentive to integrate the hinterland transportation operation and run in the carrier haulage mode.

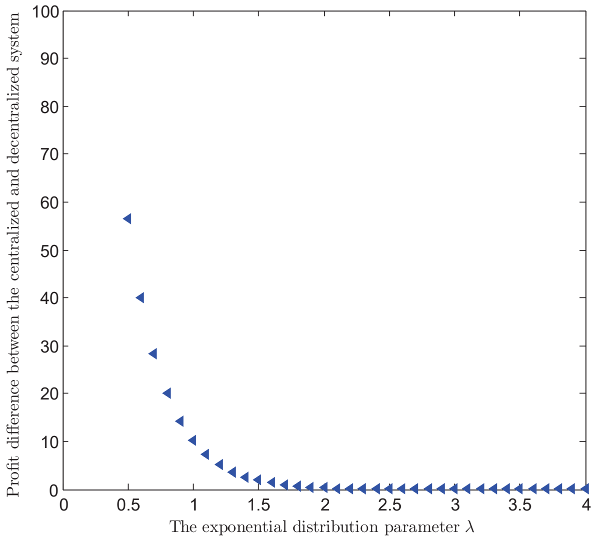


Figure 2: Sensitivity analysis about system coordination

From the above analysis, the decentralized system is not coordinated all the time. This provides theoretical support to some extend for the ocean carrier's incentive to integrate the full container supply chain and construct centralized system in the multi-modal transportation, especially in the port area that is not very short of empty containers. Our paper aims to study the decisions of the ocean carrier and the hinterland container operator in the decentralized system so as to provide reference for related decision makers.

Exploration for coordinated and practical new contract system could be future research directions.

6 Loss-averse Ocean Carrier

Recent years, customer behavior attracted attention in economics and management science research (Wang and Webster, 2009; Lee et al., 2015). Many decision makers are loss averse. Namely, decision makers prefer avoiding losses to acquiring equivalent gains. In the hinterland container transportation system we studied, we recall the objective function of the ocean carrier as follows.

$$\max_{F \geq 2a_1+2a_2} \quad \widetilde{\Pi}(F) = \int_{\tau}^{t} rs(T + a_1 + 2a_2 - F)^+ f(T)dT + \int_{t}^{\infty} rs(t + a_1 - F)^+ f(T)dT \\ -h\int_{t}^{\infty} (T - t - a_1)^+ f(T)dT - \int_{t}^{\infty} pf(T)dT.$$

In some cases (e.g., in the export-oriented hinterland area where the exported cargo volume is much higher than the imported cargo volume and empty containers are in shortage), in order to enhance the utilization rate of the empty container, the ocean carrier could be loss averse so as to avoid the holding cost and opportunity cost of the empty container in hinterland. To represent the loss aversion tendency of the ocean carrier, we assume that a loss is multiplied by a factor $\alpha \geq 1$, while a positive gain is taken as is. The larger the α, the more loss averse the ocean carrier. When $\alpha = 1$, we have the risk-neural case. Note that the loss averse ocean carrier model is not applied to the import-oriented hinterland area where the imported cargo volume is much higher than the exported cargo volume and the ocean carrier is eager to get the empty container back as soon as possible to satisfy the oversea loading areas demand.

The objective function of the ocean carrier under the loss-averse phenomenon is

$$\max_{F \geq 2a_1+2a_2} \quad \widetilde{\Pi}(F) = \int_{\tau}^{t} rs(T + a_1 + 2a_2 - F)^+ f(T)dT + \int_{t}^{\infty} rs(t + a_1 - F)^+ f(T)dT \\ -\alpha h\int_{t}^{\infty} (T - t - a_1)^+ f(T)dT - \alpha\int_{t}^{\infty} pf(T)dT. \tag{6.1}$$

By replacing p and h with αp and αh, we use computational study to analyze the impact of the loss-averse degree (α) on the ocean carrier's decision under the exponential distributed container called time. In the computational experiment, we set $a_1 = 2\ days$, $a_2 = 0.5\ days$, $M = \$50$, $N = \$10$, $\lambda = 5$, $s = \$260/day$, $\bar{F} = 10\ days$, $\tau = Day\ 4$, $h = \$10/day$, $p = \$80$, $r = 1$, $v = \$200$. As shown in Figure 3, the higher the ocean carrier's loss-averse degree, the longer free detention time he provides. When the loss-averse degree α is low, the ocean carrier cares more about the income than the holding and opportunity cost. In this case, the ocean carrier will set the free detention time as short as possible, $F^* = 2a_1 + 2a_2$, so as to maximize his income from the detention. A high loss-averse degree implies that the ocean carrier cares more about the holding and opportunity costs than the income. In this case, the hinterland container operator will dispatch the empty container back to the sea container terminal at time $F - a_1$ or $F - 2a_2 - a_1 + 2M/s$ and it will arrive at the sea terminal at the free detention time F or $F - 2a_2 + 2M/s < F$. Therefore, the ocean carrier gains no income from the detention fee. Knowing about this, the ocean carrier has incentive to set the free detention time as long as possible so as to minimize his total cost (including the holding cost and the opportunity cost).

