Traditional or Additive Manufacturing? Assessing Component Design Options through Lifecycle Cost Analysis

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A B S T R A C T

We consider an original equipment manufacturer that can either design a system component that is produced with traditional technology, or design an alternative component that is produced with additive manufacturing (AM). Designing either component requires a technology specific one-time investment and the components have different characteristics, notably in terms of production leadtime, production costs and component reliability. We support the design decision with a model that is based on evaluating the lifecycle costs of both components, covering design costs, maintenance and downtime costs, and performance benefits. We derive analytic properties of the required reliability and costs of the AM component such that its total lifecycle costs break even with that of its regular counterpart. Through our analysis, a numerical experiment and cases from two different companies, we find that component reliability and production costs are crucial to the success of AM components, while AM component design costs can be overcome to a certain degree by generating performance benefits or by using the short AM production leadtime to lower the after-sales logistics costs.

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1. Introduction

Capital goods are complex technical systems that are essential to their users’ business processes. Examples of such systems are airplanes, trains, military weapon platforms and semiconductor production machines. These systems are characterized by their high lifecycle costs, a large part of which is generated during their exploitation phase. Much of these exploitation costs are predetermined by decisions taken by the original equipment manufacturer (OEM) in the system design phase.

One interesting development, which may help reduce total lifecycle costs, is the development of additive manufacturing (AM) technology. Additive manufacturing, which is also sometimes called 3D printing, can be defined as parts fabrication by creation of successive cross-sectional layers of an object, usually based upon a three-dimensional solids model (Gao et al., 2015). These parts can be plastic parts, but also many types of metal can be used. AM offers engineers greatly improved design freedom compared with traditional manufacturing technologies. This can offer large performance benefits, for instance by reducing fuel consumption for airplanes through reduced component weight or lowering the power consumption of pumps by optimizing fluid flow through cooling channels. Furthermore, AM offers much shorter leadtimes for small production series, which can reduce required spare part investments and increase the responsiveness of after-sales service supply chains.

At the same time, AM components currently require high development costs compared to their regular counterparts. While the regular component is usually an adapted version of a component installed in earlier systems, the AM component often requires new design features, especially if there is a wish to capture potential performance benefits by making use of AM’s design freedom. Further complicating the shift to AM components is the fact that design rules for production engineers are still under development and that preliminary rules are not easily generalized over different products and different AM systems (Yang et al., 2017, p. 83). This complicates the design process and increases development costs. When a design has been decided upon, trial production runs are required to test product reliability and to fine-tune production parameter settings, such as laser intensity or layer thickness. Such fine-tuning must be done in great detail, as each of the different geometric properties of complex products may require individual attention. Laser intensity, for example, may need to change repeatedly when the laser passes over alternating sequences of solid material and cavities of different sizes. This detailed testing can further increase the development costs compared to traditional manufacturing, such as machining, for which material properties are standardized and remain largely constant during the production...
process (Yang et al., 2017, p. 83). Note that there are exceptions for which the development costs of an AM part may be smaller than those for developing an traditional part, for example in cases where expensive tooling such as casting or injection molds are required. Our model takes this into account by including a development cost difference that is unrestricted in sign.

When evaluating the potential of AM, many product developers currently focus on weighing off the potential performance benefits against the required design investment. This means that potential cost reductions in the after-sales service supply chain, which are due to changes in reliability, production costs and production leadtime, are neglected. In some cases, opting for AM over traditional manufacturing is obvious. GE’s new jet engine fuel injection nozzle, for example, exhibits improved characteristics over the former traditional design in terms of reliability, lead time, performance and production costs (GE Aviation, 2014). However, there are many components for which the characteristics of AM components are not all favorable. In fact, a current limitation of AM technology is that there is often uncertainty concerning the mechanical properties of such parts (Bikas, Stavropoulos, & Chryssolouris, 2016). This may have a large negative effect on the maintenance and repair costs that are incurred over the course of an asset’s lifecycle.

Potential negative characteristics may be offset by a reduction of the average production leadtime, which is expected to be much shorter for small series of spare parts when AM is employed as opposed to traditional manufacturing methods. This is the key assumption that we make for our analytical model. The question remains what properties the AM component must have in terms of reliability and production costs in order to be preferred over its regular counterpart. We introduce a model that compares the total lifecycle costs of the regular part with those of the AM part, taking into account design costs, performance benefits and all spare part related costs, including maintenance and downtime costs. This model is used to evaluate the break-even component production costs and the break-even component reliability, such that the total lifecycle costs of the regular part equal those of its AM counterpart. For these break-even characteristics we derive analytic properties, conduct numerical experiments and we present two case studies to gain insight into the conditions under which an AM component outperforms a regular component.

The design decision that we consider takes place during the system design phase. The OEM can either design a regular component, which is usually based on a component that was used for earlier versions of the system, or he can design a completely new component and make use of the capabilities of AM to capture performance benefits and profit from a much reduced production lead time. Note that not all component types are suitable for AM, so a pre-selection of candidate components can be made based on methods that evaluate AM suitability based on basic component characteristics (e.g., Knofius, van der Heijden, & Zijm, 2016). We support the decision to design a regular component or an AM version by developing a model that considers either reliability or unit production costs as given and provides the values of the other parameters for which one design option dominates the other in terms of lifecycle costs. This is useful in practice if it is hard for design engineers to find good estimates for these parameters. In that case, optimizing a component’s design would be impossible as that involves taking into account the even more difficult to characterize relationships between the design investment, the unit production costs and the component reliability. Each of these relationships, such as the one modeled in Mettas (2000) between design investment and component reliability, is very difficult to parameterize. Doing this properly for all the relationships involved is even more difficult. If engineers estimate that the break-even properties provided by our model will be comfortably met, then AM is preferable. In cases where estimated properties are much worse than the break-even properties, it is better to opt for traditional production technology. If the estimated properties are similar to the break-even properties, more research must be done to more accurately determine the eventual AM component characteristics, or an organization may consider more qualitative reasons to opt for the AM version, for example to gain experience with the technology.

Our model can also be applied to redesign decisions that are taken during the exploitation phase. In this case, we require a negligible transition period for replacing the old component with new versions, to avoid a period of the lifecycle where two component types are operating in the field simultaneously. Such fast transitions can occur when there is a large performance benefit to exploit in combination with ample opportunity to upgrade to the AM part, for example in the aviation industry when lighter AM components become available. Another application of our model during the exploitation phase, is when the OEM must redesign a poorly designed component. Earlier studies have shown that in such upgrade situations, it is often advantageous to preventively replace all components directly after redesign, instead of replacing them one-for-one at the time of failure (e.g., Clavareau & Labeau, 2009; Öner, Kiesmüller, & van Houtum, 2015).

We determine the lifecycle costs that are generated by all parties in the supply chain, from the OEM to the end user. If the AM component is preferable due to lower lifecycle costs, and there are multiple parties involved in generating the lifecycle costs, including potential benefits, then a method is required to determine in which way each party benefits, for example via game-theoretical methods. Developing such a framework, however, is beyond the scope of this paper. If the level of cooperation in the supply chain is limited, our model can still be used by individual parties, who must then recognize which parts of the lifecycle costs and potential performance benefits apply to their situation. In summary, our contribution is as follows:

1. We develop an original model for a component design decision, based on the evaluation of the total lifecycle costs of two competing types of components, one produced with traditional technology and one produced via additive manufacturing. We take into account design costs, performance benefits and after-sales service logistics costs.
2. We generate analytic insights into the relationship between design costs, performance benefits and the minimally required AM component characteristics. We conduct a numerical experiment to generate additional insight into situations where AM can likely be successfully applied to component design.
3. Two case studies are conducted to test the current applicability of AM in a component design setting.

The remainder of this paper is organized as follows. In Section 2, we survey the literature on related system design problems and on spare parts related to AM. Next, Section 3 contains the model formulation. Section 4 contains the analysis of our model, and Section 5 contains a numerical experiment that is used to generate managerial insight into the potential of AM for spare part supply. In Section 6 we present the two case studies and Section 7 includes some extensions in which we add stochasticity to two of the variables in our model. Section 8 contains our conclusions.

2. Literature review

We evaluate the design decision to opt for either a regular part or an AM part based on its effect on total lifecycle costs. In this section, we first review literature related to such design decisions in reliability allocation problems and then in warranty problems. Our total lifecycle cost model also includes a spare part inventory
system, so we also briefly review literature on spare parts management in relation to additive manufacturing. Finally we review a case study on AM component redesign.

In the literature on reliability allocation problems, typically, the dependency is modeled between a design decision, which is to select a certain component or a certain reliability level, and total lifecycle costs. Reliability allocation literature deals with selecting an optimal reliability level for a particular component. In the case of spare parts, the analysis also requires modeling an inventory system. The lifecycle costs then consist of design, production, inventory holding and repair/downtime costs. A typical objective is to maximize system availability given a budget constraint. Or alternatively, to minimize total lifecycle costs under a system availability constraint. Öner, Kiesmüller, and van Houtum (2010) and Selçuk and Ağralı (2013) optimize a reliability allocation decision in combination with a spare part inventory system. Both models assume that the number of systems in the field is constant. A different model formulation is provided by Jin and Tian (2012), who optimize the one-time reliability allocation decision and the periodic inventory control decisions under the assumption that the installed base increases randomly over time.

Our work is different from reliability allocation problems in terms of the approach that we follow. Due to the practical motivation of our work, we do not optimize over the design investment but we incorporate predetermined development costs and a performance benefit that one component may have over the other. We deploy a model similar to Öner et al. (2010) and use this to find a break-even point where the total lifecycle costs of the regular part and its AM counterpart are equal. This requires a different solution approach than traditional cost function minimization.

Another literature stream that deals with evaluation/optimization of total lifecycle costs through design decisions, are warranty models. These often have a separate design decision related to the warranty type or warranty period length, which impacts total lifecycle costs. Several examples of warranty models that also deal with reliability allocation, and that are, therefore, the most related to our work within the warranty literature stream, are Huang, Liu, and Murthy (2007), Wang, Huang, and Du (2010), Chattopadhyay and Rahman (2008) and Hussain and Murthy (2003). In the warranty literature, however, it is common not to take into account spare part holding costs and system downtime, which is essential for our evaluation, since AM has the potential to significantly reduce these cost components.

