Perpetual growth, distribution, and robots

Working Paper Series

#2018-023

Perpetual growth, distribution, and robots

Önder Nomaler and Bart Verspagen

Maastricht Economic and social Research institute on Innovation and Technology (UNU-MERIT)
email: info@merit.unu.edu | website: http://www.merit.unu.edu

Maastricht Graduate School of Governance (MGSoG)
email: info-governance@maastrichtuniversity.nl | website: http://www.maastrichtuniversity.nl/governance

Boschstraat 24, 6211 AX Maastricht, The Netherlands
Tel: (31) (43) 388 44 00

UNU-MERIT Working Papers
ISSN 1871-9872

Maastricht Economic and social Research Institute on Innovation and Technology
UNU-MERIT

Maastricht Graduate School of Governance
MGSoG

UNU-MERIT Working Papers intend to disseminate preliminary results of research carried out at UNU-MERIT and MGSoG to stimulate discussion on the issues raised.

Perpetual Growth, Distribution, and Robots

Önder Nomaler[1] and Bart Verspagen[2]

[1]Eindhoven Centre for Innovation Studies (ECIS), o.nomaler@tue.nl and

[2]UNU-MERIT, verspagen@merit.unu.edu

Version of 17 May 2018

Abstract

The current literature on the economic effects of machine learning, robotisation and artificial intelligence suggests that there may be an upcoming wave of substitution of human labour by machines (including software). We take this as a reason to rethink the traditional ways in which technological change has been represented in economic models. In doing so, we contribute to the recent literature on so-called perpetual growth, i.e., growth of per capita income without technological progress. When technology embodied in capital goods are sufficiently advanced, per capita growth becomes possible with a non-progressing state of technology. We present a simple Solow-like growth model that incorporates these ideas. The model predicts a rising wage rate but declining share of wage income in the steady state growth path. We present simulation experiments on several policy options to combat the inequality that results from this, including a universal basic income as well as an option in which workers become owners of "robots".

Keywords: perpetual economic growth; economic effects of robots; income distribution

JEL Codes: O41, O33, E25, P17

We thank Luc Soete, Thomas Ziesemer, Adriaan van Zon and participants at the UN Expert Group Meeting on "Accelerated Technological Change, Artificial Intelligence, Automation and their Implications for Sustainable Development Targets" (Mexico City, 26-27 April 2018) for comments on an earlier version. The views expressed and any remaining errors remain solely our own.

1. Introduction

Discussions about new technologies replacing labour are as old as the economics academic discipline. Frey and Osborne (2017, p. 256) quote Queen Elizabeth I of England speaking out against an innovative knitting machine, in support of her "poor subjects". Freeman and Soete (1994) quote David Ricardo entertaining the view that "the employment of machinery is frequently detrimental to [the] interests" of the working class. In this sense, the recent debate on the expected impact of advanced computer technologies replacing labour is not new (e.g., Frey and Osborne).

The effects of new technology on employment will likely play out in terms of the quantity of labour demanded (employment) and the price paid for labour (wages) (Katsoulacos, 1986). The total employment effects of technological innovation will consist of both direct effects (e.g., introduction of labour-saving machines) and indirect effects (e.g., increased demand for labour due to increased product demand as a result of a fall in prices due to the innovation, or as a result of product innovation). Economic theory and models exist to analyse the interplay (and net effect) of these direct and indirect effects, as well as the impact of wages of workers of various skill-levels (Vivarelli, 1995; Freeman and Soete, 1994; Katsoulacos, 1986).

If "robots" and similar technologies are similar to previous waves of innovation with respect to their impact on the labour market, we may well apply this body of theory to the recent debate. By and large, we could then conclude that even if labour market adjustment will take a fair amount of time (up to decades), jobs will not disappear in the long run, because of indirect compensation effects of various kinds (Freeman and Soete, 1994; Autor, 2015; Bessen, 2016). However, as we will argue in the next section, there are reasons to assume that the impact of technologies such as machine learning, robots and artificial intelligence will be different from previous waves of technological change, because their potential to substitute labour for capital is very high, and they may have adverse effects on labour income. One the other hand, it is also clear that these technologies have a high potential to create (extra) economic growth. As we will show in the model that we develop below, they may in fact give rise to so-called perpetual growth, which takes place by investment in capital only (i.e., no further investment in technology).

This clearly puts the issue of replacement of labour by technology (embodied in capital) in the realm of economic growth. Obviously, technological change has long been considered a main determinant of economic growth (e.g., Nelson and Winter, 1982; Lucas, 1988; Romer, 1990; Silverberg and Verspagen, 1994). Acemoglu and Restrepo (2017) present a model of economic growth in which technology can either take the form of automating (replacing) human labour, or create new "tasks" that contribute to production and can only be performed by humans. Although their model potentially yields a growth path in which all human labour is replaced by capital, the emphasis of their analysis is on a growth path in which the two types of technological change balance each other out. In their model, "there are powerful self-correcting market forces pushing the economy towards balanced growth (…) for example the arrival of a series of new automation technologies, will set in motion self-correcting forces (…) there will be an adjustment process restoring the level of employment and the labour share back to their initial values" (p. 28).

Our interest, on the other hand, is in the situation where the labour-replacing technologies come to dominate the economy, and create perpetual growth as described above. Whether this will be the case is a matter of technological foresight, in which we do not engage beyond a brief summary of some of the debate that we will provide in the next section. The interest of our analysis is therefore mainly in exploring the potential consequences of one of the possible technological scenarios, i.e., that in which "robots"[1] will replace not only human labour but also technological development as the main source of growth. We will ask what growth looks like in such a scenario, what the consequences for labour income and inequality will be, and which potential policies may be effectuated for combatting inequality.

The remainder of this paper is organised as follows. In the next section, we will briefly discuss the existing literature on the economic impact of robot-technology. We will focus on the work that tries to assess the degree to which human labour will become substitutable by robots and related technologies. We will also briefly touch upon the way in which this form of technological change can be analysed in economic models. Section 3 will present the production function that will be the basis of our model. It will describe how the micro foundations of the production function may be related to the findings in the literature surveyed in Section 2, and how this production function changes our traditional outlook on the way technological progress influences production.

Section 4 will show how the production function is embedded in a simple growth model. In Section 5, we analyse the model to investigate under what conditions growth will emerge, and how the growth rate can be quantified. This analysis suggests that that growth can indeed be "perpetual". Section 6 will also look at the dynamics of the labour income share and the wage rate, both along the steady state of perpetual growth, and in the transient towards the steady state. Section 7 explores some options for combatting inequality that may result from perpetual growth, by means of a social protection policy. The analysis here is by means of numerical simulations. Section 8 summarises the argument, draws some conclusions, and outlines some avenues for further research.

2. Capital, labour, and robots

The pre-publication version of Frey and Osborne (2017) sparked a debate about the extent of the future impact of automation on employment and labour markets in general. They conclude that "47% of total US employment is in the high-risk category, meaning that associated occupations are potentially automatable over some unspecified number of years, perhaps a decade or two" (p. 265). Here, "automatable" or "automation" means that human labour is substituted by capital goods, in particular computers including the software that makes them run. Frey and Osborne identify machine learning (ML) and mobile robotics (MR) as two main technological developments with a potentially high impact. They view ML as a way to automate cognitive tasks previously performed by human labour, and MR as a way of automating manual work (p. 258). They follow Autor et al. (2003) in applying a two-way categorisation of labour tasks, i.e., routine vs. non-routine tasks and cognitive vs. manual tasks, and argue that the domain of non-routine (i.e., non-automatable) tasks is rapidly shrinking.

[1] We use the terms "robots" to colloquially refer to a set of technologies that Frey and Osborne (2017) describe as "machine learning". See the next section.

Arntz et al. (2016) modify the Frey and Osborne methodology of estimating automation risk for employment. Frey and Osborne estimate the risk for each of a set of 702 occupations, and then count how many people are employed in the occupations that emerge as "high risk". Arntz et al. argue that this approach pays too little attention to the fact that although people may be employed in the same occupation, they may still spend different amounts of time on specific tasks. For example, one lawyer may spend 50% of her time assessing legal documents in a way that could be done by an intelligent computer programme, while another lawyer may spend only 10% of his time on this task. They have data at the level of the individual worker, and hence are able to estimate the automation risk for every worker in their dataset, rather than for the standard set of occupations that these workers have. Their results show a much lower share of workers at high risk, which is defined similarly to Frey and Osborne as a 70% probability of being automated. Their estimate for the US is less than 10% of employment being at high risk, vs. the 47% of Frey and Osborne. The countries with maximum risk in the results by Arntz et al. (2016) are Austria and Germany, with 12%.

