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Determination of α_s from jet multiplicities measured on the Z^0 resonance

L3 Collaboration

B. Adeva ^a, O. Adriani ^b, M. Aguilar-Benitez ^c, H. Akbari ^d, J. Alcaraz ^c, A. Aloisio ^e,
 G. Alverson ^f, M.G. Alvisi ^g, Q. An ^g, H. Anderhub ^h, A.L. Anderson ⁱ, V.P. Andreev ^j,
 T. Angelov ⁱ, L. Antonov ^k, D. Antreasyan ^l, P. Arce ^c, A. Arefiev ^m, T. Azemoon ⁿ, T. Aziz ^o,
 P.V.K.S. Baba ^g, P. Bagnaia ^p, J.A. Bakken ^q, L. Baksay ^r, R.C. Ball ⁿ, S. Banerjee ^{o, g}, J. Bao ^d,
 L. Barone ^p, A. Bay ^s, U. Becker ⁱ, J. Behrens ^h, S. Beingessner ^t, Gy.L. Bencze ^{u, r}, J. Berdugo ^c,
 P. Berges ⁱ, B. Bertucci ^p, B.L. Betev ^k, A. Biland ^h, R. Bizzarri ^p, J.J. Blaising ^t, P. Blömeke ^v,
 B. Blumenfeld ^d, G.J. Bobbink ^w, M. Bocciolini ^b, W. Böhlen ^x, A. Böhm ^v, T. Böhringer ^y,
 B. Borgia ^p, D. Bourilkov ^k, M. Bourquin ^s, D. Boutigny ^t, J.G. Branson ^z, I.C. Brock ^a,
 F. Bruyant ^a, C. Buisson ^g, A. Bujak ^y, J.D. Burger ⁱ, J.P. Burq ^g, J. Busenitz ^g, X.D. Cai ^g,
 C. Camps ^v, M. Capell ⁿ, F. Carbonara ^e, F. Carminati ^b, A.M. Cartacci ^b, M. Cerrada ^c,
 F. Cesaroni ^p, Y.H. Chang ⁱ, U.K. Chaturvedi ^g, M. Chemarin ^g, A. Chen ^e, C. Chen ^g,
 G.M. Chen ^g, H.F. Chen ⁿ, H.S. Chen ^g, M. Chen ⁱ, M.L. Chen ⁿ, G. Chiefari ^e, C.Y. Chien ^d,
 C. Civinini ^b, I. Clare ⁱ, R. Clare ⁱ, G. Coignet ^t, N. Colino ^a, V. Commichau ^v, G. Conforto ^b,
 A. Contin ^a, F. Crijns ^w, X.Y. Cui ^g, T.S. Dai ⁱ, R. D'Alessandro ^b, R. de Asmundis ^e, A. Degré ^t,
 K. Deiters ^{a, g}, E. Dénes ^u, P. Denes ^q, F. DeNotaristefani ^p, M. Dhina ^h, D. DiBitonto ^g,
 M. Diemoz ^p, F. Diez-Hedo ^a, H.R. Dimitrov ^k, C. Dionisi ^p, F. Dittus ^k, R. Dolin ⁱ, E. Drago ^e,
 T. Driever ^w, D. Duchesneau ^s, P. Duinker ^{w, a}, I. Duran ^{a, c}, H. El Mamouni ^g, A. Engler ^a,
 F.J. Eppling ⁱ, F.C. Erné ^w, P. Extermann ^s, R. Fabbretti ^h, G. Faber ⁱ, S. Falciano ^{a, p}, Q. Fan ^{g, g},
 S.J. Fan ^g, M. Fabre ^h, J. Fay ^g, J. Fehlmann ^h, H. Fenker ^f, T. Ferguson ^a, G. Fernandez ^c,
 F. Ferroni ^{p, a}, H. Fesefeldt ^v, J. Field ^s, G. Finocchiaro ^p, P.H. Fisher ^d, G. Forconi ^s,
 T. Foreman ^w, K. Freudenreich ^h, W. Friebel ^g, M. Fukushima ⁱ, M. Gailloud ^y,
 Yu. Galaktionov ^m, E. Gallo ^b, S.N. Ganguli ^o, P. Garcia-Abia ^c, S.S. Gau ^g, S. Gentile ^p,
 M. Gettner ^f, M. Glaubman ^f, S. Goldfarb ⁿ, Z.F. Gong ^{g, n}, E. Gonzalez ^c, A. Gordeev ^m,
 P. Göttlicher ^v, D. Goujon ^s, C. Goy ^t, G. Gratta ^k, A. Grimes ^f, C. Grinnell ⁱ, M. Gruenewald ^k,
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 A. Hervé ^a, K. Hilgers ^v, H. Hofer ^h, H. Hoorani ^g, L.S. Hsu ^e, G. Hu ^g, G.Q. Hu ^g, B. Ille ^g,
 M.M. Ilyas ^g, V. Innocente ^{e, a}, E. Isiksal ^h, E. Jagel ^g, B.N. Jin ^g, L.W. Jones ⁿ, P. Kaaret ^q,
 R.A. Khan ^g, Yu. Kamyshkov ^m, D. Kaplan ^f, Y. Karyotakis ^{t, a}, M. Kaur ^g, S. Khokhar ^g,
 V. Khoze ^j, D. Kirkby ^k, W. Kittel ^w, A. Klimentov ^m, A.C. König ^w, O. Kornadt ^v,
 V. Koutsenko ^m, R.W. Kraemer ^a, T. Kramer ⁱ, V.R. Krastev ^k, W. Krenz ^v, J. Krizmanic ^d,
 A. Kuhn ^x, K.S. Kumar ^g, V. Kumar ^g, A. Kunin ^m, S. Kwan ^f, A. van Laak ^v, V. Lalieu ^s,
 G. Landi ^b, K. Lanus ^{a, g}, W. Lange ^g, D. Lanske ^v, S. Lanzano ^c, P. Lebrun ^g, P. Lecomte ^h,
 P. Lecoq ^a, P. Le Coultre ^h, I. Leedom ^f, J.M. Le Goff ^a, A. Leike ^g, L. Leistam ^a, R. Leiste ^g,
 J. Lettry ^h, P.M. Levchenko ^j, X. Leytens ^w, C. Li ⁿ, H.T. Li ^g, J.F. Li ^g, L. Li ^h, P.J. Li ^g, Q. Li ^g,
 X.G. Li ^g, J.Y. Liao ^g, Z.Y. Lin ⁿ, F.L. Linde ^a, D. Linnhofer ^a, R. Liu ^g, Y. Liu ^g, W. Lohmann ^g,
 S. Lökös ^r, E. Longo ^p, Y.S. Lu ^g, J.M. Lubbers ^w, K. Lübelmeyer ^v, C. Luci ^a, D. Luckey ^{g, i},
 L. Ludovici ^p, X. Lue ^h, L. Luminari ^p, W.G. Ma ⁿ, M. MacDermott ^h, R. Magahiz ^r, M. Maire ^t,