Our work is also related to spare parts inventory management for complex systems. For a general review on this topic, see Basten and Van Houtum (2014). The scope of our work is the entire system lifecycle, including the after-sales service logistics, which is typically mentioned as an application where AM can have a large impact (Ben-Ner & Siemsen, 2017). There is, however, only a limited amount of literature that explores the effect of AM on after-sales service logistics. Sirichakwal and Conner (2016) model a single stock point that follows a continuous review base-stock policy with lost sales to numerically investigate the effect of shortened lead times on the optimal base stock level. They conclude that lower lead times indeed lower the optimal base stock levels. Their analysis, however, assumes equal reliability for both part types, ignoring its effect on system downtime costs. Furthermore, it does not take into account investment cost differences related to using different technologies. These assumptions are also made by Liu, Huang, Mokasdar, Zhou, and Hou (2014), who compare the safety stock levels as a function of the desired service level for a supply chain network with a centralized traditional manufacturing center to one with a centralized AM facility and one with decentralized AM facilities. Khajavi, Partanen, and Holmström (2014) investigate the positioning of AM capacity in a multi-echelon supply chain by conducting scenario simulation, in order to gain insight into when AM will outperform traditional production methods. Their model also assumes equal reliability and it requires assumptions related to the future state of technology. We avoid such assumptions, and allow for different reliability levels of both components, by focusing on identifying break-even characteristics of AM components.

In terms of the insights that we provide, our approach is related to that of Atzeni and Salmi (2012), who compare the production costs for a traditional, high-pressure die-cast landing gear structure, to the production costs for a redesigned version that is produced via AM. With their detailed production cost model, they establish the break-even point in terms of volume produced, beyond which the traditionally produced part is cheaper. Their analysis, however, does not take into account design costs, or costs accumulated during the exploitation phase related to inventory holding, downtime and repair. We also provide analytic properties of the break-even point, which Atzeni and Salmi (2012) do not provide.

Like Öner et al. (2010) and Selçuk and Ağralı (2013), we assume that the installed base size remains constant over the product lifecycle, so that we can focus on the effect of the AM lead time reduction during this time period. We do not consider end-of-life decisions for spare parts inventory management, which would involve demand forecasting for a decreasing installed base, like, e.g., Kim, Dekker, and Heij (2017) and Hong, Koo, Lee, and Ahn (2008) do, as well as final-order decisions, like those that Teunter and Foruini (1999) make. Applying AM to such challenges is certainly interesting for future studies.

### 3. Model

In this section we introduce our modeling assumptions and define cost expressions related to the development, production and exploitation of regular and AM parts. A complete overview of all model variables and all model input parameters can be found in Tables 1 and 2, respectively.

An OEM designs a critical component for one of its next generation capital goods, to which we refer as the system. The OEM estimates that it will sell N units of the system and the time until the systems are phased out is T months. We assume that the N systems are sold at time t = 0, at which point also the design costs and the production costs are incurred. We follow the approach of Öner et al. (2010) and Selçuk and Ağralı (2013) by assuming that

### Table 1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>R(·)</td>
<td>Total performance benefits from using AM parts</td>
</tr>
<tr>
<td>Ĉ(·)</td>
<td>Total costs for production, inventory holding and downtime/repair</td>
</tr>
<tr>
<td>D(·)</td>
<td>Total downtime and repair costs</td>
</tr>
<tr>
<td>g(·)</td>
<td>Erlang loss probability for an inventory system</td>
</tr>
<tr>
<td>H(·)</td>
<td>Total inventory holding costs</td>
</tr>
<tr>
<td>K(·)</td>
<td>Net difference in investment costs between AM and regular part</td>
</tr>
<tr>
<td>P(·)</td>
<td>Initial part production costs</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ</td>
<td>Load on the inventory system</td>
</tr>
<tr>
<td>b_p</td>
<td>Performance benefit per AM component per unit time</td>
</tr>
<tr>
<td>c_d</td>
<td>Downtime and repair costs incurred per failure when a part is available</td>
</tr>
<tr>
<td>c_e</td>
<td>Emergency downtime and repair costs incurred in an out-of-stock situation</td>
</tr>
<tr>
<td>c_S</td>
<td>Component production costs</td>
</tr>
<tr>
<td>h</td>
<td>Holding cost rate in Euro per Euro per year</td>
</tr>
<tr>
<td>p</td>
<td>Investment costs related to developing a part</td>
</tr>
<tr>
<td>L</td>
<td>Mean component production leadtime</td>
</tr>
<tr>
<td>N</td>
<td>Installed base size</td>
</tr>
<tr>
<td>T</td>
<td>Time horizon length</td>
</tr>
<tr>
<td>τ</td>
<td>Component mean time between failure</td>
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the size of the installed base remains constant during the exploitation phase. This is a reasonable assumption given that the ramp-up phase of the installed base is typically short compared to the total time that the systems are in use.

For the system, the OEM can design a component that is produced via traditional production technology, or he can design a component that is produced via AM. We call these components the regular part and the AM part, respectively. Variables for which the AM and regular characteristics can differ, receive a superscript R or A to denote characteristics of a regular part and an AM part, respectively. Superscript x is used to denote a general characteristic that holds for both a regular and an AM component.

We assume that the characteristics of the regular component are known early in the design phase. This can realistically be expected, since such components are often upgraded from designs that were incorporated in previous system versions. The regular component would also be manufactured with a technology that is more mature, so that accurate estimations of its characteristics can be given.

Designing either a regular part or an AM part requires an investment in terms of design and testing costs. The investment costs for regular and AM components is denoted by R and P, respectively. We denote the expected difference between these two investment costs by I:

\[ I = R - P. \]

We generally expect the investment costs for the AM part to exceed the investment costs for the regular part, due to unfamiliarity with AM technology and the design principles involved. However, I can be negative when large investment costs are associated with using traditional production technology. For instance in the case of expensive tooling, such as casting molds, that is not required when producing with AM.

From the investment costs, we subtract a potential performance benefit, B(·), to take into account potentially beneficial effects of using an AM part. One example of such benefits is in aviation, where AM components with a honeycombed interior can greatly reduce weight compared to regular components. This decreases the plane’s fuel consumption, which leads to considerable savings. Such efficiency improvements result in a performance benefit over the product lifetime that is typically linear in the number of systems that benefit and their usage period:

\[ B(\cdot) = b_p N T, \]

with \( b_p \) being defined as the value of the performance benefit per AM part per unit time. While \( b_p \) is typically positive, we do not require this in our analysis. Throughout our paper, for functions like \( B(\cdot) \), we only explicitly write down arguments when they are required to denote dependencies, and we stick to the use of \((-\cdot-)\) otherwise. For instance, \( B(N) \) denotes the performance benefits for an installed base of size \( N \).

We define the net value of the investment costs and performance benefits as

\[ K(\cdot) = I - B(\cdot). \]

For the same reasons that \( I \) can be negative, it is possible for \( K \) to be negative. Additionally, a negative value of \( K(\cdot) \) can occur when the total performance benefits exceed the net difference between investment costs for the AM part and the regular part. Note that not all benefits may be expressed as a function of time, for example in the case of expected competitive advantages. In such cases it may be possible to assess the expected benefit and include it in the value for \( K(\cdot) \).

Besides the investment costs and performance benefits, there are also costs related to the production and maintenance of the systems. These costs depend on the respective parameters of the part that is installed. The production costs depend directly on the component production costs, \( c^p_x \geq 0 \), which can differ for the regular part and the AM part. This leads to the following expression for the unit production costs:

\[ P^e(\cdot) = c^p_x N. \]

In reliability allocation models it is common to make unit production costs dependent on the product’s reliability (e.g., Mettas, 2000) and the associated design investment. As mentioned in the introduction, this relationship is very difficult to characterize in practice. For this reason, we do not model this dependency, but focus on providing decision makers with the break-even characteristics of an AM component for a given value of the design investments and expected performance benefits.

The production leadtimes \( L^e \) are independent and identically distributed, with a constant mean over time, with \( L^e \sim L^0 \). The systems are supplied with spare parts from a single stock point that follows a continuous review \( S^e = 1 \) base stock policy. Inventory holding costs are incurred at a rate of \( h \in \mathbb{R}^+ \) per unit time, also for parts that are on order. Hence, the inventory holding costs are:

\[ H^e(\cdot) = h c^e_x T S^e. \]

When a part fails it is replaced by a spare part from inventory, if one is available. In that case, a new part is ordered immediately. Otherwise, an emergency shipment is conducted and the demand is lost to the stock point. In the former case, repair and downtime costs \( c^r \) are incurred, which includes the costs for handling and failure diagnostics. In the latter case, emergency system repair and downtime costs \( c^d \) are incurred, with \( c^d \leq c^r \). We assume that the downtime costs \( c^d \) are equal for regular and AM parts, which is generally the case in practice because the time and the resources required for failure diagnosis and repair are equal for both parts. Further, we assume that \( c^d \leq c^r \) because expediting a printed part at a local supplier is most likely faster and less expensive than expediting a regular part from a central warehouse.