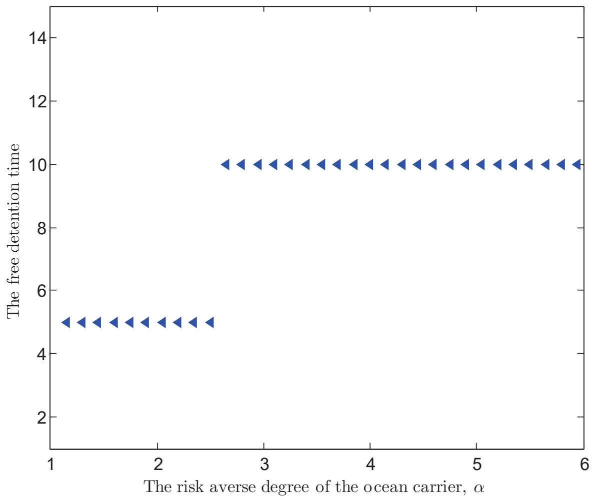


Figure 3: The impact of the ocean carrier's loss-averse degree on the free detention time

7 Sensitivity analysis

In this section, we conduct theoretical analysis and numerical studies to reveal the impact of the parameter change on the system outcomes. Firstly, we conduct computational experiments to study the effect of the empty container demand distribution on the system outcomes. Secondly, we analyze how the ocean carrier's emphasizing on the revenue affects the two players' decisions. At last, we investigate how the change of the hinterland transportation time a_1 and a_2 affects the profits of the hinterland container operator and the ocean carrier.

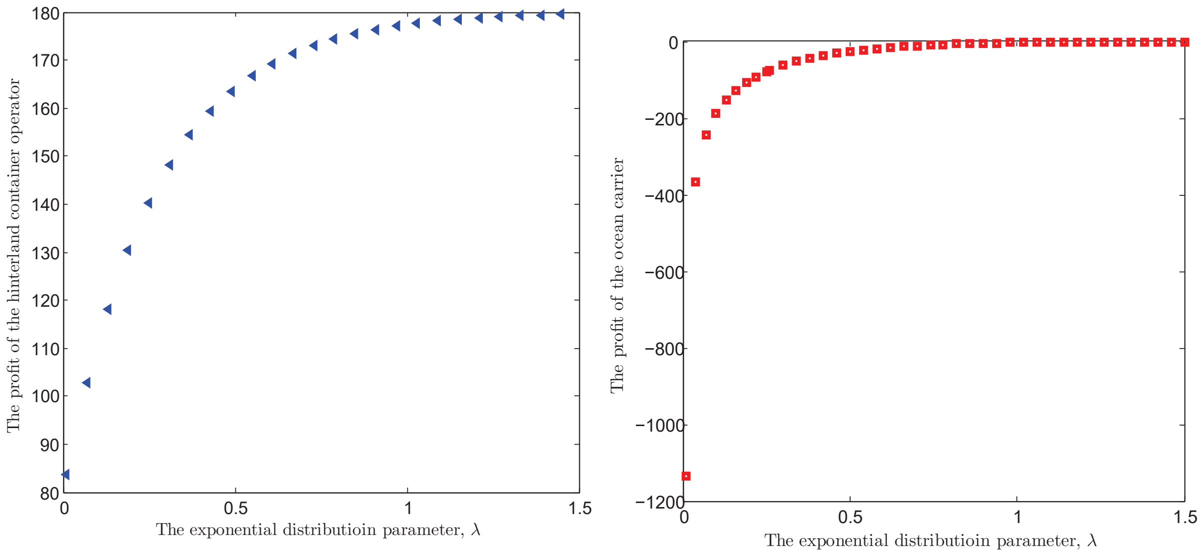
7.1 The effect of the empty container demand rate, λ

We now use numerical experiments to investigate how the change of λ (a high λ indicates that empty containers at the hinterland inland terminals are likely to be called by the consignors shortly) affects the system outcomes. We set the common parameters as in Table 2.

Table 2: Parameter setting for contract system comparison

Parameters	**Applied Value**
Transportation time between sea terminal and inland terminal, a_1	2 days
Transportation time between the consignor and inland terminal, a_2	0.5 days
The maximal free detention time, $\bar{F}$	10 days
The detention fee rate, s	\$60
Transportation cost between sea terminal and inland terminal, M	\$50
Transportation cost between the consignor and inland terminal, N	\$10
The arrival time of the empty container at the inland terminal, τ	day 4
The parameter of exponential distribution, λ	5
The empty container holding cost at the sea terminal, h	\$10/day
The empty container opportunity cost, p	\$200
The revenue of the hinterland container operator, v	\$250
The ocean carrier's emphasizing on the revenue, r	1