P.K. Malhotra ^o, R. Malik ^g, A. Malinin ^m, C. Maña ^{a, c}, D.N. Mao ⁿ, Y.F. Mao ^ζ,
M. Maolinbay ^h, P. Marchesini ^g, A. Marchionni ^b, J.P. Martin ^β, L. Martinez ^c, F. Marzano ^p,
G.G.G. Massaro ^w, T. Matsuda ⁱ, K. Mazumdar ^o, P. McBride ^μ, T. McMahon ^γ, D. McNally ^h,
Th. Meinholz ^v, M. Merk ^w, L. Merola ^c, M. Meschini ^b, W.J. Metzger ^w, Y. Mi ^g, M. Micke ^v,
U. Micke ^v, G.B. Mills ⁿ, Y. Mir ^g, G. Mirabelli ^p, J. Mnich ^v, M. Möller ^v, L. Montanet ^a,
B. Monteleoni ^b, G. Morand ^s, R. Morand ^t, S. Morganti ^p, V. Morgunov ^m, R. Mount ^κ,
E. Nagy ^{u, a}, M. Napolitano ^c, H. Newman ^κ, M.A. Niaz ^g, L. Niessen ^v, W.D. Nowak ^θ,
H. Nowak ^θ, S. Nowak ^θ, D. Pandoulas ^v, G. Paternoster ^e, S. Patricelli ^c, Y.J. Pei ^v,
D. Perret-Gallix ^t, J. Perrier ^s, A. Pevsner ^d, M. Pieri ^b, P.A. Piroué ^q, V. Plyaskin ^m, M. Pohl ^h,
V. Pojidaev ^m, N. Produit ^s, J.M. Qian ^{i, g}, K.N. Qureshi ^g, R. Raghavan ^o, G. Rahal-Callot ^h,
P. Raziš ^h, K. Read ^q, D. Ren ^h, Z. Ren ^g, S. Reucroft ^f, A. Ricker ^v, T. Riemann ^θ, C. Rippich ^α,
H.A. Rizvi ^g, B.P. Roe ⁿ, M. Röhner ^v, S. Röhner ^v, Th. Rombach ^v, L. Romero ^c, J. Rose ^v,
S. Rosier-Lees ^t, R. Rosmalen ^w, Ph. Rosselet ^y, J.A. Rubio ^{a, c}, W. Ruckstuhl ^s,
H. Rykaczewski ^h, M. Sachwitz ^θ, J. Salicio ^c, J.M. Salicio ^c, G. Sartorelli ^g, G. Sauvage ^t,
A. Savin ^m, V. Schegelsky ^j, D. Schmitz ^v, P. Schmitz ^v, M. Schneegans ^t, M. Schöntag ^v,
H. Schopper ^ξ, D.J. Schotanus ^w, H.J. Schreiber ^θ, R. Schulte ^v, S. Schulte ^v, K. Schultze ^v,
J. Schütte ^μ, J. Schwenke ^v, G. Schwering ^v, C. Sciacca ^c, R. Sehgal ^g, P.G. Seiler ^h, J.C. Sens ^w,
I. Sheer ^z, V. Shevchenko ^m, S. Shevchenko ^m, X.R. Shi ^α, K. Shmakov ^m, V. Shoutko ^m,
E. Shumilov ^m, N. Smirnov ^j, A. Sopczak ^{κ, z}, C. Souyri ^t, C. Spartiotis ^d, T. Spickermann ^v,
B. Spiess ^x, P. Spillantini ^b, R. Starosta ^v, M. Steuer ^{g, i}, D.P. Stickland ^q, B. Stöhr ^h, H. Stone ^s,
K. Strauch ^μ, B.C. Stringfellow ^γ, K. Sudhakar ^{o, z}, G. Sultanov ^a, R.L. Sumner ^q, H. Suter ^h,
R.B. Sutton ^α, J.D. Swain ^g, A.A. Syed ^g, X.W. Tang ^ζ, E. Tarkovsky ^m, J.M. Thenard ^t,
E. Thomas ^g, C. Timmermans ^w, Samuel C.C. Ting ⁱ, S.M. Ting ⁱ, Y.P. Tong ^g, F. Tonisch ^θ,
M. Tonutti ^v, S.C. Tonwar ^o, J. Tòth ^u, G. Trowitzsch ^θ, K.L. Tung ^ζ, J. Ulbricht ^x, L. Urbàn ^u,
U. Uwer ^v, E. Valente ^p, R.T. Van de Walle ^w, H. van der Graaf ^w, I. Vetlitsky ^m, G. Viertel ^h,
P. Vikas ^g, U. Vikas ^g, M. Vivargent ^{t, i}, H. Vogel ^α, H. Vogt ^θ, M. Vollmar ^v, G. Von Dardel ^a,
I. Vorobiev ^m, A.A. Vorobyov ^j, An.A. Vorobyov ^j, L. Vuilleumier ^y, M. Wadhwa ^g, W. Walk ^a,
W. Wallraff ^v, C.R. Wang ⁿ, G.H. Wang ^α, J.H. Wang ^ζ, Q.F. Wang ^μ, X.L. Wang ⁿ, Y.F. Wang ^b,
Z. Wang ^g, Z.M. Wang ^{g, n}, J. Weber ^h, R. Weill ^y, T.J. Wenaus ⁱ, J. Wenninger ^s, M. White ⁱ,
R. Wilhelm ^w, C. Willmott ^c, F. Wittgenstein ^a, D. Wright ^q, R.J. Wu ^ζ, S.L. Wu ^g, S.X. Wu ^g,
Y.G. Wu ^ζ, B. Wysłouch ^{i, a}, Z.Z. Xu ⁿ, Z.L. Xue ^λ, D.S. Yan ^λ, B.Z. Yang ⁿ, C.G. Yang ^ζ,
G. Yang ^g, K.S. Yang ^ζ, Q.Y. Yang ^ζ, Z.Q. Yang ^λ, Q. Ye ^g, C.H. Ye ^g, S.C. Yeh ^g, Z.W. Yin ^λ,
J.M. You ^g, C. Zabounidis ^f, C. Zaccardelli ^κ, L. Zehnder ^h, M. Zeng ^g, Y. Zeng ^v, D. Zhang ^z,
D.H. Zhang ^w, S.Y. Zhang ^ζ, Z.P. Zhang ⁿ, J.F. Zhou ^v, R.Y. Zhu ^κ, A. Zichichi ^{a, g} and J. Zoll ^a