Nedelkoska and Quintini (2018) extend the analysis of Arntz et al. (2016) and find that Slovakia is the country with the largest share of jobs at high risk, at 33%. They also provide more detailed numbers of the actual probability of automation risk, and find that the median is at 48% risk of automation (the mean is 47%, and the standard deviation is 20%). Chui et al. (2016) focus entirely on tasks carried out in the work environment, rather than on jobs and occupations (which are essentially packages of tasks), and estimate that in the US, the three sets of tasks that are at highest risk of being automated comprise a total of 51% of total time spent. Their risk of automation varies between 64% and 78%.

Thus, although the actual extent of automation risk clearly depends on the method used to estimate this risk, we may still conclude that this risk is substantial. Even the conservative method (Nedelkoska and Quintini) finds a median or mean risk that is near to about half, which means that one out of every two workers in their set of developed countries could be automated in the not-so-far-away future.

How should we think about this form of technological change and the impact it may have on future economic relations? One way that economists have looked at technological progress is that it is a major force for increasing the productivity of human labour, which in technical parlour is called factor-enhancing technological progress. Although we may identify numerous examples of this tendency in the history of technological change, Frey and Osborne and other contributors to the debate clearly have something different in mind. The risk of automation that they consider is the risk of human labour being completely substituted by technology (embodied in machines), i.e., a specific production task being carried out without any labour input. Peretto and Seater (2013) call this factor-eliminating technical change.

We may consider a basic example to illustrate this distinction further. A basic shovel, probably made of some kind of metal, greatly improved human productivity in the production task of moving earth (digging). The invention of mechanical diggers, first powered by steam and later by fossil fuels, again greatly improved productivity in this task. A human operating a mechanical digger can move much larger amounts of soil in a fixed amount of time than a single human using a shovel, who can in turn move larger amounts than a human using just bare hands. The shovel and the mechanical digger are pieces of capital that embody technology that complements human labour by making it more productive. Some workers may lose their jobs as shovel operators when mechanical diggers are introduced, or as bare hand diggers when shovels are introduced. Therefore these pieces of equipment may, at the same substitute for human labour, depending on total demand in the industry. Workers that are substituted in this way may, in the longer run, as a result of indirect effects such as an increased demand for digging services, be re-employed (after acquiring new skills) as operators of the new technology. In this case, they will tend to end up being better-off due to higher wages that reflect higher productivity.

Under the influence of machine learning, however, the mechanical digger could be made self-operating, just as autonomous (self-driving) cars are now being developed. Such an autonomous digger would substitute entirely for human labour, and the task of moving earth could, under at least some circumstances, be carried out without any input of humans. This ultimate step in technological progress (robotisation) is very different than those before, since it puts the worker out of a job without any perspective of being re-employed in the digging business. Therefore, it also reduces workers' welfare (income), unless the worker has some share in the ownership of the mechanical diggers or the robot operators (we will return to this ownership issue below).

Our analysis here will be entirely aimed at exploring the consequences of widespread automation for human material welfare. The analysis will consider the consequences of the form of technological progress that substitutes for labour. We will not consider how this form of technological progress comes about endogenously, but instead assume that it will (soon) reach a state in which it can start to play a major role in the economy, i.e., what Pratt (2015) calls a "Cambrian explosion for robotics".

Two main issues motivate our work. First, factor-eliminating technological change creates a strong potential for economic growth. In particular, it may change the game of technological progress completely. Factor-augmenting technological progress requires a constant investment in research and development or science (e.g., Romer, 1990), or human capital (e.g., Lucas, 1988). But after reaching a critical level of advancement in factor-eliminating technologies, investment in technology or knowledge will no longer be necessary to keep growth going. Instead, growth can be sustained merely by further investment in the ultimately-conceived capital vintage. In other words, the constant state of technological knowledge is enough to fuel growth by capital investment alone.

This kind of growth has been called perpetual growth (e.g., Prettner, 2017). Although we (and Prettner) will associate it to what Frey and Osborne call machine learning, it is hardly a new idea. Solow (1956, p. 78) already coined the possibility when discussing the constant elasticity of substitution (CES) production function in the context of his growth model: under certain parameter conditions, "… the capital-labor ratio increases indefinitely and so does real output per head. The system is highly productive and saves-invests enough at full employment to expand very rapidly."

The idea of perpetual growth is, however, just a footnote in the contribution of Solow (1956). His main contribution was to show that in the presence of a non-reproducible production factor (i.e., labour) that has somewhat limited substitutability with a reproducible production factor (capital), growth of per capita output would eventually halt, unless the productivity of the non-reproducible factor could be augmented in some way (by technological progress). The model that was presented by Solow depends heavily on a so-called Cobb-Douglas aggregate production function, which excludes the possibility of perpetual growth (Solow, 1956, p. 76).

In terms of our digging example, giving a single worker more than one shovel does not lead to higher quantities of earth moved by that worker (i.e., the marginal returns to capital are decreasing). With a given demand for digging services, growth of the diggers' income is not possible by endowing them with more shovels. The introduction of mechanical diggers, on the other hand, would enable fewer workers to do the job, which means rising productivity and rising income for the workers that remain. But again, after all qualified operators are provided with one mechanical digger, there is no point in investing further in more mechanical diggers. It is this kind of technological progress (i.e., factor augmenting) that can be accommodated in Solow's Cobb-Douglas production function. On the other hand, as illustrated by the above Solow quotation, a CES production function opens up a wider range of growth opportunities.

Solow's CES corollary was not taken up by the endogenous growth literature that developed in the 1980s and 1990s. Instead, the focus was on formal conceptual frameworks (models) where sustainability of growth is strongly dependent on technological change that counteracts diminishing marginal returns to capital, for example by a succession of labour-augmenting inventions embodied in intermediate goods (Romer, 1990), or investment in human capital (Lucas, 1988). However, now that we are beginning to realise that cumulative technological progress has enormously increased the substitutability of labour (even its cognitive faculties) with certain types of capital, there is a need to develop a new formal conceptual framework to study growth (of the perpetual type).

Solow's corollary points to the CES-based aggregate production functions as a way forward on this path. Acemoglu and Restrepo (2017, p. 10) show that an aggregate CES production function of the type that we will be using (i.e., with labour and "robot capital" as the two production factors) may be derived from a micro foundation in which human labour tasks can be automated, in the sense of Frey and Osborne. Our approach below will combine automation with factor-augmenting technological progress and hence will require distinguishing between two basic types of capital: one that embodies each form of technological change that we identified.

The second issue that motivates our approach is that under a state of perpetual growth, labour as a production factor will potentially receive an ever-declining share of total income, which leads to high inequality. We are interested here both in the question whether and how quickly this inequality arises, and in how it can be combatted. To investigate the latter issue, we will build into our model a number of experiments that mimic social protection policies, including a universal basic income.

What we will not investigate in the model that we present below are the employment consequences of factor-eliminating technological progress. In fact, we will assume that the labour force is constant and will always be fully employed (this is also the standard assumption in Solow's growth model). We do not necessarily believe that this is a realistic assumption, but nevertheless make it because we feel that in the context of perpetual growth, the income distribution issue is much more salient than the employment issue. After all, perpetual growth ultimately makes employment superfluous, because factor-eliminating technological progress reduces the role of labour in the production process greatly.

The occurrence of perpetual growth in our model hinges on three major factors: the share of human work that can potentially be automated, the relative costs of labour-replacing capital vs. labour, and the elasticity of substitution between labour and labour-replacing capital. If labour-replacing capital ("robots") gets cheaper, if more labour tasks can be substituted by robots, and if human and robot labour can be substituted more easily, perpetual growth is more likely to occur. This is similar to Prettner's (2017) model of perpetual growth, although he assumes perfect substitutability between labour and robots. Our results show that perpetual growth may already arise when substitution is far from perfect, i.e., at an earlier stage of development of machine learning technology. DeCanio (2016) employs a production function similar to ours, in an analysis aimed at identifying the impact of robotisation on wages. The approach in Peretto and Seater's (2013) model is different and does not involve either costs or the elasticity of substitution.

3. A production function for robotisation

We formalise the ideas from the previous section in the following production function:

$$Q = \left(\alpha(b_o K_o)^{\mu} + (1-\alpha)[\beta(b_S K_S)^{\rho} + (1-\beta)(b_L L)^{\rho}]^{\frac{\mu}{\rho}}\right)^{\frac{1}{\mu}} \qquad \text{(1a)}$$

Q is output, L is labour input (employment), K_o and K_s are different kinds of capital that we will discuss in detail below, μ and ρ are parameters related to substitutability in the production process, and α, β, b_o , b_s and b_L are other parameters to be discussed below. We consider a homogenous labour input, i.e., do not distinguish between different kinds of labour, such as high-skilled and low-skilled, or blue and white collar. The empirical work on the automation risk surveyed above suggests that labour of all kinds may be automated, although it may also be true that some kinds of labour have a higher risk than others. We deliberately abstract, however, from making such a distinction, to keep the analysis simple, and in order to bring out the results of the analysis in a sharp way.