We assume that the mean lifetime of components is generally distributed and that the number of systems that is served from the single stock point is sufficiently large. Hence, we may assume that the total demand process for spare parts follows a Poisson process with rate \( N/\tau^e \) (see Van Houtum & Krakenburg, 2015, p. 14). Our assumptions imply that the on-hand stock process is identical to the process of the number of free servers of an M|G|c queue with \( S^e \) parallel servers, arrival rate \( N/\tau^e \) and service time \( L^e \), i.e., an Erlang loss system. The emergency shipment probability, i.e., the probability of being out of stock, is identical to the Erlang loss probability, \( g^e(\cdot) \), for a system with \( S^e \) servers and a system load \( \alpha^e = NL^e/\tau^e \):

\[ g^e(\cdot) = \sum_{i=0}^{S^e} \frac{(\alpha^e)^i}{i!}. \]

The Erlang loss rate is used to calculate downtime and repair costs, which also consist of component production costs that are linear in the number of failures due to the fact that the inventory system follows a base-stock policy with one-for-one replenishment:

\[ D^e(\cdot) = (1 - g^e(\cdot)) \frac{NT(c^e_x + c^r_x)}{\tau^e} + g^e(\cdot) \left( \frac{NT(c^d_x + c^r_x)}{\tau^e} + \frac{NT(c^d_x + c^p_x)}{\tau^e} \right). \]

We now provide the cost function for the sum of the production costs, inventory holding costs and downtime and repair costs. This function, \( C(\cdot) \), holds for both the regular and the AM parts. Note that \( C(\cdot) \) does not represent the total lifecycle costs, as it does not include the investment costs and potential performance benefits that AM can bring during the exploitation phase. It holds that:

\[ C(\cdot) = P^e(\cdot) + H^e(\cdot) + D^e(\cdot). \]
Disposal costs are excluded from the analysis, as these are expected to be much smaller compared to the other cost factors for high-tech systems (e.g. Öner, Franssen, Kiesmüller, & Van Houtum, 2007). To find the optimal base-stock level that minimizes total lifecycle costs, we can limit ourselves to minimizing $C_i(\cdot)$, since $K(\cdot)$ is independent of the base stock level $S^*$. Hence, we solve the optimization problem $(Y^*)$: 

$$
(Y^*) \quad \min_{S^*} C_i(S^*)
$$

There is no closed-form solution for the optimal base-stock level $S^*(\cdot)$. However, only the holding and downtime costs depend on $S^*$. The holding costs, $h^p(\cdot)$, linearly depend on $S^*$, and the Erlang loss function is convex in $S^*$ for a fixed $x^*$. Thus, minimizing $C_i(\cdot)$ is convex in $S^*$, and we can easily find the optimal base stock level $S^*(\cdot)$ via a numerical search procedure. We define the optimized cost function:

$$
\hat{C}_i(\cdot) = C_i(S^*(\cdot)).
$$

While one could simplify our model by incorporating $K(\cdot)$ into the unit production costs, we choose not to do so and explain the reasons behind this choice in the next section.

4. Analysis

To identify when AM is preferable over traditional production, we require the break-even point where the total lifecycle costs for both components are equal. Such break-even values can be used by decision makers to determine whether or not to opt for AM. In Section 3 we introduced $K(\cdot)$ as a measure that includes performance benefits and the difference in development costs for AM and regular parts. Given that these play a large role in deciding whether or not to opt for an AM component, we evaluate how $K(\cdot)$ influences break-even characteristics in terms of component reliability in Section 4, and in terms of component production costs in Section 4.2.

Another reason for exogenizing $K(\cdot)$ in this manner, is that $K(\cdot)$ can be estimated quite accurately in the early design stage. It is common for an OEM to estimate the costs associated with design activities, for example by assigning engineering hours to a design project. Estimating attainable performance benefits can be done in cooperation with an AM service provider who is knowledgeable on the design freedom that AM offers. The remainder of total lifecycle costs, especially those related to the reliability of AM parts and their production costs, are much more difficult to estimate, which is why our model provides decision makers with the minimally required values for production costs and reliability of an AM part for a given value of $K(\cdot)$. Before we proceed, we provide several properties of the optimal cost function $\hat{C}_i(\cdot)$ in Lemma 1.

**Lemma 1.** The optimal cost function $\hat{C}_i(\cdot)$ has the following properties:

(i) $\hat{C}_i(x^*)$ is strictly decreasing in $x^*$.

(ii) $\hat{C}_i(N)$ is strictly increasing in $N$.

(iii) $\hat{C}_i(c^*_p)$ is strictly increasing in $c^*_p$.

The proof of Lemma 1 and all further proofs can be found in the appendix.

4.1. Properties of the break-even reliability levels under equal production costs

In this section, we investigate the behavior of the break-even reliability characteristics of AM components in relation to several key parameters, most notably the value of $K(\cdot)$. For this part of the analysis, we evaluate the scenario where the component production costs are equal for regular and AM parts, i.e. $c^*_p = c^*_R = c_p$. In practice, this may occur, for example, when many production or assembly steps are required to produce a part. In that case, more expensive hours of the AM machine are offset by the production speed with which complex geometry is achieved or assembly steps are skipped. We formally introduce the break-even reliability, $\tau^{B^*}(\cdot)$, in Definition 1.

**Definition 1.** The break-even reliability of an AM component is $\tau^{B^*}(\cdot)$ such that $\hat{C}_i(\tau^*) = C_i(\tau^{B^*}(\cdot)) + K(\cdot)$.

Since $\hat{C}_i(x^*)$ is independent of $K(\cdot)$, as explained in Section 3, the value of $K(\cdot)$ determines $\tau^{B^*}(\cdot)$ through Definition 1. Lemma 2 shows that $\tau^{B^*}(K(\cdot))$ exists only up to a certain value of $K(\cdot)$ and that if it exists, it is unique for that value of $K(\cdot)$.

**Lemma 2.** $\tau^{B^*}(K(\cdot))$ has the following properties:

(i) $\tau^{B^*}(K(\cdot))$ does not exist if $K(\cdot) > K_{lim}$.

(ii) For $K(\cdot) < K_{lim}$, $\tau^{B^*}(K(\cdot))$ is uniquely defined.

The intuition behind $K_{lim}$ is as follows: The regular component costs related to inventory holding, downtime and repair are finite, thus limiting the maximum savings that an AM component can achieve. Therefore, once $K(\cdot)$ exceeds the regular system's inventory holding, downtime and repair costs, i.e., $K(\cdot) > K_{lim}$, these investment costs cannot be compensated anymore, i.e., $\tau^{B^*}(K(\cdot))$ does not exist. This means that $K_{lim}$ can be used as an early go or no-go decision during the design process, as AM cannot be preferable when $K(\cdot)$ is such that $K(\cdot) > K_{lim}$. For $K(\cdot) < K_{lim}$, Lemma 2(ii) states that $\tau^{B^*}(K(\cdot))$ is unique for a given value of $K(\cdot)$. Its implication, in combination with Lemma 1(i), is that if the OEM can design an AM component at cost $K(\cdot)$, that is expected to attain $\tau^{B^*}(K(\cdot))$, that AM part is preferable over the regular part.

Unfortunately, there is no closed-form solution to $\tau^{B^*}(\cdot)$, mainly because it is integrated into the Erlang loss probability. Therefore, we can only evaluate it numerically up to an arbitrary accuracy, $\varepsilon$, defined as:

$$
\varepsilon = \left| \frac{\hat{C}_i(\cdot) - \hat{C}_i(\cdot) - K(\cdot)}{\hat{C}_i(\cdot)} \right| = 0.000001.
$$

We use binary search to determine $\tau^{B^*}(\cdot)$, knowing that $\hat{C}_i(\tau^*)$ is decreasing in $\tau^*$ (Lemma 1(ii)). Doing so for a range of values for $K(\cdot)$ yields a curve such as the one shown in Fig. 1, which we use to illustrate our results. The values used to generate Fig. 1 can be found in Table 3. Note that $K(\cdot)$, which is shown on the bottom axis of Fig. 1 is a lump-sum, i.e., it can represent any combination of $p^h$, $p^k$ and $K_{lim}$.

We refer to the area above the break-even curve in Fig. 1 as the AM region and to the area below the curve as the regular region. We see that $\tau^{B^*}(\cdot)$ is increasing in $K(\cdot)$ and that $\tau^{B^*}(\cdot)$ goes to infinity when $K(\cdot)$ approaches $K_{lim}$, beyond which no break-even values exist. This illustrates the practical use of visualizing the entire break-even curve by exogenizing $K(\cdot)$, since the entire curve, including the asymptotic behavior around $K_{lim}$ is required to assess how decisively a part is located in the AM region. We also see that the intersection point of the break-even reliability curve with the regular reliability occurs at a positive value of $K(\cdot)$. We show in Theorem 1 that these are structural properties of $\tau^{B^*}(\cdot)$ as a function of $K(\cdot)$, and that they provide us with general insights into the required reliability of an AM component.

**Theorem 1.** $\tau^{B^*}(K(\cdot))$ has the following properties:

(i) $\tau^{B^*}(K(\cdot))$ is strictly increasing in $K(\cdot)$ for $K(\cdot) < K_{lim}$.

(ii) $\lim_{K(\cdot) \to K_{lim}^{-}} \tau^{B^*}(K(\cdot)) = \infty$. 

(iii) If $K(\cdot) \leq 0$ then $\tau^{\text{IM}}(K(\cdot)) < \tau^R$.

The implication of Theorem 1(i) is that a higher design investment requires a higher AM part reliability in order to break even. Theorem 1(ii) implies that the required increase in reliability grows to infinity as $K(\cdot)$ approaches $K_{\text{IM}}$. When an AM component creates large performance benefits, $K(\cdot)$ can become negative. Theorem 1(iii) describes that when differences in investment costs between the regular and AM part are at least canceled out by the benefits (i.e., $K(\cdot) = L^A - L^R - b_2 NT \leq 0$), the break-even reliability is strictly below the reliability of the regular component. This also implies that when benefits are expected to equal the difference in required investment, i.e. $K(\cdot) = 0$, any AM part with reliability at least equal to that of the regular part is preferred over the regular part. This is especially useful for future applications of AM, since it is expected that investment costs for AM components will decrease as engineers become more familiar with its design principles and trial production costs decrease due to a better understanding of AM process parameters. This implies that in many future cases, $L^A$ will be similar to $L^R$. This is explored further in the numerical experiment in Section 5.

So far, we have considered the behavior of $\tau^{\text{IM}}(\cdot)$ as a function of $K(\cdot)$. Other interesting behavior of $\tau^{\text{IM}}(\cdot)$ relates to the size of the installed base and the length of the remaining time horizon. We would expect that an increase in the installed base size, or a larger remaining time horizon, has a positive effect on the size of the AM region, as it allows us to spread the investment costs over more parts or a longer time period. While this does seem to be the case when $K(\cdot)$ is large, we find that when performance benefits outweigh investment costs (i.e., $K(\cdot) < 0$), an increase in the remaining time horizon actually leads to a higher required reliability, thus decreasing the AM region. Fig. 2, for which we use the parameters of Table 3, provides an example of this behavior, where on the right side of the graph, the break-even curve goes down when $T$ increases from 60 months to 240 months, while on the left side, the break-even curve goes up although it will never exceed $\tau^R$ for negative $K(\cdot)$ by Theorem 1(ii).