^a European Laboratory for Particle Physics, CERN, CH-1211 Geneva 23, Switzerland

^b INFN – Sezione di Firenze and University of Firenze, I-50125 Florence, Italy

^c Centro de Investigaciones Energeticas, Medioambientales y Tecnologicas, CIEMAT, E-28040 Madrid, Spain

^d Johns Hopkins University, Baltimore, MD 21218, USA

^e INFN – Sezione di Napoli and University of Naples, I-80125 Naples, Italy

^f Northeastern University, Boston, MA 02115, USA

^g World Laboratory, FBLJA Project, CH-1211 Geneva, Switzerland

^h Eidgenössische Technische Hochschule, ETH Zürich, CH-8093 Zurich, Switzerland

ⁱ Massachusetts Institute of Technology, Cambridge, MA 02139, USA

^j Leningrad Nuclear Physics Institute, SU-188 350 Gatchina, USSR

^κ Central Laboratory of Automation and Instrumentation, CLANP, Sofia, Bulgaria

^l INFN – Sezione di Bologna, I-40126 Bologna, Italy

^m Institute of Theoretical and Experimental Physics, ITEP, SU-117 259 Moscow, USSR

ⁿ University of Michigan, Ann Arbor, MI 48109, USA

^o Tata Institute of Fundamental Research, Bombay 400 005, India

^p INFN – Sezione di Roma and University of Rome “La Sapienza”, I-00185 Rome, Italy

