Effective second moment of area of uniform built-up members

Effective second moment of area of uniform built-up members
Effektive Flächenträgheitsmomente gleichförmiger mehrteiliger Stäbe

Prof. ir. H.H. Snijder

Eindhoven University of Technology, Department of the Built Environment, Structural Design, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

Summary

For built-up compression members, Eurocode 3 in part EN 1993-1-1 [1] includes the shear stiffness to account for the effect of shear deformations directly into the amplification factor when determining second-order forces. Built-up members loaded in bending are not treated at all in EN 1993-1-1 [1]. This article is based on a study [2] performed at Eindhoven University of Technology and introduces the concept of effective second moment of area to account for the influence of the finite shear stiffness of built-up members. This article is largely based on [3] but is an updated and improved version. The effective second moment of area is checked by numerical models. The shear stiffness is derived for a simple configuration of a battened built-up member and for more complex configurations and for laced built-up columns shear stiffnesses are presented. The design rules for built-up compression members in EN 1993-1-1 [1] are analysed and suggestions for modifications are made to include the concept of the effective second moment of area. It is advantageous that the concept of the effective second moment of area can be used for both built-up columns and beams. For built-up beams, the effective second moment of area including the shear stiffness allows the deflections to be calculated with the standard rules for flexural bending.

Zusammenfassung

Bei mehrteiligen Druckstäben wird im Eurocode 3 Teil 1-1 [1] der Einfluss der Schubverformungen bei der Berechnung der Schnittgrößen nach Theorie II. Ordnung in einem Vergrößerungsfaktor berücksichtigt. Mehrteilige Bauteile unter reiner Biegebeanspruchung werden mit dieser Norm nicht erfasst. Der vorliegende Artikel basiert auf einer Studie an der Eindhoven University of Technology. Es wird das Konzept der effektiven Flächenträgheitsmomente vorgestellt, mit dem der Einfluss der endlichen Schubsteifigkeit mehrteiliger Stäbe erfasst wird. Der Artikel basiert größtenteils auf [3], entspricht jedoch einer aktualisierten und überarbeiteten Fassung. Die Formeln zur Berechnung der effektiven Flächenträgheitsmomente wurden mit Hilfe numerischer Modelle überprüft. Die Schubsteifigkeiten werden für einfache und komplexere Ausführungen von Rahmenstäben sowie für Gitterstäbe abgeleitet. Die Bemessungsregeln für mehrteilige Druckstäbe in EN 1993-1-1 werden untersucht und Empfehlungen zur Anpassung an das Konzept der effektiven Flächenträgheitsmomente erarbeitet. Es ist vorteilhaft, dass dieses Konzept sowohl bei mehrteiligen Stützen als auch Trägern angewendet werden kann. Bei letzteren können die Schnittgrößen und Verformungen mit den Formeln berechnet werden, die für reine Biegeverformungen abgeleitet wurden.

Keywords: built-up, beams, columns, members, shear stiffness, second moment of area
Keywords: Mehrteilig, Träger, Stützen, Bauteile, Schubsteifigkeit, Trägheitsmoment

1 Introduction

Uniform built-up members consist of two or more steel sections connected at a constant spacing. The connected sections are called chords. These chords act together to form more or less one cross-section by connecting them discretely at a limited number of points through coupling elements. Depending on their shape and the way of force transfer, battened and laced built-up members are distinguished, Fig. 1.

In the past, built-up steel members were applied due size limits of available sections. It was necessary to build up heavily loaded columns using chords interconnected by lacings or battens. In this way, a member section was constructed with material at a certain distance from the neutral axis to obtain improved stiffness and strength properties of the section using the scarce and valuable material effectively. This resulted in built-up steel members with an industrial look based on craftsmanship: built-up members with often riveted connections between chords on the one hand and battens and lacings on the other. Nowadays, in modern steel construction, a revival of built-up members can be observed: hollow sections as chords and as battens and lacings, interconnected by welding, are used for architectural reasons if an open column is required combined with a smooth appearance. Of course, this type of columns is also applied when the columns are heavily loaded for their favourable stiffness and strength properties.

The chords can be interconnected in two different ways: either by battens or by lacings. An example of both ways is shown in Fig. 1. In this figure the distance a is the module length between the positions where the chords are connected and h_0 is the distance between the neutral axes of the chords. The member x-axis is chosen in length direction. The z-axis of these members is the so-called ‘material-free axis’. This axis does not intersect any material of the chords. With respect to flexural buckling around the material-free axis, the member behaves as a built-up member where the chords do not act fully together in the cross-section

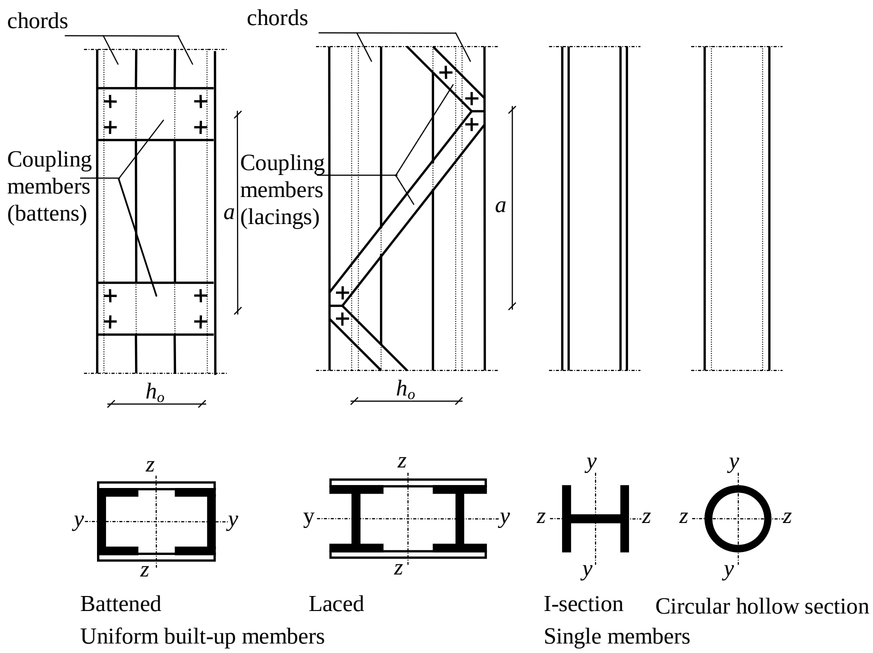


Figure 1: Uniform built-up members (left) and single members (right)
Bild 1: Gleichförmige mehrteilige (links) und einteilige Stäbe (rechts)

due to finite shear stiffness. The y-axis does intersect the material of the chords and is called the 'material axis'. With respect to flexural buckling around the material axis the member behaves as a single member where the chords act fully together in the cross-section.