The reason for assuming two different kinds of capital goods lies in the variety of productive services that can jointly produce output Q. Some of these services can be produced by tasks carried out by human labour L (in a combination of physical and cognitive activities). Examples of these kinds of services are bricklaying or composing music. Some of these kinds of "human-like" activities can also be performed by machines (including software), but the key element of their definition is that all of them can in principle be performed by the human body and/or mind. The kind of capital that may replace these human-like activities will be referred to as K_s. It is what is often referred to as "robots", but includes a broad set of technologies such as artificial intelligence or machine learning. Note that the activities that are produced by either L or K_s are the kinds of activities that Frey and Osborne argue may be automated in the (near) future.

The production factors K_s and L may use varying amounts of a different kind of capital, which is denoted by K_o. The key characteristic of this kind of capital is that it produces services that a human body or mind cannot deliver. As an example of capital type K_o, one may think of a blast furnace, an office building, or a lithography machine used to produce silicon chips. All of these types of capital perform services that a human body or brain cannot provide.

Capital goods are measured in monetary terms (dollars), and labour in time (such as person-hours). The productivity parameters b_o, b_s and b_L convert these units into output Q. The production function is nested. Labour L and capital K_s form a composite production factor in the "inner part" of the function. This composite production factor follows a logic similar to Acemoglu and Restrepo's (2017) production function. It consists of a fixed number of tasks that can notionally be performed by human labour alone. A share β of these tasks can be automated, i.e., can also be produced by capital of type K_s alone.

We assume that there are T such tasks, so that βT $(0 \leq \beta \leq 1)$ of these can be performed both by humans and robots, i.e., are automatable. The ease of substitution between the $(1 - \beta)T$ human-critical tasks (non-automatable) and the βT potentially automatable tasks is determined by the parameter ρ, which implies the elasticity of substitution to be $\sigma_\rho \equiv 1/(1-\rho)$. Strong complementarity (low ρ) means that the T tasks are equally needed to produce output. In this case, even when all of the βT tasks are actually automated (which will happen if the cost of production with robots are lower than production with humans), the economy will have robotised only βT of all tasks and $(1 - \beta)T$ tasks remain produced by human labour, which is the limit to robotisation. On the other hand, a high value of ρ means that the production process is able to intensify the provision of the βT subset of tasks while relying less on the subset $(1 - \beta)T$, without loss of output. In this case, the economy may be fully robotised (i.e., no human labour used in production) despite $\beta < 1$, by only performing the βT tasks that are automatable.

The ability of the economy to specialise in a subset of all tasks (i.e., ρ) is clearly determined by factors such as demand (i.e., the relative economic value of the respective tasks) and openness to international trade. If some tasks are relatively more valuable than others, the economy can choose to increase its productivity by producing more of the relatively more valuable ones while decreasing (or terminating) the production of the less valuable tasks and perhaps importing them. In our model, we will abstract from these demand issues, and also from international trade. Instead, we will model the substitution between human labour and robot capital K_s only in terms of the production function.

So far, we discussed the role of the parameters β and ρ in determining the possibilities of substitution between labour L and robot capital K_s. β indicates the degree of potential automation, while ρ measures the ease of substitution between automatable and non-automatable tasks. The third and final parameter that plays a role in robotisation is b_s, which measures the productivity of K_s. Similarly, b_L measures the productivity of human labour. These parameters will determine whether or not a potentially automatable task will actually be automated. A higher (lower) b_s/b_L ratio will provide incentives for a relatively robot (human) intensive production.

Similar to the way we define services to be provided by humans and robots as tasks, we define total economic output as a collection of productive activities. The outer level of the production function, with relevant parameters α and μ, defines how the two broad types of activities (i.e., human-like services, and pure capital services) can be combined to produce output. The parameter α indicates the share of activities that are intensive in pure-capital services while $1 - \alpha$ is the share of activities that are intensive in human-like services.

The key parameter in the outer part of the production function is μ, which determines the degree of substitution or complementarity between the activities intensive in the composite K_s – L factor on the one hand, and K_o on the other hand. The parameter μ can be seen as specifying the limitations of a process of structural change from productive activities that are intensive in K_o (such as metal making, which requires blast furnaces), to a service-intensive economy that relies much more on activities that are intensive in the composite K_s – L factor. The elasticity of substitution in this outer part is defined as $\sigma_\mu \equiv 1/(1-\mu)$.

Our analysis of the economic effects of robotisation relies on the assumption that $0 < \rho \leq 1$ or $\sigma_\rho > 1$, which emphasises a state where automatable tasks are fairly well able to substitute non-automatable tasks, and vice versa. If $\rho < 0$, robotisation becomes much less of an issue. We will also assume $\rho > \mu$, and hence $\mu < 0$ and $\sigma_\mu < 1$. This means that labour L can be substituted more easily with K_s than the composite K_s – L factor can be substituted with K_o. This assumption essentially follows from the particular way in which we break down aggregate capital into two categories (i.e., K_o as inherently complementary to labour, and K_s as inherently substitutable). The parameter b_o (i.e., the productivity of K_o) plays a similar role to b_s and b_L, i.e., it determines the cost-effectiveness of K_o relative to the composite K_s – L factor, and hence has an impact on the relative use of these two factors.

We do not analyse the parameters b_o, b_s, b_L α, β, μ and ρ as a function of time. This implies that the state of technology is given. Robot technology is embodied in capital K_s, which can substitute for labour. b_s and β are a reflection of how far robot-technology has progressed (productivity-wise and substitutability-wise respectively), while ρ is a reflection of the ability of the production process to (relatively) specialise in robotised subset of tasks. The analysis will be aimed at finding out how these parameters influence growth, and what this implies for the resulting income distribution.

This production function may be used to represent a wide range of secular phenomena that may be observed over (recent) economic history. For example, we may interpret the rise of the services sector as a decrease of α (which implies an intensification of activities that need human-like tasks), combined with an increase in b_L (the modernisation of the services sector leading to higher productivity of not yet robot-substitutable human labour). Frey and Osborne's increased automation risk means that more and more tasks (also in the services sector) are becoming automatable (β increasing), and also that "robots" are becoming cheaper (higher b_s).

While the latter (increasing β and b_s) may lead to worries of job loss and a decline of labour income, a continuing decrease of α may offset this tendency. However, the process of decreasing α ("servicification" of the economy) may well have come close to an end. If this is the case, the "new mode" of technological progress (increasing β and b_s) may well have entirely different consequences for employment and wages than an "old mode" of technological progress, which may be represented by increasing b_o and b_L without a corresponding increase in β and b_s. In other words, the way in which our production function embodies different forms of technological change suggests that "this time it may be different" in terms of the consequences of technological progress for labour markets. This is what we will investigate in the model that will now be presented.

There doesn't seem to be an emerging sector in the horizon that can achieve what services sector achieved in the past (1-α cannot increase). This might mean that this time it is different.

4. A simple growth model

With the assumed values of the substitution parameters $\mu < 0 < \rho \leq 1$, (and associated $\sigma_\mu \leq 1 < \sigma_\rho$), two special and extreme cases are of interest. One is where μ approaches $-\infty$. This is the case where capital K_o and the composite factor K_s – L are completely complementary (σ_μ = 0), so that the outer form of the production function turns into the familiar Leontief form, as in

$$Q = min\left[b_s K_s, [\beta(b_s K_s)^\rho + (1-\beta)(b_L L)^\rho]^{\frac{1}{\rho}}\right] \tag{1b}$$

where the α parameter disappears.

The other case is when μ approaches zero, and the elasticity of substitution σ_μ converges to 1. In this case, the outer part of the production function (1a) becomes the familiar Cobb-Douglas form, as in

$$Q = A(b_o K_o)^\alpha \left([\beta(b_s K_s)^\rho + (1-\beta)(b_L L)^\rho]^{\frac{1}{\rho}}\right)^{1-\alpha} \tag{1c}$$

where A is an arbitrary new parameter that we set to 1 without loss of generality.

We use the production function (1a), and in particular (1b) and (1c), in a simple Solow-like growth model (Solow, 1956) with exogenous savings, and without technological progress and without population growth. A first assumption in this model is that the production factor labour is always fully employed:

$$L = N \tag{2}$$

In this equation N is the labour force, which is exogenous.

In order to be able to analyse the income distribution, we distinguish between income from capital, and income from labour. The latter is earned solely by the members of the labour force N, while income from capital is earned by capital-owners. For most of the analysis, we will assume that capital owners and workers are separate groups, i.e., nobody is both a capital owner and a worker. Only at the last stage of the analysis will we relax that assumption.