- ^a Princeton University, Princeton, NJ 08544, USA
^r Union College, Schenectady, NY 12308, USA
^s University of Geneva, CH-1211 Geneva 4, Switzerland
^t Laboratoire de Physique des Particules, LAPP, F-74519 Annecy-le-Vieux, France
^u Central Research Institute for Physics of the Hungarian Academy of Sciences, H-1525 Budapest 114, Hungary
^v I. Physikalisches Institut, RWTH, D-5100 Aachen, FRG¹
 and III. Physikalisches Institut, RWTH, D-5100 Aachen, FRG¹
^w National Institute for High Energy Physics, NIKHEF, NL-1009 DB Amsterdam, The Netherlands
 and NIKHEF-H and University of Nijmegen, NL-6525 ED Nijmegen, The Netherlands
^x Paul Scherrer Institut (PSI), Würenlingen, Switzerland
^y University of Lausanne, CH-1015 Lausanne, Switzerland
^z University of California, San Diego, CA 92182, USA
^{aa} Carnegie Mellon University, Pittsburgh, PA 15213, USA
^{bb} Institut de Physique Nucléaire de Lyon, IN2P3-CNRS/Université Claude Bernard, F-69622 Villeurbanne Cedex, France
^{cc} Purdue University, West Lafayette, IN 47907, USA
^{dd} University of Alabama, Tuscaloosa, AL 35486, USA
^{ee} High Energy Physics Group, Taiwan, ROC
^{ff} Institute of High Energy Physics, IHEP, Beijing, P.R. China
^{gg} Chinese University of Science and Technology, USTC, Hefei, Anhui 230 029, P.R. China
^{hh} High Energy Physics Institute, DDR-1615 Zeuthen-Berlin, GDR
^{kk} California Institute of Technology, Pasadena, CA 91125, USA
^{ll} Shanghai Institute of Ceramics, SIC, Shanghai, P.R. China
^{mm} Harvard University, Cambridge, MA 02139, USA
ⁿⁿ University of Hamburg, D-2000 Hamburg, FRG

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We present a study of jet multiplicities based on 37 000 hadronic Z^0 boson decays. From this data we determine the strong coupling constant $\alpha_s = 0.115 \pm 0.005$ (exp.) $\pm_{0.012}^{0.013}$ (theor.) to second order QCD at $\sqrt{s} = 91.22$ GeV.

1. Introduction

The observation of jets produced in e^+e^- annihilation [1,2] opened an important area for tests of perturbative quantum chromodynamics (QCD) [3]. Hard quarks and gluons produced in hadronic events form jets, which preserve the energy and direction of the primary partons. Perturbative QCD predicts the fraction of events with two, three or more hard partons as a function of the parameter $A_{\overline{MS}}$, which determines the strong coupling constant α_s at a given scale μ^2 . QCD predicts that α_s decreases logarithmically with increasing energy.

The Z^0 resonance is ideal for a determination of α_s from the measured jet multiplicities for the following reasons: (1) Hadronization effects are small at such a high center of mass energy. Jets are more collimated than at lower energies. (2) The hadronic cross

section is large. (3) Initial state hard proton radiation is strongly suppressed.

We report here on measurements of jet multiplicities at the Z^0 resonance using the L3 detector at LEP. Comparing our data to the predictions of perturbative QCD in second order we derive a value for $A_{\overline{MS}}$ and $\alpha_s(\sqrt{s} = M_Z)$. In order to explore the energy dependence of α_s the 3-jet rates measured at different center of mass energies are compared to the QCD calculations.

2. The L3 detector

The L3 detector covers 99% of 4π . The detector includes a central vertex chamber, a precise electromagnetic calorimeter composed of bismuth germanium oxide crystals, a uranium and brass hadron calorimeter with proportional wire chamber readout, a high accuracy muon chamber system, and a ring of

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scintillation triggers counters. These detectors are installed in a magnet with an inner diameter of 12 m. The magnet provides a uniform field of 0.5 T along the beam direction. The luminosity is measured with two small angle electromagnetic calorimeters. A detailed description of each detector subsystem, and its performance, is given in ref. [4].

The fine segmentation of the electromagnetic detector and the hadron calorimeter allows us to measure the axis of jets with an angular resolution of 2.5° , and to measure the total energy of hadronic events from Z^0 decay with a resolution of 12%.

For the present analysis, we used the data collected in the following ranges of polar angles:

- for the electromagnetic calorimeter, $42.4^\circ < \theta < 137.6^\circ$,
- for the hadron calorimeter, $5^\circ < \theta < 175^\circ$.

3. Selection of hadronic events

Events collected at a center of mass energy of $\sqrt{s} = 91.22 \pm 0.03$ GeV from the 1990 (March–June) LEP running period are used for this analysis.

The primary trigger for hadronic events requires a total energy of 15 GeV in the central region of the calorimeters ($|\cos \theta| < 0.74$), or 20 GeV in the whole detector. This trigger is in a logical OR with a trigger using the barrel scintillation counters and with a charged track trigger. The combined trigger efficiency for selected hadronic events exceeds 99.95%.

The selection of $e^+e^- \rightarrow$ hadrons events is based on the energy measured in the electromagnetic detector and in the hadron calorimeter:

$$0.6 < E_{\text{vis}}/\sqrt{s} < 1.4,$$

$$|E_{\parallel}|/E_{\text{vis}} < 0.40, \quad E_{\perp}/E_{\text{vis}} < 0.40,$$

$$N_{\text{cluster}} > 12,$$

where E_{vis} is the total energy observed in the detector, E_{\parallel} is the energy imbalance along the beam direction, and E_{\perp} is the transverse energy imbalance. An algorithm was used to group neighbouring calorimeter hits, which are probably produced by the same particle, into clusters. Only clusters with a total energy above 100 MeV were used. The algorithm normally reconstructs one cluster for each particle produced near the interaction point. Thus the cut on the

number of clusters rejects low multiplicity events (e^+e^- , $\mu^+\mu^-$, $\tau^+\tau^-$).