If the battens or lacings between the chords would not deform, the second moment of area of the built-up member would increase with increasing distance h_0 between the chords. Then, with the same amount of chord material, a greater bending stiffness can be achieved.

In Fig. 1, also single members are shown. Both principal axes are material axes. The shear force in an I-section loaded in bending around the y-axis is largely carried by the web. In general, the shear stiffness of the web is substantial and the associated shear deformations are negligibly small such that plane sections remain plane when bent: Bernoulli's hypothesis applies. Therefore, shear deformations are not accounted for, only deformations due to bending are taken into account.

In bending around the material-free axis of a built-up member the web is not fully present. The shear force needs to be transferred by the battens or lacings. A continuous connection between the chords is not present, as is the case for a single I-section where the web is continuously connected to the flanges. For this reason, in built-up members, the shear deformations cannot be neglected. Bernoulli's hypothesis no longer applies. The stiffer the battens or lacings behave, and the more battens and lacings there are, the smaller the shear deformations are. Then, the extent to which the chords act together in the cross-section increases.

2 Effective second moment of area

In this article, the effect of finite shear stiffness and so the presence of shear deformations, is incorporated in the second moment of area of the built-up member. This second moment of area including the effect of shear deformations is called the effective second moment of area. This allows for taking into account the extent to which the chords act together in the cross-section not only for built-up members in compression but also for built-up members in bending.

Using the effective second moment of area I_{eff} has a number of advantages. The effective second moment of area can be used in existing design rules for deflection to take shear deformations implicitly into account. The effective second moment of area also indicates the effectiveness of modifications if the bending stiffness needs to be increased. The effective second moment of area can be compared to the second moment of area for the chords acting fully together in the cross-section neglecting the shear deformations, I_1. If I_{eff} and I_1 are of the same order of magnitude, the effective second moment of area can be increased by choosing heavier chords or increasing the distance between the chords. However, increasing the number of battens or lacings or increasing their size has limited effect since the extent to which the chords act together is already almost maximum. If it is chosen to increase the distance between the chords, the shear deformations increase as well so it may be necessary then to increase the number or size of battens or lacings. If the difference between I_{eff} and I_1 is substantial, increasing the number and/or size of the battens and lacings is useful to reduce the influence of shear deformations.

3 Built-up members in bending

The effective second moment of area I_{eff} can be determined by adding the deformations due to bending and shear and setting the result equal to the bending deformation of a beam with an effective second moment of area I_{eff}. From this equation, I_{eff} can be solved. For a simply supported beam with span L, loaded by a uniformly distributed load q, the deflection at mid span is:

$$f = f_b + f_v = \frac{5qL^4}{384EI_1} + \frac{qL^2}{8S_v} \tag{1}$$

In this equation the first term represents the deflection due to bending and the second term the deflection due to shear. The Young's modulus of steel is denoted as E and the shear stiffness of the built-up bending member as S_v. The second moment of area for the chords acting fully together in the cross-section when the shear stiffness is infinite is:

$$I_1 = 0{,}5h_0^2A_{ch} + 2I_{ch} \tag{2}$$

Where A_{ch} and I_{ch} are the area and the second moment of area of one chord respectively. The deflection at mid span can be expressed as a function of I_{eff} instead of I_1 and S_v as follows:

$$f = \frac{5qL^4}{384EI_{eff}} \tag{3}$$

Setting the Eqns. (1) and (3) equal results in the following expression for I_{eff}:

$$I_{eff} = \frac{I_1}{1 + \frac{48}{5}\frac{EI_1}{S_vL^2}} \tag{4}$$

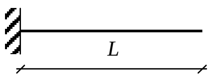
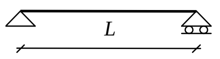
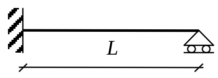
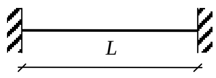
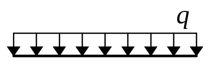
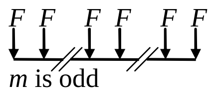
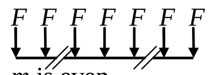
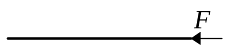
The effective second moment of area is determined based on the deflection at mid span. For the case considered of a simply supported beam loaded by a uniformly distributed load the reference point is at mid span where the deflection is maximum and where therefore the deflection has to be determined. In general the following equation holds for the effective second moment of area:

$$I_{eff} = \frac{I_1}{1 + \alpha\frac{EI_1}{S_vL^2}} \tag{5}$$

The factor α depends on the load case and the support conditions of the built-up member. For standard load cases and supports the factor α for built-up members loaded in bending is given in Table 1 (first four rows) [2]. In Table 1, m is the number of modules of the built-up member and F is a concentrated load. The factor α in Table 1 is given for the reference point. For simply supported beams and for beams fully clamped at both sides, the reference point is located at mid span where the deflection is maximum. For beams which are clamped at one side and have a roller support at the other, also mid span is taken as reference point, despite that the defection is not maximum here. For cantilever beams, the reference point is the free beam end where the deflection is maximum.

Table 1: Factor α for standard load cases and support conditions

Tabelle 1: Faktor α für häufige Lastfälle und Lagerungsbedingungen

$\boldsymbol{\alpha}$				
	4	$\frac{48}{5}$	36	48
 m is odd	$\frac{12m}{3m+1}$	$\frac{48m^2}{5m^2+1}$	$\frac{72m^2}{2m^2+1}$	$\frac{48m^2}{m^2+1}$
 m is even	$\frac{12m}{3m+1}$	$\frac{48m^2}{5m^2-4}$	$\frac{72m^2-96m+24}{2m^2-1}$	48
	$\frac{\pi^2}{4}$	π^2	$2\pi^2$	$4\pi^2$