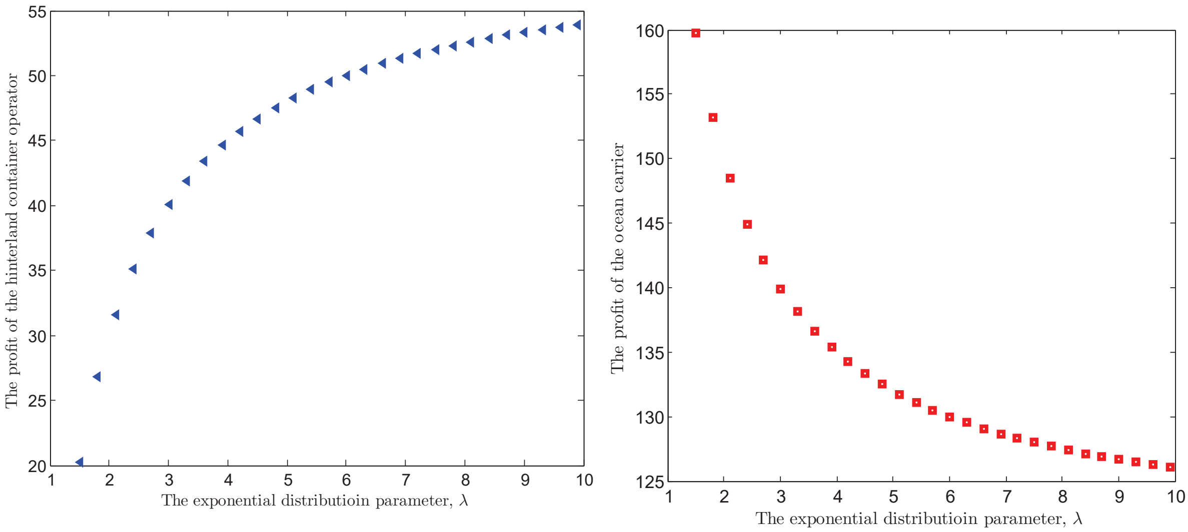
As shown in Figure 4 (a) and Figure 5 (a), the optimal profit of the hinterland container operator increases with the exponential distribution parameter λ, which indicates that the hinterland container operator's cost is lower in the export-oriented hinterland area where the empty containers will be called by the consignors shortly. As shown in Figure 4 (b), if the detention fee s is relatively high (namely $\lambda < s/2(M - a_2 s)$), the ocean carrier's optimal profit is less or equal to zero. When $\lambda < s/2(M - a_2 s)$, the empty container is dispatched from the inland terminal to the sea terminal such that the free detention time is exactly enjoyed. Therefore, the ocean carrier earns no revenue in this case and may occur holding and opportunity costs. However, if $\lambda \geq s/2(M - a_2 s)$, the hinterland container operator will keep the empty container at the inland terminal till



(a) The hinterland container operator's optimal profit

(b) The ocean carrier's optimal profit

Figure 4: The effect of exponential distribution parameter, λ ($\lambda < s/2(M - a_2 s)$)



(a) The hinterland container operator's optimal profit

(b) The ocean carrier's optimal profit

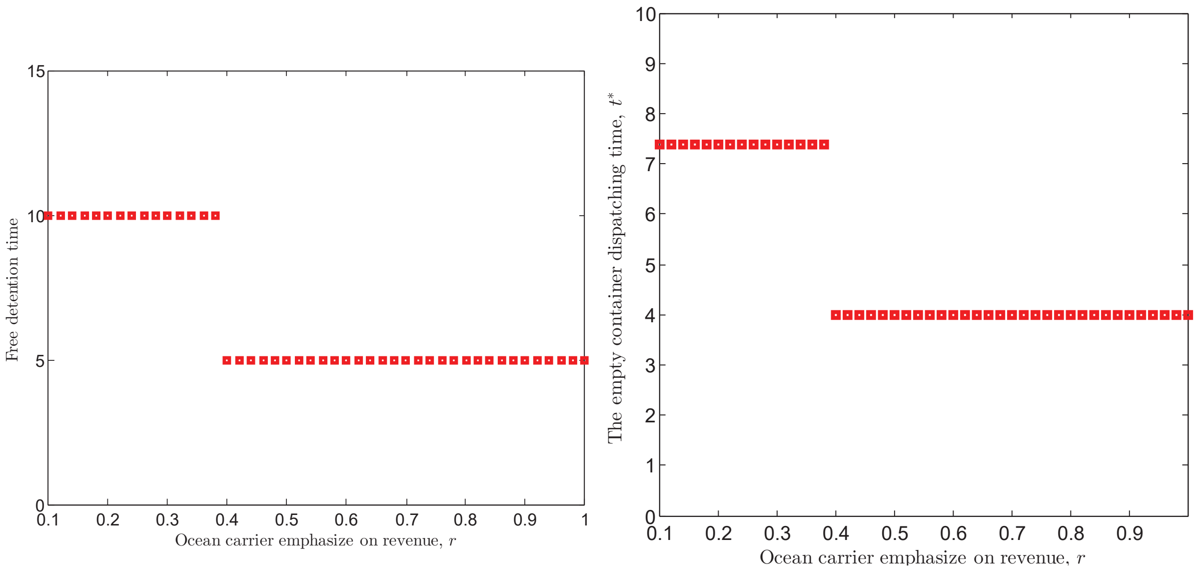
Figure 5: The effect of exponential distribution parameter, λ ($\lambda \geq s/2(M - a_2 s)$)