Since there is no closed-form solution to $\tau^{\text{IM}}(\cdot)$, we cannot explicitly evaluate its sensitivity to $N$ or $T$ in general. Therefore, we focus on two special cases that we can evaluate. These points are $K_1(\cdot)$ and $K_2(\cdot)$ and they are depicted in Fig. 3, for which we use the parameters of Table 3, except that we used $L^R = 1.5$ months as will be clarified later in this section.

We first analyze the behavior of $K_1(\cdot)$ and after that proceed with properties of the behavior of $K_2(\cdot)$.

**Definition 2.** $K_1(\cdot)$ is the value of $K(\cdot)$ such that $\tau^{\text{IM}}(K_1(\cdot)) = \tau^R$.

$K_1(\cdot)$ is defined by the intersection point of $\tau^{\text{IM}}$ with $\tau^R$. This intersection point of the break-even reliability curve with the regular part’s reliability is of particular practical significance, since it gives us insight into the amount by which the AM part’s development costs, minus its performance benefits, may exceed those of the regular part, before we require it to be technologically superior in terms of reliability. We formalize several properties of $K_1(\cdot)$ in Theorem 2.

**Theorem 2.** $K_1(\cdot)$ has the following properties:

(i) $K_1(\cdot) > 0$.
(ii) $K_1(T)$ is increasing in $T$.
(iii) $\tau^{\text{IM}}(K_1(T); T + \varepsilon) \leq \tau^{\text{IM}}(K_1(T); T)$, with $\varepsilon > 0$.

The implication of Theorem 2(ii) and Theorem 2(iii) is that when the remaining time horizon increases, the break-even curve shifts below and to the right, increasing the AM region. This means that the OEM can then spend more on developing an AM component and that the AM part requires a lower reliability in order to break even with the regular part as $T$ increases. Since $K_1(\cdot)$ is always positive, these findings imply that switching to AM components becomes more attractive for longer remaining time horizons, when there are no, or relatively small, performance benefits involved.

**Remark 1.** In our numerical experiment (Section 5), we see that for installed base sizes that are normally encountered in practice, an increase in $N$ has the same implications as an increase in $T$. 

---

**Table 3**

Numerical example parameters.

<table>
<thead>
<tr>
<th>$b_2$ [€/month]</th>
<th>$c_2$ [€]</th>
<th>$c_3$ [€]</th>
<th>$L^A$ [months]</th>
<th>$L^R$ [months]</th>
<th>$\tau^R$ [months]</th>
<th>$N$ [dimensionless]</th>
<th>$T$ [months]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>40</td>
<td>200</td>
<td>800</td>
<td>0.5</td>
<td>3</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>

**Fig. 1.** Break-even reliability as a function of $K(\cdot)$ for the numerical example of Table 3.
Counter examples to this behavior exist, however, for very small values of \(N\). For example, using the parameters from Table 3, but setting \(L^A = 2.9\), increasing \(N\) from 3 units to 4 units results in a decrease of \(K_1(\cdot)\) from 8.45 to 8.14 Euro. In our numerical experiment (see Section 5), we examine the behavior of \(K_i(N)\) in more detail for more realistic values of \(N\).

Theorem 2 describes a positive influence of an increase in \(T\) on the AM region, i.e., a decrease in break-even reliability as \(T\) increases. We have also observed earlier, that there are cases where an increase in \(T\) has the opposite effect, see Fig. 2, where on the left side the break-even reliability increases as \(T\) increases. We next examine this behavior in more detail. To do this, we first define a specific value \(K_2(\cdot)\) for which we prove that an increase in \(T\) corresponds to an increase in required reliability.

Definition 3. \(K_2(\cdot)\) is the value of \(K(\cdot)\) such that \(\frac{NL^A}{\tau(\tau^A[K_2(\cdot)])} = \frac{NL^R}{\tau^R} \).

Recall that \(\tau^A(\cdot)\) is decreasing as \(K(\cdot)\) decreases (Theorem 1(i)). When we decrease \(K(\cdot)\) far enough, we encounter a value \(\tau^A[K_2(\cdot)]\) where the break even reliability exactly off-sets the reduced production lead time: at this point, it holds that the load on the Erlang loss inventory system is equal for the regular and AM part, i.e., \(a^R = a^A\). In the example of Fig. 3 we used \(L^R = 2L^A\), in which case we know that \(a^R = a^A\) holds when \(\tau^A[K(\cdot)] = \tau^R/2 = 5\) months. This allows us to formalize the behavior of \(\tau^A[K_2(\cdot)]\) as a function of \(T\):

Theorem 3. For \(\tau^A[K_2(\cdot)] \) it holds that \(\tau^A(K_2(T); T < T^A(K_2(T); T + \varepsilon) < \tau^R\), when \(c^R_2 = c^R\).

We set \(c^R_2 = c^R\) so that we know that the base stock levels for the regular and AM part are equal when \(a^R = a^A\). Although we only prove Theorem 3 for a specific point \(K_2(\cdot)\), and for \(c^R_2 = c^R\), experiments indicate that an increase in \(T\) typically results in greater \(\tau^A(\cdot)\) when \(K(\cdot) < 0\) and when \(c^R_2 \geq c^A\). This implies that when AM performance benefits are expected to outweigh the required investment, we must also meet a larger reliability to break even as \(T\) increases, although \(\tau^A(K(\cdot) < 0)\) will never exceed \(\tau^R\) (see Theorem 1(iii)).

Remark 2. Numerical experiments indicate that for practical examples, an increase in \(N\) has the same implications for \(\tau^A(K_2(\cdot))\) as an increase in \(T\). However, counter examples to this behavior exist. For example, using the parameters from Table 3, but setting...
\( L^R = 1, L^A = 0.9, c_d = 1 \) and \( h = 0.5 \), increasing \( N \) from 1 unit to 2 units results in a decrease of \( r^d(K_2(\cdot)) \) from 9 months to 8.95 months.

### 4.2. Properties of the break-even component costs under equal reliability

In this section, we investigate the behavior of the break-even component production costs. We consider a scenario where the reliability of an AM component is equal to that of the regular component, i.e., \( r^R = r^A = r \). This is a scenario that often occur in practice, especially for components that are less technically challenging to print, or that are not subjected to direct mechanical loads. We formally introduce the break-even component production cost, \( c^b_p(\cdot) \), in Definition 4. In Lemma 4, we next show that \( c^b_p(\cdot) \) exists only up to a certain value of \( K(\cdot) \), and that if it exists, it is unique for that value of \( K(\cdot) \).

**Definition 4.** The break-even condition for the AM component’s production costs is defined as \( c^b_p(\cdot) \) such that \( \hat{C}^R(c^b_p(\cdot)) = \hat{C}^A(c^b_p(\cdot)) + K(\cdot) \).

**Theorem 4.** \( c^b_p(K(\cdot)) \) has the following properties:

(i) \( c^b_p(K(\cdot)) \) does not exist if \( K(\cdot) > \hat{C}^B(c^b_p) \).

(ii) For \( K(\cdot) \in (-\infty, \hat{C}^B(c^b_p)) \), \( c^b_p(K(\cdot)) \) is uniquely defined.

(iii) \( c^b_p(K(\cdot)) \) is strictly decreasing in \( K(\cdot) \).

We are able to separate \( c^b_p(\cdot) \) from the break-even equation of Definition 4 by reordering:

\[
c^b_p(\cdot) = \frac{c^b_p(N + hTS^p(\cdot) + \frac{NT(c_p^b - c_d)}{2}) + g^R(\cdot) \frac{NT(c_p^b - c_d)}{2} - g^A(\cdot) \frac{NT(c_p^b - c_d)}{2} - K(\cdot)}{N + hTS^p(\cdot) + \frac{NT(c_p^b - c_d)}{2}}.
\]  \( \tag{3} \)

Unfortunately, we can only evaluate \( c^b_p(\cdot) \) numerically, since \( S^p(\cdot) \) depends on \( c^b_p(\cdot) \). There is also no closed-form expression for the optimal base stock level \( S^p(\cdot) \). We can still, however, obtain some insight into the behavior of the break-even component costs. Similar to the previous section, we manipulate \( K(\cdot) \) to obtain insight into how \( c_p^b(\cdot) \) behaves under the influence of required design investments and expected performance benefits. Fig. 4 shows an example of typical behavior of \( c_p^b(K(\cdot)) \). The parameters used are in Table 3.

Because \( \hat{C}^R(c^b_p) \) is increasing in \( c^b_p(\cdot) \) (Lemma 1), we require \( c^b_p \) to be below the break-even point in order for AM to be preferable. From Fig. 4 we observe that \( c^b_p(\cdot) \) is decreasing in \( K(\cdot) \) and that at some point, see Lemma 4(i), the break-even production costs cease to exist, which is due to our requirement that \( c^b_p(\cdot) \geq 0 \). The value of \( K(\cdot) \) where \( c^b_p(\cdot) \) ceases to exist can be found via Eq. (3):

\[ c_p^b(\cdot) = 0 \Rightarrow c_p^b(N + hTS^p(\cdot) + \frac{NT(c_p^b - c_d)}{2}) + g^R(\cdot) \frac{NT(c_p^b - c_d)}{2} = K(\cdot). \]

That the break-even production costs cease to exist at \( \hat{C}^R(c^b_p) \) is due the fact that AM can save no more than the life cycle costs of the regular component related to production, inventory holding and emergency shipments. This occurs when AM production costs are zero. Hence, if \( K(\cdot) \) increases beyond \( \hat{C}^R(c^b_p) \), no break-even values can be found. This bound on \( K(\cdot) \) can be used as an early go or no-go for the evaluation of AM as a production option, because the value of \( K(\cdot) \) can be estimated early in the design process. We also see that when \( K(\cdot) = 0 \), the break-even production costs of the AM component are greater than the production costs of the regular part. These observations are similar to the properties of \( r^d(K(\cdot)) \).

Finally, we note that the intersection point of \( c^b_p(\cdot) \) with \( c_p^b(\cdot) \) occurs at the exact same value of \( K(\cdot) \) as \( \hat{K}_i(\cdot) \) from Section 4.1, since we have assumed equal reliability and because the component production costs are also equal at this intersection point. Hence, both models are equivalent at \( K(\cdot) = \hat{K}_i(\cdot) \), and the insights from Theorem 2 also hold for the intersection point where \( c_p^b(\cdot) = c^b_p(\cdot) \).