4 Built-up members in compression

4.1 Effective second moment of area

The effective second moment of area for built-up compression members is determined using the elastic critical force for built-up compression members, as originally derived by Engesser in 1891 [4, 5]. The elastic critical force of a simply supported column in compression with length L is:

$$N_{cr} = \frac{\pi^2 EI_1}{L^2 + \frac{\pi^2 EI_1}{S_v}} \tag{6}$$

Eqn. (6) can be rewritten as follows:

$$\frac{1}{N_{cr}} = \frac{1}{N_{cr,1}} + \frac{1}{S_v} \tag{7}$$

Where:

$$N_{cr,1} = \frac{\pi^2 EI_1}{L^2} \tag{8}$$

The elastic critical force using I_{eff} is:

$$N_{cr} = \frac{\pi^2 EI_{eff}}{L^2} \tag{9}$$

The expression for I_{eff} is now obtained by setting the Eqns. (6) and (9) equal, resulting in:

$$I_{eff} = \frac{I_1}{1 + \pi^2 \frac{EI_1}{S_v L^2}} \tag{10}$$

The equation for I_{eff} for a built-up compression member has the same format as for a built-up bending member, compare Eqns. (10) and (5), but now the factor α equals π^2 This holds true for a simply supported column with buckling length L_{cr} equal to L. Other support conditions result in other values for the factor α, see Table 1 (fifth row). As an example, for $L_{cr} = 2L$ (last column in Table 1) it holds that $\pi^2/L_{cr}^2 = \pi^2/4L^2$ and so $\alpha = \pi^2/4$ The factor α can be written for compression members as $\alpha = (\pi L/L_{cr})^2$. The influence of the buckling length is taken into account in the factor α. The effective second moment of area can be used in Eqn. (9) to obtain the elastic critical force of a built-up column.

4.2 Effective buckling length

As an alternative to taking the effect of the shear stiffness into account in the (effective) second moment of area, its effect can also be taken into account in the (effective) buckling length. Eqn. (6) is then set equal to:

$$N_{cr,1} = \frac{\pi^2 EI_1}{L_{cr,eff}^2} \tag{11}$$

This results in:

$$L_{cr,eff} = \sqrt{L^2 + \frac{\pi^2 EI_1}{S_v}} \tag{12}$$

5 Shear stiffness

To determine the effective second moment of area I_{eff}, the shear stiffness S_v of the built-up member must be known.

In case of battened built-up members, the shear stiffness depends on the stiffness of the chords, the battens and their connections. In Fig. 2 a battened built-up member is schematically shown.

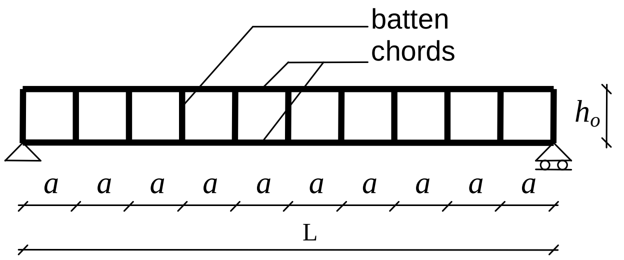


Figure 2: Battened built-up member
Bild 2: Rahmenstab

The shear stiffness can be determined by considering a module of the built-up member with length a, see Fig. 3. In Fig. 3, I_b is the second moment of area of a batten and the angle γ represents the shear strain. This module is subsequently loaded by a shear force V. It is assumed that:

- the points of inflection in the chords, where the bending moments are zero, are located at distance $a/2$ from the batten;
- a point of inflection is present half way the batten;
- the connections between chords and batten are rigid
- the shear force V is equally divided over the two chords.

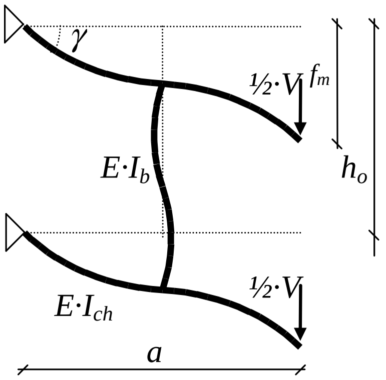


Figure 3: Module of a built-up member loaded by shear force V
Bild 3: Durch Schub V beanspruchter Abschnitt eines Rahmenstabes

Due to the shear force V, the module of the built-up member deforms. To reduce the complexity of the equations, only the bending deformations in chords and batten are is considered. The deformation f_m of the module of Fig. 3 is then:

$$f_m = \frac{Va^3}{24EI_{ch}} + \frac{Va^2h_0}{12EI_b} \tag{13}$$

The bending deformations in chords and batten result in the shear strain:

$$\gamma = \frac{f_m}{a} = \frac{Va^2}{24EI_{ch}} + \frac{Vah_0}{12EI_b} \tag{14}$$

The shear stiffness is obtained by dividing the shear force V by the shear strain resulting in:

$$S_v = \frac{V}{\gamma} = \frac{1}{\frac{a^2}{24EI_{ch}} + \frac{ah_0}{12EI_b}} = \frac{24EI_{ch}}{a^2\left[1 + \frac{2I_{ch}h_0}{I_b a}\right]} \tag{15}$$

Eqn. (15) corresponds with the left part of Eqn. (6.73) of EN 1993-1-1 [1] if there the number of planes of battens is taken as $n = 1$. If also shear deformations in chords and battens are taken into account and the rotations in the connections between them, the general Eqn. (16) for the share stiffness can be derived [2], based on the model of Fig. 4:

$$S_v = \frac{1}{\frac{(a-r_b)^3}{24aEI_{ch}} + \frac{a(h_0-r_{ch})^3}{12h_0^2nEI_b} + \frac{(a-r_b)}{2aGA_{w,ch}} + \frac{a(h_0-r_{ch})}{h_0^2nGA_{w,b}} + \frac{a(h_0-r_{ch})^2}{2h_0^2nC}} \tag{16}$$

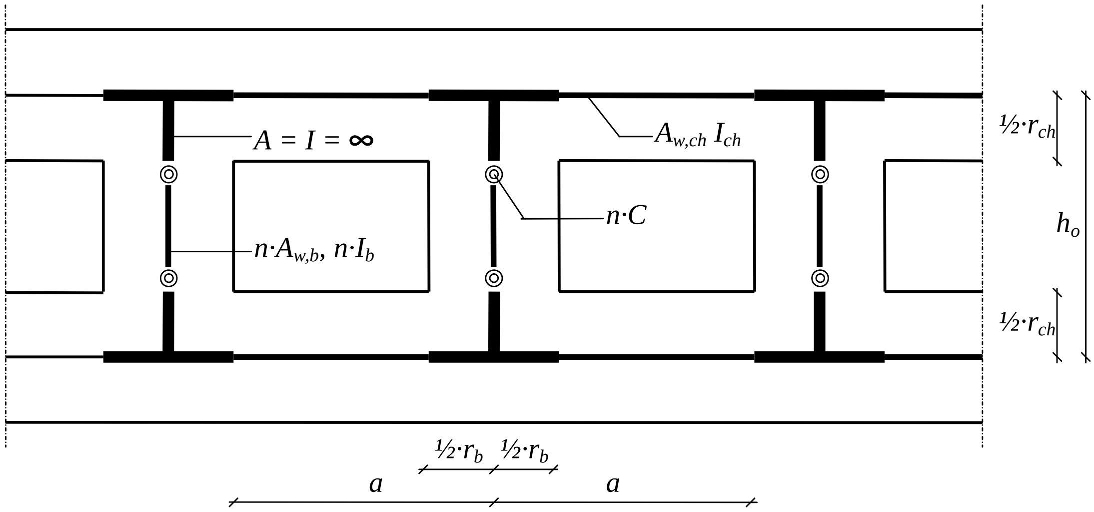


Figure 4: Model with symbols used
Bild 4: Verwendetes Berechnungsmodell mit Formelzeichen