Related to this, we also implement separate savings rates for labour income and capital income. This requires us to first define these two types of income. We obtain the (real) factor prices of the three production factors by their marginal products, and use these to obtain shares of the factor in total income (denoted by ϖ with appropriate subscripts), which for the general form of the production function (1a) looks as follows:

$$\varpi_L \equiv \frac{p_L L}{Q} = \frac{\partial Q}{\partial L}\frac{L}{Q} = (1-\alpha)(1-\beta)\frac{(b_L L)^\rho \Theta_{LS}^{\frac{\mu}{\rho}}}{\Theta_{LS}\Theta_T} \tag{3a}$$

$$\varpi_S \equiv \frac{p_S K_S}{Q} = \frac{\partial Q}{\partial K_S}\frac{K_S}{Q} = (1-\alpha)\beta\frac{(b_S K_S)^\rho \Theta_{LS}^{\frac{\mu}{\rho}}}{\Theta_{LS}\Theta_T} \tag{3b}$$

$$\varpi_O \equiv \frac{p_O K_O}{Q} = \frac{\partial Q}{\partial K_O}\frac{K_O}{Q} = \alpha\frac{(b_O K_O)^\mu}{\Theta_T} \tag{3c}$$

In these equations, we define $\Theta_T \equiv \alpha(b_o K_o)^\mu + (1-\alpha)[\beta(b_s K_s)^\rho + (1-\beta)(b_L L)^\rho]^{\frac{\mu}{\rho}}$ and $\Theta_{LS} \equiv \beta(b_s K_s)^\rho + (1-\beta)(b_L L)^\rho$, while p_L, p_c and p_s are the prices of labour, K_s and K_o, respectively. When the outer-part of the production function is Cobb-Douglas (equation 1c), this yields a share α for capital K_o, and a share 1 – α for the composite K_s – L factor (sum of 3a and 3b). We also define the shares of L and K_s in joint income of the composite factor, as $\nu_L \equiv \frac{\varpi_L}{\varpi_L+\varpi_s}$ and $\nu_s \equiv \frac{\varpi_s}{\varpi_L+\varpi_s}$.

Next, we assume that savings out of labour income are zero, while a fraction s_K of capital income (both robot capital and complementary capital) is saved. This makes total savings (S) equal to

$$S = s_K Q(\varpi_o + \varpi_s) \tag{4}$$

All savings are invested. The investment process is described by the following two equations:

$$S = I = I_o + I_s \tag{5}$$

$$\dot{K}_o = I_o - \delta K_o, \dot{K}_s = I_s - \delta K_s \tag{6}$$

Equation (5) specifies that all savings are used for investment (I) in capital. Equation (6) specifies the motion of both capital stocks, where we use a dot above the capital stock variables to denote their change over time (time derivative). Capital stocks decline by a fixed depreciation rate δ (which for simplicity we set equal between the two capital types) and increase by investment. In order to determine how investment is distributed over the two types of capital, we assume profit maximisation leading to the marginal products of the two types of capital being equal to each other. The (formal) implications of this assumption will be investigated below.

Finally, we add a last equation that defines consumption by a national income identity:

$$C = Q - S \tag{7}$$

We now proceed to analyse the model in terms of its implications for growth and distribution.

5. Perpetual growth

The basic insight about growth in our model is similar to that of the original Solow-model without technological change (Solow, 1956): growth of output per worker can increase as long as the amount of capital per worker keeps growing. Without population growth (as we assume), this is the case as long as net investment (I – δK) is positive.

To derive the conditions under which this will happen, we start by factoring out K_s from the inner part of the production function (1a), which gives rise to

$$Q = \left(\alpha(b_o K_o)^\mu + (1-\alpha)(K_s)^\mu \left[\beta b_s^\rho + (1-\beta) b_L^\rho \left(\frac{L}{K_s}\right)^\rho\right]^{\frac{\mu}{\rho}}\right)^{\frac{1}{\mu}} \tag{8}$$

If the capital stock K_s keeps growing, and with constant $N = L$, the term $\frac{L}{K_s}$ goes to zero, and hence equation (8) will converge to

$$Q = \left(\alpha(b_o K_o)^\mu + (1-\alpha)\left(\beta^{1/\rho} b_s K_s\right)^\mu\right)^{\frac{1}{\mu}} \quad (9)$$

We assume that the allocation of investment over the two types of capital is profit maximising, which implies that the marginal products of the two capital stocks are equal ($dQ/dK_s = dQ/dK_o$). These marginal products are obtained by differentiating the function (9) with respect to K_o and K_s. Then, with some rearranging, the profit maximisation condition yields

$$K_o = K_s \left(\frac{\alpha b_o^\mu}{(1-\alpha) b_s^\mu \beta^{\mu/\rho}}\right)^{\frac{1}{1-\mu}} \quad (10)$$

We now focus on the special case where capital K_c and the composite K_s – L factor are pure complements. This implies that μ approaches $-\infty$, and that $\sigma_\mu = 0$. We leave details of the alternative assumption $\sigma_\mu = 1$ to the appendix. Under the assumption $\sigma_\mu = 0$, equation (10) will reduce to

$$K_o = K_s \frac{\beta^{1/\rho} b_s}{b_o} \quad (10a)$$

Now we substitute equation (10a) into the production function (9), which after some rearranging will yield a relation between output and only one type of capital:

$$Q = b_s K_s \beta^{1/\rho} \quad (11)$$

Net investment ($I - \delta K$) can be written as $s_K Q - \delta(K_o + K_s)$. Growth requires this to be positive, and using equations (10a) and (9), we arrive at the growth condition

$$b_s \beta^{\frac{1}{\rho}} > \frac{\delta b_o}{s_K b_o - \delta} = \frac{1}{\frac{s_K}{\delta} - \frac{1}{b_o}} \quad (12a)$$

If this condition is satisfied, growth is perpetual, and it arises only from investment in K_s (robots), without improvement in technology (robot or otherwise). Once a parameter state is reached in which robots are cost-effective enough (b_s high enough), are sufficiently able to substitute human labour (β and/or ρ large enough), and the savings rate s_K is high enough, growth will be self-propelling by investment in K_s alone. A higher b_o (i.e., higher productivity of the non-robot capital) also contributes positively to the likelihood of perpetual growth.

The appendix shows that if we use the production function (1c), i.e., when the elasticity of substitution $\sigma_\mu = 1$ instead of 0, the growth condition becomes

$$b_s \beta^{1/\rho} > \frac{1}{1-\alpha}\left(\frac{\delta}{s_K}\frac{1}{(\alpha b_o)^\alpha}\right)^{1/1-\alpha} \quad (12b)$$

The growth condition (12a) is more stringent than (12b), i.e., perpetual growth is harder to achieve when the outer part of the production function allows no substitution (i.e., lower opportunities for structural change). For example, when $s_K = \delta\,(b_s + b_o)/(b_s b_o)$ even $\beta = 1$ will not allow perpetual growth in equation (12a) (net investment is zero in this case), while (12b) will still allow growth (this holds for a wide range of b_s and b_o values). However, numerical analysis suggests that these differences are not very large beyond the threshold $s_K = \delta\,(b_s + b_o)/(b_s b_o)$.

Figure 1 shows how the growth condition depends on ρ, β and b_s. The lines labelled as "CD" refer to the production function (1c), i.e., condition (12b) and σ_μ = 1, while the lines labelled as "Leon" refer to σ_μ = 0 (condition 12a and production function 1b). δ has been set to 0.05 in this figure, and b_o = 1 and s_K = 0.4. Separate curves for different values of b_s have been drawn. When ρ = 0, β always needs to be 1 for the growth equation to be satisfied, irrespective of b_s (and other parameters). For larger values of ρ, the threshold value of β becomes lower, ceteris paribus (and vice versa). Increasing the value of b_s (i.e., making robot capital more productive) will make the growth condition less stringent, for either form of the production function. The figure also confirms the earlier point that the Leontief outer-form yields a more stringent growth condition that the Cobb-Douglas form.