In total 36 728 events were selected.

Applying the same cuts to simulated events, we find that 97% of the hadronic decays from the Z^0 are accepted.

The contamination from e^+e^- and $\tau^+\tau^-$ final states in the hadronic event sample is below 0.2% and can be neglected. Also the contribution to the event sample from the “two photon process” $e^+e^- \rightarrow e^+e^- +$ hadrons is found to be negligible after the above cuts.

Monte Carlo distributions were generated by the parton shower program JETSET 7.2 [5] with $A_{\text{LL}} = 290$ MeV and string fragmentation. The b quark fragmentation function was adjusted to match our measured inclusive muon data [6]. The generated events were passed through the L3 detector simulation [7] which includes the effects of energy loss, multiple scattering, interactions and decays in the detector materials and beam pipe.

The measured distributions in the cut quantities and in event shape variables agree very closely with the corresponding simulated distributions [8].

4. Measurement of jet multiplicities

Jets are reconstructed out of clusters in the calorimeters by using the “JADE” version [9] of an invariant mass jet algorithm. In this recombination scheme there is a close agreement between jet rates on parton and detector level. First the energy and direction of all clusters are determined. For each pair of clusters i and j the scaled invariant mass squared

$$y_{ij} = (2E_i E_j / E_{\text{vis}}^2) (1 - \cos \theta_{ij})$$

is then evaluated. E_i and E_j are the cluster energies and θ_{ij} is the angle between clusters i and j . The cluster pair for which y_{ij} is smallest is replaced by a pseudo-cluster k with four-momentum

$$p_k = p_i + p_j.$$

This procedure is repeated until all y_{ij} exceed the jet resolution parameter y_{cut} . The remaining (pseudo)-clusters are called jets. Increasing y_{cut} lowers the fraction of multijet events but increases the separation of the jets.

Fig. 1 shows the measured distributions of scaled

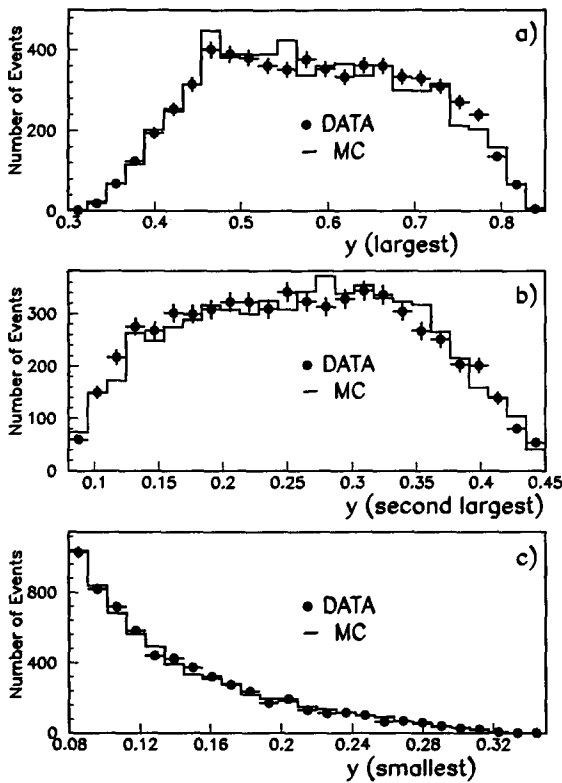


Fig. 1. Measured y distributions for jet pairs in 3-jet events at $y_{\text{cut}}=0.08$ in comparison with the Monte Carlo predictions (parton shower, $A_{\text{LL}}=290$ MeV) [5,7]. The scaled invariant mass squared values $Y=(2E_m E_n/E_{\text{vis}}^2)(1-\cos\theta_{mn})$ for all three pairs of reconstructed jets m and n were calculated and ordered. (a) Distribution of the largest scaled invariant mass squared. (b) Distribution of the second largest scaled invariant mass squared. (c) Distribution of the smallest scaled invariant mass squared.

invariant mass squared values for the three pairs of jets in 3-jet events reconstructed using a jet resolution parameter $y_{\text{cut}}=0.08$. The simulated distributions are in good agreement with the experimental ones.

The relative jet production rates $f_i = \sigma_{i\text{-jets}}/\sigma_{\text{tot}}$, where i is the number of jets, are then determined as a function of the jet resolution y_{cut} . The data sample was subdivided into different subsamples to study a possible time dependence. In addition the jet multiplicities were analyzed as a function of the polar and azimuthal angle of the event thrust axis with respect to the beam line. No deviation from uniformity was observed within the statistical error of 3%. We con-

servatively assign a systematic relative error of 3% to the measured three jet rate f_3 . Our jet multiplicities are shown in fig. 2a together with their statistical and systematic errors combined quadratically.