Where:

r_b	width of the batten (see Fig. 4);
r_{ch}	width of the chord (see Fig. 4);
n	number of planes of battens (see Fig. 5);
$A_{w,ch}$	shear area of the chord;
$A_{w,b}$	shear area of the batten;
C	rotational stiffness of the connection between batten and chord (see Fig. 4);
G	shear modulus for which:

$$G = \frac{E}{2(1+\nu)} \tag{17}$$

ν Poisson's ratio

n=2 *n=1*

Figure 5: Factor n
Bild 5: Faktor n

In Eqn. (16), the first two terms in the denominator represent the bending stiffnesses of the chords and the battens respectively, the third and the fourth term take the influence of the shear stiffnesses of chords and battens respectively into account and the fifth term is for the rotational stiffness of the connections between chords and battens. The part r_{ch} of the chord has a larger stiffness and is taken infinitely stiff in the model of Fig. 4. The same is assumed for the part r_b of the batten. Note that if in Eqn. (16) the rotational stiffness C is taken equal to infinity, the brace and chord widths r_{ch} and r_b are taken as zero, the shear stiffnesses of braces and chords GA are infinite and the number of planes of battens n is one, Eqn. (16) reduces to Eqn. (15). If in Eqn. (16) the rotational stiffness C is taken equal to infinity and the brace and chord widths r_{ch} and r_b are taken as zero, Eqn. (16) reduces to:

$$S_v = \frac{1}{\frac{a^2}{24EI_{ch}} + \frac{ah_0}{12nEI_b} + \frac{1}{2GA_{w,ch}} + \frac{a}{h_0 nGA_{w,b}}} \tag{18}$$

Eqn. (18) can be rewritten as follows:

$$S_v = \frac{24EI_{ch}}{a^2\left[1 + \frac{2I_{ch}h_0}{nI_b a} + \frac{12I_{ch}}{A_{w,ch}a^2}\frac{E}{G} + \frac{24I_{ch}}{nA_{w,b}ah_0}\frac{E}{G}\right]} \tag{19}$$

If now the shear stiffness of the chords is taken infinite, Eqn. (19) reduces to:

$$S_v = \frac{24EI_{ch}}{a^2\left[1 + \frac{2I_{ch}h_0}{nI_b a} + \frac{24I_{ch}}{nA_{w,b}ah_0}\frac{E}{G}\right]} \tag{20}$$

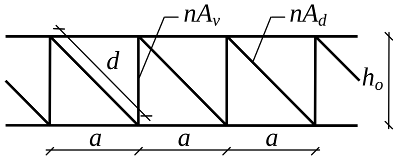
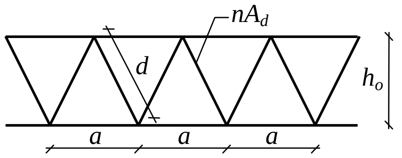
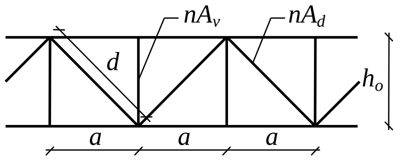
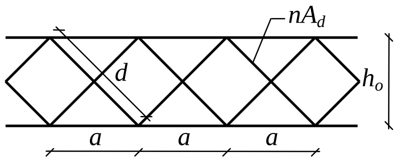
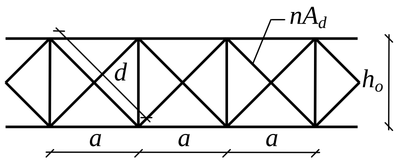
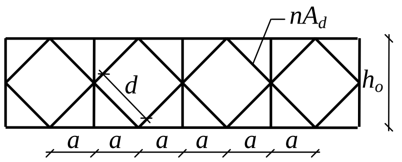
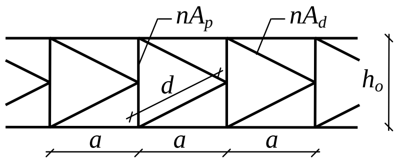
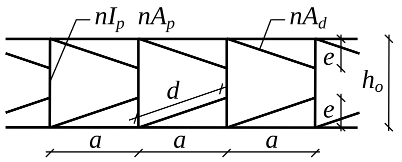
Eqn. (20) will be included in the next version of EN 1993-1-1 and takes the finite bending stiffness of chords and battens into account as well as the finite shear stiffness of the battens.

In case of laced built-up members, the shear stiffness depends on the stiffness of the chords and the lacings. The derivation occurs in a similar manner as for the battened built-up members above. For laced built-up members the shear stiffnesses were determined for different lacing configurations in amongst others [3-7]. These shear stiffnesses are shown in Table 2, where:

d length of a diagonal;
A_d area of the cross-section of a diagonal;
e eccentricity of the connection of the diagonal;
A_p area of the cross-section of a post (transverse element);
I_p second moment of area of a post (transverse element).

The shear stiffnesses for the first three rather common lacing configurations of Table 2 are included in EN 1993-1-1 [1]. The shear stiffnesses for the more complicated subsequent five lacing configurations of Table 2 are not given in EN 1993-1-1 [1] and are added here for convenience.