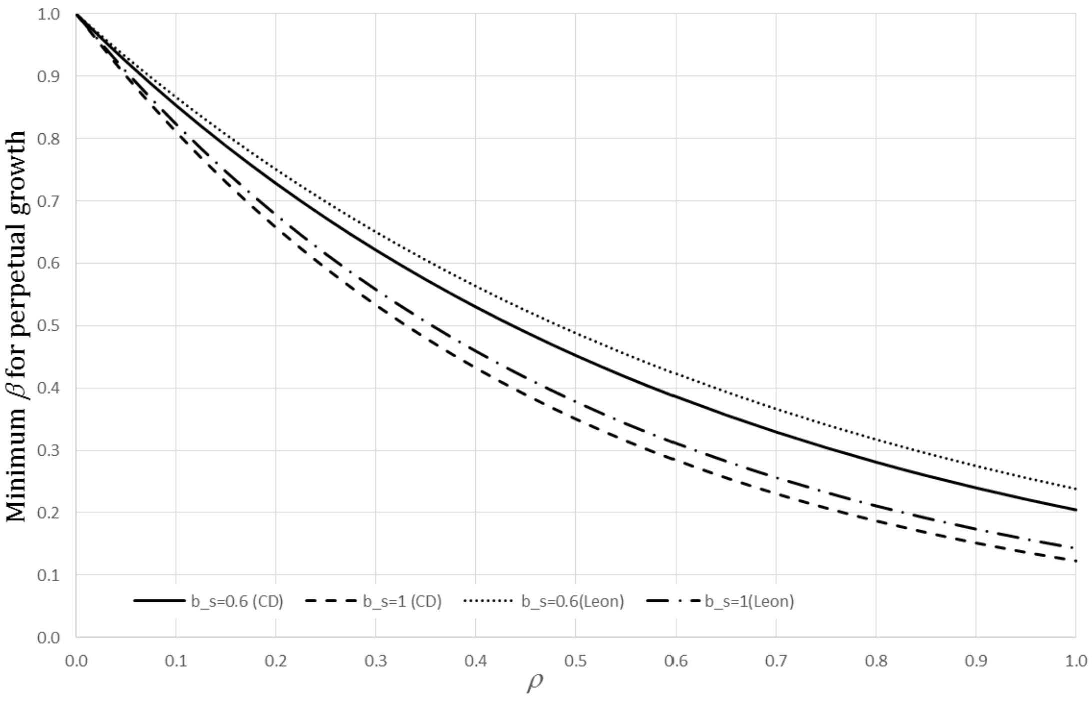

Figure 1. The growth condition in terms of β and ρ

Equation (11) implies that the growth rate of output in the steady state, i.e., where L/K_s goes to zero, and with σ_μ = 0, is equal to the growth rate of the capital stock K_s. Without population growth (as assumed), this growth rate, which will be denoted by g_Q, is also equal to the growth rate of per capita output. The growth rate of the capital stock – and hence output – is equal to net investment divided by (K_c + K_s). To obtain this, we use equations (10a) and (11) to write:

$$g_Q = \frac{I-\delta(K_o+K_s)}{K_o+K_s} = \frac{s_K Q}{K_o+K_s} - \delta = s_K \frac{b_o b_s \beta^{1/\rho}}{b_o+b_s\beta^{1/\rho}} - \delta \qquad (13a)$$

In the appendix, we show that using similar logic, the growth rate in case of production function (1c) and $\sigma_\mu = 1$ is equal to

$$g_Q = s_K(\alpha b_o)^\alpha \left(b_s(1-\alpha)\beta^{1/\rho}\right)^{1-\alpha} - \delta \qquad (13b)$$

These expressions show that the steady-state growth rate depends positively on b_s, β and ρ (the "robot parameters"), as well as on the savings rate. The depreciation rate has a negative impact on the growth rate, while the effect of α (in case of 13b) on the growth rate is ambiguous.

Figure 2 illustrates the growth rate as a function of ρ. The figure uses $b_o = 1$, $b_s = 0.6$, $\beta = 0.5$ and $\alpha = 0.3$. The solid line represents the growth rate when $\sigma_\mu = 0$ (equation 13a), the dotted line is for $\sigma_\mu = 1$ (equation 13b). Growth takes off after the threshold values for ρ consistent with growth conditions (12a) and (12b). Higher substitutability between K_o and the composite $K_s - L$ factor influences the growth rate positively, as the dotted line is always above the solid line. Plotting the growth rate with alternative parameter values suggests that this is a fairly general result, although the growth rates converge for high values of b_s and β.

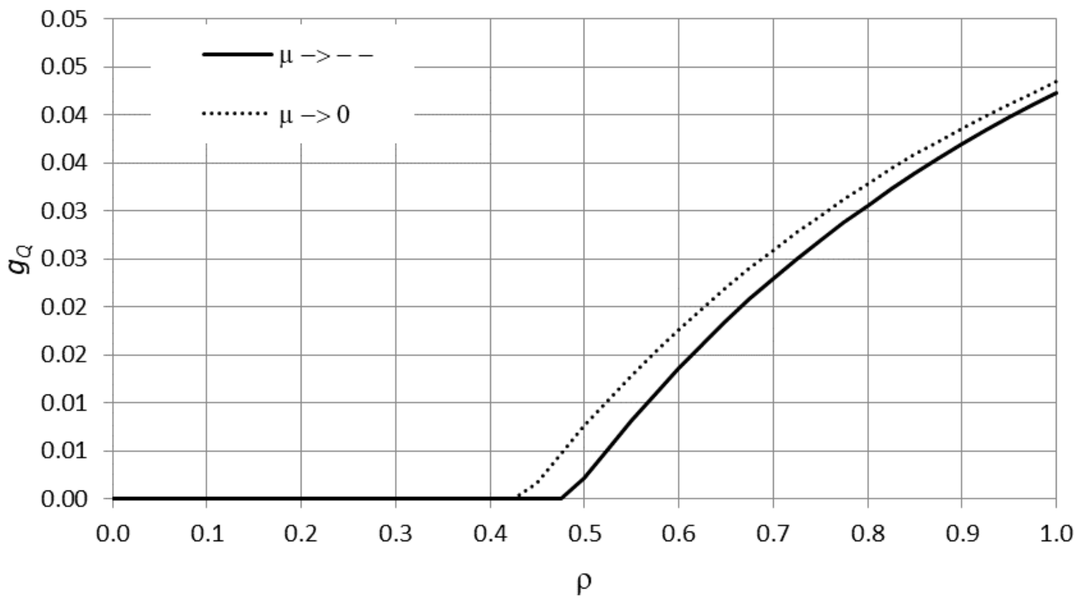

Figure 2. The perpetual growth rate

6. Perpetual growth and labour income

How does labour income evolve in a perpetually growing economy, both in terms of its share of total income, and in terms of the wage rate? The key element of perpetual growth is capital accumulation, both in terms of K_o and K_s. Our question is therefore how the wage rate and the share of wages in total income are affected by the growth rates of both capital stocks. As DeCanio (2016) shows, the answer to this question strongly depends on the kind of production function that is applied, and in particular on the second derivative of output with respect to the production factors. If the production function contains just two production factors, the price paid for one production factor will never be influenced negatively by the other factor. In other words, if homogenous labour is combined with just one kind of capital, the questions we are asking here would be of little relevance.

However, in the kind of nested production function with three production factors that we use, dynamics of factor payments are more complicated. These production functions support the idea (which, as DeCanio (2016) briefly summarises, has been around at least since the since the days of Marx and Ricardo) that technological change by accumulation of capital may affect labour income in a negative way. To investigate this, we derive the (partial) elasticities of the labour income share and of the wage rate, with respect to growth of the capital stock. We will denote these elasticities by ε (appropriately subscripted), and start with the elasticity of the labour income share with respect to the capital stock K_o, which we derive for the general form of the production function (1a):

$$\varepsilon_o \equiv \frac{\partial \varpi_L}{\partial K_o}\frac{K_o}{\varpi_L} = -\mu\frac{(b_o K_o)^{\mu}}{\Theta_T} = -\mu\varpi_o \qquad (14)$$

Because $\mu < 0$ and $\varpi_o > 0$, this is strictly positive, which implies that the partial effect of accumulation of the capital stock K_o on the share of labour income is positive. In this sense, capital K_o is truly labour augmenting.

Similarly, we derive the elasticity of the labour income share with respect to the capital stock K_s as

$$\varepsilon_s \equiv \frac{\partial \varpi_L}{\partial K_s}\frac{K_s}{\varpi_L} = \frac{\beta(b_s K_s)^{\rho}}{\Theta_{LS}}\left[\mu\left(1 - \frac{(1-\alpha)\Theta_{LS}^{\frac{\mu}{\rho}}}{\Theta_T}\right) - \rho\right] = \nu_s(\varpi_o\mu - \rho) \qquad (15)$$

With $\mu < 0$, $\rho > 0$ and both income shares positive, this expression is strictly negative, i.e., the accumulation of the robot capital stock K_s will have a negative partial effect on the share of labour income. Thus, we see an interesting contradiction: capital accumulation of type K_o augments labour income, while accumulation of K_s affects it negatively. This strong conclusion is a result of the labour-substituting nature of robot capital, and the labour-augmenting (complementary) nature of other capital.