We have corrected our measurements for the detector effects, resolution and acceptance. We used the JETSET 7.2 Monte Carlo program as described

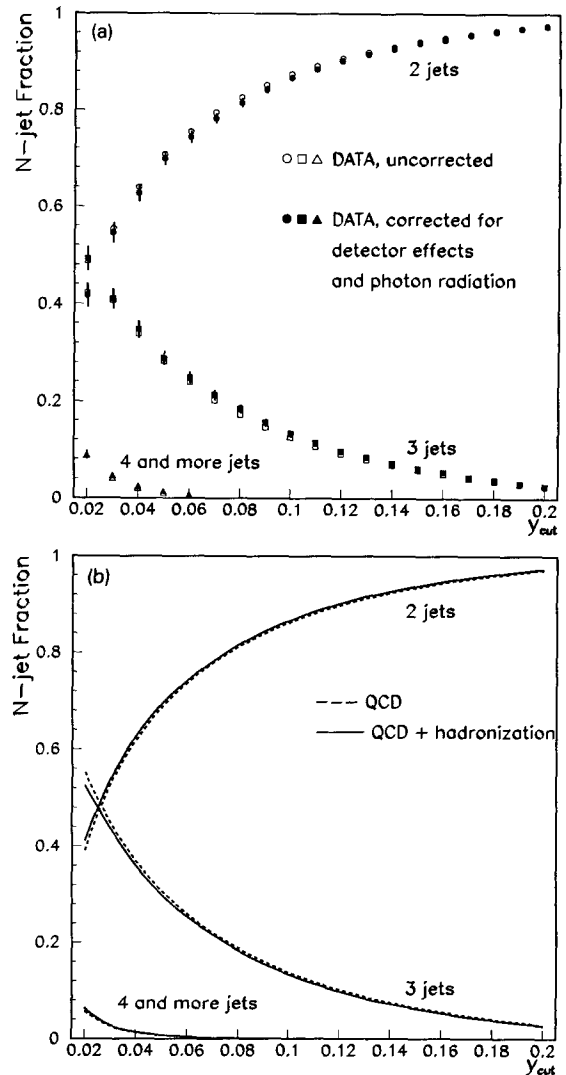


Fig. 2. (a) Measured jet fractions before and after corrections for detector effects and photon radiation. The smoothness of the y_{cut} dependence comes from the fact that all points are measured with the same data sample, and are therefore correlated. (b) Multijet rates predicted by second order QCD [10] for $\mu^2/s=0.08$ and $A_{\text{MS}}=190$ MeV without and with hadronization correction.

above. To correct for detector resolution, we determined the probabilities D_{ij} for an event with jet multiplicity j on the generator level to appear as an i -jet event after all particles were passed through the detector simulation for each value of y_{cut} . The number of jets on the generator level was calculated by applying the jet algorithm to the generated particles (after hadronization and decays). The raw jet fractions were then corrected using the inverse of the matrix D . The corrections amount to only a few percent due to the good angular and energy resolution of the L3 detector. The effects of the detector acceptance are also very small, since the polar angular range $-0.996 < \cos \theta < 0.996$ is covered. The detector effects change the 3-jet rate by typically $\Delta f_3/f_3 = -(5-10)\%$.

The uncertainties of the detector correction were studied by changing the energy response in different detector components in the Monte Carlo simulation by up to 10%. Larger variations are incompatible with the measured energy distributions in the calorimeters. We find a systematic uncertainty in the 3-jet fraction of 4%. Combined with the 3% error on the uncorrected jet multiplicities we estimate the total experimental uncertainty in the determination of f_3 to be $\delta f_3/f_3 = 5\%$. The relative error of f_4 is found to be 10%.

In addition a small correction for initial and final state photon radiation was applied which changes the

3-jet fraction by typically +3%.

Table 1 shows the corrected multijet fractions for jet resolution parameters y_{cut} in the range 0.02–0.20, together with the combined statistical and systematic errors. The same numbers are also displayed in fig. 2a in comparison with the uncorrected jet rates.

5. Comparison to perturbative QCD

For a given parton recombination scheme, QCD (calculated to second order) predicts the rate of 2-, 3- and 4-jet events as a function of the parameter $A_{\overline{\text{MS}}}$, the center of mass energy squared s ($\approx M_Z^2$), the scale μ^2 and the jet resolution y_{cut} . The dependence of the 3- and 4-jet fractions on α_s [10] is given by

$$f_i(A, s, \mu^2, y_{\text{cut}}) = A_i(y_{\text{cut}}) \alpha_s(A, \mu^2) + B_i(y_{\text{cut}}, \mu^2/s) \alpha_s^2(A, \mu^2),$$

where $A_4 \equiv 0$. The 2-jet rate is given by $f_2 = 1 - f_3 - f_4$. The renormalisation scale μ^2 is not fixed in second order QCD.

For the functions A_i and B_i we use the parametrizations for the "E₀" recombination scheme by Kunzst and Nason [10], which are based on the second order QCD calculations of Ellis, Ross and Terrano [11]. They are in good agreement with the multijet fractions calculated by Kramer and Lampe [12]. The E₀ scheme is equivalent to the JADE jet algorithm as described above for up to four massless partons. The dependence of α_s on $A_{\overline{\text{MS}}} \equiv A_{\overline{\text{MS}}}^{(5)}$ is computed using the relation given in ref. [13] for 5 quarks.