Table 2: Shear stiffnesses S_v for laced built-up members
Tabelle 2: Schubsteifigkeiten S_v von Gitterstäben

Lacing configuration	Shear stiffness S_v
	$\frac{nEA_d a h_0^2}{d^3\left[1+\frac{A_d h_0^3}{A_v d^3}\right]}$
	$\frac{nEA_d a h_0^2}{2d^3}$
	$\frac{nEA_d a h_0^2}{d^3}$
	$\frac{2nEA_d a h_0^2}{d^3}$
	$\frac{nEA_d a h_0^2}{d^3}$
	$\frac{nEA_d a h_0^2}{2d^3}$
	$\frac{nEA_d a h_0^2}{2d^3\left(1+\frac{h_0^3 A_d}{d^3 A_p}\right)}$
	$\frac{2nEa}{\frac{d^3}{rA_d}+\frac{r}{A_p}+\frac{a^2(h_0-2e)^2}{12h_0 I_p}}$

6 Comparative calculations

6.1 Bending

Using the Finite Element Method, the theoretically derived effective second moment of area including the effect of shear stiffness is checked. Fig. 6 shows the FEM model for the built-up beams in bending. The model consists of beam elements. They all have the section properties of an HE100A section. Shear deformation is not included in the beam elements used. The distance h_0 between the neutral axes of the chords is 500mm and the module length a is 1000mm. The number of modules m was varied to obtain different lengths. The minimum number according to EN 1993-1-1 [1] is three.

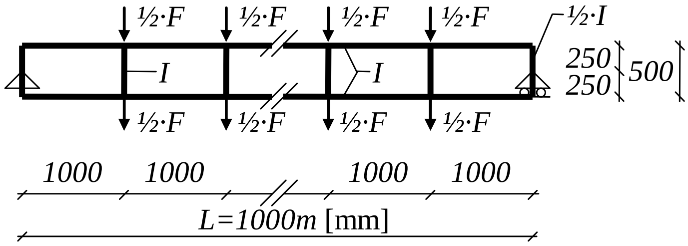


Figure 6: FEM model for a built-up beam in bending
Bild 6: FEM-Modell für einen mehrteiligen Träger unter Biegebeanspruchung

Point loads have been applied to both chords as indicated in Fig. 6. From the calculated deflection f_{FEM} at mid span, the second moment of area can be determined as follows:

$$I_{FEM}=\frac{\beta F L^3}{384 E f_{FEM}} \tag{21}$$

Where $\beta = (5m^4 - 4m^2 - 1)/m^3$ for m is odd and $\beta = (5m^2 - 4)/m$ for m is even.

This numerically determined effective second moment of area is compared with the theoretically derived effective second moment of area of Eqn. (5). The FEM model only considers bending deformations of the chords and battens so that Eqn. (15) applies for the shear stiffness. In Fig. 7, the second moment of area is shown on the vertical axis against the beam length on the horizontal axis. If the beam length increases the effective second moment of area increases. This is because at increasing length the influence of the shear stiffness decreases and bending dominates, see Eqn. (1). The effective second moment of are approaches for longer beam lengths to the second moment of area I_1 of Eqn. (2) for the chords acting fully together in the cross-section. Fig. 7 shows that the agreement between numerical and theoretical effective second moment of area is well. Agreement is better for longer than for shorter beams. This can be explained as follows. For short beams, in the limit of the length approaching to zero, Eqn. (5) modifies into:

$$I_{eff} = \frac{S_v L^2}{\alpha E} \tag{22}$$

Only the shear stiffness plays a role now and the shear stiffness is therefore dominant for shorter lengths. In the deflection shape the shear deformations dominate. The middle batten in Fig. 6 remains straight in case of an even number of modules. If this number is odd, the module in the middle does not act in shear. In both cases the model therefore acts stiffer than according to theory, resulting in a greater numerical than theoretical effective second moment of area. This effect is stronger for short beams.

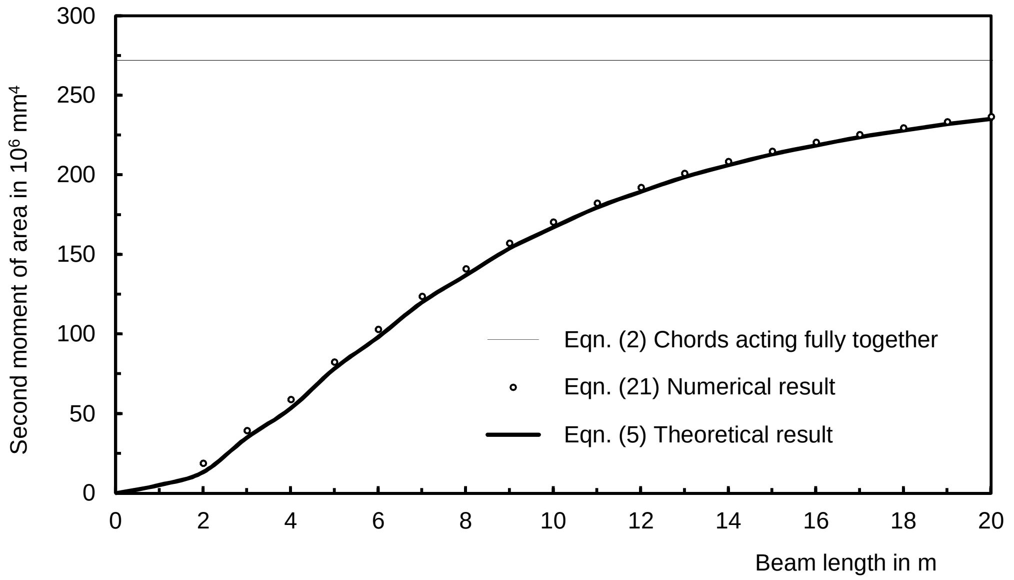


Figure 7: Effective second moment of area versus beam length - comparative calculations for the beam of Fig. 6 with constant module length

Bild 7: Effektives Flächenträgheitsmoment in Abhängigkeit von der Trägerlänge – Vergleichsrechnung für den Träger nach Bild 6 mit konstanter Abschnittslänge

6.2 Compression

The model of Fig. 8 is used to determine the elastic critical force by FEM running a linear buckling analysis. The same data as mentioned for the bending model applies.

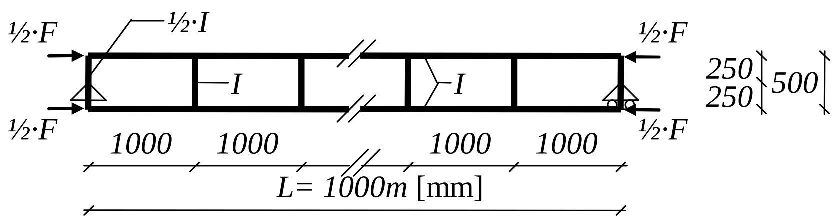


Figure 8: FEM model for a built-up column in compression

Bilde 8: FEM-Modell für einen Rahmenstab unter Druckbeanspruchung

In Fig. 9 the elastic critical force is shown on the vertical axis against the column length. The numerically determined elastic critical force is compared to the theoretically derived one of Eqn. (9) using Eqn. (10) for the effective second moment of area and Eqn. (15) for the shear stiffness. The elastic critical force decreases with increasing column length. The agreement between the numerical and the theoretical results is good for greater lengths. For shorter lengths the theoretical result is substantially lower than the numerical results. For longer lengths, the elastic critical force approaches the value valid for the chords acting fully together, Eqn. (8). The influence of the finite shear stiffness of shorter built-up columns is substantial.