it is called by the consignor and move it back to the sea terminal after it is filled by the consignor. The ocean carrier will set the free detention time F as short as possible so as to maximize his income from the detention. In this case the ocean carrier earns positive profit and the profit decreases with λ (as shown in Figure 5 (b)). Namely, the more eager the consignor calls the empty container, the lower profit the ocean carrier obtains. In summary, the ocean carrier should not set the detention fee rate s too high. Otherwise, he may produce negative profit.

Figure 4 and Figure 5 reveal that, given the values of detention fee rate s, the transportation cost M and other parameters, the hinterland container operator's decision about sending the empty container back to the sea terminal (send or not, and when to send) depends on the exponential distribution parameter λ and $\lambda = s/2(M - a_2 s)$ is the critical points of the ocean carrier's and the hinterland container operator's optimal objective function (refer to the optimal objective functions in Section 4). Since the distribution parameter λ is mainly obtained by the estimation from historical data, it is critical for the hinterland container operator to get precise estimation so as to make reasonable decisions.

7.2 The effect of the ocean carrier's emphasizing on revenue, r

Based on the optimal decisions in Section 4, we further conduct sensitivity analysis about the system outcomes with the change of the ocean carrier's emphasizing on the revenue.



(a) The ocean carrier's free detention time

(b) The hinterland container operator's empty container dispatching time

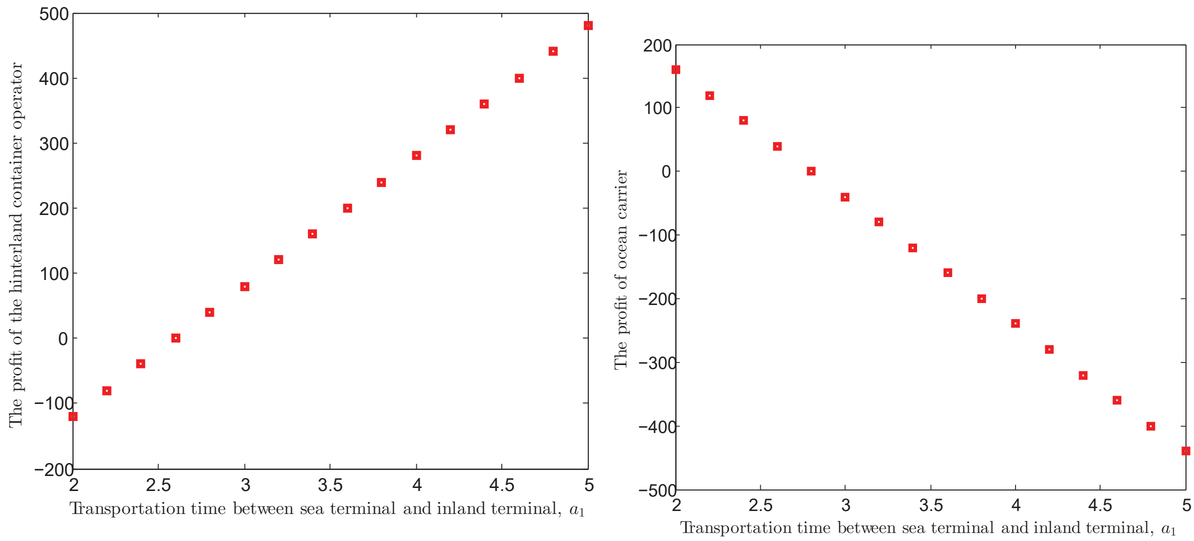
Figure 6: The effect of the ocean carrier's emphasizing on revenue, r

As shown in Figure 6 (a), the more the ocean carrier emphasizes on his revenue, the shorter free detention time he will set. With the increase of the ocean carrier's focus on his revenue, the hinterland container operator will send the empty container back to the sea terminal earlier (as shown in Figure 6 (b)). Therefore, the ocean carrier can not only increase his profit but also get the empty container back earlier by setting a higher value of r.

7.3 The effect of the hinterland transportation time a_1 and a_2

Based on the optimal decisions in Section 4, we further conduct sensitivity analysis about the system outcomes with the change of the hinterland transportation time under the exponential distributed empty container called time.

We set the common parameters as, $a_1 = 2\ days$, $a_2 = 0.5\ days$, $M = \$50$, $N = \$10$, $\tau = Day\ 4$, $\lambda = 5$, $\bar{F} = 10$, $s = \$200/day$, $h = \$5/day$, $p = \$40$.