Which of these two effects will dominate depends on the growth rates of both capital stocks, and the income shares and parameters in equations (14) and (15). In particular stages of the transient towards the steady state growth path, either one effect may dominate, and as a result the labour income share may either increase or decrease. For example, if the rate of accumulation of K_o is high (low) as compared to that of K_s, we may expect that the share of labour income will rise (fall). However, in the steady state, the growth rates of both types of capital are equal to each other (by equation 10), and hence we may add up equations (14) and (15) to yield the elasticity of the labour income share with respect to total capital accumulation:

$$\varepsilon_{os} \equiv \varepsilon_o + \varepsilon_s = -\mu\varpi_o + \nu_s(\varpi_o\mu - \rho) = -\mu\varpi_o(1-\nu_s) - \nu_s\rho \qquad (16)$$

For $\nu_s < 1$ and $\mu < 0$, this may either be positive or negative, while for $\mu = 0$ (which is the special case of production function 1c), or $\nu_s = 1$, the expression will become strictly negative. To see that the latter ($\nu_s = 1$) is indeed the case along the steady state growth path, we rewrite equation (3a) as

$$\varpi_L = (1-\alpha)(1-\beta)\frac{\left(\frac{L}{K_s}\right)^{\rho} b_s^{\rho} K_s^{\mu}\left(\beta b_s^{\rho}+(1-\beta)b_L^{\rho}\left(\frac{L}{K_s}\right)^{\rho}\right)^{\frac{\mu-\rho}{\rho}}}{\Theta_T} \quad (17)$$

This shows that in the steady state, when $\frac{L}{K_s} \to 0$, ϖ_L will also converge to zero. Consequently, ν_s will converge to one in the steady state, which implies that equation (16) will eventually become negative. Thus, the share of labour income will decline towards zero in the steady state.[2]

Does the same hold for the wage rate (which we will denote as $W = \partial Q/\partial L$)? To answer this question, we derive similar elasticities as before, this time for the wage rate with respect to both capital stocks. We denote these elasticities by ϵ, and find them to be equal to

$$\epsilon_o \equiv \frac{\partial W}{\partial K_o}\frac{K_o}{W} = (1-\mu)\frac{(b_o K_o)^{\mu}}{\Theta_T} = (1-\mu)\varpi_o \quad (18)$$

$$\epsilon_s \equiv \frac{\partial W}{\partial K_o}\frac{K_s}{W} = \frac{\beta(b_s K_s)^{\rho}}{\Theta_{LS}}\left[\frac{(1-\alpha)\Theta_T}{\Theta_T} + \mu\left(1-\frac{(1-\alpha)\Theta_{LS}^{\frac{\mu}{\rho}}}{\Theta_T}\right) - \rho\right] = \nu_s\big(1-\rho-\varpi_o(1-\mu)\big) \quad (19)$$

$$\epsilon_{os} \equiv \epsilon_o + \epsilon_s = \nu_s(1-\rho) + \varpi_o(1-\mu)(1-\nu_s) \quad (20)$$

Equation (18) is strictly positive, i.e., the partial effect of capital accumulation of type K_o on the wage rate is positive, either in the steady state or in a transient towards it. Hence the analysis for the wage rate confirms the conclusion of the labour-augmenting nature of Ko that we arrived at in the analysis for the labour income share (equation 14).

Equation (19) may either be positive or negative, depending on parameter values and income shares. For the special case $\mu = 0$ (production function 1c), we have $\varpi_0 = \alpha$, and the sign of equation (19) depends on the sign of $(1 - \rho - \alpha)$: if $\rho > 1 - \alpha$, the partial effect of capital accumulation of type K_s on the wage rate will be negative, otherwise it will be positive.

Equation (20), which holds only in the steady state when the rates of accumulation of both types of capital are equal, is strictly positive (it can be shown that the threshold value for ρ to make this equation negative lies above 1, which is ruled out). Hence in the steady state, the wage rate will rise, although the share of wages in income will decline. The conclusion is that perpetual growth by robots makes labour better off in absolute terms (the wage rate), but in relative terms (income distribution), workers clearly are victims of robots.

7. Distributional policies

The remainder of the paper will be devoted to a discussion of two possible ways of fighting inequality that results from growth by robots. The first is a social protection policy that taxes income from capital ownership and re-distributes the revenue of this tax to wage earners. We implement this by a fixed tax rate τ raised every period. The second option introduces savings

[2] Using a production function very similar to ours, DeCanio (2016) estimates the elasticity of substitution between robots and human labour at which the labour share begins to decline. His estimations point to a value of the substitution parameter (ρ in our case) between 0.4 and 0.5.

out of wage income, thus enabling wage earners to invest in capital (of both types) and complement their wages by capital income.

In order to undertake this analysis, we numerically simulate the model rather than provide analytical solutions. We use production function (1c) in these simulations, i.e., the elasticity of substitution $\sigma_\mu = 1$. We use discrete time, i.e., for every period in the simulation, we (i) calculate output using the existing capital stocks and available labour, (ii) calculate the income shares and associated savings, (iii) numerically solve the investment allocation problem and assign total savings to the two different kinds of investment, (iv) update the capital stocks, and return to step (i) for a new period. We set initial values of K_o and K_s to 100 and 0.1 respectively, and N = 100 throughout all simulations. Parameter values are $\alpha = 0.3$, $\delta = 0.05$, $s_K = 0.4$, $b_o = b_s = b_L = 1$, $\beta = 0.4$, $\rho = 0.7$. These parameter values ensure perpetual growth (without taxes). We simulate 400 periods and assure that convergence in the growth rates is achieved at that point.

We start the discussion with the tax option, which can be seen as the implementation of a universal basic income (Van Parijs and Vanderborght, 2017). The obvious drawback of this is that taxes will hurt capital accumulation by reducing the funds available for investment, and thereby lowers the perpetual growth rate. Figure 3 shows the result. Without any taxes and re-distribution, we observe a growth rate of about 3.7%, and a share of labour income in total income that is very close to zero. When the tax rate rises, the growth rate declines, until it becomes very close to zero at $\tau = 0.5$.

The impact of the re-distributed tax on the share of income is about equal to the tax rate τ itself, because it is in effect almost a tax on total income (wage income is very small when substantial growth occurs). Thus, in order to reach a labour income share of about 0.4 – 0.6, , we need a tax rate of about 40%. But this essentially makes growth go away.

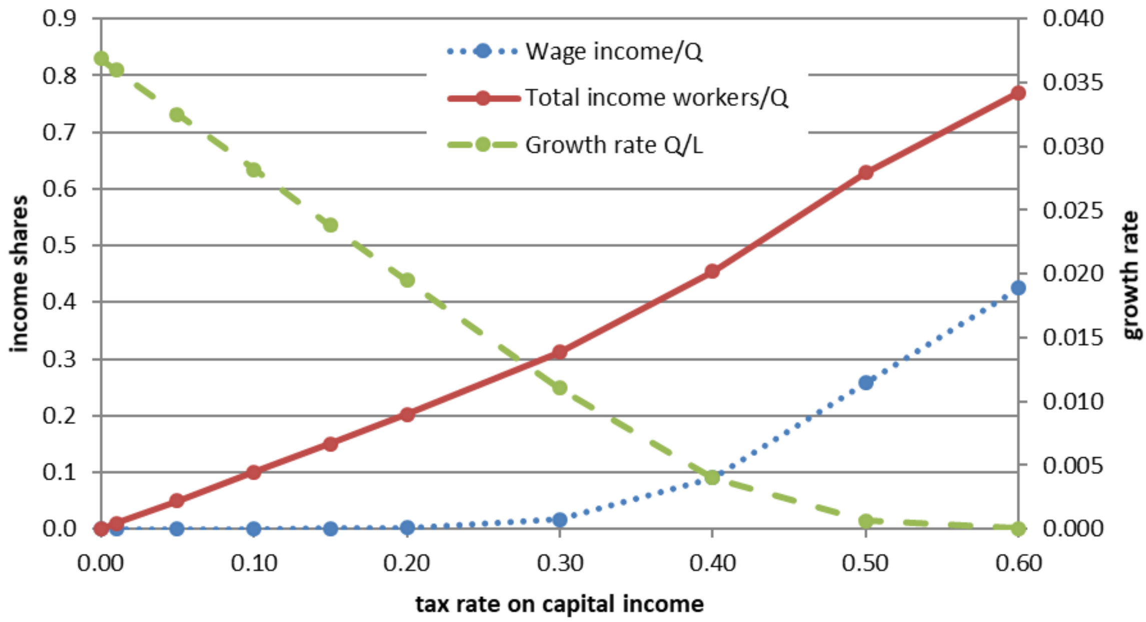

Figure 3. The effects of income re-distribution based on a capital income tax

Next, we look at a case where workers save and invest in capital, following Freeman's (2014) idea that workers "can rule the world" by owning robots. To this end, we modify equation (4) as follows (note that we also substitute income shares that apply to the case of production function 1c):

$$S = s_K Q\left[\alpha + (1-\alpha)\frac{b_1 K_s^{\rho}}{b_1 K_s^{\rho} + b_2 L^{\rho}}\right] + s_L(1-\alpha)Q\frac{b_2 L^{\rho}}{b_1 K_s^{\rho} + b_2 L^{\rho}} \qquad \text{(4a)}$$

where s_L is a new parameter that represents the savings rate out of labour income. Note that for s_L = 0 and production function (1c), equation (4a) reduces to equation (4), i.e., the model is as before. Note also that equation (4a) assumes that savings rates are specific to income category, i.e., when a worker has both labour income and capital income, that worker applies two distinct savings rates (s_L and s_K). All savings are invested. We keep track of which part of the two capital stocks is owned by workers, and which part by capital owners. The ownership of capital does not impact the growth rate in any way, but s_L > 0 does imply that the total amount of savings is larger, and hence the growth rate is also larger. But this is a small effect, because savings out of wages is a small amount, because wages as a share of income (the source of extra savings) are near zero when growth is substantial.