### 5. Numerical experiment

To generate insight into the applicability of AM in practice we conduct a numerical experiment on a range of input parameters. Our goal is to provide managerial insights into situations where AM is most suitable to replace traditional technology, and to provide insights into AM characteristics that require the most attention for the technology to become more widely applicable. We report the following outcome variables:

(i) \( \hat{K}_i(\cdot) \): AM is not a mature manufacturing technology yet, and high investment costs are often required for its application. Investigating \( \hat{K}_i(\cdot)/c_p \) gives insight into how much higher \( \hat{p} \) may be relative to \( p \) in the absence of large performance benefits. A high ratio of \( \hat{K}_i(\cdot)/c_p \) indicates that a
significant investment can be made towards the development of an AM component, if that component’s reliability is comparable to that of its regular counterpart.

(ii) \( t^\alpha_K(K(\cdot) = 0)/t^\beta_K \): As AM matures, design engineers will gain experience with its application and the AM development costs will decrease. Ultimately, we expect that the AM and regular investment costs to be balanced, i.e., \( \beta \equiv \alpha \), which makes the investigation of \( t^\alpha_K(K(\cdot) = 0)/t^\beta_K \) relevant.

Lemma 2 states that this ratio is strictly less than one. The closer this ratio is to one, the closer the reliability of the AM component must be to that of its regular counterpart in the absence of performance benefits.

(iii) \( c^\alpha_p(K(\cdot) = 0)/c^\beta_p \): This outcome variable shows the allowed extra production costs for an AM component when the regular and AM design effort is balanced.

To investigate the behavior of the output variables mentioned above, we set up a full factorial experiment over the parameter values defined in Table 4. Each parameter has three possible values, the middle values being commonly encountered in practice. The other values may apply in specific cases. For instance, an installed base size of 25 units can apply to radar installations for a specific class of naval vessels. For all three outcome variables, we set \( c^\alpha_p = c^\beta_p = c_p \). For the first and the second outcome variable, we set \( c^\alpha_p = c^\beta_p = c_p \) in order to determine \( t^\alpha_K(\cdot) \). For the third outcome variable we set \( t^\alpha_p = t^\beta_p = \tau \) and \( c^\alpha_p = c^\beta_p = c_p \) in order to determine \( c^\alpha_p(K(\cdot) = 0)/c^\beta_p(K(\cdot) = 0) \).

The remaining parameters are set at the following levels: \( h = 0.02e \$/month \) and \( L^k = 3 \) months. All variations provide a total of 2187 combinations. Table 5 contains the most interesting results from the experiment. The remaining results can be found in Appendix 8.1.

From Table 5 we identify interesting behavior for the application of AM in the near to mid-term future, as described by the ratio \( K_1(\cdot)/c_p \). Firstly, \( c_2/c_p \) has almost no effect on the value of \( K_1(\cdot)/c_p \), indicating that small or large downtime costs relative to the component production costs has little effect on the application of AM. \( K_1(\cdot)/c_p \) is much more sensitive to an increase in \( t^\alpha_k \) or \( L^k \). For \( t^\alpha_k \), this is due to the fact that a reliable component implies a low base-stock level and few emergency shipments, thus limiting the costs that can be saved due to the short AM lead time. An increase in \( L^k \) similarly decreases the ratio of \( K_1(\cdot)/c_p \), as this limits the cost savings in the after-sales service supply chain, which then diminishes the allowable AM design investment. Finally, an increase in \( N \) allows for a greater AM design investment, as this is spread out over a larger installed base. However, we also observe that the reliability of the AM component has to also increase when \( N \) increases. This is due to the fact that failure and downtime costs become much too large if an unreliable AM part is installed in a large number of systems.

We observe that AM is currently most suitable for components with a sizable, but not too large, installed base and long system lifetime. This fits well with the capital goods setting and matches with the findings described in Theorem 2. We observe that the additional costs for developing an AM component, compared to the costs for developing the regular component, may often be several hundred times larger than the component production costs. However, we also find that the reliability of the regular component that is considered should not be too large, as this diminishes the potential cost savings in terms of after-sales service logistics costs. If inventory holding costs and emergency shipment costs cannot be sufficiently decreased, then the AM investment costs quickly become a deterrent to apply AM.

In the long term, we expect \( K_1(\cdot) \) to be closer to zero in the absence of performance benefits, as investment costs for AM decrease. Investigating \( K_1(\cdot) = 0 \), we observe several effects on the required reliability that an AM component must have. We find that when component production costs are small compared to costs related to asset downtime, i.e., the ratio of \( c_2/c_p \) is high, then the AM component must be almost as reliable as the regular component. This effect is due to the downtime costs becoming a more influential component of the total life cycle costs, so that any increase in failure rate also has a high impact. We also see that changes in \( t^\alpha_k, L^k \) or \( N \) have a small effect on the break-even reliability at \( K_1(\cdot) = 0 \).

Furthermore, even in the most extreme cases, the AM component may only be 13% less reliable than the regular component. This indicates that the reliability that an AM part can achieve is crucial for its application. In a capital goods setting, the costs related to machine downtime are simply too big to allow for substantial reductions in reliability, even with the logistical benefits that AM offers. This also implies that even in the absence of performance benefits, it is very beneficial to apply AM if there is an opportunity to use AM to increase a component’s reliability. For example by integrating multiple components into one part, and thus removing potential failure modes. In such cases, a small increase in reliability compared to the regular component can have a large effect on total life cycle costs.

### Table 4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Average</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_2/c_p )</td>
<td>2</td>
<td>107</td>
<td>5</td>
<td>676</td>
</tr>
<tr>
<td>4</td>
<td>109</td>
<td>5</td>
<td>685</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>111</td>
<td>6</td>
<td>693</td>
<td></td>
</tr>
<tr>
<td>( t^\alpha_k )</td>
<td>12</td>
<td>176</td>
<td>15</td>
<td>693</td>
</tr>
<tr>
<td>24</td>
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<td>48</td>
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<td>( L^k )</td>
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<td>1</td>
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<tr>
<td>( N )</td>
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### Table 5

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Average</th>
<th>Min</th>
<th>Max</th>
</tr>
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<tbody>
<tr>
<td>( K_1(\cdot)/c_p )</td>
<td>2</td>
<td>107</td>
<td>5</td>
<td>676</td>
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<td>4</td>
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<td>8</td>
<td>111</td>
<td>6</td>
<td>693</td>
<td></td>
</tr>
<tr>
<td>( t^\alpha_K(K(\cdot) = 0)/t^\beta_K )</td>
<td>0.943</td>
<td>0.872</td>
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</tr>
<tr>
<td>0.969</td>
<td>0.928</td>
<td>0.986</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.103</td>
<td>1.064</td>
<td>1.171</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c^\alpha_p(K(\cdot) = 0)/c^\beta_p(K(\cdot) = 0) )</td>
<td>0.943</td>
<td>0.872</td>
<td>0.973</td>
<td></td>
</tr>
<tr>
<td>0.969</td>
<td>0.928</td>
<td>0.986</td>
<td></td>
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<tr>
<td>1.103</td>
<td>1.064</td>
<td>1.171</td>
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</tr>
</tbody>
</table>
Similar effects are observed for the required production costs of AM components. Table 5 shows that, on average, AM parts are allowed to be 11% more expensive than regular parts when development costs are balanced and no performance benefits are involved. Even in the best case scenario, an AM part is only allowed to be 18% more expensive than its regular counterpart. This implies that the required production costs, like the required reliability levels, are likely to limit the application of AM parts in the near future.

6. Case studies

We perform case studies to illustrate the practical applicability of our model, as well as the current performance of AM compared with traditional technology. The first case study is performed at a company that manufactures access equipment and its spare parts. The evaluated component is a stainless steel hydraulic valve block that is used to control the bucket movement of a 60-meter boom lift (see Fig. 5). This component must be able to withstand large hydraulic pressure, but the amount of pressure is also predictable and the AM component is expected to cope with this type of load as well as the regular version does, i.e., the reliability of the AM version is expected to be equal to that of the regular version. Therefore, we focus on determining the required production costs for the AM component, using the method described in Section 4.2. The data for the analysis is shown in Table 6. Downtime costs $c_d$ is estimated based on one day of downtime for a large rental company at a cost of $€\,475$ in lost revenue per failure. Emergency downtime and shipment costs $c_d^e = 4c_d$. This ratio of $c_d^e/c_d$ is fairly low, but reasonable for such types of equipment, which are typically not crucial to entire business processes.

The AM lead time of two weeks is typical for the service that third party AM service providers guarantee. The other parameters come from company records, with all data provided in Euros and months.

The data in Table 6 is used to generate the break-even curve that is shown in Fig. 6. To estimate whether or not this component is suitable for AM we must also determine its position on the graph. To do this, we require estimates for the values of $c_d^e$ and $K(\cdot)$. The AM production costs are largely determined by the component weight. The regular part weighs 8 kilograms and to serve as an indication of the weight of the AM component, we refer to a similar valve block that has previously been designed for AM in a cooperation between Layerwise, an AM service provider, and the VTT research center of Finland. Using a topologically optimized AM design resulted in a 76% weight reduction over the regular version. A similar weight reduction for our component would result in a weight of 1.92 kilograms. The density of stainless steel 316 is 7860 kilograms per cubic centimeter, which results in a component volume of 244 cubic centimeter. The AM cost of stainless steel is estimated to be $\€\,3.14$ per cubic centimeter (see Roland Berger Strategy Consultants, 2013), bringing our estimate of $c_d^e$ to $€\,767$.

To estimate $K(\cdot)$, we require an estimate of the difference between the regular and AM design costs. The regular part is a typical valve block that will be easy to design, while the design engineers can use off-the-shelf software to optimize the topology of the AM part. We estimate that the difference between these design costs is negligible. However, the AM part requires extensive testing to fine-tune production parameters. Typically, at least ten trial production runs are required to find the correct combination of production parameters and subsequent testing of the products is required. Based on the estimate for $c_d^e$, which largely determines the costs of a trial production run, we estimate that $l = €\,10.000$ is realistic.