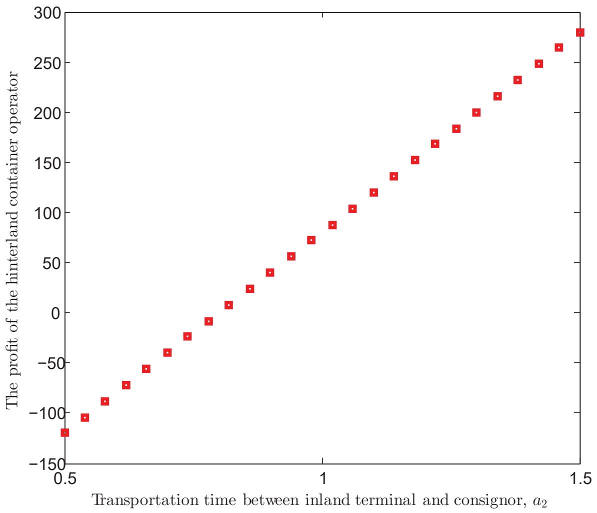


(a) The hinterland container operator's optimal profit

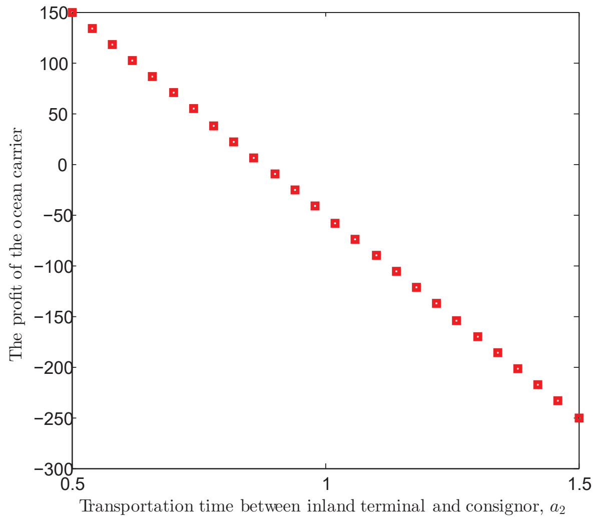
(b) The ocean carrier's optimal profit

Figure 7: The effect of transportation time between sea and inland terminal, a_1

As shown in Figure 7 (a) and Figure 8 (a), the hinterland container operator's optimal profit increases with a_1 (the transportation time between the sea container terminal and the inland container terminal) and a_2 (the transportation time between the inland terminal and the consignor). Namely, the longer time the empty container spends in the hinterland, the more profit the hinterland container operator makes. Since $F^* = 2a_1+2a_2$ or $F^* = \bar{F}$, the longer time the empty container spending in the hinterland makes the ocean carrier set longer free detention time, which makes the hinterland container operator's cost lower. As a reslut, by setting a higher free detention time, the ocean



(a) The hinterland container operator's optimal profit



(b) The ocean carrier's optimal profit

Figure 8: The effect of transportation time between inland terminal and consignor, a_2

carrier gets lower profit (as shown in Figure 7 (b) and Figure 8 (b)).

Figure 7 and Figure 8 reveal that, in the port area where the hinterland transportation system is weak such that containers spend much time in hinterland, the ocean carrier gets hurt in the detention profit.

8 Conclusions and Remarks

In this paper, we have studied the ocean carrier's decision about the container free detention time and the hinterland container operator's decision about the time to dispatch the empty container back to the sea terminal from the inland terminal. We proposed a two-stage game model to characterize the players' decisions under exponentially distributed empty container called time by the consignor. We derived the optimal decisions of the two players and provided property analysis about the optimal solutions. By checking the coordination of the system, we find that in a export-oriented port area where the empty containers in the inland terminal are likely to be called by the consignors shortly, the two-stage system is coordinated. The ocean carrier has incentive to integrate the hinterland transportation operation in the port area which is not very short of empty containers. A counter-intuitive finding is the loss-averse ocean carrier tends to provide a long free detention time so as to minimize the holding cost and opportunity cost of the empty container.

The numerical studies reveal following insights: (1) A higher detention fee rate does not guarantee higher profit for the ocean carrier. It is not recommended for the ocean carrier to set an arbitrary high detention fee rate. (2) The ocean carrier's profit may get hurt in the port area where the hinterland transportation system is weak such that containers spend too much time in hinterland.

Although we made some assumptions in the model in order to make the problem tractable, we tackled the detention decisions in the hinterland container supply chain and the analysis results should provide managerial guidance to practitioners. We are considering a new research direction about the optimal decision of the ocean carrier on the detention fee rate which will be affected by the locations of the consignees in the hinterland he serves and the hinterland empty container sharing strategy of the hinterland container operators. A possible extension of our model is to consider the utilization and the transportation starting time of barges or trains. The multi-segment price after the free detention time is also a possible future research direction.