The QCD predictions can be compared to the measured multijet rates after corrections for hadronization have been applied. For this purpose we generated events using the GKS [14] matrix element generator implemented in the JETSET 7.2 Monte Carlo program together with fragmentation parameters determined from a comparison of predicted and measured distributions for several event shape variables. Transition probabilities H_{ij} for a transformation of an event with jet multiplicity j on the parton level into an i -jet event after hadronization were evaluated. The second order QCD jet multiplicities were then corrected using the matrix H . The relative correction in the 3-jet rate amounts to about 1 to 5%

Table 1
Measured multijet rates in percent corrected for detector effects and photon radiation. The errors are the combined statistical and systematic uncertainties. Jet fractions > 4 are 0.36% at $y_{\text{cut}} = 0.02$ and below 0.1% for $y_{\text{cut}} \geq 0.03$. For $y_{\text{cut}} \geq 0.08$ the 4-jet rate drops below 0.1%.

y_{cut}	2 jets	3 jets	4 jets
0.02	41.2 ± 2.5	49.8 ± 2.5	8.6 ± 0.9
0.03	53.9 ± 2.1	41.5 ± 2.1	4.6 ± 0.46
0.04	62.2 ± 1.8	35.4 ± 1.8	2.4 ± 0.25
0.05	69.6 ± 1.5	29.1 ± 1.5	1.3 ± 0.15
0.06	74.4 ± 1.2	24.8 ± 1.2	0.72 ± 0.08
0.07	78.4 ± 1.1	21.2 ± 1.1	0.39 ± 0.05
0.08	81.5 ± 0.9	18.4 ± 0.9	0.12 ± 0.02
0.10	86.4 ± 0.7	13.6 ± 0.7	–
0.12	90.1 ± 0.5	9.86 ± 0.52	–
0.14	92.7 ± 0.4	7.35 ± 0.39	–
0.16	94.8 ± 0.3	5.25 ± 0.29	–
0.18	96.4 ± 0.2	3.64 ± 0.21	–
0.20	97.5 ± 0.2	2.46 ± 0.15	–

in the y_{cut} range 0.05–0.20. Fig. 2b compares the jet multiplicities before and after hadronization as a function of y_{cut} .

The theoretical uncertainty was estimated by changing the fragmentation parameters. Replacing those optimised for the matrix element generator by the JETSET default values (for parton shower) modifies the 3-jet rate by only 3%. To study the theoretical uncertainties further the whole analysis was repeated using the ‘‘E’’ recombination scheme [10]. In this scheme the scaled invariant mass squared y is calculated taking into account the masses of the (pseudo)clusters to be recombined: $y_{ij} = (p_i + p_j)^2 / E_{\text{vis}}^2$. In the E scheme fragmentation effects in the jet rate are much larger than in the JADE scheme. The α_s value found in the E scheme analysis was larger by about 0.008 than in the JADE scheme. We have assigned half of this difference as a theoretical uncertainty on the α_s value derived from the JADE scheme analysis. For the theoretical uncertainty due to fragmentation and recombination scheme dependence in the 3-jet fraction we obtain $\delta f_3 / f_3 = 5\%$.

To interpret the measured jet rates in the framework of QCD the renormalisation scale μ^2 needs to be fixed. It is not possible to restrict the renormalisation scale experimentally for the following reason: Jet multiplicities of four are predicted by second order QCD only on the tree level, higher jet fractions are not calculated at all. Therefore a meaningful comparison between data and theory can be performed only for values of y_{cut} where the 4-jet rate is small. However, in that case a change in μ^2 can be compensated by a corresponding change in $A_{\overline{\text{MS}}}$, so that both parameters cannot be determined simultaneously. We fixed μ^2 to the central value $y_{\text{cut}}s$, corresponding to the typical momentum $\sqrt{y_{\text{cut}}s}$ transferred to the hard gluons radiated. We took into account the uncertainty in our measured value of $A_{\overline{\text{MS}}}$ induced by a variation in μ^2/s in the wide range 0.001–1. This covers the results of various theoretical and experimental investigations aiming at a determination of the renormalisation scale [15,16].

The measured multijet fractions for different values of y_{cut} are strongly correlated, and the statistical errors are negligible for this data sample. Therefore we used only one value to derive $A_{\overline{\text{MS}}}$. The comparison for all other values of the jet resolution parameter can then be considered as a test of QCD. We have

chosen $y_{\text{cut}} = 0.08$, so that the 4-jet fraction is negligible ($\approx 0.1\%$) while the 3-jet rate is still large ($18.4\% \pm 0.9\%$). We find

$$A_{\overline{\text{MS}}}^{(s)} = 190_{-50}^{+60} (\text{exp.})_{-90}^{+170} (\text{theor.}) \text{ MeV}$$

for $\mu^2/s = y_{\text{cut}} = 0.08$. The theoretical error includes uncertainties due to fragmentation and recombination scheme dependence (± 60 MeV) and due to the renormalisation scale (± 80 MeV). This translates into

$$\alpha_s(\sqrt{s} = 91.22 \text{ GeV}) = 0.115 \pm 0.005 (\text{exp.})_{-0.010}^{+0.012} (\text{theor.}) .$$

This result is in agreement with those given in refs. [16,17].