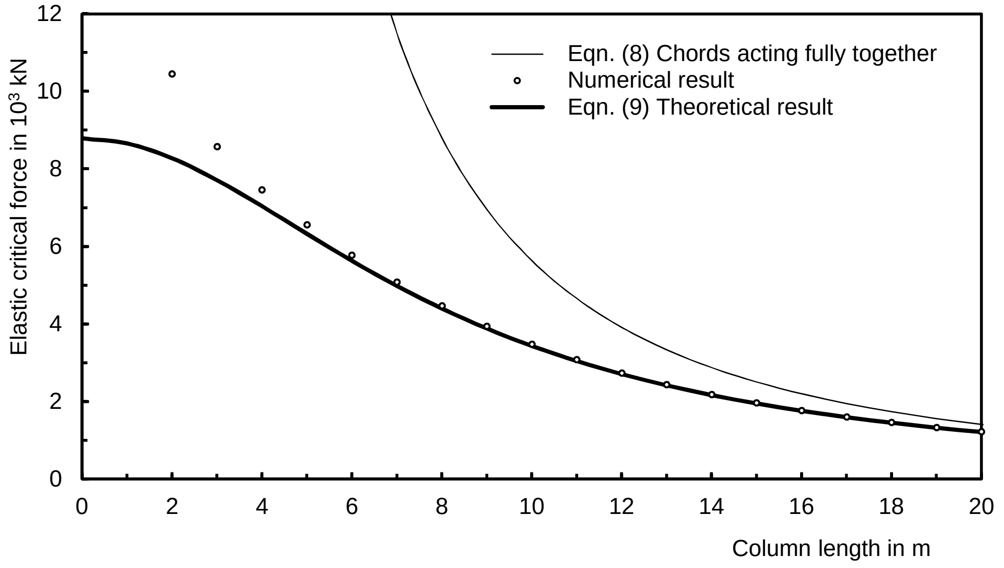


Figure 9: Elastic critical force versus column length - comparative calculations for the column of Fig. 8 with constant module length

Bild 9: Knicklast in Abhängigkeit von der Stützenlänge – Vergleichsrechnung für die Stütze nach Bild 8 mit konstanter Abschnittslänge

The decreasing difference between the numerical results and the theoretical results with increasing column length can be explained as follows. Using the Eqns. (9) and (10) it can be derived that, in the limit of the column length approaching to zero, the elastic critical force is:

$$N_{cr} = S_v \tag{23}$$

Then, in the buckling mode the shear deformations dominate. So, for lower lengths the shear deformations dominate. The middle batten in Fig. 8 remains straight in case of an even number of modules. If this number is odd, the module in the middle does not act in shear. In both cases the model therefore acts stiffer than according to theory resulting in a greater numerical than theoretical elastic critical buckling force. This effect is stronger for short columns.

Fig. 10 shows the elastic critical force against the number of modules m for a 10m long column. The number of modules increases up to 20 and thus the module length a decreases. Again the numerically determined elastic critical force is compared to the theoretically derived one of Eqn. (9) using Eqn. (10) for the effective second moment of area and Eqn. (15) for the shear stiffness. The elastic critical force increases with increasing number of modules m. This is as expected since the built-up column gets stiffer with increasing number of battens as the number of modules increases. The agreement between the numerical and the theoretical results is good, even for low numbers of modules. For low numbers of modules, the shear stiffness of Eqn. (15) is dominated by bending of the chords; for high numbers of modules bending of the battens dominates. For very high numbers of modules, the elastic critical force approaches the value valid for the chords acting fully together, Eqn. (8).

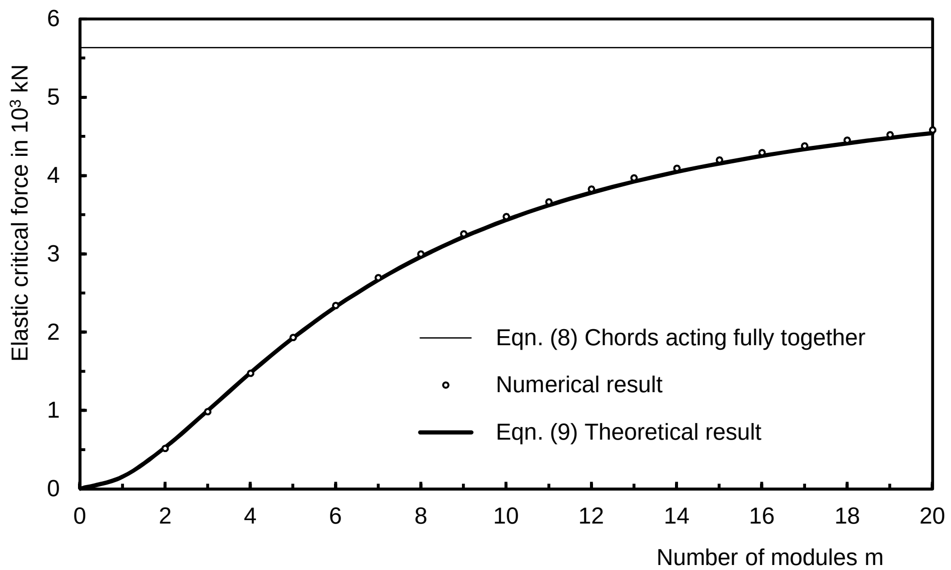


Figure 10: Elastic critical force versus number of modules - comparative calculations for the column of Fig. 8 with variable module length and $\mathrm{L} = 10$ m

Bild 10: Knicklast in Abhängigkeit von der Anzahl der Abschnitte – Vergleichsrechnung für die Stütze nach Bild 8 mit einer Länge $L = 10$ m und veränderlicher Abschnittslänge

7 Code provisions

7.1 Design rules in the current version of EN 1993-1-1 [1]

EN 1993-1-1 [1] treats uniform built-up compression members in clause 6.4. There, also an effective second moment of area is introduced. For laced and battened compression members the Eqns. (24) and (25) are given respectively:

$$I_{eff,EN} = 0{,}5h_0^2A_{ch} \tag{24}$$

$$I_{eff,EN} = 0{,}5h_0^2A_{ch} + 2\mu I_{ch} \tag{25}$$

Where μ is an efficiency factor, see Table 3. In Table 3, λ is the slenderness defined as $\lambda = L/i_0$ with $i_0 = \sqrt{I_1/2A_{ch}}$. Note that 'EN' has been added to the index of the symbol for the effective second moment of area in Eqns. (24) and (25) to indicate that this is the formula of EN 1993-1-1 [1].