The company indicates that weight reduction of the bucket construction creates some performance benefit, as it creates a competitive advantage due to increased weight carrying capacity. The exact value of this advantage, however, is difficult to determine. Therefore, we ignore this and assume that $K(\cdot) = l$. Note that these are rough estimates for $c_d^e$ and $K(\cdot)$, which is common business
practice when evaluating new products or components early in the development process. We will see that these rough estimates, in combination with the model's break-even curves are sufficient to draw conclusions about this case.

Fig. 6 clearly shows that the estimation for $c_p^A$ is much higher than the break-even production costs, indicating that traditional technology is preferred in this case. One reason for this is that the component's reliability relative to the system lifetime is large, which limits the impact of lead time reduction (Section 5). We also observe that much of the potential benefit of AM in terms of after-sales costs has been attained, as a further reduction of the AM leadtime from two weeks to one week has little effect. Further improvement must come from an increase in reliability, which is unlikely for this component type, or a decrease in AM production costs. In this case, $c_p^A$ must decrease by approximately 40% for the AM part to become preferable, which may well occur as AM technology continues to develop. The clear difference also illustrates why rough estimates for $c_p^A$ and $K(\cdot)$ will often be sufficient to draw conclusions from our model output.

The second case we examine is one from the aviation industry. Fig. 7 shows an aileron bracket used to control the roll of an aircraft, in this case a business jet. Each jet has two of these parts, situated at the end of each wing. The characteristics for the regular part come from company records and are shown in Table 7.

The engineering department of the company performed a redesign of the regular bracket. The resulting AM part is made of titanium, instead of aluminium. Titanium is approximately 60% heavier compared to aluminium, but due to design improvements the AM part is 25% lighter than the regular part, saving 80 gram per component. A recent report (Wren, 2011) estimates that one kilogram of weight saved results in € 183.60 in fuel costs saved per year per airplane. Savings of 80 gram per component, for 170 airplanes with 2 components each ($N = 340$) that are used for a period of 15 years implies $B(\cdot) = € 75K$. Designing the AM component cost € 5K more than the regular part. We, very conservatively, estimate the one-time part certification costs for this safety-critical part at € 100K. This brings the final value of $K(\cdot)$ to € 30K. The production costs for the AM part are € 1000, compared to € 450 for the regular part.

The result of the analysis is shown in Fig. 8. We see that also in this second case, traditional technology is still preferred. As in case study 1 this is mainly due to the high production costs of AM components, although this is expected to decrease substantially in the coming years. Another reason is that the component is featured twice per airplane for a fleet of only 170 business jets. Since every airplane features such brackets, we have included the result that is obtained when this redesign is conducted for Boeing’s 767. We assume that each of the 1100 767’s (Wikipedia, 2017) in operation features two brackets. We also conservatively assume that 80 grams are saved per bracket, even though these brackets are likely heavier than those of the business jet of the original case. We then obtain $B(\cdot) = 485K$, in which case the decision design is then decisively in favor of the AM part, as Fig. 8 shows. We note that a positive business case requires a method for dividing the total benefits over the separate parties involved, as in this case study an OEM is involved in the development of the part and the airline companies benefit from the fuel savings.

Table 7

<table>
<thead>
<tr>
<th>$c_p$</th>
<th>$c_d$</th>
<th>$c_p^A = c_p^R$</th>
<th>$h$</th>
<th>$L^A$</th>
<th>$L^R$</th>
<th>$N$</th>
<th>$r^A = r^R$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>450</td>
<td>600</td>
<td>25,000</td>
<td>0.018</td>
<td>0.5</td>
<td>2.25</td>
<td>340</td>
<td>120</td>
<td>180</td>
</tr>
</tbody>
</table>

7. Extensions

Some of the variables that we have so far treated as being deterministic may be stochastic in practice. As extensions to our
model, we consider the impact of stochasticity in $N$ and in $T$. We still assume that all systems are both installed and taken out of service at the same time.

We first add uncertainty in $T$ to our model. We assume $T$ to be uniformly distributed on the interval $[a_T, b_T]$, i.e., its pdf $f(t) = 1/(b_T - a_T)$ and its expectation $\mathbb{E}(T) = 0.5 \cdot (a_T + b_T)$. Our optimized cost function is:

$$
\begin{align*}
\tilde{C}^c(\cdot) & = \int_{a_T}^{b_T} \left(c^p c_T^* \mathbb{E}(S^c(\cdot)) + g^c(\cdot) \frac{N \mathbb{E}(T) (c^2_T - c_d)}{\tau^x} + \frac{N \mathbb{E}(T) (c_d + c^2_d)}{\tau^x} \right) f(t) dt \\
& = c^p c_T^* \mathbb{E}(T) S^c(\cdot) + g^c(\cdot) \frac{N \mathbb{E}(T) (c^2_T - c_d)}{\tau^x} + \frac{N \mathbb{E}(T) (c_d + c^2_d)}{\tau^x}.
\end{align*}
$$

Similarly, $\tilde{R}(\cdot) = \int_{a_T}^{b_T} \left(1 - b_T N\right) f(t) dt = 1 - b_T \mathbb{E}(T)$.

We thus see that the distribution of $T$ only has an impact on the total costs through its mean, implying that this type of uncertainty does not impact the design decision that our model supports.

We continue by assuming a uniformly distributed installed base size $\tilde{N} = \mathbb{U}[a_N, a_N + 1, \ldots, b_N]$, with $a_N$ and $b_N$ being non-negative integer values. This leads to the following optimized cost function:

$$
\begin{align*}
\tilde{C}^c(\cdot) & = \sum_{n=\text{int}(\tilde{N})}^{b_N} \mathbb{P}\{\tilde{N} = n\}\left(c^p n + h c^p \mathbb{E}(T) S^c(\cdot) \right) + g^c(\cdot) \frac{n \mathbb{E}(T) (c^2_T - c_d)}{\tau^x} + \frac{n \mathbb{E}(T) (c_d + c^2_d)}{\tau^x}
\end{align*}
$$

We assume that $N$ is revealed at the start of the time horizon, after which $S^c$ is optimized for that realization of $N$. Note that the expectation for the optimized life cycle costs $\tilde{C}^c(\cdot)$ consists of a linear combination of $b_N - a_N + 1$ cost functions $\tilde{C}^c(\cdot)$ (see Eq. (1)). As each separate term is convex in $S^c(\cdot)$, we can easily obtain the expectation of the optimized life cycle costs by numerically optimizing $S^c(\cdot)$ for each realization of $N$.

Next, we define $\tilde{R}(\cdot) = \sum_{n=\text{int}(\tilde{N})}^{b_N} \mathbb{P}\{\tilde{N} = n\}(1 - b_T \mathbb{E}(T)n) = 1 - b_T \mathbb{E}(T)\mathbb{E}\{\tilde{N}\}$ and we define $\tilde{R}^+\{\tilde{R}(\cdot)\}$ such that $\tilde{C}^c(\tilde{R}(\cdot)) = \tilde{C}^c(\tilde{R}^+\{\tilde{R}(\cdot)\}) + \tilde{K}$. We observe that Eq. (5) possesses many of the same properties of Eq. (1), due to the fact that the former consists of a linear combination of the latter. Two of those properties are that $\tilde{C}^c(\cdot)$ is strictly decreasing in $\tau^x$ and that $\tilde{R}^+\{\tilde{R}(\cdot)\}$, when it exists, is uniquely defined. Both properties are used to determine break-even reliability values in the next part of our analysis.

To show the impact of uncertainty in $N$ we conduct a copy of the numerical experiment of Section 5 with $\tilde{N} = \mathbb{U}[0.8N, \ldots , 1.2N]$. We define $\Delta_{\tilde{N}} = \tilde{K} - \tilde{K}_0$. as the change in the intersection point of regular and required reliability for the deterministic case ($\tilde{K}_0$) and the stochastic case ($\tilde{K}$), with $\tilde{K}_0$ such that $\tilde{C}^c(\tilde{R}(\cdot)) = \tilde{C}^c(\tilde{R}^+\{\tilde{R}(\cdot)\}) + \tilde{K}_0$. Table 8 shows that the difference is on average a little more than 2%, which will typically not impact the design decision supported by our model. There is almost no impact on the other outcome variable, $\tilde{R}^+\{\tilde{R}(\cdot)\} = 0$, which is why this part is omitted from the results.

Fig. 9 shows the model deviation for the case with the largest absolute $\Delta_{\tilde{N}} = 6.7\%$ from Table 8. The parameters of this case are shown in Table 9. The figure illustrates that uncertainty in $N$ has

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_d/c_p$</td>
<td>2</td>
</tr>
<tr>
<td>$\tau^x$</td>
<td>12</td>
</tr>
<tr>
<td>$L^c$</td>
<td>0.25</td>
</tr>
<tr>
<td>$N$</td>
<td>25</td>
</tr>
</tbody>
</table>

### Table 8

Chains of events $K_i$ under stochastic $N$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_d/c_p$</td>
<td>2</td>
</tr>
<tr>
<td>$\tau^x$</td>
<td>12</td>
</tr>
<tr>
<td>$L^c$</td>
<td>0.25</td>
</tr>
<tr>
<td>$N$</td>
<td>25</td>
</tr>
</tbody>
</table>

### Table 9

Parameters to illustrate the impact of uncertainty in $N$.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>250</td>
<td>500</td>
<td>1</td>
<td>3</td>
<td>48</td>
<td>25</td>
<td>120</td>
</tr>
</tbody>
</table>
little impact on most of the break-even curve, except for very high values of $K(\cdot)$, where the AM region shrinks slightly. The reason why uncertainty in $N$ does impact the break-even curve, as opposed to uncertainty in $T$, is that $N$ has a non-linear impact on the cost function through the optimal base-stock level $S^\ast(n)$, which in turn impacts the Erlang loss rate. The resulting inaccuracy will typically not impact the design decision, since the high required reliability of AM components compared to the regular parts at high levels of $K(\cdot)$ are very unlikely to be achieved in practice.