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Appendix

Proof of Proposition 1:

Proof. Recall that the objective function of the hinterland container operator under the exponentially distributed container called time is

$$\Pi^E(t) = \begin{cases} \Pi_1^E(t) = v - \int_\tau^t (M+2N)\lambda e^{-\lambda(T-\tau)} dT - \int_t^\infty (3M+2N)\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad\qquad if \quad t \le F - 2a_2 - a_1; \\ \Pi_2^E(t) = v - \int_\tau^t \left[s(T+2a_2+a_1-F) + M + 2N\right] \lambda e^{-\lambda(T-\tau)} dT \\ \quad - \int_t^\infty (3M+2N)\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad\qquad if \quad F-2a_2-a_1 < t \le F - a_1, \tau \ge F - 2a_2 - a_1; \\ \Pi_3^E(t) = v - \int_{F-2a_2-a_1}^t s(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)} dT - \int_\tau^t (M+2N)\lambda e^{-\lambda(T-\tau)} dT \\ \quad - \int_t^\infty (3M+2N)\lambda e^{-\lambda(T-\tau)} dT, \\ \qquad\qquad if \quad F-2a_2-a_1 < t \le F - a_1, \tau < F - 2a_2 - a_1; \\ \Pi_4^E(t) = v - \int_\tau^t \left[s(T+2a_2+a_1-F) + M + 2N\right] \lambda e^{-\lambda(T-\tau)} dT \\ \quad - \int_t^\infty \left[s(t+a_1-F) + 3M + 2N\right] \lambda e^{-\lambda(T-\tau)} dT, \\ \qquad\qquad if \quad t > F - a_1, \tau \ge F - 2a_2 - a_1; \\ \Pi_5^E(t) = v - \int_{F-2a_2-a_1}^t s(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)} dT - \int_\tau^t (M+2N)\lambda e^{-\lambda(T-\tau)} dT \\ \quad - \int_t^\infty \left[s(t+a_1-F) + 3M + 2N\right] \lambda e^{-\lambda(T-\tau)} dT, \\ \qquad\qquad if \quad t > F - a_1, \tau < F - 2a_2 - a_1. \end{cases}$$

It is easy to derive that $\Pi_1^E(t)$ increases in t. The first order derivatives of $\Pi_2^E(t)$ and $\Pi_3^E(t)$ are as follows:

$$\frac{\partial \Pi_2^E(t)}{\partial t} = \frac{\partial \Pi_3^E(t)}{\partial t} = [2M - s(t + 2a_2 + a_1 - F)]\lambda e^{-\lambda(t-\tau)}.$$

By letting the first order derivatives equal to 0, we get the stationary point, $t = F - 2a_2 - a_1 + 2M/s$. We have,

$$\frac{\partial^2 \Pi_2^E(t)}{\partial t^2} = \frac{\partial^2 \Pi_3^E(t)}{\partial t^2}|_{t=F-2a_2-a_1+2M/s} = -s\lambda e^{-\lambda(t-\tau)} < 0.$$

Therefore,

(1) If $F - 2a_2 - a_1 + 2M/s \le F - a_1$, $\Pi_2^E(t)$ and $\Pi_3^E(t)$ are quasi-concave functions in their feasible regions, and $t = F - 2a_2 - a_1 + 2M/s$ is the maximizer;

(2) If $F - 2a_2 - a_1 + 2M/s > F - a_1$, $\Pi_2^E(t)$ and $\Pi_3^E(t)$ are increasing functions in their feasible regions, and $t = F - a_1$ is the maximizer. □

Proof of Proposition 2:

Proof. It is easy to derive the first order derivatives of $\Pi_4^E(t)$ and $\Pi_5^E(t)$ are as follows:

$$\frac{\partial \Pi_4^E(t)}{\partial t} = \frac{\partial \Pi_5^E(t)}{\partial t} = (2M\lambda - 2a_2 s\lambda - s)e^{-\lambda(t-\tau)}.$$

Since $e^{-\lambda(t-\tau)} > 0$, it is obvious that

(1) If $M > a_2 s + s/2\lambda$, $\Pi_4^E(t)$ and $\Pi_5^E(t)$ are increasing functions in their feasible regions;

(2) If $M = a_2 s + s/2\lambda$, $\Pi_4^E(t)$ and $\Pi_5^E(t)$ are independent of t;

(3) If $M < a_2 s + s/2\lambda$, $\Pi_4^E(t)$ and $\Pi_5^E(t)$ are decreasing functions in their feasible regions.