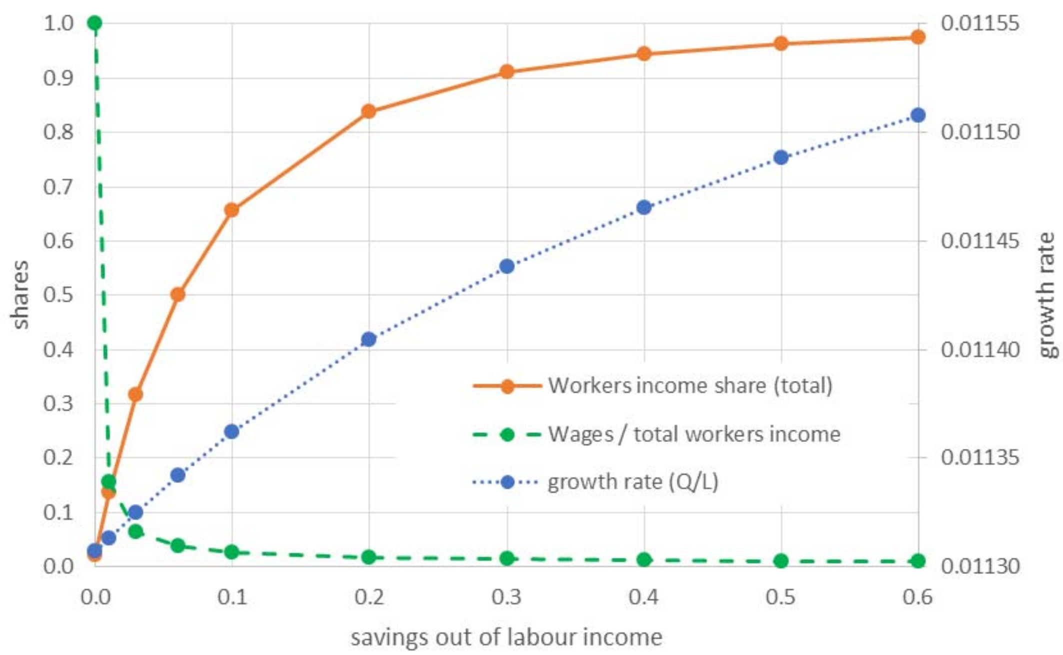

Figure 4. Distributional and growth effects of labour income saving

Figure 4 presents the results of the experiment, using values for s_L from 0 to 0.6. We set ρ = 0.5 for this analysis, and all other parameters as before. Growth increases with the savings rate, but this effect is very small as indicated by the scale of the vertical axis on the right, which covers a difference of only 0.00025. The effects on distribution are much more substantial. Where workers' income consists of wages only, without saving, it is near to zero as a share of total income, as expected. With positive savings out of labour income, this increases rapidly for small savings rates s_L, to about 65% when s_L = 0.1, and well above 95% for s_L = 0.6. Wages as a share of total workers' income decline rapidly for positive values of s_L, and are negligible already for s_L = 0.1.

Concluding from the two experiments aimed at realising a reduction inequality, it seems that taxing and re-distributing is much less effective than workers' savings and investment. Perpetual growth by robots causes inequality between workers and capital owners, and the key to reducing this inequality lays in transforming workers to capital owners. In this model, robots practically make human labour go away, but provide a source of riches for their owners.

8. Summary and Conclusions

Our analysis, in particular the production function that we use, starts from the assumption that technological progress in the form of machine learning and artificial intelligence is of a fundamentally new nature. This form of technological development produces a form of capital that substitutes for human labour rather than augments it. Our model is aimed at investigating the consequences of this for labour markets, in particular wages and the wage share of total income (our model assumes full employment and hence has little to say about the employment consequences of technological change).

Our analysis illustrates that labour-substituting technological progress can generate perpetual growth, i.e., sustained growth of per capita output with a given state of technology. Based on empirical work by, among others, Frey and Osborne (2017), Arntz et al. (2017) and Nedelkoska and Quintini (2018), we argue that technologies such as machine learning, mobile robotics and artificial intelligence (in short, "robots") may soon be able to provide this kind of labour-substituting technology. Our analysis suggests three main factors in determining whether the robot-technology will generate perpetual growth: the extent to which the technology can automate human labour tasks, how cheap the technology can be implemented, and how well automated tasks can substitute tasks performed by humans in the aggregate production process.

Our model was able to derive conditions for perpetual growth to occur, and expressions for the perpetual growth rate. Perpetual growth is more likely to occur, and the growth rate will be higher, when robot technology is more advanced. In particular, we show that robot-technology does not have to offer complete or even near-complete substitutability for human labour in order to generate perpetual growth.

In a future where robots generate perpetual growth, income from human labour will fetch a negligible part of total income in the economy, even though the absolute wage rate will rise as a result of robot-based perpetual growth. This means that perpetual growth makes workers better off in an absolute way, but worse off in a relative way. This implies that without social protection policy, income inequality may rise to unprecedented levels in a future economy. We implemented some simulation experiments in which two potential kinds of social protection are considered.

In one experiment, income from robots was taxed and transferred to those who are living only on a wage income. This can be seen as the implementation of a universal basic income, in a budget-neutral way. The drawback of the tax is that it reduces funds available for investment in capital, which is the source of perpetual growth. We assumed that the tax does not influence the savings rate (i.e., that it is paid proportionally out of consumption and investment). The main finding was that in order to reduce inequality in a substantial way, i.e., to keep the income share of wage owners (including the tax transfer or universal basic income) at a level comparable to a pre-perpetual growth state of the economy, growth will be reduced substantially. This suggests that it may be difficult to fight inequality by a universal basic income if growth becomes very much dependent on robot-technology.

The other policy option that was considered was workers' savings, i.e., saving out of labour income. In this way, workers become also capital owners (and we assume that their savings rate on capital income is equal to that of "pure" capital income earners). This generates extra savings and investment rather than reduce it, and hence is growth-enhancing. The growth effect is small, however, because labour income remains small and hence does not generate much savings. However, "workers" are able to accumulate a capital stock, which becomes the main source of their income. Even a relatively modest savings rate for labour income (e.g., 10%) reduces inequality very significantly.

Although this savings experiment could be seen as a voluntary arrangement, not dependent on policy, it may also be implemented as a forced savings scheme, comparable to pension schemes in many (European) countries. As such, it can be seen as a form of social protection policy, and one that seems relatively effective (more so than the universal basic income). With this kind of policy, the view in Albus (1976) about a robot-society as a "peoples' capitalism" may indeed become true.

There are several open resource avenues that must be followed for the theory in this paper to become better founded and better able to think about a future economy in which robot-technology becomes dominant. For example, our growth model is relatively crude. It reflects Solow's model from the 1950s, and ignores advances made in growth theory since then. It remains to be shown whether endogenisation of important factors such as savings and investment in technology (which we ignored completely) change any of the major conclusions about perpetual growth or the income distribution.

Also, work remains to be done about social protection policy. Due to the crudeness of our model, we were not able to analyse detailed form of such policies, or look at the various ways in which these policies could be instituted. The analysis does suggest, however, some directions for research on this topic, such as the relationship between a universal basic income and aggregate savings and investment. Relatively little seems to be known about this relationship, but our results suggest that it may be crucial for how well such a scheme could work.

In summary, our analysis points to the possibility that robot-technology will be a truly revolutionising force, and calls for more research on the implications and policy processes that will be necessary to make it a soft revolution.

References

Acemoglu, D. and P. Restrepo, 2017, The Race Between Machine and Man: Implications of Technology for Growth, Factor Shares fnd Employment, NBER Working Paper 22252, National Bureau of Economic Research

Albus, J.S., 1976, Peoples' Capitalism: the Economics of the Robot Revolution, New World Books.