8. Conclusions

In this paper, we introduce and develop a model for evaluating two production methods that can be used to produce two differently designed, but functionally the same system components. The practical motivation for this model is the potential that additive manufacturing offers compared to traditional technology, which in our case is increased design freedom and reduced production lead times. The former can create performance benefits, while the latter is beneficial to the after-sales service logistics. We evaluate the OEM's design decision to opt for either the regular component or its AM counterpart by modeling total lifecycle costs, taking into account design costs, logistical costs, including maintenance and downtime costs, and performance benefits. The break-even characteristics that our model generates allow the OEM to decide which design option to select early in the design process.

Through our model analysis, a numerical experiment, two case studies and our extensions, we gain managerial insights into the applicability of AM in a component design setting. We find that AM component development costs are sometimes allowed to be high relative to the regular component development costs, for example when the installed base size is large, or when the system lifetime is long, as this allows the additional development costs to be spread out. Interestingly, when the installed base size is small, such development costs become a major detriment to AM components. This is due to the fact that the savings created by the smaller production lead time are finite. If these logistical savings are small, and there are no performance benefits to offset the development costs, then AM can easily be at a disadvantage when production volumes over the course of the system lifecycle are small. So while spare parts typically involve production of small series over time, the most suitable candidate parts for AM for the foreseeable future are those that strike a balance between spreading out the investment costs over a significant amount of parts, while not requiring the type of production volume that favors traditional manufacturing technology.

Our results indicate that some logistical savings can be generated by the reduced production lead time that AM offers. Based on our numerical experiment, we conclude that typically allowed deficits compared to regular parts in terms of reliability and production costs are approximately 5% and 10%, respectively. This indicates that the logistics benefits of AM offer some slack in terms of component characteristics, which may serve to offset the increased design effort that AM parts currently require. Potential performance benefits, however, will be the more attractive reason to switch to AM for the foreseeable future.

We also find that the same conditions that enable large investments, such as a large number of systems and a long system lifecycle, require the reliability of AM parts to at least approach the reliability of the regular parts, as increased downtime and repair costs can otherwise offset the logistics benefits that are gained. In this complex setting, our model facilitates a careful consideration of the design options, so that OEMs can quantify in which cases additive manufacturing should be selected over traditional manufacturing.

Acknowledgment

The research leading to this paper has been supported by NWO under project number 438-13-207. For providing details regarding the two case studies, the authors thank Jorn Jansman, Stijn Verputten, JLG Ground Support, specifically Ton Wolters, and Fokker Aerostructures, specifically Marko Bosman. The authors also gratefully acknowledge two anonymous referees for their role in improving this paper.

Appendix A

Appendix A.1 contains additional results from the numerical experiment. A separate lemma that is used in one of the proofs is presented in Appendix A.2. Appendices A.3 through A.8 contain the proofs.

A1. Additional numerical experiment results

This appendix contains additional results from the numerical experiment of Section 5.

A2. Lemma 3

For some proofs, we require a property that relates to the optimal base-stock levels, which we formally define in the following lemma:

Lemma 3. Under optimal base-stock level $S^\ast(\cdot)$, it holds that $(g^\prime(S^\ast(\cdot)) - g^\prime(S^\ast(\cdot) + \delta)) \frac{N(c^2_p - c_d)}{\tau^x} \leq \delta h_c^p$, for $\delta \in \mathbb{N}^+$.

Proof. We know, due to the optimality of $S^\ast(\cdot)$ that $\Delta(C(S^\ast(\cdot))) = C(S^\ast(\cdot) + \delta) - C(S^\ast(\cdot)) \geq 0$, for $\delta \in \mathbb{N}^+$:

$$\Delta(C(S^\ast(\cdot))) = \left[ c_p^2 N + h_c^p T(S^\ast(\cdot) + \delta) + g^\prime(S^\ast(\cdot) + \delta) \right]$$

$$= \delta h_c^p T + \left[ g^\prime(S^\ast(\cdot) + \delta) - g^\prime(S^\ast(\cdot)) \right] \frac{N(c^2_p - c_d)}{\tau^x}.$$

From this, it follows that:

$$\left( g^\prime(S^\ast(\cdot)) - g^\prime(S^\ast(\cdot) + \delta) \right) \frac{N(c^2_p - c_d)}{\tau^x} \leq \delta h_c^p.$$

Note that the non-strict inequality is the result of the possibility that it may occur that base stock level $S^\ast(\cdot)$ and $S^\ast(\cdot) + 1$ are both optimal. In all cases where there is a unique optimal base stock level, or when $\delta > 1$, the inequality is strict. □

A3. Proof of Lemma 1

The proofs for Lemma 1(ii) and Lemma 1(iii) follow the same procedure as the proof of Lemma 1(i). Let $\varepsilon > 0$. We omit the superscript $x$ in this proof for ease of notation.

(i)

$$\hat{C}(\tau + \varepsilon) - \hat{C}(\tau) = C(S(\tau + \varepsilon); \tau + \varepsilon) - C(S(\tau); \tau)$$

$$\leq C(S^\ast(\tau; \tau + \varepsilon) - C(S^\ast(\tau; \tau))$$

$$= \left[ g(S^\ast(\tau; \tau + \varepsilon)) \frac{N(c_p^2 - c_d)}{\tau + \varepsilon} \right]$$
\[-g(S'(\tau); \tau) \frac{NT(c_p - c_d)}{\tau} + \left[ \frac{NTc_d}{\tau + \epsilon} - \frac{NTc_d}{\tau} \right] \leq 0.\]

The weak inequality is obtained since \(C(S'(\tau); \tau \epsilon) \geq C(S'(\tau + \epsilon); \tau + \epsilon)\). In the next equality the holding costs cancel out because both cost functions have the same base stock level. The strict inequality follows from the fact that \(g(S'; \tau)\) is strictly decreasing in \(\tau\) for a fixed base-stock level (see Öner et al., 2010).

(ii) Let \(N \in \mathbb{N}^+\), then:
\[
\hat{C}(N + 1) - \hat{C}(N)
\]
\[
= C(S'(N + 1); N + 1) - C(S'(N); N)
\]
\[
\geq C(S'(N + 1); N + 1) - C(S'(N + 1); N)
\]
\[
= \left[ c_p(N + 1) + h c_p T S'(N) + g(S'(N); N + 1) \right] \frac{(N + 1) T (c_p - c_d)}{\tau} + \frac{(N + 1) T c_d}{\tau}
\]
\[
- \left[ c_p(N) + h c_p T S'(N) + g(S'(N); N) \frac{N T (c_p - c_d)}{\tau} + \frac{N T c_d}{\tau} \right]
\]
\[
= c_p + \left[ \frac{g(S'(N); N + 1)}{(N + 1) T (c_p - c_d)} + \frac{T c_d}{\tau} \right]
\]
\[
- \frac{g(S'(N); N)}{(N + 1) T (c_p - c_d)} + \frac{T c_d}{\tau}
\]
\[
> 0.
\]

The strict inequality follows from the fact that the Erlang loss probability, \(g(S'; N)\), is strictly decreasing in the service rate (see Harel, 1990) and hence strictly increasing in \(N\).

(iii)
\[
\hat{C}(S'(c_p + \epsilon); c_p + \epsilon) - \hat{C}(S'(c_p); c_p) \geq \hat{C}(S'(c_p + \epsilon); c_p + \epsilon) - C(S'(c_p + \epsilon); c_p)
\]
\[
\geq \epsilon \left( N + h f S'(c_p + \epsilon) \right)
\]
\[
> 0.
\]

### Table A1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Average</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_p/c_d)</td>
<td>4</td>
<td>107</td>
<td>5</td>
<td>673</td>
</tr>
<tr>
<td>8</td>
<td>109</td>
<td>5</td>
<td>683</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>111</td>
<td>6</td>
<td>692</td>
<td></td>
</tr>
<tr>
<td>(T)</td>
<td>60</td>
<td>63</td>
<td>5</td>
<td>261</td>
</tr>
<tr>
<td>120</td>
<td>98</td>
<td>8</td>
<td>405</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>167</td>
<td>13</td>
<td>693</td>
<td></td>
</tr>
<tr>
<td>(c_p)</td>
<td>250</td>
<td>109</td>
<td>5</td>
<td>693</td>
</tr>
<tr>
<td>1000</td>
<td>109</td>
<td>5</td>
<td>693</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>109</td>
<td>5</td>
<td>693</td>
<td></td>
</tr>
</tbody>
</table>

Results from the numerical experiment.

\[\hat{C} (\tau_k) = \left[ c_p N + h c_p T S_k^0 (\tau_k) + g_k^0 (\tau_k) \frac{N T (c_p - c_d)}{\tau_k} + \frac{N T c_d}{\tau_k} \right] \leq \left[ \hat{C}_k (\tau_k) \right].\]

which holds since both \(S_k^0 (\tau_k)\) and \(g_k^0 (S_k^0, \tau_k)\) go to zero. In other words, \(\hat{C}(\tau_k)\) is strictly larger than \(c_p N\) for every possible value of \(\tau_k\). If we next assume that \(K(\cdot) \geq K_{\text{lim}} = h c_p T S^0_0 + g^0 (\cdot) \frac{N T (c_p - c_d)}{\tau_k} + \frac{N T c_d}{\tau_k}\), we see that there exists no break-even reliability \(\tau_k (\cdot)\):

\[\hat{C} (\tau_k (\cdot)) = \hat{C}_k (\tau_k) - K(\cdot) \leq c_p N + h c_p T S_k^0 (\tau_k) + g_k^0 (\tau_k) \frac{N T (c_p - c_d)}{\tau_k} + \frac{N T c_d}{\tau_k} \]

For \(\tau_k (\cdot)\) to exist on the entire interval \(K(\cdot) \in (-\infty, K_{\text{lim}})\), we require that \(\hat{C}(\tau_k (\cdot))\) can take on any value on the interval \((0, \infty)\). We find:

\[\lim_{\tau_k \to 0} \hat{C} (\tau_k) = \lim_{\tau_k \to 0} \left[ c_p N + h c_p T S_k^0 (\tau_k) + g_k^0 (\tau_k) \frac{N T (c_p - c_d)}{\tau_k} + \frac{N T c_d}{\tau_k} \right] = c_p N.\]

(ii) This follows from the fact that \(\hat{C} (\tau_k)\) is strictly decreasing in \(\tau_k\) (see Lemma 1(i)).