Table 3: Efficiency factor μ

Table 3: Wirkungsgrad μ

Criterion	Efficiency factor μ
$\lambda \geq 150$	0
$75 < \lambda < 150$	$\mu = 2 - \frac{\lambda}{75}$
$\lambda \leq 75$	1,0

In the Eqns. (24) and (25) the effect of the shear stiffness is not included and these equations are similar but not equal to Eqn. (2). This raises two questions:

- where in EN 1993-1-1 [1] is the effect of shear stiffness included?
- why was the second moment of area of the chords acting fully together in the cross-section by infinite shear stiffness (Eqn. (2)) not used?

To answer the first question, it should be noted that EN 1993-1-1 prescribes the chords to be designed for the design compression force $N_{ch,Ed}$ at mid-length of the built-up member according to:

$$N_{ch,Ed} = 0{,}5N_{Ed} + \frac{M_{Ed}h_0A_{ch}}{2I_{eff,EN}} \tag{26}$$

Where:

$$M_{Ed} = \frac{N_{Ed}e_0 + M_{Ed}^I}{1 - \frac{N_{Ed}}{N_{cr,EN}} - \frac{N_{Ed}}{S_v}} \tag{27}$$

$N_{cr,EN}$ elastic critical force of the built-up member given by:

$$N_{cr,EN} = \frac{\pi^2 EI_{eff,EN}}{L^2} \tag{28}$$

N_{Ed} design value of the compression force to the built-up member

M_{Ed} design value of the maximum moment in the middle of the built-up member considering second-order effects

M_{Ed}^I design value of the maximum moment in the middle of the built-up member without second-order effects

Note that the effect of the shear stiffness is taken into account in Eqn. (27). The general expression for the amplification factor, used for estimating and including the second-order effect, is given by the following expression, which is Eqn. (5.4) of EN 1993-1-1 [1]:

$$\frac{\alpha_{cr}}{\alpha_{cr} - 1} = \frac{1}{1 - \frac{1}{\alpha_{cr}}} \tag{29}$$

Where α_{cr} is the load multiplication factor to obtain the elastic critical force, so $\alpha_{cr} = N_{cr}/N_{Ed}$. Using Eqn. (7) for built-up columns, Eqn. (29) can be rewritten as follows:

$$\frac{1}{1 - \frac{N_{Ed}}{N_{cr,1}} - \frac{N_{Ed}}{S_v}} \tag{30}$$

The denominators of the Eqns. (30) and (27) are the same, except that both equations use a different elastic critical force, which relates to the second question. So the effect of the shear stiffness is taken into account via the amplification factor in Eqn. (27).

The effect of the shear stiffness is taken into account when evaluating the design compression force $N_{ch,Ed}$ at mid-length of the built-up member including the second-order effect. If this design force is smaller than the chord cross-sectional resistance, the overall stability of the built-up member is guaranteed. However, this check is not written down in EN 1993-1-1 [1] since the flexural buckling check of the chord is always decisive:

$$\frac{N_{ch,Ed}}{N_{b,Rd}} \leq 1{,}0 \tag{31}$$

Where $N_{b,Rd}$ is the design value of the flexural buckling resistance of the chord taking the appropriate buckling length into account which is the module length a for battened built-up columns and which depends on the lacing configuration for laced built-up columns. This answers the first question.

Intermezzo

Before answering the second question, first a personal intermezzo. End of the 1980's I worked as a your research assistant on the next version of the Dutch steel design codes. I considered the first drafts of Eurocode texts which were then just available and the DIN from which the Eurocode design rules for built-up columns were borrowed, and was confronted with exactly that second question. My superiors at TNO advised me to ask Prof. Lindner in Berlin. This was the time before email and making an international telephone call was not easy at all. So I decided to write a letter. And to my surprise within two weeks or so I received the answer from Prof. Lindner, which you can read below. That was my first contact with Prof. Lindner. Many more would follow: at conferences but above all at meetings, e.g. those of ECCS TC8 Stability which he chaired for a long time with great dedication, or at meetings of the European code drafting committees where he leads the German delegation and always speaks with great authority. I would never have believed some 30 years ago, that I would be in the position now of writing this article in his honour on his 80th anniversary.

Now to the second question. For laced built-up columns, normally the distance h_0 between the neutral axes of the chords is substantial meaning that the Steiner term (first term) in Eqn. (2) is dominant and the second term for the chords themselves can be neglected. For that reason Eqn. (2) is modified into Eqn. (24) in the code, which is a safe sided modification. For battened built-up columns the distance h_0 between the neutral axes of the chords is usually limited, meaning that fully neglecting the second term for the chords themselves in Eqn. (2) is very uneconomical. For battened built-up columns with stocky chords, $\mu = 1$ and Eqn. (25) modifies into Eqn. (2). However for slender chords $\mu = 0$ and the second term in Eqn. (25) vanishes such that Eqn. (25) modifies into Eqn. (24). For intermediate slender chords, the second moment of area of the chords themselves is taken into account in a reduced way with μ as given by the formula in Table 3. Reason for the decreasing contribution of the second moment of area of the chords themselves with increasing chord slenderness is that compression members are not only loaded in compression but also in bending due to imperfections. In [6] it is reported that parts of the built-up column may yield before the ultimate resistance of the built-up column is reached, decreasing its stiffness. Therefore, the second moment of area of the chords themselves may not be fully taken into account. This answers the second question.

The equation in EN 1993-1-1 [1] for the shear stiffness of battened built-up columns is:

$$S_v = \frac{24EI_{ch}}{a^2\left[1 + \frac{2I_{ch}h_0}{nI_b a}\right]} \leq \frac{2\pi^2 EI_{ch}}{a^2} \tag{32}$$

The first part corresponds to Eqn. (15). Remarkable is the second part containing an upper bound which limits the shear stiffness to the sum of the elastic critical forces of two individual chords. If the battens are infinitely stiff, Eqn. (15) results in:

$$S_v = \frac{24EI_{ch}}{a^2} \tag{33}$$

Since 24 is greater than $2\pi^2$, the shear stiffness could become greater than the sum of the elastic critical forces of the individual chords. However, if a built-up column with infinitely stiff battens is assumed to behave fully elastic, the compression force on it can never exceed the sum of the elastic critical forces of the individual chords. This is reflected by limiting the shear stiffness as in Eqn. (32).