Arntz, M., Gregory, T. and U. Zierahn (2016), "The Risk of Automation for Jobs in OECD Countries: A Comparative Analysis", OECD Social, Employment and Migration Working Papers, No. 189, OECD Publishing, Paris

Autor, D., Levy, F. and R.J. Murnane, 2003, The skill content of recent technological change: an empirical exploration, Quarterly Journal of Economics, vol. 118, pp. 1279–1333.

Autor, D., 2015, Why Are There Still So Many Jobs?, Journal of Economic Perspectives, vol. 29, pp. 3–30

Bessen, J., 2016, How Computer Automation Affects Occupations: Technology, Jobs, and Skills, Boston University School of Law, Law & Economics Working Paper No. 15-49

Chui, M., Manyika, J. and M. Miremadi, 2016, Where machines could replace humans – and they can't (yet), McKinsey Quarterly, July.

DeCanio, S.J., 2016, Robots and humans –complements or substitutes?, Journal of Macroeconomics, vol. 49, pp. 280–291

Freeman, C. and L. Soete, 1994, Work for All or Mass Unemployment, London: Pinter, 1994

Freeman, R.B. (2014), "Who owns the robots rules the world", IZA World of Labor 2014: 5

Frey, C.B. and M.A. Osborne, 2017, The future of employment: How susceptible are jobs to computerisation?, Technological Forecasting and Social Change, vol. 114, pp. 254–280.

Katsoulacos, Y., 1986, The Employment Effect of Technical Change. A theoretical study of new technology and the labour market, Brighton, Wheatsheaf

Lucas, R., 1988, On the mechanics of economic development, Journal of Monetary Economics, vol. 22, pp. 3-42

Nedelkoska, L. and G. Quintini, 2018, Automation, skills use and training, OECD Social, Employment and Migration Working Papers, 14 March, OECD Publishing, Paris.

Nelson, R.R. and S.G. Winter. (1982), An Evolutionary Theory of Economic Change, Harvard University Press.

Peretto, P.F. and J.J. Seater, 2013, Factor-eliminating technical change, Journal of Monetary Economics, vol. 60, pp. 459–473

Pratt, G.A., 2015, Is a Cambrian Explosion Coming for Robotics?, Journal of Economic Perspectives, vol. 29, pp. 51–60

Prettner, K., 2017, A Note on the Implications of Automation for Economic Growth and the Labor Share, Macroeconomic Dynamics, advance copy.

Romer, P., 1990, Endogenous Technological Change, Journal of Political Economy, 1990, vol. 98, pp. S71-102

Silverberg, G. and B. Verspagen, 1994, Collective Learning, Innovation and Growth in a Boundedly Rational, Evolutionary World, Journal of Evolutionary Economics, vol 4, 207-226.

Solow, R.M., 1956, A Contribution to the Theory of Economic Growth, The Quarterly Journal of Economics, vol. 70, pp. 65–94

Van Parijs, Ph. and Y. Vanderborght, 2017, Basic Income. A Radical Proposal for a Free Society and a Sane Economy, Harvard University Press.

Vivarelli, M., 1995, The Economics of Technology and Employment: Theory and Empirical Evidence, Aldershot: Elgar.

Appendix – Additional derivations

We present here some additional derivations on the growth properties of the model when we use production function (1b), i.e., where the outer part of the production function is Cobb-Douglas. We start with the condition for perpetual growth, which, as in the main text, is based on net investment ($I - \delta K$) being positive. As in the main text, we start by factoring out K_s from the CES part of the production function (1b), yielding

$$Q = A(b_o K_o)^{\alpha} K_S^{1-\alpha} \left(\left[\beta b_S^{\rho} + (1-\beta) \left(\frac{b_L L}{K_S} \right)^{\rho} \right]^{\frac{1}{\rho}} \right)^{1-\alpha} \quad \text{(A1)}$$

With the term $\frac{L}{K_S}$ going to zero, equation (A1) will converge to

$$Q = A(b_o K_o)^{\alpha} K_S^{1-\alpha} \left(\beta^{\frac{1}{\rho}} b_S \right)^{1-\alpha} \quad \text{(A2)}$$

Profit maximising investment implies $dQ/dK_S = dQ/dK_O$, which, based on the production function (1b) and keeping in mind $\frac{L}{K_S} \to 0$, yields

$$K_o = K_S \frac{\alpha}{1-\alpha} \quad \text{(A3)}$$

Substituting (A3) in (A2), we obtain

$$Q = K_S \left(\frac{\alpha}{1-\alpha} b_o \right)^{\alpha} \left(\beta^{1/\rho} b_S \right)^{1-\alpha} \quad \text{(A4)}$$

Using equations (A3) and (A4), the growth condition becomes

$$\beta > \left(\frac{\delta}{s_K} \frac{1}{(\alpha b_o)^{\alpha}} \frac{1}{\left(b_S(1-\alpha)\right)^{1-\alpha}} \right)^{\frac{\rho}{1-\alpha}} \quad \text{(A5)}$$

The growth rate of the capital stock K_s, which by equation (A4) is equal to the growth rate of output, is equal to net investment divided by the capital stock itself. Using also equation (A3), we obtain

$$g_Q = \frac{I - \delta(K_o + K_S)}{K_o + K_S} = \frac{s_K Q}{K_o + K_S} - \delta = s_K (\alpha b_o)^{\alpha} \left(b_s (1-\alpha) \beta^{1/\rho} \right)^{1-\alpha} - \delta \quad \text{(A6)}$$

The UNU-MERIT WORKING Paper Series

2018-01 *The serendipity theorem for an endogenous open economy growth model* by Thomas Ziesemer

2018-02 *Instability constraints and development traps: An empirical analysis of growth cycles and economic volatility in Latin America* by Danilo Sartorello Spinola

2018-03 *Natural, effective and BOP-constrained rates of growth: Adjustment mechanisms and closure equations* by Gabriel Porcile and Danilo Sartorello Spinola

2018-04 *The relevance of local structures for economic multiplier effects of social pensions in Uganda* by Maria Klara Kuss, Franziska Gassmann and Firminus Mugumya

2018-05 *Regulating the digital economy: Are we moving towards a 'win-win' or a 'lose-lose'?* by Padmashree Gehl Sampath

2018-06 *The economic impacts of a social pension on recipient households with unequal access to markets in Uganda* by Maria Klara Kuss, Patrick Llewellin and Franziska Gassmann

2018-07 *The effect of weather index insurance on social capital: Experimental evidence from Ethiopia* by Halefom Y. Nigus, Eleonora Nillesen and Pierre Mohnen

2018-08 *Evaluating intergenerational persistence of economic preferences: A large scale experiment with families in Bangladesh* by Shyamal Chowdhury, Matthias Sutter and Klaus F. Zimmermann

2018-09 *Agricultural extension and input subsidies to reduce food insecurity. Evidence from a field experiment in the Congo* by Koen Leuveld, Eleonora Nillesen, Janneke Pieters, Martha Ross, Maarten Voors and Elise Wang Sonne

2018-10 *Market integration and pro-social behaviour in rural Liberia* by Stephan Dietrich Gonne Beekman, and Eleonora Nillesen

2018-11 *Sanctioning Regimes and Chief Quality: Evidence from Liberia* by Gonne Beekman, Eleonora Nillesen and Maarten Voors

2018-12 *Chinese development assistance and household welfare in sub-Saharan Africa* by Bruno Martorano, Laura Metzger, Marco Sanfilippo

2018-13 *Foreign direct investment in sub-Saharan Africa: Beyond its growth effect* by Hassen Abda Wako

2018-14 *Fluctuations in renewable electricity supply: Gains from international trade through infrastructure?* by Thomas Ziesemer

2018-15 *What is the potential of natural resource based industrialisation in Latin America? An Input-Output analysis of the extractive sectors* by Beatriz Calzada Olvera and Neil Foster-McGregor

2018-16 *Global Value Chains and Upgrading: What, When and How?* by Padmashree Gehl Sampath and Bertha Vallejo

2018-17 *Institutional factors and people's preferences in the implementation of social protection: the case of Ethiopia* by Vincenzo Vinci and Keetie Roelen

2018-18 *Towards a European full employment policy* by Jo Ritzen and Klaus F. Zimmermann

2018-19 *Drivers of growth in Tunisia: Young firms vs incumbents* by Hassan Arouri, Adel Ben Youssef, Francesco Quatraro and Marco Vivarelli

2018-20 *Africa Sector Database (ASD): Expansion and update* Emmanuel Buadi Mensah and Adam Szirmai

2018-21 *Combatting corruption in higher education in Ukraine* by Anna Vasylyeva and Ortrun Merkle

2018-22 *What more can we learn from R&D alliances? A review and research agenda* by Andrea Martinez-Noya and Rajneesh Narula
2018-23 *Perpetual growth, distribution, and robots* Önder Nomaler and Bart Verspagen