### A5. Proof of Theorem 1

(i) By Definition 1, it holds that \(\hat{C}_k (\tau_k (\cdot)) = \hat{C}_k (\tau_k) - K(\cdot)\). This implies that if \(K(\cdot)\) increases, \(\hat{C}_k (\tau_k (\cdot))\) decreases and from Lemma 1(i) we know that \(\hat{C}_k (\tau_k)\) is strictly decreasing in \(\tau_k\).

(ii) From the proof of Lemma 2(i), we know that \(\lim_{\tau_k \to \infty} \hat{C}_k (\tau_k) = c_p N\) and that \(\hat{C}_k (\tau_k) - K_{\text{lim}} = c_p N\), which, in combination with Theorem 1(i) implies the result.

We assume that \(\tau_k (\cdot) \geq \tau^k\) and show that then \(K(\cdot) > 0\), which proves by contradiction that if \(K(\cdot) \leq 0\), then \(\tau_k (\cdot) < \tau^k\).

\[K(\cdot) = \hat{C}_k (S_k^0 (\tau^k); \tau^k) - \hat{C}_k (S_k^0 (\tau^k); \tau^k) \geq \hat{C}_k (S_k^0 (\tau^k); \tau^k) - \hat{C}_k (S_k^0 (\tau^k); \tau^k) \geq \left[ c_p N + h c_p T S_k^0 (\tau_k) + g_k^0 (S_k^0 (\tau^k); \tau^k) \frac{N T (c_p - c_d)}{\tau^k} + \frac{N T c_d}{\tau^k} \right] \leq \left[ c_p N + h c_p T S_k^0 (\tau_k) + g_k^0 (S_k^0 (\tau^k); \tau^k) \frac{N T (c_p - c_d)}{\tau^k} + \frac{N T c_d}{\tau^k} \right].\]

### A4. Proof of Lemma 2

(i) We are interested in finding \(\tau_k (\cdot)\) such that \(\hat{C}_k (\tau_k) = \hat{C}_k (\tau_k (\cdot)) + K(\cdot)\). If \(\hat{C}_k (\tau_k)\) is given and \(K(\cdot)\) increases, then \(\hat{C}_k (\tau_k (\cdot))\) must decrease. From Lemma 1(i), we know that \(\hat{C}_k (\tau_k)\) is decreasing in \(\tau_k\), so in order to find the minimal possible life cycle costs when using an AM component, we are interested in:

\[\lim_{\tau_k \to \infty} \hat{C}_k (\tau_k) = \lim_{\tau_k \to \infty} \left[ c_p N + h c_p T S_k^0 (\tau_k) + g_k^0 (\tau_k) \frac{N T (c_p - c_d)}{\tau_k} + \frac{N T c_d}{\tau_k} \right] c_p N.\]
The weak inequality is obtained since $C^d(S^b_k(t); \tau^A(t)) \leq C^d(S^b_0(\tau^A))$. The second weak inequality is obtained by substituting $c^d_k$ for $c^d$. The strict inequality follows from the fact that the Erlang Loss probability is strictly increasing in the system load (see Harel, 1990), so that $g^d(S^b_k(\tau^A); \tau^A(t)) < g^d(S^b_0(\tau^A))$, since $L^A < L^R$ and $\tau^A(t) \geq \tau^R$.

A6. Proof of Theorem 2

(i) This follows from Theorem 1(i) and Theorem 1(iii).

(ii) Let $\varepsilon > 0$ and $\Delta K_i(T) = K_i(T + \varepsilon) - K_i(T)$. We then need to prove that $\Delta K_i(T) > 0$.

From Definition 2, we know that $\tau^A(K_i(T + \varepsilon)) = \tau^A(K_i(T)) = \tau^R$ and we denote this value by $\tau$. From Definition 1, we know that $C^d(T - \tau(T)) = K_i(T)$. This means that:

$$
\Delta K_i(T) = \left[ C^d(T + \varepsilon) - C^d(T) \right] - \left[ C^d(T) - C^d(T) \right] = \left[ C^d(T - \tau(T)) - C^d(T - \tau(T)) \right]
$$

$$
= \left( h_{c^d}(T + \varepsilon)C^d(S^b_k(\tau)) + g^d(S^b_k(\tau)) \right) \frac{N(c^d_k - c_d)}{\tau} \frac{N(T + \varepsilon)c^d_k}{\tau} + N(T + \varepsilon)c_d
$$

which means that:

$$
\left( g^d(S^b_k(\tau)) - g^d(S^b(\tau)) \right) \frac{N(c^d_k - c_d)}{\tau} \frac{N(T + \varepsilon)c^d_k}{\tau} \leq \left( S^b_k(\tau) - S^b(\tau) \right) h_{c^d}.
$$

This in turn implies that:

$$
\Delta K_i(T) = \varepsilon \left( h_{c^d}(S^b_k(\tau) - S^b(\tau)) + \frac{g^d(S^b_k(\tau) - g^d(S^b(\tau)))}{\tau} \frac{N(c^d_k - c_d)}{\tau} \right)
$$

$$
= \varepsilon \left( h_{c^d}(S^b_k(\tau) - S^b(\tau)) - \frac{g^d(S^b_k(\tau) - g^d(S^b(\tau)))}{\tau} \frac{N(c^d_k - c_d)}{\tau} \right)
$$

$$
\geq \varepsilon \left( h_{c^d}(S^b_k(\tau) - S^b(\tau)) - \frac{g^d(S^b(\tau) - g^d(S^b(\tau)))}{\tau} \frac{N(c^d_k - c_d)}{\tau} \right)
$$

$$
= 0.
$$

Note that equality can only occur when the optimal base stock level is not unique, i.e., costs are equal under base stock level $S^b_k(\tau)$ and base stock level $S^b(\tau) + 1$. Furthermore, it must be the case that $S^b_k(\tau) = S^b(\tau) - 1$, otherwise $\delta > 1$ and we regain a strict inequality as a result of Lemma 3. In all other cases, it will hold that $\Delta K_i(T) > 0$.

(iii) Theorem 2(ii) states that $K_i(\cdot)$ is increasing in $T$. As Theorem 1(i) states that $\tau^A$ is increasing in $K_i(\cdot)$, this implies that at $K_i(T)$, $\tau^A(K_i(T))$ is less than $\tau^A(K_i(T))$. Thus, we prove that $\Delta K_i(T) = 0$.

A7. Proof of Theorem 3

To prove the first inequality, we define the term $\Delta T$, as the difference between two break-even equations according to Definition 1, one for $T$ and one for $T + \varepsilon$, and both for $K_i(T)$ as defined in Definition 3. Both equations should be equal to zero, so that also $\Delta T = 0$. We prove for the case where $\tau^A(K_i(T) + \varepsilon) \leq \tau^A(K_i(T) + \varepsilon)$. Then $\Delta T > 0$. Since we know that $h_{c^d}(K_i(T) + \varepsilon) = h_{c^d}(K_i(T) - \varepsilon) + h_{c^d}(K_i(T) + \varepsilon)$. Then $\Delta T > 0$, we know that it must hold that $\tau^A(K_i(T) + \varepsilon) > \tau^A(K_i(T) + \varepsilon)$, which completes our proof.

$$
\Delta T = \left[ C^{d^e}(T; \tau) - C^{d^e}(T; \tau) \right] - \left[ C^{d^e}(T; \tau) - C^{d^e}(T; \tau) \right]
$$

First, we provide both break-even equations and we let $K_i(T)$ cancel out from both break-even equations. Then, we substitute $\tau^A(K_i(T) + \varepsilon) = \tau^A(K_i(T) + \varepsilon) + \varepsilon$. This results in the first inequality, because $C^{d^e}(T; \tau)$ is decreasing in $\tau$ (see Lemma 1(i)), which
results in a cost decrease if we increase \( \tau^{A_4}(K_2(T); T + \varepsilon) \) up to \( \tau^{A_4}(K_2(T); T) \). The inequality is non-strict to also include the case where \( \tau^{A_4}(K_2(T); T + \varepsilon) \) is equal to \( \tau^{A_4}(K_2(T); T) \). Then we write the entire equation. For the final equality, note that we have equal loads on the inventory systems, i.e., \( a^R = a^A \), and we have equal cost structures. This implies that \( S^{A_4}(\cdot) = S^{A_4}(\cdot) \) and \( g^A(\cdot) = g^A(\cdot) \). We also know that \( S^{A_4}(T) = S^{A_4}(T + \varepsilon) \), since Lemma 3 shows that the optimal base stock level is independent of \( T \). Hence, many terms cancel out. The final inequality then follows from the fact that \( K_2(\cdot) < K_1(\cdot) \), such that \( \tau^{A_4}(K_2(T); T) < \tau^{A_4}(K_2(T); T + \varepsilon) \).

A8. Proof of Theorem 4

(i) Suppose \( K(\cdot) > \tilde{C}^A(c^p) \), then the following holds via Eq. (3):

\[
\begin{align*}
\tau^{A_4} = & \frac{g^R(c^p) + g^A(\cdot) \cdot \frac{N(T \varepsilon \alpha - c^p)}{1 - \varepsilon}}{N + h T S^{A_4}(c^p) + \frac{N T c^p}{1 - \varepsilon}} \\
< & \frac{\tilde{C}^R(c^p) - K(\cdot)}{N + h T S^{A_4}(c^p) + \frac{N T c^p}{1 - \varepsilon}} < 0,
\end{align*}
\]

where the first inequality is obtained by eliminating the negative cost term related \( g^A(\cdot) \). Hence, we require a negative production cost of the AM component in order to meet break-even, which is clearly not feasible.

(ii) This follows from the fact that \( \tilde{C}(\cdot) \) is increasing in \( c^p \) (Lemma 1).

(iii) Let \( \varepsilon > 0 \) and assume that \( c^p < \tilde{C}^R(c^p) \) for some \( K(\cdot) \). By Definition 4 we find the following expression:

\[
\tilde{C}^R(c^p) > \tilde{C}^R(c^p) + \varepsilon = \tilde{C}^R(c^p) + \varepsilon
\]

implies that in order to compensate for the increase in \( K(\cdot) \) and meet the break-even condition of Definition 4, \( c^p \) must be smaller than \( c^p + \varepsilon \), since \( \tilde{C}^R(c^p + \varepsilon) \) is increasing in \( c^p \) (Lemma 1(iii)).

References