The errors for $A_{\overline{\text{MS}}}$ and α_s are dominated by theoretical errors, in particular by the renormalisation scale uncertainties. It can be expected that they can be reduced significantly should a complete $O(\alpha_s^3)$ QCD calculation become available.

Fig. 3 compares the jet multiplicities calculated in second order QCD with $\mu^2/s = 0.08$ and $A_{\overline{\text{MS}}} = 190$ MeV to our measurements. The agreement is excellent for $y_{\text{cut}} \geq 0.05$, where the 4-jet rate is below 1%. For smaller jet resolution parameters the measured

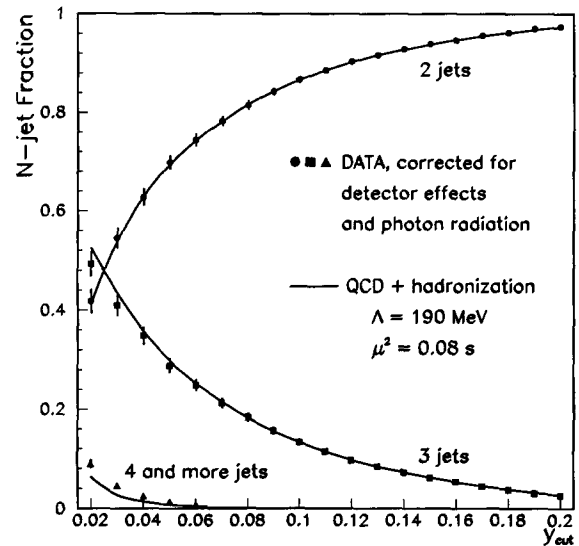


Fig. 3. Comparison of corrected measured jet fractions and predicted jet rates in second order QCD [10] (including hadronization correction) for $\mu^2/s = 0.08$ and $A_{\overline{\text{MS}}} = 190$ MeV.

number of events with high jet multiplicity exceeds the predicted rate. This difference indicates the importance of higher order contributions which have not yet been calculated.

6. Energy dependence of the 3-jet fraction

3-jet fractions for $y_{\text{cut}}=0.08$ measured in e^+e^- annihilation for center for mass energies between 14 and 91 GeV [9,16,18] are shown in fig. 4. Other measurements of α_s in e^+e^- annihilation are compatible with these 3-jet rates [19]. The energy dependence is reproduced by QCD for our measured value of $\Lambda_{\overline{\text{MS}}}=190$ MeV and $\mu^2/s=y_{\text{cut}}=0.08$ for $\sqrt{s}>20$ GeV. The QCD prediction shown in fig. 4 is corrected for hadronization effects which are assumed not to vary with energy. This approximation is good to a few percent for $\sqrt{s}>20$ GeV [9,18]. From the comparison of all measured 3-jet fractions an energy

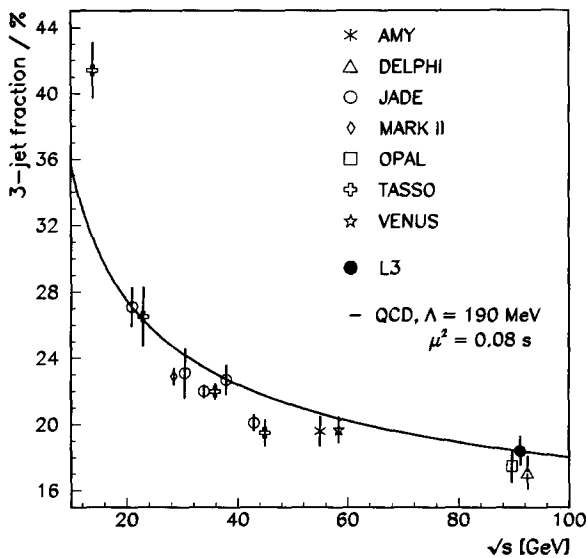


Fig. 4. Energy dependence of the 3-jet fraction measured in e^+e^- annihilation at different center of mass energies for $y_{\text{cut}}=0.08$ [9,16,18] in comparison with the second order QCD predictions [10] (including hadronization correction) for $\mu^2/s=0.08$ and our measured value of $\Lambda_{\overline{\text{MS}}}=190$ MeV. For the points below the Z^0 resonance only statistical errors are shown, while the three error bars for the 3-jet rates measured by DELPHI, OPAL and L3 include also systematic uncertainties. For better readability of the graph different points at the same center of mass energy were slightly shifted horizontally.

independent strong coupling constant can be ruled out.

7. Conclusions

From the measured jet multiplicities in 37 000 hadronic Z^0 decays we determine the strong coupling constant $\alpha_s=0.115 \pm 0.005$ (exp.) $^{+0.012}_{-0.010}$ (theor.) to second order QCD at $\sqrt{s}=91.22$ GeV. The errors are dominated by renormalisation scale uncertainties. The running of α_s as predicted by QCD is confirmed by a comparison of 3-jet multiplicities measured at different center of mass energies.

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