□

Proof of Proposition 3:

Proof. Recall that the objective function of the ocean carrier under the exponentially distributed container called time is

$$\widetilde{\Pi}^E(F) = \begin{cases} \widetilde{\Pi}_1^E(F) = -h\int_{t+a_1}^{\infty}(T-t-a_1)\lambda e^{-\lambda(T-\tau)}dT - \int_t^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\ \qquad\qquad if\ t \le F-2a_2-a_1; \\ \widetilde{\Pi}_2^E(F) = \int_{\tau}^{t} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT \\ \qquad -h\int_{t+a_1}^{\infty}(T-t-a_1)\lambda e^{-\lambda(T-\tau)}dT - \int_t^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\ \qquad\qquad if\ F-2a_2-a_1 < t \le F-a_1, \tau \ge F-2a_2-a_1; \\ \widetilde{\Pi}_3^E(F) = \int_{F-2a_2-a_1}^{t} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT \\ \qquad -h\int_{t+a_1}^{\infty}(T-t-a_1)\lambda e^{-\lambda(T-\tau)}dT - \int_t^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\ \qquad\qquad if\quad F-2a_2-a_1 < t \le F-a_1, \tau < F-2a_2-a_1; \\ \widetilde{\Pi}_4^E(F) = \int_{\tau}^{t} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT + \int_t^{\infty} rs(t+a_1-F)\lambda e^{-\lambda(T-\tau)}dT \\ \qquad -h\int_{t+a_1}^{\infty}(T-t-a_1)\lambda e^{-\lambda(T-\tau)}dT - \int_t^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\ \qquad\qquad if\quad t > F-a_1, \tau \ge F-2a_2-a_1; \\ \widetilde{\Pi}_5^E(F) = \int_{F-2a_2-a_1}^{t} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT + \int_t^{\infty} rs(t+a_1-F)\lambda e^{-\lambda(T-\tau)}dT \\ \qquad -h\int_{t+a_1}^{\infty}(T-t-a_1)\lambda e^{-\lambda(T-\tau)}dT - \int_t^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\ \qquad\qquad if\quad t > F-a_1, \tau < F-2a_2-a_1. \end{cases}$$

Based on the optimal strategy of the hinterland container operator, we can get the best response function of the ocean carrier as follows:

$$
\widetilde{\Pi}^E(F) = \begin{cases}
\widetilde{\Pi}_4^E(F)|_{t=\infty} = \int_{\tau}^{\infty} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT, \\
\qquad\qquad\qquad if\ M > a_2 s + s/2\lambda,\ \tau \geq F-2a_2-a_1; \\
\widetilde{\Pi}_5^E(F)|_{t=\infty} = \int_{F-2a_2-a_1}^{\infty} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT, \\
\qquad\qquad\qquad if\ M > a_2 s + s/2\lambda,\ \tau < F-2a_2-a_1; \\
\widetilde{\Pi}_2^E(F)|_{t=F-2a_2-a_1+2M/s} = \int_{\tau}^{F-2a_2-a_1+2M/s} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT \\
\quad -h\int_{F-2a_2+2M/s}^{\infty}(T-F+2a_2-2M/s)\lambda e^{-\lambda(T-\tau)}dT - \int_{F-2a_2-a_1+2M/s}^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\
\qquad\qquad\qquad if\ M < a_2 s,\ F-2a_2-a_1 \leq \tau \leq F-2a_2-a_1+2M/s; \\
\widetilde{\Pi}_3^E(F)|_{t=F-2a_2-a_1+2M/s} = \int_{F-2a_2-a_1}^{F-2a_2-a_1+2M/s} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT \\
\quad -h\int_{F-2a_2+2M/s}^{\infty}(T-F+2a_2-2M/s)\lambda e^{-\lambda(T-\tau)}dT - \int_{F-2a_2-a_1+2M/s}^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\
\qquad\qquad\qquad if\ M < a_2 s,\ \tau < F-2a_2-a_1; \\
\widetilde{\Pi}_2^E(F)|_{t=\tau} = -h\int_{\tau+a_1}^{\infty}(T-\tau-a_1)\lambda e^{-\lambda(T-\tau)}dT - \int_{\tau}^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\
\qquad\qquad\qquad if\ M < a_2 s,\ F-2a_2-a_1+2M/s < \tau \leq F-a_1; \\
\widetilde{\Pi}_4^E(F)|_{t=\tau} = \int_{\tau}^{\infty} rs(\tau+a_1-F)\lambda e^{-\lambda(T-\tau)}dT - h\int_{\tau+a_1}^{\infty}(T-\tau-a_1)\lambda e^{-\lambda(T-\tau)}dT \\
\quad -\int_{\tau}^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \\
\qquad\qquad\qquad if\ M < a_2 s, \tau > F-a_1, \\
\qquad\qquad\qquad if\ a_2 s \leq M \leq a_2 s + s/2\lambda, \tau > F-a_1; \\
\widetilde{\Pi}_2^E(F)|_{t=F-a_1} = \int_{\tau}^{F-a_1} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT - h\int_{F}^{\infty}(T-F)\lambda e^{-\lambda(T-\tau)}dT \\
\quad -\int_{F-a_1}^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \quad if\ a_2 s \leq M \leq a_2 s + s/2\lambda, F-2a_2-a_1 \leq \tau \leq F-a_1; \\
\widetilde{\Pi}_3^E(F)|_{t=F-a_1} = \int_{F-2a_2-a_1}^{F-a_1} rs(T+2a_2+a_1-F)\lambda e^{-\lambda(T-\tau)}dT - h\int_{F}^{\infty}(T-F)\lambda e^{-\lambda(T-\tau)}dT \\
\quad -\int_{F-a_1}^{\infty} p\lambda e^{-\lambda(T-\tau)}dT, \quad if\ a_2 s \leq M \leq a_2 s + s/2\lambda, \tau < F-2a_2-a_1.
\end{cases}
$$