7.2 Suggestions for modifications

The concerns for battened built-up columns that compression members are not only loaded in compression but also in bending due to imperfections and that parts of the built-up column may yield before the ultimate resistance of the built-up column is reached, seem to be already covered by the flexural buckling check of the chord of Eqn. (31). Moreover, these concerns, if relevant, would apply to any column. For laced built-up columns there seems to be nothing against inclusion of the second moment of area of the chords themselves in Eqn. (24). Therefore it is suggested to use Eqn. (2) instead of the Eqns. (24) and (25) in the Eqns. (26) and (28).

Finally it is suggested to use Eqn. (9) for the elastic critical force of a built-up column with Eqn. (10) for the effective second moment of area including the effect of the shear stiffness. This instead of Eqn. (28) for the elastic critical force with Eqns. (24) and (25) for the effective second moment of area. Instead of Eqn. (10) for the effective second moment of area including the effect of the shear stiffness, also the more general form of Eqn. (5) can be used. This would yield the same approach for built-up columns and beams. If Eqns. (9) or (10) are used for the elastic critical force of built-up columns, Eqn. (27) requires modification as follows:

$$M_{Ed} = \frac{N_{Ed}e_0 + M_{Ed}^I}{1 - \frac{N_{Ed}}{N_{cr}}} \tag{34}$$

Further research is recommended to substantiate the suggested modifications to EN 1993-1-1 [1].

8 Concluding remarks

For the design of built-up steel members, apart from the bending stiffness also the shear stiffness is important. The finite shear stiffness reduces the member stiffness and affects the deflection of beams and the elastic critical force of columns. Currently, EN 1993-1-1 [1] only provides design rules for built-up columns and the effect of the finite shear stiffness is included in the amplification factor to calculate second-order forces in the chords of the built-up column. The paper introduces the concept of the effective second moment of area to include the effect of finite shear stiffness and it proposes to use this concept for the design of built-up members. This concept can conveniently be used for built-up members loaded in compression as well as for those loaded in bending, the format of the effective second moment of area being the same, its coefficients being different. For built-up beams, the deflections can then be calculated with the standard applied mechanics rules. For built-up columns, the design rules then need to be slightly modified as indicated in the paper. Further research is needed to substantiate the proposed modifications. The paper also provides background information to the design rules in EN 1993-1-1 [1] and provides equations for the finite shear stiffness of various battened and laced built-up members, which were numerically checked for a simple battened built-up member.

References

[1] EN 1993-1-1, Eurocode 3: Design of steel structures- part 1-1: General rules and rules for buildings, CEN, Brussels, 2006.

[2] Schroijen, P.W.M., Snijder, H.H., Hoenderkamp, J.C.D., Bakker, M.C.M., *Chords in built-up steel members acting together* (in Dutch), report TUE/CO/98.04, Eindhoven University of Technology, Dept. of the Built Environment, Structural Design, Eindhoven, February 1998.

[3] Schroijen, P.W.M., Snijder, H.H., Hoenderkamp, J.C.D., Effective second moment of area of built-up steel members (in Dutch), Part 1 in *Bouwen met Staal*, 157, November/December 2000, pp.50-56, Part 2 in *Bouwen met Staal*, 158, February 2001, pp. 48-53.

[4] Timoshenko, S.P. en Gere, J.M., *Theory of elastic stability*, McGraw-Hill Book Company, Inc., New York, 2nd edition, 1961.

[5] Bleich, F., *Buckling Strength of Metal Structures*, McGraw-Hill, 1952.

[6] Ramm, W., Uhlmann, W., Zur Anpassung des Stabilitätsnachweises für mehrteilige Druckstäbe an das europäische Nachweiskonzept, *Der Stahlbau*, Vol. 50, No. 6, June 1981, pp. 161-172.

[7] Hoenderkamp, J.C.D., A general method of analysis for tall building structures subject to lateral loads, PhD thesis, Department of Civil Engineering and Applied Mechanics, McGill University, Montreal, Quebec, May 1983.

Figure captions
Bildunterschriften

Figure 1: Uniform built-up members (left) and single members (right)
Bild 1: Gleichförmige mehrteilige (links) und einteilige Stäbe (rechts)

Figure 2: Battened built-up member
Bild 2: Rahmenstab

Figure 3: Module of a built-up member loaded by shear force V
Bild 3: Durch Schub V beanspruchter Abschnitt eines Rahmenstabes

Figure 4: Model with symbols used
Bild 4: Verwendetes Berechnungsmodell mit Formelzeichen

Figure 5: Factor n
Abbildung 5: Faktor n

Figure 6: FEM model for a built-up beam in bending
Abbildung 6: FEM Modell für einen mehrteiligen Träger unter Biegebeanspruchung

Figure 7: Effective second moment of area versus beam length - comparative calculations for the beam of Fig. 6 with constant module length
Bild 7: Effektives Flächenträgheitsmoment in Abhängigkeit von der Trägerlänge – Vergleichsrechnung für den Träger nach Bild 6 mit konstanter Abschnittslänge

Figure 8: FEM model for a built-up column in compression
Bilde 8: FEM-Modell für einen Rahmenstab unter Druckbeanspruchung

Figure 9: Elastic critical force versus column length - comparative calculations for the column of Fig. 8 with constant module length
Bild 9: Knicklast in Abhängigkeit von der Stützenlänge – Vergleichsrechnung für die Stütze nach Bild 8 mit konstanter Abschnittslänge

Figure 10: Elastic critical force versus number of modules - comparative calculations for the column of Fig. 8 with variable module length and $L = 10$m
Bild 10: Knicklast in Abhängigkeit von der Anzahl der Abschnitte – Vergleichsrechnung für die Stütze nach Bild 8 mit einer Länge $L = 10$ m und veränderlicher Abschnittslänge

Table captions

Table 1: Factor α for standard load cases and support conditions
Tabelle 1: Faktor α für häufige Lastfälle und Lagerungsbedingungen

Table 2: Shear stiffnesses S_v for laced built-up members
Tabelle 2: Schubsteifigkeiten S_v von Gitterstäben

Table 3: Efficiency factor μ
Tabelle 3: Wirkungsgrad μ