The first order derivatives (and second order derivatives if necessary) of the ocean carrier's best response functions are as follows.

$$\frac{\partial \widetilde{\Pi}_4^E(F)|_{t=\infty}}{\partial F} = -\int_{\tau}^{\infty} rs\lambda e^{-\lambda(T-\tau)} < 0,$$

which indicates that $\widetilde{\Pi}_4^E(F)|_{t=\infty}$ decreases with F.

$$\frac{\partial \widetilde{\Pi}_5^E(F)|_{t=\infty}}{\partial F} = -\int_{F-2a_2-a_1}^{\infty} rs\lambda e^{-\lambda(T-\tau)} < 0,$$

which indicates that $\widetilde{\Pi}_5^E(F)|_{t=\infty}$ decreases with F.

$$\frac{\partial \widetilde{\Pi}_2^E(F)|_{t=F-2a_2-a_1+2M/s}}{\partial F} = (rs + 2rM\lambda + he^{-\lambda a_1} + p\lambda)e^{-\lambda(F-2a_2-a_1+2M/s-\tau)} - rs,$$

$$\frac{\partial^2 \widetilde{\Pi}_2^E(F)|_{t=F-2a_2-a_1+2M/s}}{\partial F^2} = -\lambda(rs + 2rM\lambda + he^{-\lambda a_1} + p\lambda)e^{-\lambda(F-2a_2-a_1+2M/s-\tau)} < 0,$$

which indicates that $\widetilde{\Pi}_2^E(F)|_{t=F-2a_2-a_1+2M/s}$ deceases in his feasible region and $F = \tau + 2a_2 + a_1 - 2M/s$ is its maximizer.

$$\frac{\partial \widetilde{\Pi}_3^E(F)|_{t=F-2a_2-a_1+2M/s}}{\partial F} = (rs - rse^{2M\lambda/s} + 2rM\lambda + he^{-\lambda a_1} + p\lambda)e^{-\lambda(F-2a_2-a_1+2M/s-\tau)},$$

which indicates that, if $B = rs - rse^{2M\lambda/s} + 2rM\lambda + he^{-\lambda a_1} + p\lambda \geq 0$, $\widetilde{\Pi}_3^E(F)|_{t=F-2a_2-a_1+2M/s}$ increases with F; otherwise, $\widetilde{\Pi}_3^E(F)|_{t=F-2a_2-a_1+2M/s}$ decreases with F.

$$\frac{\partial \widetilde{\Pi}_2^E(F)|_{t=\tau}}{\partial F} = 0,$$

which indicates that, $\widetilde{\Pi}_2^E(F)|_{t=\tau}$ is independent of F.

$$\frac{\partial \widetilde{\Pi}_4^E(F)|_{t=\tau}}{\partial F} = -\int_{\tau}^{\infty} rs\lambda e^{-\lambda(T-\tau)} < 0,$$

which indicates that, $\widetilde{\Pi}_4^E(F)|_{t=\tau}$ decreases with F.

$$\frac{\partial \widetilde{\Pi}_2^E(F)|_{t=F-a_1}}{\partial F} = (rs + 2a_2 rs\lambda + he^{-\lambda a_1} + p\lambda)e^{-\lambda(F-a_1-\tau)} - rs,$$

$$\frac{\partial^2 \widetilde{\Pi}_2^E(F)|_{t=F-a_1}}{\partial F^2} = -\lambda(rs + 2a_2 rs\lambda + he^{-\lambda a_1} + p\lambda)e^{-\lambda(F-a_1-\tau)} < 0,$$

which indicates that $\widetilde{\Pi}_2^E(F)|_{t=F-a_1}$ deceases in his feasible region and $F = \tau + a_1$ is its maximizer.

$$\frac{\partial \widetilde{\Pi}_3^E(F)|_{t=F-a_1}}{\partial F} = (rs - rse^{2a_2\lambda} + 2rsa_2\lambda + he^{-\lambda a_1} + p\lambda)e^{-\lambda(F-a_1-\tau)},$$

which indicates that, if $C = rs - rse^{2a_2\lambda} + 2rsa_2\lambda + he^{-\lambda a_1} + p\lambda \geq 0$, $\widetilde{\Pi}_3^E(F)|_{t=F-a_1}$ increases with F; otherwise, $\widetilde{\Pi}_3^E(F)|_{t=F-a_1}$ decreases with F. □